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Growing electric field parallel to magnetic field due to transverse kinetic drift waves in inhomogeneous corona

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ABSTRACT

The transverse drift wave, which is unstable due to purely kinetic effects, and driven by the density and magnetic field gradients, is discussed in context of its application to the solar corona. The gradients of the two quantities are opposite to each other, as required by the equilibrium pressure balance, and they are in the plane perpendicular to the magnetic field vector. The transverse drift wave has such properties that it propagates strictly perpendicularly to both the magnetic field vector and the mentioned gradients. It is electromagnetic, with the perturbed electric field in the direction of the equilibrium magnetic field, while the perturbed magnetic field vector is in the direction of the equilibrium gradients. Such an orientation of the electric field implies a possibility of acceleration of coronal plasma particles along the background magnetic field, in both directions, outward and inward. In the case of locally open magnetic structures, the outwardly moving particles should contribute to the solar wind. Those moving inwards eventually arrive in the lower solar atmosphere where the mean free path is short and, due to collisions, they should disperse their energy to the surrounding plasma and contribute to heating. It is also shown that accelerated particles can additionally be stochastically heated by the wave. This completely new stochastic heating mechanism is found here for the first time. It takes place provided that the particles are simultaneously accelerated by the wave to large enough velocities in the parallel direction. The model is applicable to any inhomogeneous coronal environment, like magnetic loops, coronal holes and the so-called EIT waves, named after the Extreme-ultraviolet Imaging Telescope (EIT) used for their first detection.

Key words: Sun: corona – Sun: oscillations.

1 INTRODUCTION

Electric fields in the solar atmosphere have been in the focus of researchers for many decades. This is mainly because of their possible role in the acceleration and heating of plasma particles in the corona and solar wind; see Fletcher & Hudson (2008) and references cited therein. Numerous models dealing with the coronal heating are nicely described in the review papers of Klimtchuk (2006), and Narain & Ulmschneider (1990). In many situations it is in fact the parallel (to the magnetic field) electric field that is of major importance. Electric fields in the solar atmosphere have been reported in a large number of studies in the past; to mention just a few of those who observed very strong fields, Davis (1977) and Zhang & Smartt (1986) and references cited therein.

However, predicting the (parallel) electric field by appropriate physical models has turned out to be a great challenge, partly due to the fact that the most common description of the solar atmosphere is based on the one-fluid magnetohydrodynamics (MHD) model. The MHD theory ignores the electron mass and, as a result, in the low-frequency limit, $\omega \ll \Omega_i$, the basic equation for the parallel component of the electric field amplitude, $\varepsilon_{zz} E_z = 0$, has only trivial solutions. On the other hand, within the two-component (or in general multicomponent) description, regardless of whether it is fluid or kinetic, the parallel electric field appears naturally, and various plasma modes in multicomponent plasma theory predict its presence.

As an example, in our recent papers dealing with drift waves in inhomogeneous solar corona (Vranjes & Poedts 2009a,b,c,d, 2010a,b), and where a new paradigm for the coronal heating has been put forward, the electric field appears due to growing electrostatic drift modes that propagate obliquely to the magnetic field vector. Typical scales for those instabilities are such that $k_\perp \gg k_\parallel$, where $k_\perp$, $k_\parallel$ are, respectively, the perpendicular and parallel wavenumber components with respect to the magnetic field. The drift wave instabilities were shown to be able to produce the electric field in the perpendicular direction that was measured in tens

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of kV m^{-1}. This was accompanied by strong stochastic heating of ions. Details are available in the references given above. Although, in view of $k_\perp \gg k_\parallel$, this implies relatively weak simultaneous electric fields along the magnetic field vector, a considerable particle acceleration in the parallel direction was demonstrated (Vranjes & Poedts 2009a).

Except for in Vranjes & Poedts (2009b), in all these works the driving source for the instability was the density gradient. Hence, the mode requires no external driver, and it develops primarily in the area with the strongest density gradient. So the energy stored in such plasma inhomogeneities is released through the growing drift-wave mode, resulting eventually in the particle heating and acceleration. On the other hand, in Vranjes & Poedts (2009b) all the three background plasma quantities (density, temperature, magnetic field) were inhomogeneous, resulting in even stronger instability.

A basic common feature of these studies is that the drift wave is essentially *longitudinal* electrostatic mode [though in principle coupled to the Alfvén wave (Vranjes & Poedts 2010a)]. In a geometry with the magnetic field given by $\boldsymbol{B}_0 = B_0\boldsymbol{e}_z$, and the equilibrium gradients in the x-direction, the mode was propagating nearly perpendicularly to both the magnetic field vector and the density gradient vector, i.e. predominantly in the y-direction, hence the wave electric field was $\boldsymbol{E} = E_y\boldsymbol{e}_y + E_z\boldsymbol{e}_z$, $|E_y| \gg |E_z|$. This further implies predominantly perpendicular displacements of the plasma particles, due to the leading-order $\boldsymbol{E} \times \boldsymbol{B}$ drift, that is in the x-direction.

