Chapter 1
Introduction

Norma B. Crosby

> In turn, these invariance properties suggest that the presence of scaling distributions in data obtained from complex natural or engineered systems should be considered the norm rather than the exception and should not require "special" explanations.
>
> Willinger et al. (2004)

1.1 A New Theory Emerges

In the mid-1980s a new theory emerged that aimed to explain how complex nonlinear systems with many degrees of freedom observed in nature are able to produce powerlaw relationships from simple redistribution rules of nearest-neighbor interactions. The incentive behind this theory was that it could be the underlying concept for temporal and spatial scaling observed in a wide class of dissipative systems with extended degrees of freedom. It was the Danish theoretical physicist Per Bak and his co-authors who introduced this concept that became known as SOC (Bak et al. 1987; 1988). This acronym stands for Self-Organized Criticality and since its birth this concept has been applied to a wide range of disciplines covering solar physics, astrophysics, magnetospheric physics, geophysics, biophysics, and social sciences.

Many systems consist of a large number of entities that interact in a complex way and exhibit nonlinear behavior; they are called nonlinear dissipative systems. Indeed, the Solar System is full of multi-scale phenomena that obey nonlinear spatio-temporal scaling laws. On Earth, such extreme nonlinear events are known as earthquakes, landslides, wildfires, volcanoes, snow avalanches, rock-falls, crashes in the stock market, etc. Their counterparts

Norma B. Crosby
Belgian Institute for Space Aeronomy, Ringlaan-3-Avenue Circulaire, B-1180 Brussels, Belgium, e-mail: Norma.Crosby@aeronomie.be

in space - solar storms - range from solar flares, coronal mass ejections, substorms in the magnetosphere to solar energetic particle events. What do all these natural dynamic phenomena (examples are displayed in Fig. 1.1), have in common?

1. They cover a large range of temporal, as well as spatial scales.
2. The most extreme events, known as "black swans" (Taleb 2007), are of concern to society.
3. There are large databases so that statistical approaches can be used for interpreting the data characterizing the phenomena.
4. Size distributions (on log-scale) of parameters describing the phenomena (volumes, energies, etc.) cover many orders of magnitude.
5. Powerlaw-like behavior has been found to be a universal characteristic of such phenomena.

Though it is most often the largest "avalanche" events that make the headlines in the newspapers, the myriads of smaller events share the same statistical properties. With databases becoming larger and larger (covering long time spans) it is not always possible to study each single event and a statistical approach such as a frequency distribution can be used instead. Powerlaw behavior is systematically observed when frequency distributions are constructed of both measured (e.g. peak count rate, total duration) and theoretical (e.g. total energy released) parameters describing the events (identified as "avalanches" in this Chapter) constituting the "avalanche" database. Performing a frequency distribution on an "avalanche" database is a valid approach if the phenomenon being studied results from the same mechanisms of energy release on all scales.

Why does nature produce powerlaw behavior? What is (are) the mechanism(s) responsible for powerlaws observed in nature as well as in social sciences? Various concepts (models) have been proposed to explain this observed powerlaw signature. One of these is SOC that characterizes the behavior of dissipative systems that contain a large number of elements interacting via nearest-neighbor interactions over short and long ranges. The systems evolve to a critical state in which a minor event starts a chain reaction that can affect any number of elements in the system. Frequency distributions of the output parameters from the chain reaction taken over a period of time can be represented by powerlaws.

Several reviews, textbooks, and monographs have been written on SOC regarding phenomena that display this behavior as well as models that simulate SOC; see for example Bak (1996), Jensen (1998), Turcotte (1999), Charbonneau et al. (2001), Hergarten (2002), Sornette (2004), Christensen and Moloney (2005), Aschwanden (2011), Crosby (2011), and Pruessner (2012). In the current book an inter-disciplinary approach is applied to the SOC concept. The book covers all types of naturally occurring "avalanche" phenomena where SOC behavior has been studied and presents both observational results as well as results obtained by theoretical models.

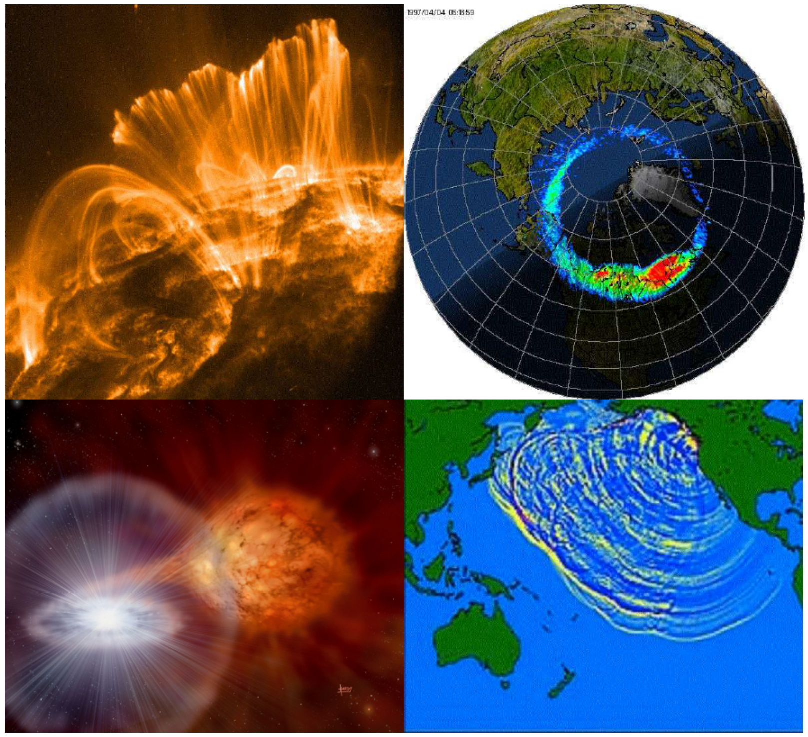

Fig. 1.1: Examples of natural phenomena in space and on Earth that exhibit powerlaw behavior. Upper left-hand corner: Solar flare of 2000 Nov. 9 observed in EUV with the TRACE spacecraft in 171 Å (credit: NASA, TRACE), upper right-hand corner: Global image of the auroral oval observed by the Ultraviolet Imager (UVI) onboard the NASA satellite "Polar" (credit: NASA, Polar/UVI Team, George Parks), lower left-hand corner: Artistic rendering of the cataclysmic variable star RS Ophiuchi, which exhibits a nova outburst about every 20 years. This binary system contains a white dwarf and a red giant with mass transfer (credit: PPARC, David A. Hardy), lower right-hand corner: Satellite recording of tsunami waves produced by one of the 10 largest earthquakes, originating in North America (credit: NOAA).

