An estimate of the solar radiation incident at the top of Pluto's atmosphere

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Abstract. — Calculations of the daily solar radiation at the top of the atmosphere of Pluto are presented in a series of figures giving the seasonal and latitudinal variation for three fixed values of the obliquity ($\varepsilon = 60$, 75 and 90°). It is shown that the maximum daily insolation is incident at the poles at solar longitudes near 110 and 250° with values ranging from 11 to 13 cal cm^{-2} $(day)^{-1}$ (North pole) and from 13.5 to 15.5 cal cm^{-2} $(day)^{-1}$ (South pole). At the equator, maxima of the order of 6 cal cm^{-2} $(day)^{-1}$ are found around autumnal equinox. The solar longitude intervals where the polar solar energy exceeds the equatorial one extend from 20 to 160° and from 200 to 340° respectively. The steady increase of the polar insolation with increasing obliquity is accompanied by a corresponding loss of the equatorial solar radiation. The large eccentricity of Pluto produces significant north-south seasonal asymmetries in the daily insolation, whereas the change in the obliquity causes mainly a global latitudinal redistribution, although the general pattern of the contour maps illustrating the variability of the daily solar radiation with latitude and season is nearly similar. It is also interesting to note that in the equatorial region summer and winter are, roughly speaking, repeated twice a year. In addition, we also numerically studied the latitudinal variation of the mean daily insolation. It is found that in summer if ε varies from 60 to 90°, the mean summer insolation increases by about 15 % at the poles, remains approximately constant at a latitude of 20°, but decreases at the equator also by approximately 15 %. In winter, an increase of the obliquity yields a mean daily insolation which is reduced at all latitudes (maximally by about 20 % near the equator), the influence of ε being of decreasing significance at polar region latitudes. As for Uranus, the equator receives less annual average energy than the poles ; the ratio of both insolations amounts to about 0.9, 0.7 and 0.6 for an obliquity equal to 60, 75 and 90° respectively. Our calculations also reveal that at a latitude close to 30°, the dependence of the mean annual daily insolation on the obliquity is either unimportant.

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1. INTRODUCTION

The upper-boundary insolation of the atmospheres of the outer planets, excluding Pluto, has been computed by e.g. Vorob'yev and Monin [1975] and Levine *et al.* [1977]. To the best of our knowledge, similar calculations for Pluto have never been published.

Pluto, the most distant member of the solar system so far known, was discovered on February 18, 1930 after 25 years of intense and systematic search. To date and taking into account the very long orbital period of approximately 248 years, observations of the planet cover only a little more than 20 % of its orbit. This limited time period led to some very diverse determinations of some orbital elements needed, for example, for the calculation of the solar radiation reaching the top of the planet's atmosphere. Actually, the orbital plane is relatively accurate determined, so all values of the inclination (i), the heliocentric longitude of the planet's perihelion (π_0) and the ascending node (Ω_0) are more or less compatible [see e.g. Duncombe *et al.*, 1972 ; Newburn and Gulkis, 1973 ; Nacozy and Diehl, 1974, 1978a, b ; Nacozy, 1980 ; Seidelman *et al.*, 1980]. Some other orbital parameters have been more difficult to measure ; particularly the angle between the planet's spin axis and its orbit normal (ε) is very poorly determined [Davies *et al.*, 1980]. Even recent measurements vary strongly and consequently, it is not possible at this time to have sufficient confidence in any particular value. Although the discovery of the satellite of Pluto [Christy and Harrington, 1978, 1980 ; Harrington and Christy, 1980, 1981] has obviously stimulated the observations, it should be emphasized that a continued research program is absolutely needed in order to improve the Pluto ephemerides.

In spite of the fact that the obliquity (ε) of Pluto is so questionable, we calculated the solar radiation incident on the planet for three fixed values of ε ranging from 60° [Anderson and Fix, 1973] over 75° [Golitsyn, 1979] to 90° [The Handbook of the British Astronomical Association, 1980]. Although the two extreme values of the obliquity used span 30°, the calculations presented in this study enable us to have a quantitatively good estimate of the solar insolation. Moreover, they illustrate fairly well the sensitivity of the daily and the seasonal average insolation to changes in the obliquity in the above mentioned interval.

In a first section, we present expressions for the upper-boundary insolation of the outer planets. Then, taking into account the orbital and planetary data of Pluto, we calculate the planetocentric longitude of its perihelion (λ_P) and the length of the northern (and southern) summer and winter (T_S and T_W). The results of the daily insolation are presented in the form of three contour maps giving the incident solar radiation in calories per square centimeter per planetary day as a function of latitude and solar longitude and in two figures illustrating the seasonal variation of the equatorial and polar solar energy.

In addition, the latitudinal variation of the mean daily solar radiations are included in a series of three figures.

2. Solar radiation incident on a planetary atmosphere

The instantaneous insolation (I) at the upper-boundary of the atmosphere of a planet of the solar system can be expressed as [see e.g. Ward, 1974 ; Vorob'yev and Monin, 1975 ; Levine *et al.*, 1977 ; Van Hemelrijck and Vercheval, 1981 ; Van Hemelrijck, 1982a, b, c :

$$I = S \cos z \qquad (1)$$

with :

$$S = S_0/r_\odot^2 \qquad (2)$$

and :

$$r_\odot = a_\odot(1 - e^2)/(1 + e \cos W) \qquad (3)$$

In expressions (1) to (3), S, z, S_0, $a_\odot$, e and W are respectively the solar flux at an heliocentric distance r, the zenith angle of the incident solar radiation, the solar constant at the mean Sun-Earth distance of 1 AU taken at 1.94 cal cm^{-2} (min)$^{-1}$ or 2.79 × 10^3 cal cm^{-2} (day)$^{-1}$ [Thekaekara, 1973], the planet's semi-major axis, the eccentricity and the true anomaly which is given by :

$$W = \lambda_\odot - \lambda_P \qquad (4)$$

where $\lambda_\odot$ and λ_P are the planetocentric longitude of the Sun (or solar longitude) and the planetocentric longitude of the planet's perihelion (sometimes called the argument of perihelion). It has to be mentioned that the latter parameter for each planet in the solar system (excluding Pluto) has been given by Vorob'yev and Monin [1975] and Levine *et al.* [1977].