However, there exists yet another type of drift waves, called *the transverse drift wave*, which also includes the gradient of the magnetic field (e.g. in the x-direction). A remarkable though unusual feature of the mode is that its perturbed electric field is in fact in the direction of the background magnetic field vector, while the mode propagates *strictly perpendicular* to it, in the y-direction. These features make it similar to the so-called ordinary wave (O-wave) known from basic plasma theory. However, in comparison with all other plasma waves, including the O-wave, generally speaking, the drift waves are unique in the sense that their existence implies the presence of free energy in the system (already the real part of the frequency contains the equilibrium plasma gradients). This free energy is then released through the instability of the drift wave in accordance with the mode growth rate. On the other hand, in the particular case of the transverse drift wave and the O-wave, the frequency of the former is low, and this feature appears to be essential for the acceleration and heating, as will be shown further in the text [see equation (6)]. To avoid any confusion in the terminology, we stress again that in the forthcoming text the term 'transverse' is used with respect to the wavenumber $\boldsymbol{k} \equiv k_y\boldsymbol{e}_y$, while the terms 'perpendicular' and 'parallel' are used with respect to the magnetic field vector $\boldsymbol{B}_0 = B_0\boldsymbol{e}_z$.

The transverse drift mode has been predicted long ago in Krall & Rosenbluth (1963), and also described in detail in Krall (1968) and Mikhailovskii (1992). Yet, the impression is that it is not enough exploited in the laboratory plasmas. It has been used in just a few applications to space plasmas (Chamberlain 1963; Krall & Tidman 1969), but definitely never mentioned in the context of the solar plasma. Interestingly enough, a similar model has been successfully used in Griv & Peter (1996) and in Griv, Yuan & Gedalin (1999), in a completely different physical environment, in the investigation of small-amplitude density waves in galactic discs with radial gradients.

The mode that we are going to study here is usually called in the literature the magnetic drift mode, which is a bit misleading as this puts stress on the magnetic nature of the mode. In fact it is more appropriate to call it the electromagnetic (EM) drift mode as it naturally involves both the magnetic and electric field perturbations: in the case of the equilibrium field $\boldsymbol{B}_0 = B_0\boldsymbol{e}_z$, the perturbed field components are $\boldsymbol{E}_1 = E_{z1}\boldsymbol{e}_z$ and $\boldsymbol{B}_1 = (kE_{z1}/\omega)\boldsymbol{e}_x$. Hence, the term *transverse* here describes the essential fact that both, the perturbed electric field (that is parallel to $\boldsymbol{B}_0$) and the perturbed magnetic field $\boldsymbol{B}_1$ are *perpendicular to the wavenumber* $\boldsymbol{k} = k_y\boldsymbol{e}_y$.

This plasma mode should not be confused with a mode obtained in the multicomponent fluid theory, which is usually called the magnetic (electron) drift mode (the alternative names are electron-MHD mode and magnetic electron drift vortex mode) which, however, essentially involves the equilibrium temperature gradient too. Referring to our mode, further in the text we shall use either the term EM-drift mode, or the term transverse drift mode.

2 BASICS OF THE KINETIC TRANSVERSE DRIFT WAVE

In order to describe the mode, one starts with the geometry as explained above, with inhomogeneous magnetic field and density $\boldsymbol{B}_0 = B_0(1 + \epsilon_b x)\boldsymbol{e}_z$, $n_0 = n_0(x)$, and with small *electromagnetic* perturbations propagating strictly perpendicularly to the magnetic field, that are of the form $f(x)\exp(-\mathrm{i}\omega t + \mathrm{i}k_y y)$. The plasma-$\beta$ is in principle arbitrary (i.e. finite), the wave frequency satisfies the condition $\omega/\Omega_i \ll 1$, where $\Omega_i = q_i B_0/m_i$ is the ion gyrofrequency, and small terms of the order $(k_y\rho_i)^2(\Omega_i/\omega)$ are retained. Here, $\rho_i = v_{Ti}/\Omega_i$ is the ion Larmor radius, and $v_{Ti}^2 = \kappa T_i/m_i$. The assumed linear profile of the magnetic field in the x-direction can in fact be generalized to an arbitrary $B_0(x)$, as long as $\rho_i/L_B < 1$, where $L_B = [(\mathrm{d}B_0/\mathrm{d}x)/B_0]^{-1} \equiv 1/\epsilon_b$ is the characteristic scalelength for the magnetic field inhomogeneity. In the equilibrium the distribution function reads

$$f_{j0} = N_0\left(\frac{m_j}{2\pi\kappa T_j}\right)^{3/2}\exp\left\{-\frac{m_j v^2}{2\kappa T_j}\left[1 - \epsilon_n\left(x + \frac{v_y}{\Omega_j}\right)\right]\right\}. \qquad (1)$$