In Section 2 of this Chapter it is described how frequency distributions are applied as a statistical tool. This is followed by an introduction to the SOC concept and to the models that have been built to simulate powerlaw behavior. Examples of SOC observed in natural phenomena both in space and on Earth are presented in Section 4. The end of the Chapter asks the question: What does it all mean?

1.2 Frequency distribution: a powerful tool

A frequency distribution, also known as log Number vs. log Size (log N - log S) diagram, size distribution, or occurrence frequency distribution, is a function that describes the occurrence rate of events as a function of their size. It is usually plotted as a histogram of the logarithmic number versus the logarithmic size. Input for a frequency distribution is a database (list or catalogue) of "avalanches" characterized by some size parameter. There are essentially two ways to construct a log-log histogram from a database:

1. Logarithmically binned histogram if large statistics is available (n $\geq 10^2$, ..., 10^3).
2. Rank-order plot if the size of the statistical sample is rather small.

For numerous natural phenomena it is found that the differential frequency distributions of the output parameters describing a given phenomenon taken over a period of time can be represented by powerlaws of the form:

$$N(x)dx = N_0 x^{-\alpha_x} dx \tag{1.1}$$

where N(x) is the number of events recorded with the parameter x of interest in a "differential" bin, and N_0 is a constant.

For the cumulative frequency distribution, it is the integral which expresses in each bin the sum of all events that are larger than the size parameter of the bin x:

$$N^{cum}(> x) = \int_x^\infty x^{-\alpha_x} dx \propto x^{-(\alpha_x - 1)} \tag{1.2}$$

If the differential frequency is a powerlaw function with slope α, the cumulative frequency distribution is expected to have a flatter powerlaw slope by one (α-1).

Fig. 3.1 is an example of a differential frequency distribution performed on the peak count rate solar flare data recorded by the Wide Angle Telescope for Cosmic Hard X-Rays (WATCH) experiment aboard the GRANAT satellite. WATCH measures photons in the deka-keV range and the WATCH database covers 2.5 years of observation (1990 to mid-1992). Fig. 1.3 shows the time profiles of three WATCH flares observed between 15:00 and 16:20 UT on 1990 June 19. The three peak count rates that are identified as "a", "b" and "c" in Fig. 1.3 are included in the Fig. 3.1 statistics. The distribution displayed in Fig. 3.1 follows a powerlaw with a slope of -1.58 $\pm$ 0.02 extending for almost three orders of magnitude (Crosby et al. 1998). The turn-over in the lower end of the frequency distribution may be attributed to detector sensitivity (missing the small events in the background noise).

For some "avalanche" parameters exponential turn-overs in the upper end of the frequency distribution are observed and may be due to two important issues:

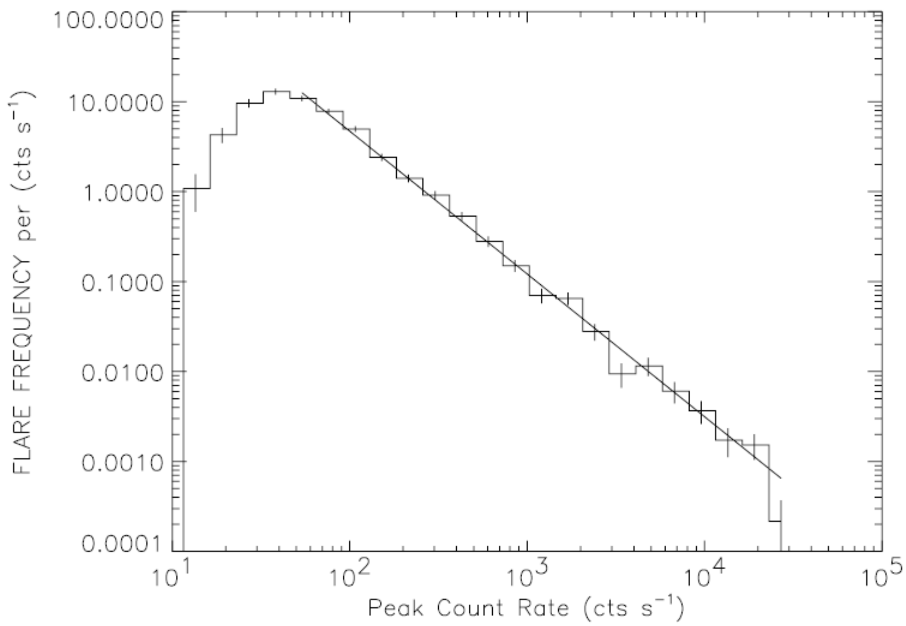

Fig. 1.2: The frequency distribution on the WATCH solar flare peak count rate data for the total observing period (1990 - mid-1992). It is well-represented by a powerlaw with a slope of -1.58 $\pm$ 0.02 extending over almost three orders of magnitude (Crosby et al. 1998).

1. Length of the dataset (missing of long-term statistics): observations have not been performed over a long enough period of time to cover all the statistics of the "avalanches".
2. Physical limit to the size of a "black swan" event: "Finite-size effects" ("avalanches" reach the boundaries of a system; Chapman et al. (1998) discusses this effect in regard to substorms).

The accuracy of a frequency distribution powerlaw fit is sensitive to the choice of dependent (measured) or independent (theoretical) parameter describing the phenomenon, as well as the statistical uncertainty of the number of events in the phenomenon database.

Identifying the underlying physics determining the exact value of the spectral index of the powerlaw, which varies for different size parameters and phenomena, is not yet well understood, but may suggest some form of universality. Since powerlaws are the only statistical distributions that are completely scale-invariant, they offer a unique way to explore the possibility of an underlying universality in nature.

1.3 Self-organized criticality and powerlaw behavior

SOC is also known as the "avalanche concept" and characterizes the behavior of dissipative systems that contain a large number of elements interacting over a short range. The systems evolve to a critical state in which a minor event starts a chain reaction that can affect any number of elements in the system. Such systems are constantly driven by some random energy input evolving into a critical state that is maintained as a powerlaw distribution. SOC theory has the following characteristics:

1. Individual events are statistically independent, spatially and temporally (leading to random waiting time distributions).
2. The size or occurrence frequency distribution is scale-free and can be characterized by a powerlaw function over some size range.
3. The detailed spatial and temporal evolution is complex and involves a fractal geometry and stochastically fluctuating time characteristics (sometimes modelled with 1/f noise, white, pink, red, or black noise).

The concept of SOC evolved from numerical simulations that utilized several relatively simple cellular-automata models. In this context the term cellular refers to the fact that the model is discrete concerning space and the term automaton means that the evolution of the system is self-operating. The traditional SOC cellular automaton model is a regular lattice grid, where redistributions occur with nearest neighbor cells.