For a special planet, the zenith angle (z) is given by:

$$\cos z = \sin \phi \sin \delta_\odot + \cos \phi \cos \delta_\odot \cos h \qquad (5)$$

where ϕ is the planetocentric (or planetographic) latitude, $\delta_\odot$ is the solar declination and h is the local hour angle of the Sun. Furthermore, the solar declination ($\delta_\odot$) can be calculated from the relation:

$$\sin \delta_\odot = \sin \varepsilon \sin \lambda_\odot \qquad (6)$$

For a rapidly rotating planet, expression (1) can be integrated to yield the amount of solar radiation incident at the top of a planetary atmosphere over the planet's day (I_D) and is given by:

$$I_D = (ST/\pi)(h_0 \sin \phi \sin \delta_\odot + \sin h_0 \cos \phi \cos \delta_\odot) \qquad (7)$$

where T is the sidereal period of axial rotation (or sidereal day) and h_0 is the local hour angle at sunset (or sunrise) and may be determined from expression (5) by the condition that at setting (or rising) $\cos z = 0$. Hence, it follows that:

$$h_0 = \text{arc cos } (-\tan \delta_\odot \tan \phi) =$$

$$\text{arc cos } [-\tan \phi \sin \varepsilon \sin \lambda_\odot/(1 - \sin^2 \varepsilon \sin^2 \lambda_\odot)^{1/2} \qquad (8)$$

if

$$|\phi| < \pi/2 - |\delta_\odot|$$

In regions where the Sun does not rise ($\phi < -\pi/2 + \delta_\odot$ or $\phi > \pi/2 + \delta_\odot$) we have $h_0 = 0$; in regions where the Sun remains above the horizon all day ($\phi > \pi/2 - \delta_\odot$ or $\phi < -\pi/2 - \delta_\odot$) we may put $h_0 = \pi$.

Finally, the mean (summer, winter or annual) daily insolation, hereafter denoted as $(\bar{I}_D)_S$, $(\bar{I}_D)_W$ and $(\bar{I}_D)_A$ respectively, may be found by integrating relation (7) within the appropriate time limits, yielding the total insolation over a season or a year and by dividing the obtained result by the corresponding length of the season (T_S or T_W) or by the sidereal period of revolution or tropical year (T_0). In our calculations and for the northern hemisphere, summer season is arbitrary defined as running from vernal equinox over summer solstice to autumnal equinox and spanning 180°; consequently, the planetocentric longitudes of the Sun $\lambda_\odot = 180°$ and $\lambda_\odot = 360°$ mark the beginning and the end of the winter period. In the southern hemisphere, the solar longitude intervals (0, 180°) and (180, 360°) divide the year into astronomical winter and summer respectively.

As an example, the mean annual daily insolation may be written under the form :

$$(\bar{I}_D)_A = (1/T_0) \int_0^{T_0} I_D dt =$$

$$(1/T_0) \int_0^{T_0} (ST/\pi)\,(h_0 \sin\phi \sin\delta_\odot + \sin h_0 \cos\phi \cos\delta_\odot) dt \qquad (9)$$

From equation (2) and by the aid of Kepler's second law :

$$(a_\odot / r_\odot)^2 dt = T_0 d\lambda_\odot / 2\pi(1 - e^2)^{1/2} \qquad (10)$$

and after some rearrangements, expression (9) can be transformed into an integral over $\lambda_\odot$ yielding :

$$(\bar{I}_D)_A =$$

$$S_0\, T \sin\phi \sin\varepsilon / [2\pi^2 (1 - e^2)^{1/2} a^2{}_\odot] \int_0^{2\pi} (h_0 - \tan h_0) \sin\lambda_\odot d\lambda_\odot \qquad (11)$$

where the dependence of h_0 in terms of $\lambda_\odot$ is given by the second equality on the right hand side of relation (8) [see e.g. Vorob'yev and Monin, 1975]. It should be pointed out that, considering the complexity of the integrand, equation (11) has generally to be integrated numerically. However, at the poles, the mean annual daily insolation can easily be obtained from (11) by putting $\phi = \pm\pi/2$ and $h_0 = \pi$ [see e.g. Murray *et al.*, 1973 ; Ward, 1974 ; Vorob'yev and Monin, 1975]. Integration yields :

$$(\bar{I}_D)_{A_P} = S_0 T \sin\varepsilon / \pi(1 - e^2)^{1/2} a_\odot^2 \qquad (12)$$

For the computations presented in this paper, the procedure to obtain the seasonal or annual average energy was as follows.

In a very good approximation, the total insolation over a season or a year ($\int I_D\, dt$) was obtained using a step-by-step summation [$\sum_i (I_D)_i (\Delta t)_i$ where the solar longitude range (0-180°, 180-360°, 0-360°) has been divided into 18 ($i = 1, ..., 18$) (summer and winter) or 36 ($i = 1, ..., 36$) (year) intervals with a bandwith $(\Delta\lambda_\odot)_i$ of 10°. The adopted constant values $(I_D)_i$ represent the calculated values in the center of each interval. Furthermore, $(\Delta t)_i$ or the length (in earth days) of the ith solar longitude interval can be determined by solving successively the following set of equations for the position of the Sun at the beginning and the end of each solar longitude interval $(\Delta\lambda_\odot)_i$. It has to be pointed out that all the relationships can be found in each text-book on spherical astronomy [see e.g. Smart, 1956].