The parameters ϵ_b and $\epsilon_n = 1/L_n = [(\mathrm{d}n_0/\mathrm{d}x)/n_0]$ are related through the plasma-β, i.e. $\epsilon_b/\epsilon_n = \beta/2$. This can be more directly seen from the two-fluid equilibrium set of equations, i.e. first by applying the vector product $\times\boldsymbol{B}_0$ on to the Ampère law, and then using the equilibrium momentum equation

$$q_j\boldsymbol{v}_{j0} \times B_0 + \nabla(n_{j0}T_{j0})/n_{j0} = 0,$$

to express the Lorentz force in the modified Ampère law through the density gradient. Here, v_{j0} is the equilibrium diamagnetic drift velocity. It is worth noting that in the theory of standard (oblique) electrostatic drift waves this same relation between ϵ_b, ϵ_n justifies the commonly used description (Weiland 2000) in which the magnetic field inhomogeneity is simply ignored as being small in comparison to the density inhomogeneity, $\epsilon_b \sim \beta\epsilon_n$, $\beta \ll 1$.

In Krall & Rosenbluth (1963), the kinetic collisionless Boltzmann–Maxwell set of equations is used in a completely general case for the perturbed EM field, i.e. $\boldsymbol{E}_1 = \boldsymbol{E}_x + \boldsymbol{E}_y + \boldsymbol{E}_z$, $\boldsymbol{B}_1 = \boldsymbol{B}_x + \boldsymbol{B}_y + \boldsymbol{B}_z$. This procedure yielded three equations for $\boldsymbol{E}_1$; two of them described two mixed longitudinal–transverse modes comprising of the components E_x, E_y. The third was an equation for the purely transverse (with respect to the wave direction) component E_z parallel to the magnetic field B_0, thus propagating perpendicular to $\boldsymbol{B}_0$. After using the assumptions $k_y^2c^2 \gg \omega^2$ and $k_y^2 v_{Te}^2 \ll \omega_{\mathrm{pe}}^2$, $\omega_{\mathrm{pe}}^2 = e^2 n_{e0}/(\varepsilon_0 m_e)$, and $k_y\rho_i < 1$, the third equation for E_z yielded the following frequency and the growth rate, respectively (Krall & Rosenbluth 1963; Weiland 2000):

$$\omega_{\mathrm{r}} = -\frac{k_y\kappa T_e}{e n_0 B_0}\frac{\mathrm{d}n_0}{\mathrm{d}x}\frac{1}{1 + k_y^2c^2/\omega_{\mathrm{pe}}^2}, \qquad (2)$$

$$\frac{\gamma}{\omega_{\rm r}} = \pi\frac{m_{\rm e}}{m_{\rm i}}\frac{\epsilon_n}{\epsilon_b}\left(1+\frac{k_y^2c^2}{\omega_{\rm pe}^2}\right)^{-2}\left(1+\frac{k_y^2c^2}{\omega_{\rm pe}^2}+\frac{T_{\rm e}}{T_{\rm i}}\right)$$
$$\times \exp\left\{-\frac{\epsilon_n}{\epsilon_b}\frac{T_{\rm e}}{T_{\rm i}}\frac{1}{1+k_y^2c^2/\omega_{\rm pe}^2}\right\}. \quad (3)$$

In the derivation of equations (2) and (3) the usual local approximation is used, implying that $k_yL_n, k_yL_b > 1$.

It is interesting to stress that in the derivation of the Cauchy principal value integral, the ion contribution to the real part of equation (2) was negligible as compared to the electron part because it was smaller by a factor of m_e/m_i, while at the same time the imaginary part in equation (3) was completely due to the ion contribution. Observe that the unstable case implies a geometry in which the gradients of the density and the magnetic field have opposite signs, i.e.

$$\frac{1}{n_0}\frac{{\rm d}n_0}{{\rm d}x}\frac{1}{B_0}\frac{{\rm d}B_0}{{\rm d}x} < 0. \quad (4)$$

Such a situation is in fact expected to be typical in inhomogeneous coronal plasma because it implies a correct balance between the gas and magnetic pressures in a plasma with spatially changing parameters.