In 1987, Per Bak and co-workers presented a model that evolves towards a critical state without any external tuning. This model is often called Per

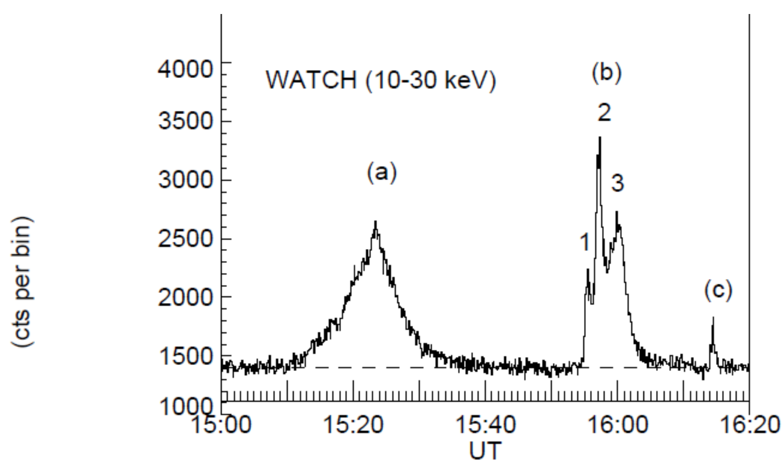

Fig. 1.3: Time profiles of the three WATCH solar flares observed between 15:00 and 16:20 UT on 1990 June 19. The peak count rates are identified as "a", "b" and "c" (Crosby et al. 1998).

Bak's sandpile model or the Bak-Tang-Wiesenfeld (BTW) model. In their first paper (Bak et al. 1987), the BTW model was derived from a model for the dynamics of an array of coupled pendulums. Thereafter the same model was interpreted in terms of sandpile dynamics (Bak et al. 1988). Sandpile avalanches are a paradigm of the SOC theory. The simplicity and beauty of the BTW sandpile model is illustrated in Fig. 1.4.

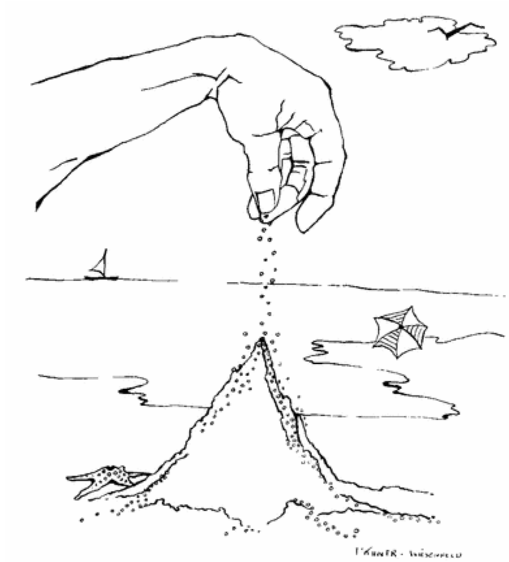

Fig. 1.4: The simplicity and beauty of the Bak-Tang-Wiesenfeld (BTW) sandpile model as drawn by Elaine K. D'Attner Wiesenfeld. Reprinted from Wiesenfeld et al. (1989) with permission.

Another model that exhibits SOC is the forest-fire model (Drossel and Schwabl 1992a; 1992b). In the simplest version of this model, a square grid of sites is considered. At each time step either a tree is planted on a randomly chosen unoccupied site or a spark is dropped on the site. If the spark is dropped on a tree, that tree and all adjacent trees are burned in a model forest-fire. The slider-block model also exhibits SOC and in this model an array of slider blocks are connected to a constant velocity driver plate by driver springs and to each other by connector springs (e.g. Carlson and Langer 1989; Carlson et al. 1994).

1.3.1 Does powerlaw behavior automatically imply SOC?

SOC always involves dynamic "avalanches" with SOC processes occurring spontaneously with an explosive evolution and multiplicative growth via next-neighbor interactions. The restriction to next-neighbor interactions in SOC processes essentially guarantees the statistical independency of individual events. However, there exist related physical processes (turbulence, Brownian motion, percolation, or chaotic systems) that share some of the characteristics of the SOC theory and thus are difficult to discriminate from a SOC process.

For example, self-organization (SO) patterns are quasi-stationary (e.g. geometric patterns in galaxy formation, granulation pattern on the solar surface, ripples on sand dunes) and SO patterns exhibit a close coupling over a large range. If dynamic processes are involved in the formation of SO patterns, they usually involve system-wide processes, such as diffusion, turbulence, convection, magneto-convection, which essentially operate with long-range interactions (via pressure, streams, flows). SO patterns can exhibit scale-free powerlaw distributions of spatial scales (e.g. the Kolmogorov spectrum in turbulent MHD cascades). Powerlaw behavior can therefore not be used as a concise distinction criterion between SO and SOC processes.

There exist also other theoretical models than SOC models that produce powerlaw behavior and many different explanations for the observed powerlaws exist. Examples include Turcotte and Malamud (2004) who related the inverse-cascade model to the results of several cellular automata models and also to real data observed for different natural hazards. Small clusters of (e.g. trees) on a grid coalesce to form larger clusters, and clusters are lost in fires that occur randomly. The result is a self-similar inverse cascade that satisfies an inverse powerlaw distribution of cluster sizes.

Rosner and Vaiana (1978) developed the stochastic relaxation model to describe solar flares. In their model flaring is a stochastic process, energy build-up is exponential between flares, and all the energy built up between flares is released by the following flare whereafter the system returns to its unperturbed "ground state" via the flare. However, their prediction that the duration of energy storage is correlated with flare size was not confirmed by observations (e.g. Lu 1995; Crosby et al. 1998; Wheatland 2000; Georgoulis et al. 2001).

Forced Self-Organized Criticality (FSOC) is an alternative concept that shares all the "avalanche" phenomenology of powerlaw distributions, but is not necessarily self-organized. The key aspect of the FSOC model is that some external dynamics exerts forces on a system to produce powerlaw like distributions of avalanches without internal self-organization (Chang 1999 and references therein).

Turbulence displays many of the common SOC observational signatures, such as the (scale-free) powerlaw distributions of spatial and temporal scales, the power spectra of time profiles, random waiting time distributions, spatial fractality, and temporal intermittency (e.g. Boffetta et al. 1999). The transition from laminar flow to turbulent flow at a critical Reynolds number represents a similar threshold instability criterion and is the analogy of the critical threshold value used in SOC systems.