The true anomaly (W) dependence of the eccentric anomaly (E) can be described by Lacaille's equation :

$$E = 2 \text{ arc tan } \{[(1-e)/(1+e)]^{1/2} \tan (W/2)\} \tag{13}$$

where W is given by expression (4).

Furthermore, the mean anomaly (M) in terms of E can be expressed by the Kepler equation :

$$M = E - e \sin E \tag{14}$$

Finally, $(\Delta t)_i$ can be written under the following form :

$$(\Delta t)_i = T_0 \; \Delta M / 2\pi \tag{15}$$

where

$$\Delta M = (M)_{i+1} - (M)_i$$

The total amount of solar energy over a season or a year being calculated, the mean daily insolations can, as already mentioned previously, be determined by dividing the obtained result by the corresponding time interval. It is obvious that T_S or T_W can also easily be found by combining (4), (13), (14) and (15). It is clear that the mean annual daily insolation $(\bar{I}_D)_A$ may also be derived from :

$$(\bar{I}_D)_A = [(\bar{I}_D)_S \; T_S + (\bar{I}_D)_W \; T_W]/T_0 \tag{16}$$

3. Planetocentric longitude of Pluto's perihelion

In the preceeding section we noted that the position of perihelion (λ_P) for the planets in the solar system is given by e.g. Vorob'yev and Monin [1975] and Levine *et al.* [1977]. To the best of our knowledge, Pluto's argument of perihelion has never been published. With this in mind, an attempt is made to derive the above mentioned parameter based on a computing algorithm discussed by Vorob'yev and Monin

[1975]. According to their paper, the argument of perihelion may be written in terms of the heliocentric longitude of the planet's perihelion (π_0) and the heliocentric longitude of the ascending node (Ω_0) as :

$$\lambda_P = \pi_0 - \Omega_0 + \Lambda \qquad (17)$$

where Λ is the planetocentric longitude of the ascending node altered by 180°. A detailed description of the procedure to calculate Λ is beyond the scope of the present work. It should however be emphasized that the element Λ can be obtained from standard spherical trigonometric relationships. It is dependent upon several orbital and planetary data and can be expressed in the general form :

$$\Lambda = f(i, \Omega_0, \pi_0, \varepsilon, \varepsilon_0, \alpha_0, \delta_0) \qquad (18)$$

where ε_0, α_0 and δ_0 are respectively the angle between the Earth's spin axis and its orbit normal ($\varepsilon_0 = 23°.45$) and the right ascension and declination of Pluto's north pole, the significance of the other quantities being already explained in the text.

The adopted planetary data for the computation of λ_P are given in Table I, whereas the results of the calculation are illustrated in Table II.

TABLE I. — Adopted planetary data for Pluto.

i (°)	Ω_0 (°)	π_0 (°)	α_0 (°)	δ_0 (°)
17.14	109.51	222.50	305	5

TABLE II. — Computed planetary data for Pluto.

ε (°)	λ_P (°)	T_S (Earth days)	T_W (Earth days)
60	192.17	48455	42128
75	191.12	48187	42396
90	190.81	48108	42475

It has to be pointed out that i, Ω_0 and π_0 were taken from the paper by Seidelman *et al.* [1980] limited to two decimals. For the direction of the north pole, we adopted the recommended values for the equatorial coordinates (α_0, δ_0) recently published by the IAU

Working Group on cartographic coordinates and rotational elements of the planets and satellites [Davies *et al.*, 1980].

In Table II one can find also the length of the seasons (T_S and T_W) for the three obliquities under consideration. From the Table it can be seen that λ_P is only weakly dependent on ε.

4. Discussion of calculation

Table III represents the numerical values of the supplementary parameters used for the computation of I_D, $(\bar{I}_D)_S$, $(\bar{I}_D)_W$ and $(\bar{I}_D)_A$.

Table III. — Adopted supplementary planetary data for Pluto.

$a_\odot$ (AU) ([a])	e ([a])	T (Earth days) ([b])	T_0 (Earth days) ([c])
39.72	0.2523	6.3867	90583

([a]) From Seidelman *et al.* [1980].
([b]) From Davies *et al.* [1980].
([c]) From Golitsyn [1979].

4.1. Daily insolation

For the daily insolation, we have followed the method adopted by Vorob'yev and Monin [1975] and Levine *et al.* [1977] in presenting our results in the form of a contour map giving the seasonal distribution in terms of the planetocentric longitude of the Sun taken to be 0° at the northern hemisphere vernal equinox. In addition we have included two figures showing the latitudinal variation of the daily insolation at the equator and the poles as a function of solar longitude.

Application of expression (7) leads to the isopleths illustrated in Fig. 1 ($\varepsilon = 60°$), Fig. 2 ($\varepsilon = 75°$) and Fig. 3 ($\varepsilon = 90°$) and to the equatorial and polar distribution plotted in Figs. 4 and 5. From the contour maps and particularly from Figs. 4 and 5 it follows that the maximum solar radiation is incident at the poles around summer solstices with values of about 11 to 13 cal cm^{-2} (day)$^{-1}$ (North pole) and 13.5 to 15.5 cal cm^{-2} (day)$^{-1}$ (South pole). Comparing this results with the corresponding intensities of the outer planets represented in Table IV [the orbital and planetary data being taken from the Hand-

TABLE IV. — Some planetary data and maximum solar radiation incident $(I_D)_{MAX}$ at the poles of the outer planets.