The normalized growth rate (3) is plotted in Fig. 1 in terms of the plasma-β for the electron–proton coronal plasma with the parameters: $n_0 = 5\times10^{14}$ m^{-3}, $T_0 = 10^6$ K. Here, we used the fact that $L_n/L_B = \beta/2$ so the fixed values for the density and temperature still imply a large span of possible values for L_n and L_B, and this at the same time determines B_0. For example, the maximum of $\gamma/\omega_{\rm r}$ in Fig. 1 is at the magnetic field of the order of a few Gauss; on the other hand, the ratio is very low for the strong magnetic field (the left-hand side of the graph).

The plot is not sensitive for changes in k_y, yet we keep it in the range that would satisfy the applicability of equations (2) and (3), i.e. $k_y\rho_i < 1$. For the given parameters and the magnetic field of a few Gauss, this implies wavelengths above 30–40 m and consequently the graph is valid for any value of L_n satisfying the local analysis used above, $k_yL_n \gg 1$. To get some feeling about wave frequencies, if we take $\lambda_y = 100$ m (implying that $k_y\rho_i = 0.3$) and assume $L_n = 10^5$ m, we have $\omega_{\rm r} = 0.27$ Hz, while for $L_n = 10^6$ m we have $\omega_{\rm r} = 0.027$ Hz. Note that, for example, in the latter case and for $B_0 = 2\times10^{-4}$ T we have $\beta = 0.43$ and consequently $L_B = 4.6\times10^6$ m.

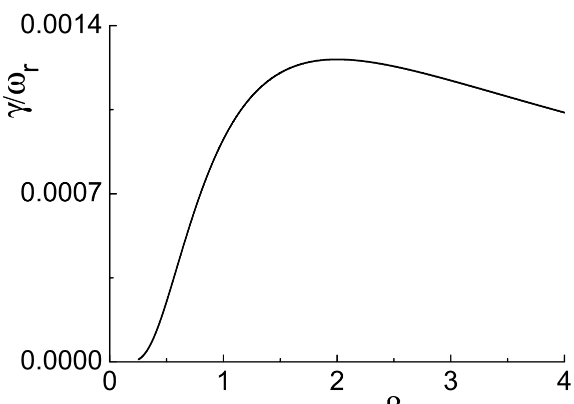

Figure 1. The ratio of the growth rate (3) and the wave frequency (2) of the transverse drift wave in terms of the plasma-β for fixed values of the plasma density and the temperature.

Compared to the standard oblique ($\boldsymbol{k}\cdot\boldsymbol{B}_0 \neq 0$) drift-wave instabilities which imply longitudinal electric field perturbations $\boldsymbol{k}\|\boldsymbol{E}_1$ (Vranjes & Poedts 2009a,b,c, 2010a,b), the presented growth for the transverse drift-wave instability ($\boldsymbol{E}_1\|B_0$, $\boldsymbol{E}_1\perp\boldsymbol{k}$) is rather small. Note that in Wu, Shi & Zhou (1986) it was shown that under certain conditions the instability may become influenced by electrons and in fact be much stronger. In their example this was due to a weak electron temperature anisotropy, $T_\perp < T_\|$, where $\perp$ and $\|$ denote, respectively, directions perpendicular to the magnetic field and parallel to it. In addition to this, the growth rate was shown to be huge, i.e. comparable to the ion gyrofrequency, when the scalelength of the density gradient L_n was comparable to the ion gyroradius.

The transverse drift-wave instability can also become stronger with the simultaneous presence of the temperature gradient. This may be found in Mikhailovskii (1992), where a similar set of equations for the transverse drift wave is derived, but in the limit of a small plasma-β. The frequency is similar to above, though it includes the pressure gradient $\omega_{\rm r} = \omega_{*\rm pe}$, while the growth rate is

$$\gamma = \pi\frac{m_{\rm e}}{m_{\rm i}}\left(1+\frac{2}{\beta}\frac{\eta}{1+\eta}\right)\frac{\omega_{*\rm pe}^2}{|\omega_{Di}|}\exp\left(-\frac{2T_{\rm e}}{\beta T_{\rm i}}\right),$$
$$\eta = \frac{1}{T_0}\frac{{\rm d}T_0}{{\rm d}x}\Bigg/\frac{1}{n_0}\frac{{\rm d}n_0}{{\rm d}x}, \quad \omega_{*\rm pe} = \frac{k_y}{en_0B_0}\frac{\partial p_{\rm e}}{\partial x},$$
$$\omega_{Di} = \frac{k_y\kappa T_0}{q_iB_0}\frac{\partial\log B_0}{\partial x}. \quad (5)$$

The contribution of the additional temperature gradient through the factor $\eta/(1+\eta)$ is obviously stronger for a smaller β.