Is it possible to distinguish between SOC and turbulence or other processes that show the same scaling? As seen in the above examples powerlaw behavior is not a sufficient argument for SOC. The following three SOC "physics-free" criteria (statistical independence, nonlinear coherent growth, random duration of rise times) have been proposed in Aschwanden (2011):

1. Statistical Independence: events that occur in a SOC system are statistically independent and not causally connected in space or time. Waiting time distributions should be consistent with a stationary or non-stationary Poisson process, in order to guarantee statistical independency by means of probabilities. Time scale separation (the time intervals of the driver are much longer than the time scale of the avalanches, at least for slowly-driven systems).
2. Nonlinear Coherent Growth: time evolution of a SOC event has an initial nonlinear growth phase after exceeding a critical threshold. The nonlinear growth of dissipated energy, or an observed signal that is approximately proportional to the energy dissipation rate, exhibits an exponential-like or multiplicative time profile for coherent processes.
3. Random Duration of Rise Times: if a system is in a state of SOC, the rise time or duration of the coherent growth phase of an avalanche is unpredictable and thus exhibits a random duration. The randomness of rise times can be verified from their statistical distributions being consistent with binomial, Poissonian, or exponential functions.

1.3.2 SOC and SOC-like models

During the last decades new SOC models, as well as non-SOC models have been proposed to explain the powerlaw behavior that is observed in large statistical datasets. These models have gone a step further so as to be able to describe additional characteristics of the observed data (e.g. slope value). For example, Georgoulis and Vlahos (1996) developed a cellular automaton SOC model that simulates flaring activity extending over an active sub-flaring background building on the work by Vlahos et al. (1995). Including both isotropic and anisotropic distributions, as well as a variable magnetic field driving mechanism, they were able to obtain two distinct powerlaw regimes representing possibly two different populations, one associated with standard flares from active regions, and the other with nanoflares from quiet-Sun regions (see Fig. 1.5).

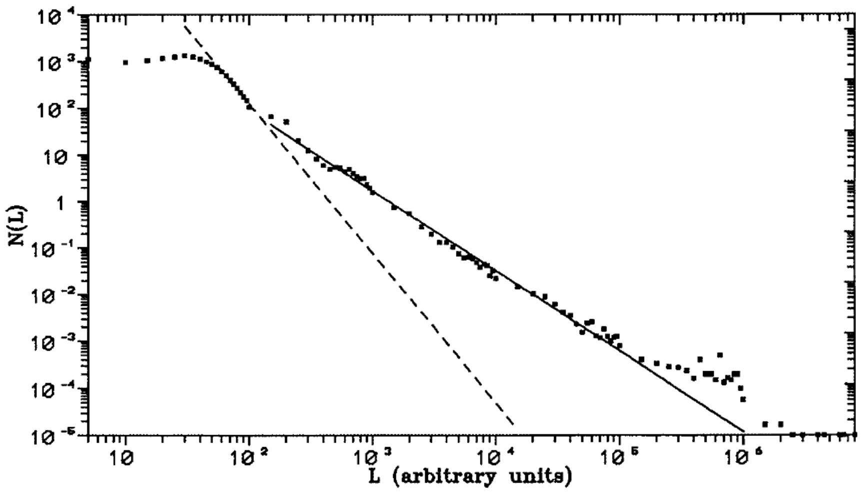

Fig. 1.5: Typical peak-luminosity frequency distribution for a 150 × 150 × 150 grid. The distribution is the average of 10 sample runs. Reprinted from Georgoulis and Vlahos (1996) with permission.

The theoretical side of SOC is covered in the first part of this book. SOC models and SOC related processes from a theoretical point of view are presented by Aschwanden in Chapter 2. He presents how SOC models have built on and been inspired by the original cellular automaton models. Furthermore, a number of alternative dynamical models that are related to SOC models or have similar scaling laws are also presented.

In some respects, chaotic systems exhibit similar complexity as SOC systems, regarding fractality and intermittency; even powerlaw distributions may result in the statistics of chaotic fluctuations. However, the difference to SOC systems is that chaotic systems have these signatures without having an intrinsic mechanism that keeps them near this critical point in a self-organizing way. The strong connections between fractal geometry and SOC from both a mathematical and conceptual understanding are described in Chapter 3 by McAteer.

Percolation controls a transport process that depends on the connectedness and propagation probability of nearest-neighbor elements. It has a lot in common with diffusion, fractal structures, as well as SOC "avalanches". Specifically, it is the fractality and intermittency of the propagating features of the percolation process that are what is in common with a SOC system. In Chapter 4 an introduction to percolation models applied to SOC phenomena is given by Milovanov. SOC-associated phenomena (self-organized turbulence in the Earth's magnetotail, phase transitions in SOC systems, mixed SOC-coherent behavior, periodic and auto-oscillatory patterns of behavior), are also discussed.

In probability theory, a branching process expresses next-neighbor interactions in terms of probabilities, a concept similar to how redistribution operates in cellular automaton models. Litvinenko (1998) applies methods of the branching theory to the Macpherson and MacKinnon (1997) cellular automation model for the occurrence of solar flares. In Chapter 5 Corral and Font-Clos present how models based on the branching process are applied to “natural hazards”. Applied to earthquakes a branching process implies the activation or slip of a fault segment that can trigger other segments to slip, with a certain probability, and so on.

In contrast to the SOC cellular automaton models that were described in the beginning of Section 3, networks are irregular nets of nodes that are interconnected in manifold patterns, containing nearest-neighbor connections and in some cases also arbitrary non-local, long-range connections. Zou, Heitzig, Small, and Kurths present in Chapter 6 recurrence networks as a novel tool of nonlinear time series analysis allowing the characterization of higher-order geometric properties of complex dynamical systems based on recurrences in phase space, which are a fundamental concept in classical mechanics. The main part of the Chapter is based on the Zou et al. (2012) paper. They demonstrate that recurrence networks obtained from various deterministic model systems as well as experimental data naturally display powerlaw degree distributions with scaling exponents that can be derived exclusively from the systems’ invariant densities.

SOC models are said to be slowly driven interaction dominated threshold systems (Jensen 1998). The scaling behavior of a SOC model can be related to some underlying continuous phase transition, which is triggered by the system self-organizing to the critical point. Pruessner presents in Chapter 7 an overview of how computer simulations are used to reproduce SOC behavior. The Chapter presents in more detail the original SOC models and shows how the observed scaling behavior can be better quantified. Furthermore, detailed functions that are used in SOC algorithms are described.

1.4 Where is SOC observed ?

Following Bak’s above mentioned pioneering work in 1987, not only was there an “avalanche” in SOC studies from the modelling side, but people started to search for signatures of SOC everywhere in natural occurring phenomena from the Sun to the Earth. In the second part of this book the Chapters concern how SOC is observed in all types of natural phenomena.