Planet	T (Earth days)	ε (°)	$a_{\odot}$ (AU)	e	λ_P (°)	$(I_D)_{MAX}$ [cal cm^{-2}(day)$^{-1}$] North pole	$(I_D)_{MAX}$ [cal cm^{-2}(day)$^{-1}$] South pole
Earth	1	23.45	1	0.01672	282.05	1070	1150
Mars	1.02	25.20	1.5237	0.09339	248	440	630
Jupiter	0.41	3.12	5.2028	0.04847	58	2.5	2.1
Saturn	0.44	26.73	9.539	0.05561	279.07	5.5	6.8
Uranus	0.45	82.14	19.18	0.04727	3.02	3.4	3.4
Neptune	0.66	29.56	30.06	0.00859	5.23	1.0	1.0

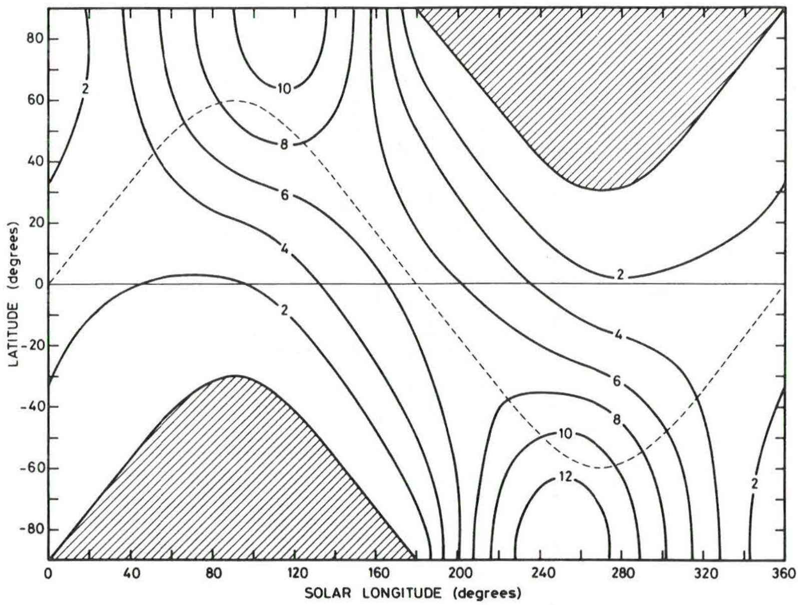

FIG. 1. — Seasonal and latitudinal variation of the daily insolation (I_D) at the top of the atmosphere of Pluto for an obliquity $\varepsilon = 60°$. Solar declination is represented by the dashed line. The areas of permanent darkness are shaded. Values of I_D, in cal cm^{-2} (planetary day)$^{-1}$ are given on each curve.

book of the British Astronomical Association [1980] and from Vorob'yev and Monin [1975], where we also have summarized the sidereal day (T), the obliquity (ε), the semi-major axis ($a_{\odot}$), the eccentricity (e) and the argument of perihelion (λ_P), it might be somewhat suprising that, taking into account the so far distance of Pluto, the maximum amount of solar energy reaching the top of its atmosphere is much higher than the solar radiation incident on Jupiter, Saturn, Uranus and Neptune. This phenomenon, however, can easily be explained by applying, for instance, expression (7) to the north pole at summer solstice. Indeed, by putting $\phi = \pi/2$, $\lambda_{\odot} = \pi/2$ and $h_0 = \pi$, relation (7) yields :

$$(I_D)_{NP(ss)} = (S_0 \text{ T } \sin \varepsilon/a_{\odot}^2)[1 + e \sin \lambda_P)/(1 - e^2)]^2 \qquad (19)$$

The term in the second bracket, representing the dependence of $(I_D)_{NP(ss)}$ on the eccentricity and the argument of perihelion, ranges from 0.9 (Saturn) over approximately unity (Uranus, Neptune, Pluto) to 1.1 (Jupiter). The above mentioned term being roughly equal for the five most remote planets it is evident that the polar solar intensity at summer solstice is mainly governed by the expression is the first bracket. Introducing the numerical values of S_0 and of the orbital elements for Pluto and the outer planets (Table IV) in expression (19) reveals that $(I_D)_{NP(ss)}$ for Pluto exceeds considerably the energy supplied to the north poles of Jupiter, Saturn, Uranus and Neptune. It is obvious that this higher maximum results from the slow axial rotation (T = 6.3867) and the very large obliquity [Pluto rotates lying practically ($\varepsilon = 60$, 75°) or really ($\varepsilon = 90°$) on its side in the plane of its orbit] which overcompensates for the extremely high value of the mean distance of Pluto from the Sun ($a_\odot \cong 39.72$).

It should be pointed out that Table IV is somewhat misleading in that it might suggest that, on one hand, the peak insolation at the north pole during summer solstice is equal to that of the south pole at its summer solstice for Uranus and Neptune and that, on the other hand, the maxima in the solar radiation incident at Jupiter occur also over the poles.

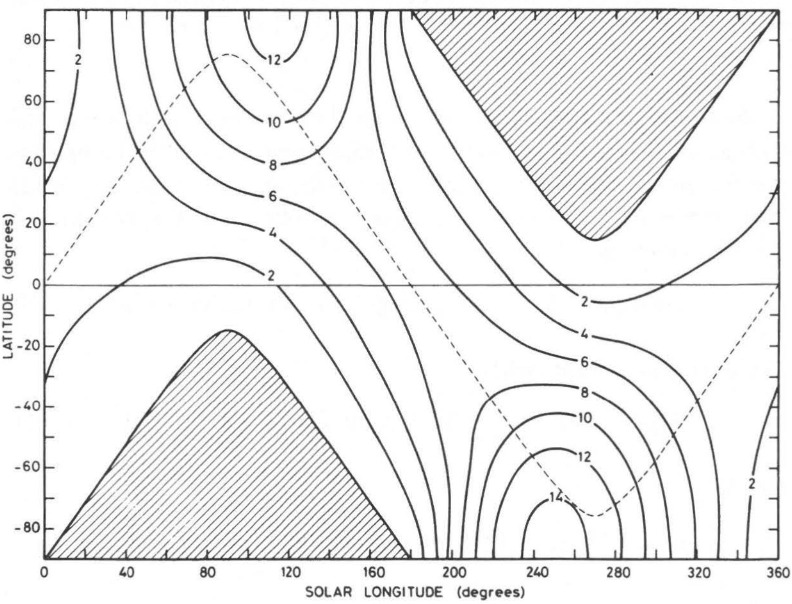

FIG. 2. — Seasonal and latitudinal variation of the daily insolation (I_D) at the top of the atmosphere of Pluto for an obliquity $\varepsilon = 75°$. See Fig. 1 for full explanation.