The drift instability is also frequently called 'universal'. The term describes its capability to develop due to various mechanisms in any plasma description, fluid or kinetic, collisional or collisionless. This is understandable in view of the fact that the mode implies a free energy in the system, which is already seen in the real part of the frequency (2) in the form of the density or magnetic field gradient. The release of the energy through those numerous instabilities is just a natural tendency of the system to relax towards a lower energy state. It is worth mentioning that even in the case when linear instabilities of the mode are absent or suppressed, this relaxation and instability may develop non-linearly. One nice example of that kind may be found in Drake, Zeiler & Biskamp (1995), where the linear collisional drift mode, although linearly stabilized by the magnetic shear, still develops some strong non-linear turbulence.

3 ACCELERATION OF PARTICLES

In view of the geometry of the mode, having a strong electric field is not so essential. Even a very small electric field due to the (weakly) growing transverse drift mode described by equations (2) and (3) can, within one half-period of the wave, easily accelerate plasma particles to velocities of the solar wind magnitude. This is easily understood because both the wave front and the electric field are parallel to the magnetic field line. A particle is continuously accelerated during one half-period of the wave because the electric field is directed along the magnetic line. This may be tested by assuming the wave $E_0\sin(\omega_{\rm r}t)$, which yields the ion velocity achieved within one half-period $T/2 = \pi/\omega_{\rm r}$ as

$$v = (eE_0/m_i)\int_0^{T/2}\sin(\omega_{\rm r}t) = 2eE_0/(m_i\omega_{\rm r}). \quad (6)$$

Hence, the lower the wave frequency the more time for the acceleration, and the larger the particle velocity. For the earlier obtained frequency $\omega_{\rm r} = 0.27$ Hz and a very weak electric field $E_0 = 0.001$ V m^{-1}, the velocity after the time $\pi/\omega_{\rm r}$ is over 700 km s^{-1}. The actual velocity of the particle may be somewhat different, yet the general picture remains.

Further, particularly in the case of locally open magnetic field lines, like in the coronal holes, the outward-moving particles will eventually become a part of the solar wind. On the other hand, the inward-moving particles will propagate towards lower layers of the solar atmosphere, where their mean free path is considerably reduced. This is due to higher density from one side, and due to lower temperature from the other (note that the mean free path of the particles j for Coulomb interaction is proportional to v_j^4/n_0). Consequently, the inward-moving particles will collide with particles in the lower layers, and they will consequently disperse their energy obtained previously by the wave. At the end this will result in an increased temperature at a certain height, possibly at the transition layer (TL). Hence, the proposed scenario implies a TL that is in fact *heated from above.*

4 DETAILS OF PARTICLE MOTION IN THE TRANSVERSE DRIFT WAVE

For the wave electric field of the form

$$E_{z1} = E_0 \exp(\gamma t) \sin(k_y y - \omega_r t),$$

where ω_r and γ are determined by equations (2) and (3) and E_0 is a constant, and expressing the perturbed magnetic field through Faraday law $\boldsymbol{B}_1 \simeq \boldsymbol{e}_x k_y E_{z1}/\omega_r$, the displacement of a particle due to the wave is described by

$$x'' = \Omega y', \tag{7}$$

$$y'' = -\Omega x' + \frac{k_y \Omega E_0 \exp(\gamma t)}{\omega B_0} \sin(k_y y - \omega_r t), \tag{8}$$

$$z'' = \frac{e E_0 \exp(\gamma t)}{m} \left(1 - \frac{k_y y'}{\omega_r}\right) \sin(k_y y - \omega_r t). \tag{9}$$

Here, the prime denotes derivative in time d/dt.

Equation (7) can be integrated and the resulting integration constant in

$$x' = \Omega y + C$$

can without loss of generality be set to zero. Using this in equations (8) and (9) and after introducing the normalization $t \to \Omega t$, x, y, $z \to k_y x$, $k_y y$, $k_y z$, one obtains the following set of equations:

$$y'' + y - a(t) z' \sin(y - bt) = 0, \tag{10}$$

$$z'' - a(t)(b - y') \sin(y - bt) = 0, \tag{11}$$

$$b = \frac{\omega}{\Omega}, \quad a(t) = \frac{k_y E_0 \exp(\gamma t)}{b B_0 \Omega}.$$

Similar to Vranjes & Poedts (2010b), equation (10) can be further transformed by introducing the new variable $\tau = bt/2$, yielding

$$\frac{\mathrm{d}^2 y}{\mathrm{d}\tau^2} + \left[\frac{4}{b^2} - \alpha(\tau)\frac{2}{b^2}\cos(2\tau)\right] y = \frac{2}{b^2}\alpha(\tau)\sin(2\tau). \tag{12}$$