1.4.1 Phenomena on Earth showing SOC behavior

Back in the early 1990s simple sandpile experiments were performed to reproduce SOC (Held et al. 1990; Bak and Chen 1991). In simple terms sand grains are added to a sandpile until the slope of the sandpile reaches a critical value and an avalanche occurs. Later ricepile experiments were also performed (e.g. Frette et al. 1996). In Chapter 8 Pruessner describes many of the SOC laboratory experiments that have been performed during these last decades.

In parallel to experiments in the laboratory numerous studies have been performed on naturally occurring phenomena on Earth. The most well-known Earth-based SOC phenomenon that has been and is being studied is the earthquake. They are triggered when a mechanical instability occurs and a fracture (the sudden slip of a fault) appears in a part of the Earth's crust. Earthquakes are associated with the slider-block model (e.g. Carlson and Langer 1989; Carlson et al. 1994).

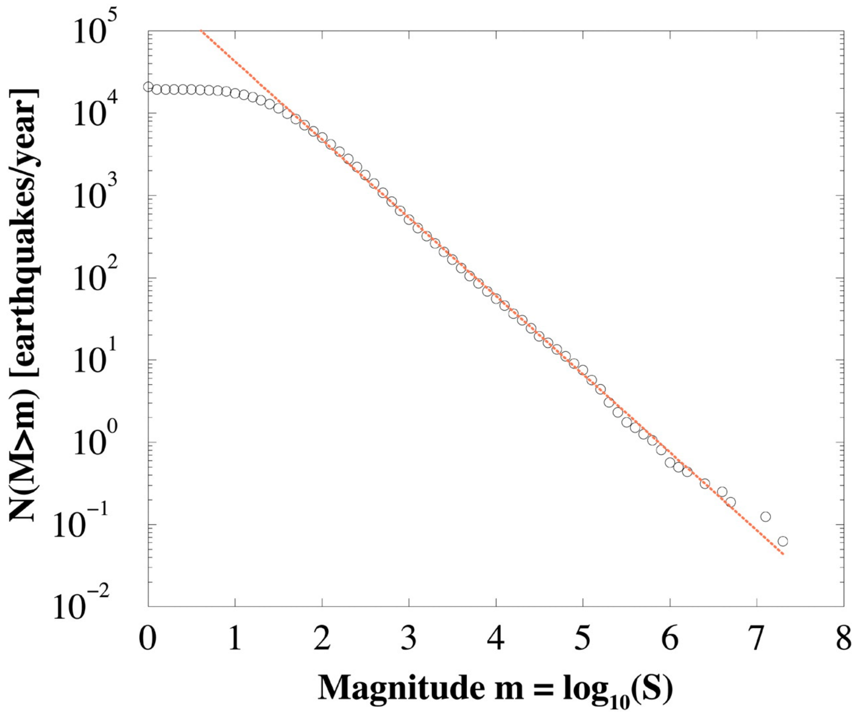

Fig. 1.6: Earthquake magnitude distribution showing a powerlaw behavior over six decades. The graph follows $log10N(M > m)\alpha - bm$, where b is the Gutenberg-Richter exponent $b = 1$ (dashed red line has a slope value of -0.95). Reprinted from Christensen et al. (2002) with permission.

Fig. 1.6 shows the earthquake magnitude distribution for the Southern California region, which is the number of earthquakes per year with magnitude (M > m). The dashed red line has a slope value of -0.95 and shows the Gutenberg-Richter Law with a gradient b $\approx$ 1 (Christensen et al. 2002; Christensen and Moloney 2005). As was seen for the solar flare distribution (Fig. 3.1) there is a turn-over in the lower end of the distribution due to problems associated with detecting small earthquakes. In Chapter 9 Sachs et al. present SOC applied to complex earthquakes and how this concept is observed in data and models, as well as how it is applied to forecasting. From a mitigation perspective the main question to answer is when will an earthquake occur? A new type of forecast based on the Natural Time Weibull (NTW) model shows promising results (Rundle et al. 2012) and is based on the idea of "filling in" a fat-tailed (scaling) distribution.

Besides seismology and earthquakes SOC behavior has been found in a wide range of geophysical systems. Forest-fires occurring on Earth are associated with the forest-fire model that also exhibits SOC behavior (Drossel and Schwabl 1992a; 1992b). In Chapter 10 Hergarten presents the forest-fire model and actual wildfires. Specifically he presents studies performed on observed wildfires and how the forest-fire model is able to reproduce the difference between natural and man-made forest-fires.

Natural landslide events are commonly associated with a trigger (e.g. earthquake (minutes after), a rapid snowmelt (hours to days), or an intense rainfall (days to weeks)) and range in size from a single landslide to many thousands. A landslide is a natural phenomenon associated with the Per Bak sandpile model. The impact of landslides is often limited to smaller areas than for example the damage caused by earthquakes. However, there is growing evidence for powerlaw size distributions in different types of landslides. In Chapter 11 Hergarten presents how SOC behavior is observed in landslides. One of the main conclusions is that scaling exponents found for regolith landslides strongly differ from those found for rockfalls and rockslides, but each of the classes may be characterized by a universal scaling exponent.

1.4.2 Phenomena in space showing SOC behavior

Besides the powerlaw behavior that is observed in natural occurring phenomena on Earth, it is also observed in a wide range of phenomena occurring in space. Like their counterparts on Earth they too come in all sizes and durations. Our closest star, the Sun, drives the continuous changing conditions in the space environment - the local space weather. It is well known that the solar corona is a very dynamic region which is the source of many phenomena (e.g. solar flares, coronal mass ejections, solar energetic particle events). It was therefore not surprising that it was on the Sun where SOC signatures were first discovered in space.

Like earthquakes are to SOC on Earth, solar flares are to SOC in space. Solar flares (including micro- and nano-flares) are one of the most compelling examples of SOC type behavior; their powerlaw distribution covers over eight orders of magnitude (Aschwanden 2011). Crosby et al. (1993) presents a summary of 13 powerlaw slopes (12 originating from observational results and 2 from modelling) found for different solar related parameters that were published over two decades ago. Since then this list has grown exponentially; see Aschwanden (2011) for an extensive overview. In Chapter 12 Charbonneau presents SOC behavior observed in solar phenomena as well as SOC models developed to describe the distribution of solar flares.

From the dawn of the space era observations in space have provided convincing evidence indicating that certain space plasma processes are in states of complexity and SOC, especially with the above discovery of the apparent powerlaw probability distribution of solar flare intensities. The most studied extra-solar SOC phenomena are stellar flares (Audard et al. 2000) and accretion or black-hole objects (Negoro et al. 1995; Mineshige and Negoro 1999). In Chapter 13 Aschwanden gives an overview of how SOC behavior has been observed in astrophysics. He compares theoretical predictions based on the fractal-diffusive self-organized criticality (FD-SOC) model (Aschwanden 2012) with observed powerlaws of size distributions observed in astrophysical systems. It is found that the generalized FD-SOC model can explain a large number of astrophysical observations (e.g. lunar craters, asteroid belts, Saturn rings, outer electron radiation belt enhancements, solar flares, soft gamma-ray repeaters, and blazars) and discriminate between different scaling laws of astrophysical observables.