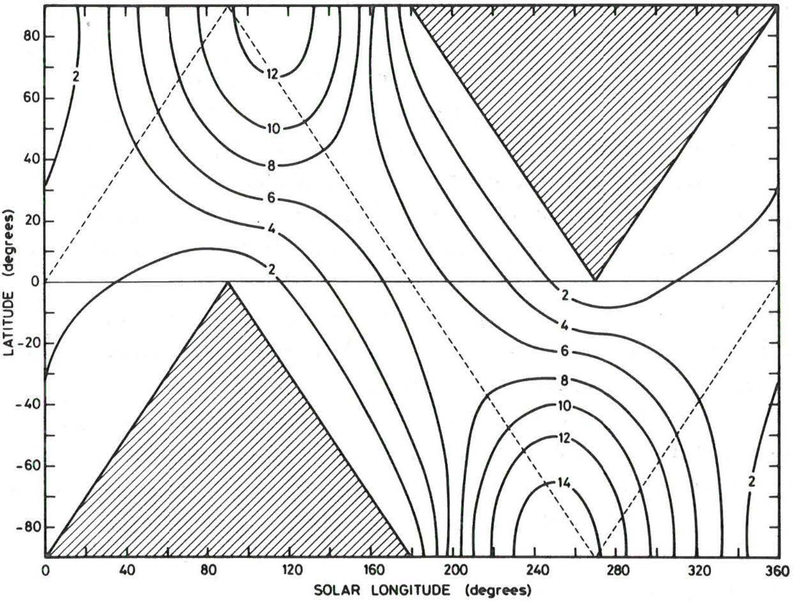

FIG. 3. — Seasonal and latitudinal variation of the daily insolation (I_D) at the top of the atmosphere of Pluto for an obliquity $\varepsilon = 90°$. See Fig. 1 for full explanation.

As to the first remark, we note that they are slightly different with $(I_D)_{NP(ss)} > (I_D)_{SP(ss)}$. This finding can easily be evaluated by computing the maximum insolation at the south pole. Indeed, from the conditions $\phi = -\pi/2$, $\lambda_\odot = 3\pi/2$ and $h_0 = \pi$ it follows that expression (7) can be written as :

$$(I_D)_{SP(ss)} = (S_0 \ T \sin \varepsilon \ a_\odot^2)[(1 - e \sin \lambda_P)/(1 - e^2)]^2 \qquad (20)$$

Dividing (19) by (20) yields :

$$(I_D)_{NP(ss)}/(I_D)_{SP(ss)} = [(1 + e \sin \lambda_P)/(1 - e \sin \lambda_P)]^2 \qquad (21)$$

Hence :

$$(I_D)_{NP(ss)} > (I_D)_{SP(ss)} \text{ if } 0 < \lambda_P < \pi \text{ (Jupiter, Uranus, Neptune)}$$

and

$$(I_D)_{NP(ss)} < (I_D)_{SP(ss)} \text{ if } \pi < \lambda_P < 2\pi \text{ (Earth, Mars, Saturn, Pluto)}$$

From (21) it is also easy to show that the ratio $(I_D)_{NP(ss)}/(I_D)_{SP(ss)}$ is equal to unity for $\lambda_P = 0$ or $\lambda_P = \pi$ and has a minimum (or maximum) value at $\lambda_P = \pi/2$ or $\lambda_P = 3\pi/2$. Uranus and Neptune, having arguments of perihelion roughly coinciding with their vernal equinoxes and their eccentricities being very small, it follows from (21) that $(I_D)_{NP(ss)} \cong (I_D)_{SP(ss)}$. Concerning more particularly Pluto, λ_P being close to π, one would expect a similar conclusion. However, the maximum difference between the peak insolations attains approximately 20 %; this effect is ascribed to the very large eccentricity of its orbit.

The equatorial insolation during summer solstice, hereafter denoted as $(I_D)_{E(ss)}$, may be obtained from relationship (7) by putting $\phi = 0$, $\lambda_\odot = \pi/2$ or $3\pi/2$ and $h_0 = \pi/2$ and is given by:

$$(I_D)_{E(ss)} = (S_0 \text{ T } \cos \varepsilon/\pi \; a_\odot^2)[(1 \pm e \sin \lambda_P)/(1 - e^2)]^2 \qquad (22)$$

where the plus sign is for the northern summer solstice and the minus sign for the southern summer solstice.