Here, $\alpha(\tau) = k_y E_0 (\mathrm{d}z/\mathrm{d}\tau) \exp(2\gamma\tau)/(B_0 \Omega)$. Equation (12), derived in the limit $\varphi = |y - bt| \ll 1$ and using $\sin(\varphi) \simeq -\sin(bt) + y\cos(bt)$, is a driven Mathieu equation with a possibility of unstable solutions for $b = 2/n$, where n is integer (Chen, Lin & White 2001; White, Chen & Lin 2002). It is particularly interesting to observe that the unstable solutions in the perpendicular y-direction (which here essentially imply a stochastic heating of particles) are in fact determined by the parallel velocity dz/dτ which is within the term $\alpha(\tau)$.

This same conclusion may be deduced also by a slightly different approach (Bellan 2006) as follows. Assume two adjacent particles in the wave field, at positions $\boldsymbol{r}_1$, $\boldsymbol{r}_2$ such that $\boldsymbol{r}_2 = \boldsymbol{r}_1 + \delta \boldsymbol{r}$, and in general having different velocities $\boldsymbol{v}_1$, $\boldsymbol{v}_2$ determined by

$$\frac{\mathrm{d}\boldsymbol{v}_1}{\mathrm{d}t} = \frac{q}{m}\left[\boldsymbol{E}(\boldsymbol{r}_1, t) + \boldsymbol{v}_1 \times \boldsymbol{B}(\boldsymbol{r}_1, t)\right],$$

$$\frac{\mathrm{d}\boldsymbol{v}_2}{\mathrm{d}t} = \frac{q}{m}\left[\boldsymbol{E}(\boldsymbol{r}_2, t) + \boldsymbol{v}_2 \times \boldsymbol{B}(\boldsymbol{r}_2, t)\right].$$

We may now investigate how the distance between the particles will change in time due to the presence of the wave. In an ordinary situation the two particles will perform a similar motion in the wave field. Otherwise the distance between them may grow in time and this is equivalent to stochastic heating (Bellan 2006). Subtracting the equations yields

$$\frac{\mathrm{d}^2 \delta \boldsymbol{r}}{\mathrm{d}t^2} = \frac{q}{m}\left[(\delta \boldsymbol{r} \cdot \nabla)\boldsymbol{E}(\boldsymbol{r}_1, t) + \frac{\delta \boldsymbol{r}}{\mathrm{d}t} \times \boldsymbol{B}_0 + \frac{\delta \boldsymbol{r}}{\mathrm{d}t} \times \boldsymbol{B}_1(\boldsymbol{r}_1, t) - \boldsymbol{v}_2 \times (\delta \boldsymbol{r} \cdot \nabla)\boldsymbol{B}(\boldsymbol{r}_1, t)\right].$$

Here, we used the following notation:

$$\boldsymbol{B}(\boldsymbol{r}_1, t) = \boldsymbol{B}_0 + \boldsymbol{B}_1(\boldsymbol{r}_1, t),$$

$$\boldsymbol{B}(\boldsymbol{r}_2, t) = \boldsymbol{B}_0 + \boldsymbol{B}_1(\boldsymbol{r}_1, t) + (\delta \boldsymbol{r} \cdot \nabla)\boldsymbol{B}(\boldsymbol{r}_1, t),$$

$$\boldsymbol{E}(\boldsymbol{r}_2, t) = \boldsymbol{E}_1(\boldsymbol{r}_1, t) + (\delta \boldsymbol{r} \cdot \nabla)\boldsymbol{E}_1(\boldsymbol{r}_1, t).$$

Bearing in mind that for the given mode in general $\boldsymbol{E}_1 = E_1 \boldsymbol{e}_z$, $\boldsymbol{B}_1 = B_1 \boldsymbol{e}_x$, we may write the following two equations for the displacement in the perpendicular (x, y) plane:

$$\frac{\mathrm{d}^2 \delta x}{\mathrm{d}t^2} = \Omega \frac{\mathrm{d}\delta y}{\mathrm{d}t}, \tag{13}$$

$$\frac{\mathrm{d}^2 \delta y}{\mathrm{d}t^2} = -\Omega \frac{\mathrm{d}\delta x}{\mathrm{d}t} + \frac{q}{m}\frac{\mathrm{d}\delta z}{\mathrm{d}t} B_1(\boldsymbol{r}_1, t) - v_{2,z}\frac{q}{m}(\delta \boldsymbol{r} \cdot \nabla) B_1(\boldsymbol{r}_1, t). \tag{14}$$