1.5 Searching for a common signature: What does it all mean?

Does there exist a common "avalanche" signature? If yes, what does this all mean in a global way? How does the world and the Universe connect? Is there a universal signature that places constraints on the energy distribution of everything in the Universe? SOC behavior provides us a unique way to interpret the behavior of "avalanches" in a global way - is there a common thread in nature?

Producing frequency distributions on observational "avalanche" data displays powerlaw behavior in most instances. Each type of phenomenon is observed to have a range of powerlaw slope values. For example, considering solar flare hard X-ray parameters, in general the powerlaw slope ranges between -1.4 through -2.4 (Crosby 2011). The difference in value is also observed on measured parameters of the same type of "avalanche" suggesting that the slope may be detector dependent. An independent parameter based on theory is therefore better for comparison purposes. The energy released in an "avalanche" is such an independent parameter and can be compared directly without considering the measured data. In this way it can be investigated if energy is released in some type of universal way. For example, powerlaw frequency distributions of the energy released in some natural phenomena (e.g. solar flares, transient brightenings, nanoflares, ionospheric emissions "auroral blobs", earthquakes) are found to be similar (slope value of the powerlaw is approximately -1.5), see Crosby (2011).

In summary, modelling a given type of "avalanche" as a complex system in a self-organized critical state provides a good context to understand the frequency distributions of the parameters describing the phenomenon. It may be SOC or another theoretical model that can account for the observed powerlaw behavior, or it may end up being a mixture of many physical processes occurring simultaneously. However, as pointed out by Crosby (2011) in a SOC review paper, whatever the theory, the observations themselves provides one with useful information when performing frequency distributions on the data. Table 4.2 summarizes the information one obtains by performing frequency distributions on "avalanches" both on data obtained by observations and on SOC model outputs. It should also be emphasized that any model, be it SOC or non-SOC, must be able to reproduce what the observations are showing (model validation).

Performing frequency distributions on "avalanches" is also a tool for practical applications in regard to mitigation and risk analysis. Indeed, the applicability of powerlaw statistics to natural hazards has important practical implications. In the paper by Sachs et al. (2012) the question "Are dragon-kings relevant to probabilistic hazard assessment?" is addressed by studying earthquakes, volcanos, etc. Sornette (2009) developed the concept of the unexpected "dragon-kings" to describe this class of extreme events that are significantly larger than the extrapolation of the powerlaw scaling of their smaller counterparts. Probabilistic seismic hazard studies often extrapolate the rate of occurrence of small earthquakes to quantify the probability of occurrence of large earthquakes. Sachs et al. (2012) argue that this extrapolation would therefore not be valid if "dragon-king" earthquakes occur and one of their examples (cumulative frequency magnitude distribution of earthquakes in the Parkfield aftershock region covering 1972 to 2009) is displayed in Fig. 1.7. They find that the Parkfield main shock with m = 5.95 (identified as the red star on Fig. 1.7) is found to lie above the extrapolation of the powerlaw correlation of the smaller earthquakes. The authors question whether the difference between m = 5.65 based on the extrapolation and m = 5.95 can be attributed to the statistical variability of the characteristic earthquakes or if this is indeed a "dragon-king" type earthquake.

Fig. 1.8 shows the cumulative number of volcanic eruptions (Nc) during the period 1800-2002 with dense rock equivalent volume (V_{DRE}) greater than V_{DRE} as a function of V_{DRE}. Sachs et al. (2012) find that global frequency size distributions of earthquakes and volcanic eruptions exhibit powerlaw behavior for small sizes but a roll over for large events similar to the behavior

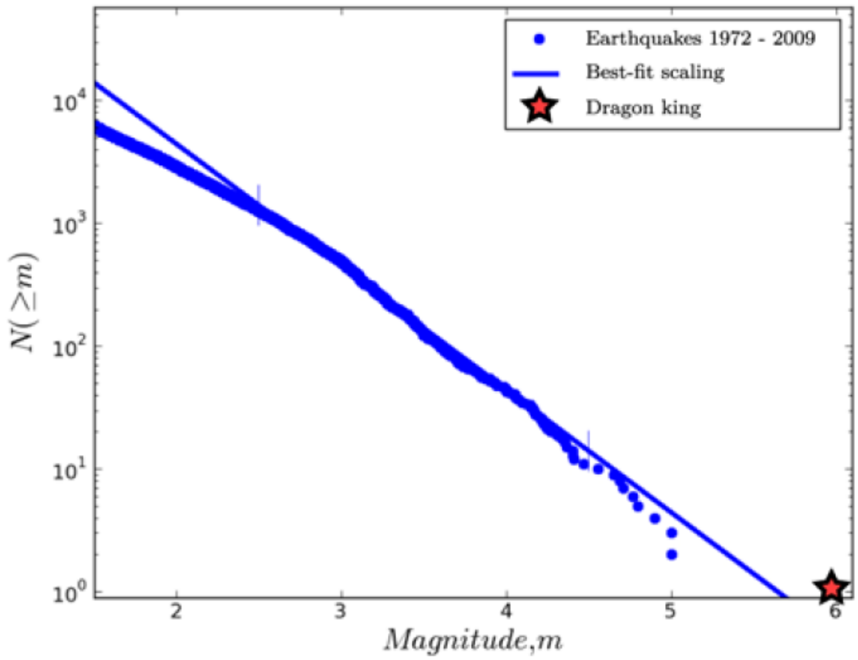

Fig. 1.7: Cumulative number of earthquakes with magnitude greater than m as a function of m for the Parkfield earthquake cycle 1972 to 2009. The best-fit scaling is shown as the blue line. The m = 5.95 Parkfield earthquake is shown as a "dragon-king" (identified as the red star). Reprinted from Sachs et al. (2012) with permission.

of the forestfire model for small firing frequencies. The Lake Toba Sumatra volcano is identified as the red star on Fig. 1.8 and is estimated to have erupted $V_{DRE} = 2{,}750 \pm 250\text{km}^3$ of dense rock equivalent 73,500 $\pm$ 500 years ago. However, the largest eruption expected in 73,500 years, extrapolating the powerlaw on Fig. 1.8, would have yielded a volume $V_{DRE} = 7.9 \times 10^6$ km^3. This is statistically not consistent and Sachs et al. (2012) therefore suggest a powerlaw with an exponential roll-over for volumes greater then $V_{DRE} \approx 10^2$ km^3 as the most realistic fit for the distribution displayed in Fig. 1.8. Such results show the important practical implications that powerlaw statistics has in regard to natural hazards and question whether a powerlaw is always the best representation for the very large "black swans" events. However, this exponential roll-over effect may also be due to observations having not been performed over a long enough period of time to cover all the statistics of the "avalanches" as mentioned in Section 2.