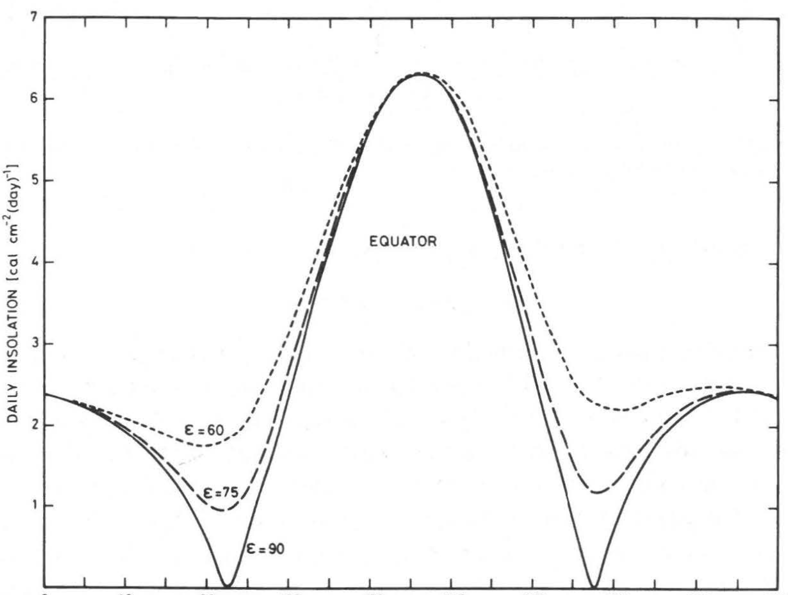

FIG. 4. — Seasonal variation of the daily insolation (I_D) at the equator of Pluto for various values of the obliquity.

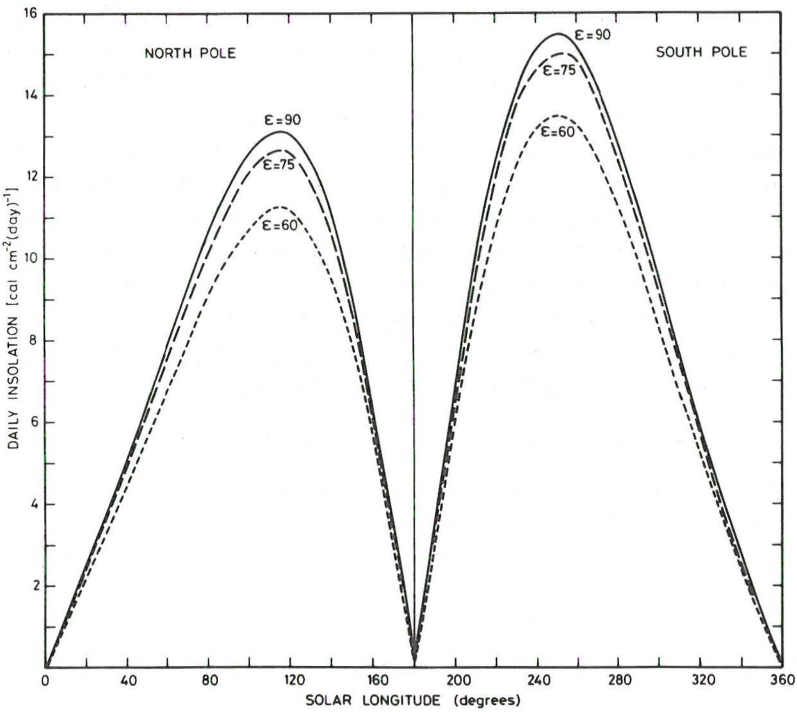

FIG. 5. — Seasonal variation of the daily insolation (I_D) at the poles of Pluto for various values of the obliquity.

Dividing (19) or (20) by (22) yields :

$$(I_D)_{P(ss)}/(I_D)_{E(ss)} = \pi \text{ tg } \varepsilon \qquad (23)$$

stating that the polar insolation at summer solstice is larger than that at the equator for $\varepsilon > 17°.7$ (all planets except Jupiter) [see e.g. Ward, 1974 ; Levine *et al.*, 1977]. As a consequence, Jupiter is the only planet in the solar system where the equator at summer solstice (and even over the entire year) receives more daily insolation than the poles.

Expression (23) clearly indicates that the value of $(I_D)_{P(ss)}/(I_D)_{E(ss)}$ is exclusively dependent on ε. For Pluto, this ratio amounts to about 5.4, 11.7 and infinity (in this case the Sun does not rise at summer solstice) respectively for obliquities equal to 60,75 and 90°.

From Figs. 1 to 3 and Fig. 5 it can be seen that $(I_D)_{NP(ss)}$ and $(I_D)_{SP(ss)}$ increase with increasing ε (Pluto's spin axis and consequently its pole tilts further away from the normal to the orbit plane so that the amount of polar energy in summertime is enhanced). This conclusion can also be derived from (19) and (20), the obliquity producing variations in the peak insolations through the sin ε dependence of the solar flux. Furthermore, according to Figs. 4 and 5, the steady increase of $(I_D)_{NP(ss)}$ and $(I_D)_{SP(ss)}$ is accompanied by a corresponding loss of the equatorial solar radiation. Evidently, this systematic decrease of insolation can also be deduced from relation (22).

More generally, one can determine the solar longitude interval where $(I_D)_P > (I_D)_E$. Indeed, from (7) it is easy to shown that :

$$(I_D)_P/(I_D)_E = \pm\pi \tan \delta_\odot = \pm \sin \varepsilon \sin\lambda_\odot/(1 - \sin^2 \varepsilon \sin^2 \lambda_\odot)^{1/2} \quad (24)$$

the plus sign being used for the north pole, the minus sign for the south pole. Introducing the numerical values for ε (60, 75 and 90°) in equation (24) it follows that, for the north pole, $(I_D)_P > (I_D)_E$ if $\lambda_\odot$ ranges from approximately 20 to about 160°. For the south pole, the polar daily insolation exceeds that of the equator in the approximate solar longitude interval (200-340°).

The contour maps and especially Fig. 5 reveal that the position of maximum solar radiation is shifted markedly, by about 20°, from the position of summer solstices. This is due to the fact that the perihelion position ($\lambda_P \cong 190°$) is located approximately 80° to the left of the south summer solstice ($\lambda_\odot = 3\pi/2$). It is worth pointing out that the solar longitude of maximum insolation can also mathematically be derived. However, taking into account the complexity of the computing algorithm, it is recommended to numerically calculate the polar insolation variability as a function of $\lambda_\odot$. From the plotted curves (Fig. 5) it can be seen that the maximum solar radiations occur near 110 and 250°.