Integrating the first equation and setting the resulting expression into the second equation, one obtains

$$\frac{\mathrm{d}^2 \delta y}{\mathrm{d}t^2} + \left[\Omega^2 - \Omega v_{2,z}\frac{\partial B_1(\boldsymbol{r}_1, t)/B_0}{\partial y}\right]\delta y = \Omega \frac{\mathrm{d}\delta z}{\mathrm{d}t}\frac{B_1(\boldsymbol{r}_1, t)}{B_0}. \tag{15}$$

Equation (15) is equivalent to equation (12). If $\xi \equiv \Omega - v_{2,z}\partial B_1/\partial y$ is positive, equation (15) describes forced harmonic oscillations due to the term on the right-hand side. Here $v_{2,z}$ is determined by equation (9) or equation (11), and both $v_{2,z}$ and B_1 periodically change sign due to the wave propagation, therefore ξ may become negative. This implies solutions for δy that grow exponentially in time. Hence, the distance between the two starting particles grows as if the medium is heated, and this is the essence of the stochastic heating by the wave, but here determined by a completely new mechanism.

Details of the particle motion in the presence of a drift wave which causes stochastic heating can be found in Vranjes & Poedts (2010b), where the drift wave in question had a completely different nature (i.e. $\boldsymbol{k} \cdot \boldsymbol{B}_0 \neq 0$). The heating in that case was entirely due to the polarization drift, and it required the perpendicular wavelength to be comparable to the gyroradius. The following condition was to be satisfied:

$$k_y^2 \rho_i^2 e\phi_1(t)/(\kappa T_i) \geq 1. \tag{16}$$

Here, $\phi_1(t)$ is the potential of the electrostatic drift wave that is growing due to any kind of the drift wave instability. Because of the great mass difference, the heating by a single mode naturally cannot act in the same time on both electrons and ions. The coupling with the Alfvén wave, which takes place for relatively larger values of the plasma-β, does not affect the heating considerably (Vranjes & Poedts 2010a).

On the other hand, the threshold for the stochastic heating that follows from equation (15),

$$|v_z| > v_c = \frac{B_0}{B_1}\frac{\Omega}{k_y}, \tag{17}$$

has completely different properties. Implicitly, it still contains the magnitude of the electric field (that is however not electrostatic in the present case) because $v_{2,z}$ is determined by it (cf. equation 9). However, no relation appears between the wavelength and the gyroradius, and it does not exclude a possibility for a simultaneous heating of both electrons and ions.

However, a more peculiar feature that follows from equations (15, 17) is that only those particles that are already accelerated in the parallel direction by the wave to the velocity v_z exceeding v_c will, in addition to acceleration, be stochastically heated. This is completely different as compared to 'standard' stochastic heating by oblique drift waves (which yields equation 16) and it follows from the *non-linear* Lorentz force terms in equations (13) and (14).

Typically, $B_0/B_1 > 1$ (or $\gg 1$), so in order to remain below the speed of light, for electrons it would require very short wavelengths. Hence, the electrons should only be subject to acceleration by this mode, and no stochastic heating is expected. For ions, the condition is more easily satisfied. Using $B_0 = 2 \times 10^{-4}$ T and assuming $B_0/B_1 = 10^2$, the threshold velocity for ions becomes $v_c = 305 \times 10^3 \lambda_y$ m s^{-1}. In the case of meter-size perpendicular wavelengths, the acceleration of ions along the magnetic field lines should develop simultaneously with the stochastic heating. However, in principle the analytical model used above is not well satisfied for this limit, which implies $k_y\rho_i < 1$. In this case the kinetic derivations for ions are performed differently (Krall & Rosenbluth 1963) and the resulting growth rate of the mode is lower, proportional to $(m_e/m_i)^{3/2}$:

$$\omega_r = -\frac{2\kappa T_e k_y \epsilon_b (k_y\rho_e)^{-2}}{m_e\Omega_e(1 + T_i/T_e)}, \tag{18}$$

$$\frac{\gamma}{\omega_r} = \frac{\pi}{k_y^2\rho_e^2}\frac{T_i}{T_e}\left(\frac{m_e}{m_i}\right)^{3/2}\left(\frac{2+2T_i}{T_e}\right)^{-1/2} \times \exp\left[-\frac{2T_e/T_i}{k_y^2\rho_e^2(1+T_i/T_e)}\right]. \tag{19}$$

In this case the (sub)meter size of the wavelength are formally analytically allowed and simultaneous acceleration and stochastic heating of ions is very likely.