During the last years there has been renewed interest in SOC and its potential applications both from theoretical and practical point-of-views. Several

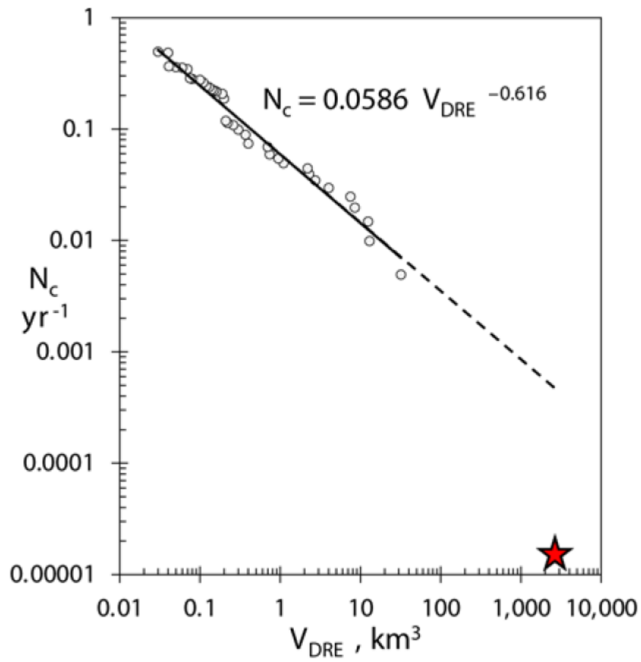

Fig. 1.8: Cumulative number of volcanic eruptions (Nc) during the period 1800-2002 with dense rock equivalent volume (V_{DRE}) greater than V_{DRE} as a function of V_{DRE}. The best-fit powerlaw scaling is also shown along with the Toba eruption in Sumatra (identified as the red star) occurring 73,500 ± 500 years ago. Reprinted from Sachs et al. (2012) with permission.

cross-disciplinary initiatives have currently been taken such as the International Space Science Institute International Team entitled "Self-Organized Criticality and Turbulence" that is made up of an interdisciplinary group of team members covering both the space- and Earth-sciences. Its aim is to cross-compare observations, to discuss SOC, SOC-related (such as turbulence), and non-SOC theoretical models, and to establish a diagnostic metrics between observations and theoretical models that yield new physical insights into SOC phenomena and complexity in nature.

As was shown in this Chapter SOC behavior has become one way of interpreting the powerlaw behavior observed in natural occurring phenomena on the Sun down to the Earth. This book, based on an inter-disciplinary approach, presents the many sides of SOC both from the theoretical side of the story as well as the observational.

Acknowledgements The author would like to thank the editor of this SOC book for asking her to write this Introduction Chapter. The author acknowledges the Inter-

Table 1.1: What do frequency distributions teach us? Based on Crosby (2011).

Observational statistics	- Frequency distributions performed on datasets describing natural dynamical phenomena (e.g. solar flares, earthquakes) exhibit powerlaw behavior.
Measurement problems and biases	- Turn-overs in the lower end of the frequency distribution for various phenomena may be attributed to detector sensitivity (missing the small events in the background noise). - Exponential turn-overs in the upper end of the frequency distribution for various phenomena may be due to either the length of the dataset (missing of long-term statistics) or finite-size effects of the system. - Measured parameters are detector dependent and may give bias in the slopes of the frequency distributions for comparison purposes.
Numerical SOC observations	-Various concepts/models exist that produce powerlaw behavior such as SOC. -SOC models are able to reproduce the results found when performing frequency distributions on measured data.
Statistical commonalities in SOC statistics	-Powerlaw behavior is found to be a universal characteristic defining natural dynamic phenomena (e.g. solar flares, earthquakes). -For each type of phenomenon most distributions performed on observational data can be represented by powerlaws having a range of powerlaw slope values. -Frequency distributions of the energy released in solar flares, transient brightenings, nanoflares, ionospheric emissions and earthquakes are found to be similar (slope value of the powerlaw is approximate -1.5).
Interpretations of physical processes	-Powerlaw frequency distributions result from nonlinear or coherent processes, have no characteristic spatial scale and are the hallmark of nonlinear dissipative systems. -Powerlaw frequency distributions of the energy released in some natural phenomena are found to be similar (may suggest that energy is released in some type of universal way).
Mitigation and risk analysis	-Results from frequency distributions provide limits to the maximum strength of a phenomenon, vital for mitigation studies - probability of extreme events occurring (limit to the size of an event over a given time period). -Implementing frequency distributions into the engineering approach “empirical models” is useful for design studies as well as probabilistic hazard assessment.

national Space Science Institute “Self-Organized Criticality and Turbulence” International Team of which she is a team member and which has been an inspiration for the writing of this Introduction Chapter. She also would like to acknowledge the reviewer who took the time to provide useful feed-back thereby improving the objective of this Introduction Chapter.

References

Aschwanden, M.J.: Self-Organized Criticality in Astrophysics. The Statistics of Nonlinear Processes in the Universe, ISBN 978-3-642-15000-5, 416p. Springer-Praxis: New York (2011)

Aschwanden, M.J.: A statistical fractal-diffusive avalanche model of a slowly-driven self-organized criticality system. Astron. Astrophys. **539**, A2 (2012)

Audard, M., GĂĽdel, M., Drake, J.J., and Kashyap, V.L.: Extreme-ultraviolet flare activity in late-type stars. Astrophys. J. **541**, 396–409 (2000)

Bak, P.: How Nature works. Springer-Verlag: New York (1996)

Bak, P., Tang, C. and Wiesenfeld, K.: Self-organized criticality - An explanation of 1/f noise. Physical Review Lett. **59/27**, 381–384 (1987)

Bak, P., Tang, C. and Wiesenfeld, K.: Self-organized criticality. Physical Review A. **38/1**, 364–374 (1988)

Bak, P. and Chen, K.: Self-organized criticality. Scientific American **264**, 46–53 (1991)

Boffetta, G., Carbone, V., Giuliani, P., Veltri, P. and Vulpiani, A.: Power laws in solar flares: self-organized criticality or turbulence. Phys. Rev. Lett. **83(2)**, 4662–4665 (1999)

Carlson, J.M. and Langer, J.S.: Properties of Earthquakes Generated by Fault Dynamics. Phys. Rev. Lett. **62**, 2632–2635 (1989)