It is well known that a large eccentricity (which is the case for Pluto) produces important north-south seasonal asymmetries in the daily insolation (Figs. 1 to 3 and Fig. 5). On the other hand, a change in the obliquity causes mainly a global latitudinal redistribution (Figs. 4 and 5).

When comparing Figs. 1, 2 and 3, it is also obvious that the general pattern of the three contour maps is only slightly different. In the solar longitude interval ($\pi/2$-$3\pi/2$) and especially at equatorial and midlatitudes the isocontours closely parallel the seasonal march of the

Sun. In the region where the Sun does not set, the shape of the lines of constant daily insolation is roughly similar, although shifted with respect to the summer solstices, to the curve limiting the area of permanent sunlight. Furthermore, the isopleths clearly illustrate that for nearly half the Pluto year (approximately 124 Earth years) some parts of the planet are in permanent darkness. It is evident that the zone where the Sun does not rise increases with increasing obliquity.

Another point of interest regards the distribution of the daily solar radiation in the equatorial region. At $\varepsilon < 45°$ (all planets except Uranus and Pluto), the Arctic circles bounding the polar region in which there are days without sunset or sunrise, lie outside the tropical zone (in which there are days on which the Sun reaches the zenith) and because of the relatively small eccentricities the latitudinal insolation has one maximum and one minimum over the year if the polar night is considered as one maximum. Uranus ($\varepsilon = 82°.14$) and Pluto occupy a rather peculiar position in that they rotate lying practically or really on their sides in the orbital planes. In the polar regions the day and the night are approximately half a year long and the Sun is close to the zenith midway through the sunlit half of the year. Summer and winter are, roughly speaking, repeated twice a year in the equatorial region, the two seasons being substantially more temperate than in the polar regions. This interesting phenomenon is clearly demonstrated in Fig. 4 (contrast also this diagram with Fig. 5).

In two previous papers [Van Hemelrijck, 1982a,b] we discussed the oblateness effect [see also Brinkman and McGregor, 1979] on the solar radiation incident at the top of the atmospheres of the outer planets [all except Mars and Pluto]. An investigation related to Pluto of the influence of the flattening cannot be made, the oblateness, defined as $f = (a_e - a_p)/a_e$ where a_e and a_p signify respectively the equatorial and the polar radius, being unknown [Newburn and Gulkis, 1973].

4.2. Mean daily insolation

The mean daily insolations, taken over a season or a year, are computed by the procedure discussed in section 2, the results of which appear in Figs. 6, 7 and 8.

Fig. 6, representing the latitudinal variation of the mean summer daily insolation for the three obliquities, shows that at the poles $(\bar{I}_D)_S$ increases with increasing obliquity (ε), whereas at the equator the opposite effect is found. For comparison, if ε varies from 60 to 90°, $(\bar{I}_D)_S$ changes from 6.02 to 7.01 cal cm^{-2} $(day)^{-1}$ at the north and south pole respectively; at the equator, however, the corresponding mean summer daily insolations amount to 2.68 and 2.23 cal cm^{-2} $(day)^{-1}$. All differences are of the order of 15 %.

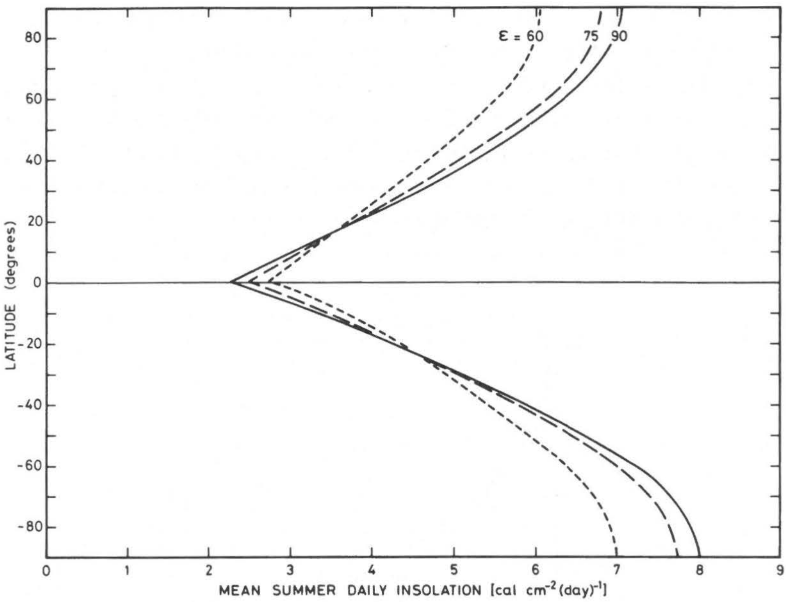

FIG. 6. — Latitudinal variation of the mean summer daily insolation $(\bar{I}_D)_S$ at the top of the atmosphere of Pluto for various values of the obliquity.

The sensitivity of the summertime polar insolation to changes in the obliquity can easily be illustrated by deriving the expression for $(\bar{I}_D)_{S_P}$. In fact, similar to relation (12) we obtain :

$$(\bar{I}_D)_{S_P} = (S_0 \ T \ T_0 \ \sin \ \varepsilon/T_S)/\pi(1 - e^2)^{1/2}a_\odot^2 \qquad (25)$$

stating that $(\bar{I}_D)_{S_P}$ is a monotonically increasing function of ε. Note also that the summertime insolation changes only by about 1 % between the two extremes of the length of the summer (T_S) (see Table II).

On the other hand, the mean summer daily insolation for the equator can only be expressed by a complete elliptical integral of the second kind [Ward, 1974; Vorob'yev and Monin, 1975], but it can mathematically be proved that it is a monotonically decreasing function of sin ε as illustrated in Fig. 6.