5 APPLICABILITY OF THE RESULTS AND SUMMARY

The growth rate and frequency of the mode discussed here are derived in Krall & Rosenbluth (1963) without too many restrictions: the plasma-β is arbitrary, and the local analysis is easily satisfied in realistic coronal situations. The solar corona is highly structured and inhomogeneous, both vertically and horizontally. Those structures imply gradients of the plasma parameters (density, temperature, magnetic field) in the direction perpendicular to the magnetic field, and this is practically all that is needed for the development of drift instabilities. Much more about the properties of these magnetic structures can be found in Vranjes & Poedts (2009a), where they are discussed in relation with the heating by ordinary (oblique) drift waves. The instability discussed in the present paper is indeed unique between all the drift instabilities as it implies *the electric field along the magnetic field lines*. The scalelengths L_n, L_B for the density and the magnetic field inhomogeneity are limitless regarding the present mode, which makes it applicable throughout the corona. In view of the values for the frequency obtained in Section 2 and the estimate of the velocity (6), it is obvious that for rather realistic plasma parameters, which also include the density scalelength $L_n = 10^6$ m, the proton energy of the order of MeV is expected for the wave electric field as small as 0.02 V m^{-1}. Measurements of strong proton fluxes from the Sun in the past several decades show that in most cases they may be associated with flares, whatever the real nature of these flares is. Analytical modelling associated with such acceleration mechanism may be found in Dalla & Browning (2008). However, such measurements also show that some proton events are not at all related to flares, i.e. they appear also when flares are completely absent. Examples of that kind, presented in Reeves et al. (1992), are assumed to be due to shock acceleration. In the present work it is shown that yet another powerful mechanism based on the transverse (electromagnetic) drift wave, which has never been discussed in this context, may in fact be behind such proton fluxes.

Very recent observations (published after the present manuscript was submitted) related to solar spicules (De Pontieu et al. 2011) revealed strong plasma flows with typical velocities of up to 100 km s^{-1}, and with bulk plasma heated up to 0.1 MK. In addition, a small fraction of the plasma in such flows appears to be heated to the coronal temperature, above 1 MK. The disturbances were also roughly quasi-periodic. These observations were presented in media as the solution for the coronal heating puzzle, although no explanation was offered for what drives and heats those flows. It was claimed that no currently available models could explain these features. However, in our previous work (Saleem, Vranjes & Poedts 2007) dealing with spicules, it was shown that the standard oblique drift wave can grow due to inhomogeneity of these flows. Such a growing mode will consequently heat the plasma in spicules due to stochastic effects described in Vranjes & Poedts (2009a,b,c,d, 2010a,b). In addition to this, the model described in the present work clearly has potential to explain such observations because it implies both acceleration and heating and, being based on the (transverse drift) wave model, it also naturally includes periodicity.

One additional particular example, where both the standard drift theory and the theory of the transverse drift wave may also work, are the so-called Extreme-ultraviolet Imaging Telescope (EIT) waves, large-scale structures propagating radially throughout the solar disc. Physically, in most cases these structures are explained in terms of magneto-acoustic waves, linear and/or non-linear. As such, they imply the density and magnetic field inhomogeneities in the direction perpendicular to the magnetic field. There are evidences of heating within the wavefront of such propagating structures (Wills-Davey & Attrill 2009). They propagate through the space, yet an observer 'surfing' on the wave peak would see a bell-shaped density profile (say in the x-direction) accompanied by a variation in the magnetic field in the same direction. This, together with the vertical magnetic field (in the z-direction) is an example of a perfect geometry for the drift wave which would then propagate in the y-direction, that is along the wave front of an EIT wave. The present transverse drift wave would have the electric field directed vertically and the 'surfer' would see streams of electrons and ions accelerated vertically in opposite directions. As the wave passes by the 'surfer' along density hump in the y-direction, the accelerated beams of electrons and ions reverse their directions. Note that even enormously large values of L_n of the order of Mm, which are expected in such density inhomogeneities associated with the EIT waves, from equation (2) may yield waves with the period of around 10 min, that are short enough for the developments of the transverse drift wave on the front of an EIT wave. As mentioned earlier, long periods imply large time-scales for the particle acceleration.

The transverse drift wave may easily develop also in coronal magnetic structures in active regions, as well as in coronal holes, and in any other area that contains the plasma inhomogeneities, and that is practically most of the solar plasma. Particles accelerated in coronal loops should contribute to heating in two ways. First, this should be due to collisions and a consequent dispersion of energy as they approach lower, more collisional layers by moving along a loop, and, second, by the new stochastic heating mechanism that is discovered here for the first time, provided that they are accelerated to high enough energies. Particles in coronal holes should behave in the same manner, yet, as the magnetic field lines are locally open, they should consequently end up in the solar wind.

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