Carlson, J.M., Langer, J.S. and Shaw, B.E.: Dynamics of earthquake faults. Rev. Modern Phys. **66**, 657–670 (1994)

Chapman, S.C., Watkins, N.W., Dendy, R.O., Helander, P. and Rowlands., G.: A simple avalanche model as an analogue for magnetospheric activity. Geophys. Res. Lett. **25/13**, 2397–2400 (1998)

Chang, T.S.: Self-organized criticality, multi-fractal spectra, sporadic localized reconnections and intermittent turbulence in the magnetotail. Phys. Plasmas. **6/11**, 4137–4145 (1999)

Charbonneau, P., McIntosh, S.W., Liu, H.L. and Bogdan, T.J.: Avalanche models for solar flares. Solar Phys. **203**, 321–353 (2001)

Christensen, K., Danon, L., Scanlon, T. and Bak, P.: Unified scaling law for earthquakes. PNAS. **99**, suppl. 1, 2509–2513 (2002)

Christensen, K. and Moloney, N.R.: Complexity And Criticality, 392p. Imperial College Press, London, U.K. (2005)

Crosby, N.B., Aschwanden, M.J. and Dennis, B.R.: Frequency Distributions and Correlations of Solar X-Ray Flare Parameters. Solar Phys. **143**, 275–299 (1993)

Crosby, N., Vilmer, N., Lund, N. and Sunyaev, R.: Deka-keV X-Ray Observations of Solar Bursts with WATCH/GRANAT: frequency distributions of burst parameters. Astronom. Astrophys. **334**, 299–313 (1998)

Crosby, N.B.: Frequency distributions: From the Sun to the Earth. Nonlinear Processes in Geophysics. **18/6**, 791–805 (2011)

Drossel, B. and Schwabl, F.: Self-organized critical forest-fire model. Phys. Rev. Lett. **69**, 11, 1629–1632 (1992a)

Drossel, B. and Schwabl, F.: Self-organized criticality in a forest-fire model. Physica A. **191**, 1-4 47–50 (1992b)

Frette, V., Christensen, K., Malthe-Sorenssen, A., Feder, J., Jessang, T. and Meakin, P.: Avalanche dynamics in a pile of rice. Nature. **379**, 49–52 (1996)

Georgoulis, M. and Vlahos L.: Coronal Heating by Nanoflares and the Variability of the Occurence Frequency in Solar Flares. Astrophys. J. Letters. **469**, L135–L138 (1996)

Georgoulis, M., Vilmer, N. and Crosby, N.: A Comparison between statistical properties of solar X-ray flares and avalanche predictions in cellular automata statistical flare models, A&A. **367**, 326–338 (2001)

Held, G.A., Solina, D.H., Solina, H., Keane, D.T., Haag, W.J., Horn, P.M. and Grinstein, G: Experimental Study of Critical mass Fluctuations in an Evolving Sandpile. Phys. Rev. Lett. **65**, 9, 1120–1123 (1990)

Hergarten, S.: Self-Organized Criticality in Earth Systems, 282p. Springer-Verlag: Germany (2002)

Jensen, H.J.:Emergent complex behavior in physical and biological systems, Cambridge University Press (1998)

Litvinenko, Y.E.: Analytical results in a cellular automaton model of solar flare occurrence. Astronomy and Astrophysics. **339**, L57–L60 (1998)

Lu, E.T.: Constraints on Energy Storage and Release Models for Astrophysical Transients and Solar Flares. Astrophys. J. **447**, 416–418 (1995)

Macpherson, K.P., and MacKinnon, A.L.: One-dimensional percolation models of transient phenomena. Physica A. **243**, 1–13 (1997)

Mineshige, S. and Negoro, H.: Accretion disks in the context of self-organized criticality: How to produce 1/f fluctuations? ASP Conf. Ser. **161**, 113–128 (1999)

Negoro H., Kitamoto S., Takeuchi M. and Mineshige S.: Statistics of X-ray fluctuations from Cygnus X-1: Reservoirs in the disk? Astrophys. J. **452**, L49–L52 (1995)

Pruessner, G.: Self-organised criticality, Cambridge University Press: Cambridge, UK (2012)

Rosner, R. and Vaiana, G.S.: Cosmic flare transients â“ Constraints upon models for energy storage and release derived from the event frequency distribution. The Astrophysical Journal. **222**, 1104–1108 (1978)

Rundle, J.B., Holliday, J.R., Graves, W., Turcotte, D.L., Tiampo, K.F. and Klein W.: Probabilities for large events in driven threshold systems. Phys. Rev. E. **86**, 021106 (2012)

Sachs, M.K., Yoder, M.R., Turcotte, D.L., Rundle, J.B. and Malamud, B.D.: Black swans, power laws, and dragon-kings: Earthquakes, volcanic eruptions, landslides, wildfires, floods, and SOC models. Eur. Phys. J. Special Topics **205**, 167–182 (2012)

Sornette, D.: Critical phenomena in natural sciences: chaos, fractals, self-organization and disorder: concepts and tools, 528p. Springer: Heidelberg (2004)

Sornette, D.: Black Swans and the Prediction of Crises. International Journal of Terraspace Science and Engineering 2 **1**, 1–18 (2009)

Taleb, N.N.: The Black Swan: The Impact of the Highly Improbable. 480p. Random House: New York (2007)

Turcotte, D.L.: Self-organized criticality. Rep. Prog. Phys. **62**, 1377–1429 (1999)

Turcotte, D.L. and Malamud, B.D.: Landslides, Forest Fires, and Earthquakes: Examples of Self-Organized Critical Behavior. Physica A. **340**, 580–589 (2004)

Vlahos, L., Georgoulis, M., Kluiving, R. and Paschos, P.: The statistical flare. Astronomy and Astrophysics. **299**, 897–911 (1995)

Wheatland, M.S.: Do solar flares exhibit an interval-size relationship? Solar Phys. **191**, 381–389 (2000)

Wiesenfeld, K., Tang, C., Bak, P.: A Physicist's Sandbox. Journal of Statistical Physics, **54**, 1441–1458 (1989)

Willinger, W., Alderson, D., Doyle, J.C. and Li, L.: More "normal" than normal: scaling distributions and complex systems. in *Proceedings of the 2004 Winter Simulation Conference*, ed. by R. G. Ingalls, M. D. Rossetti, J. S. Smith, and B. A. Peters. (IEEE Press, Piscataway, NJ), 130–141 (2004)

Zou, Y., Heitzig, J., Donner, R.V., Donges, J.F., Farmer, J.D., Meucci, R., Euzzor, S., Marwan, N. and Kurths, J.: Power-laws in recurrence networks from dynamical systems. EPL (Europhysics Letters). **98**, (2012)