Finally, it should be pointed out that in both hemispheres the intersection of the curves representing the latitudinal distribution of the mean summer daily insolation as a function of ε occurs at a latitude of approximately 20° and that $(\bar{I}_D)_S$ is only weakly dependent on ε in the neighborhood of the above mentioned latitude.

Another point about the curves is that $(\bar{I}_D)_{S_{SP}} > (\bar{I}_D)_{S_{NP}} > (\bar{I}_D)_{S_E}$. This characteristic feature follows immediately from the theoretical analysis given in section 4.1 and evidently from Figs. 4 and 5. Moreover, the first inequality also clearly indicates that the curves are not symmetric with respect to the equator.

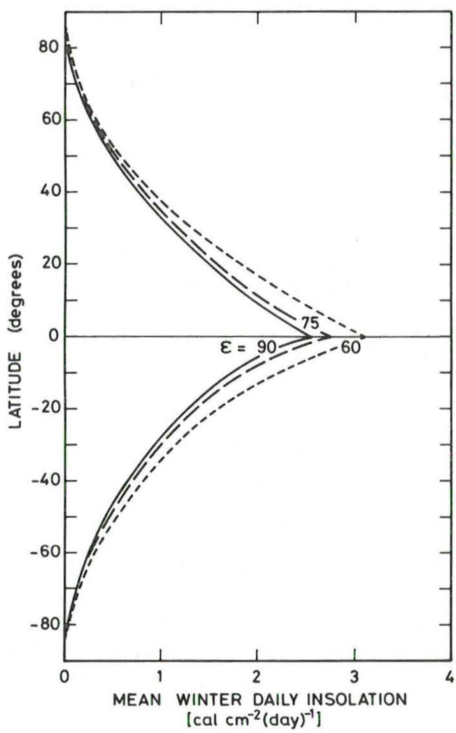

FIG. 7. — Latitudinal variation of the mean winter daily insolation $(\bar{I}_D)_W$ at the top of the atmosphere of Pluto for various values of the obliquity.

The mean winter daily insolations corresponding to the three obliquities under consideration are plotted in Fig. 7. Since in winter the Sun does not rise at the poles it is obvious that $(\bar{I}_D)_{W_P} = 0$. At all latitudes, the effect of increasing obliquity can clearly be seen to reduce the insolation. As an example, the mean wintertime insolation at the equator varies by about 15 % between the two extremes of the obliquity range and maximally by about 20 % near the equator. At high latitudes, particularly near the poles, the obliquity effect is of decreasing significance. Especially noteworthy is the fact that the loss of insolation when ε varies from 60 to 75° is much higher than the reduction in the 75-90° obliquity interval. Furthermore, the north-south seasonal asymmetry in the distribution of the incident solar flux can also be seen from Fig. 7.

For the sake of completeness, the latitudinal variation of the mean annual daily insolation is given in Fig. 8. As already previously stated,

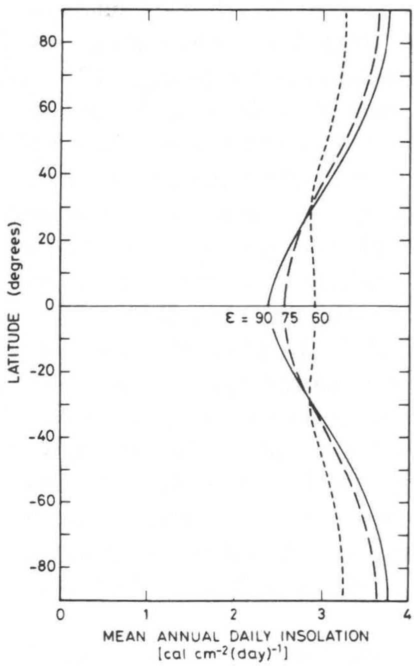

FIG. 8. — Latitudinal variation of the mean annual daily insolation $(\bar{I}_D)_A$ at the top of the atmosphere of Pluto for various values of the obliquity.

tudes, the shape of the lines of constant daily insolation is roughly similar to the curve limiting the zone of permanent sunlight.

It is also interesting to note that in the equatorial region summer and winter are repeated twice a year.

Finally, we also have studied the latitudinal variation of the mean daily insolations. It is found that in summer and if ε varies from 60 to 90°, the mean summertime insolation increases with about 15 % at the poles, but decreases at the equator with approximately the same quantity. At a latitude near 20° the variation of the mean summer daily insolation with ε is either unimportant.

In winter, and at all latitudes, an increase of the obliquity causes the mean daily insolation to reduce, the influence of the obliquity being of decreasing significance at polar region latitudes.

The seasonal north-south asymmetries in the mean daily insolation are particularly evident from Figs 6 and 7.

As stated previously, the ratio of the annual insolations at the equator and the poles is smaller than unity at $\varepsilon > 54°$. For Pluto, this ratio amounts to about 0.9 ($\varepsilon = 60°$), 0.7 ($\varepsilon = 75°$) and 0.6 ($\varepsilon = 90°$). For comparison, it is approximately equal to 2.4, 2.2, 18.4, 2.1, 0.7 and 1.9 for the Earth, Mars, Jupiter, Saturn, Uranus and Neptune respectively. Finally, Fig. 8 also clearly indicates that the hemispheric seasonal asymmetry in the solar radiation disappears when averaging the incoming solar energy over the year and that in the vicinity of $\phi = 30°$ the dependence of the mean annual daily insolation on ε is rather small.

In conclusion, we believe the calculations presented in this work could help in studies of the climatic history, the energy budget and the dynamical behavior of Pluto. It is, however, evident that in the future observations have to be carried out systematically over an extended period of time in order to improve the accuracy of some orbital elements, especially the direction of the axis of rotation which is very poorly determined.

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