# The Kinetic Approach to Model Space Plasmas

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Abstract. In space plasmas, the number density of the particles is generally very low, so that the kinetic approach is the most appropriate one to develop models. At BISA, kinetic models have been developed for planetary exospheres and solar plasmas. Recent models concern the terrestrial plasmasphere, the auroral regions, the polar wind, the solar wind and the exosphere of other planets like Saturn and Jupiter. Depending on the assumptions made in the models, the evolution equations are solved analytically or numerically with a spectral method of expansion of the solution in orthogonal polynomials. Such models are mainly developed to better understand the physical processes, but they have also practical applications for space weather prediction.

## 1. Introduction: kinetic models

In highly collision-dominated plasmas (i.e. at low altitude in the barosphere), the magnetohydrodynamic (MHD) approach gives a valid approximation of the transport equations. But in low-density plasmas at high altitude in the planetary and stellar exospheres, the mean free path of the particles becomes larger than the density scale height. A kinetic approach is then more appropriate to describe the characteristics of the plasma (Lemaire & Pierrard 2003). Figure 1 illustrates the different regions associated to collisional regimes in a planetary or a stellar atmosphere and the associated formalisms used for ionized particles.

Kinetic models provide the velocity distribution functions (VDF) of the particles as a solution of the evolution equation. The calculation of the VDF moments gives the macroscopic quantities like the number density, bulk velocity, temperatures and heat flux. The general evolution equation for a particle species with a mass m in a space plasma is given by:

$$\frac{\partial f(\mathbf{r}, \mathbf{v}, t)}{\partial t} + \mathbf{v} \cdot \frac{\partial f(\mathbf{r}, \mathbf{v}, t)}{\partial \mathbf{r}} + \frac{\mathbf{F}}{m} \cdot \frac{\partial f(\mathbf{r}, \mathbf{v}, t)}{\partial \mathbf{v}} = \left(\frac{df}{dt}\right)_c + \text{WPI}, \tag{1}$$

where the first term represents the time t dependence of the VDF, the second term corresponds to the spatial diffusion ( $\mathbf{r}$  is the position and  $\mathbf{v}$  the velocity vector of the particles), the third term gives the effects of the external forces  $\mathbf{F}$ , the first term in the right hand side of the equation represents the effects of Coulomb collisions and the last term corresponds to the effects of wave-particle interactions.

Different approximations can be used to solve the equation and they give different VDF characteristics. We are especially interested in steady state models where the time dependent term is neglected.

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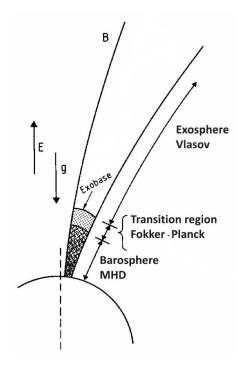


Figure 1. The different collisional regions in a planetary or stellar atmosphere and their associated equations for space plasmas.

In low-density plasmas, exospheric models use the assumption that Coulomb collisions can be neglected. If the wave-particle interactions are also neglected, one obtains the Vlasov equation for which analytic solutions can be found (Pierrard & Lemaire 1996). Such simple exospheric models are interesting because their results are easy to reproduce and they show directly the physical processes implicated in the planetary and stellar exospheres. These models have demonstrated the importance of the internal electric field in the acceleration of the solar wind (Lamy & Pierrard 2003) and polar wind (Lemaire & Pierrard 2001).

More sophisticated kinetic models including the effects of collisions have also been developed. The Boltzmann collision term was used to study the escape of hydrogen and helium atoms from the atmospheres of Earth and Mars (Pierrard 2003). In space plasmas, the Fokker–Planck Coulomb collision term was used to study the solar wind (Pierrard et al. 1999) and the terrestrial polar wind (Pierrard & Lemaire 1998). Numerical methods are then necessary to solve the evolution equation. A spectral method of expansion of the solution in Legendre and speed polynomials (Shizgal & Blackmore 1984) was used in these kinetic models. Recently, Magnus & Pierrard (2008) have improved this technique by developing orthogonal polynomials based on the Kappa function to replace the speed polynomials. Indeed, particle velocity distribution functions in space plasmas often show non-Maxwellian suprathermal tails decreasing as a power law of the velocity. Such distributions are well fitted by Kappa distributions

(Maksimovic et al. 1997) so that expansions based on such kappa polynomials converge faster.

# 2. Kinetic models of planetary plasmas

# 2.1. The plasmasphere

The plasmasphere is the high altitude extension of the ionosphere at low and mean latitudes where the plasma is trapped by the magnetic field in the inner magnetosphere. A three dimensional physical dynamic model of the terrestrial plasmasphere has been developed at BISA and constrained by realistic data (Pierrard & Stegen 2008). The core of the plasmasphere is obtained from the kinetic exospheric approach assuming a kappa velocity distribution function for the particles. The relative abundance of trapped particles is constrained so that the density profiles correspond to ISEE satellite observations. The position of the plasmapause, i.e., the limit of the plasmasphere, is determined by the interchange instability mechanism (Lemaire & Pierrard 2008). The electric field model is a combination of the corotation and the E5D convection empirical model. The convection electric field model E5D was developed by McIlwain (1986) on the basis of spectra measured by the satellites ATS5 and ATS6 and it depends on the level of geomagnetic activity Kp. The influence of different electric field empirical models has been studied in Pierrard et al. (2008). The deformation of the plasmasphere during quiet and disturbed geomagnetic periods was compared with the results of observations of IMAGE and CLUSTER satellites (Pierrard & Cabrera 2005, 2006). During quiet periods, the plasmasphere is extended to radial distances larger than 4 Re. During geomagnetic storms and substorms, the plasmasphere is eroded. Plumes are formed in the afternoon MLT (Magnetic Local Time) sector and then rotate with the Earth (Darrouzet et al. 2006). The model was also compared to other physics-based (Pierrard et al. 2009) and empirical (Reinisch et al. 2009) plasmaspheric models. Figure 2 shows the number density of the electrons obtained during a quiet period with the 3D dynamic kinetic model of the plasmasphere.

Such models have direct applications for space weather predictions since the ionosphere, the ring current and the radiation belts are influenced by the plasmaspheric particle population and by the position of the plasmapause. The model that gives the position of the terrestrial plasmapause as a function of the geomagnetic activity level can be run on the space weather portal: http://www.spaceweather.eu

# 2.2. The terrestrial polar wind

The polar wind corresponds to the plasma outflowing along open magnetic field lines and escaping from the ionosphere at high latitudes. Due to plasma transport, polar wind models are more complicated than simple hydrostatic models. Recent kinetic models of the polar wind have been reviewed in Tam et al. (2007). They take into account the effects of Coulomb collisions and are based on the solution of the Fokker–Planck equation obtained by standard finite differencing techniques (Lie-Svendsen & Rees 1996), spectral methods (Pierrard & Lemaire 1998) as well as by Monte Carlo simulations (Barghouthi et al. 2001). They showed that Coulomb collisions contribute to the formation of double-peaked

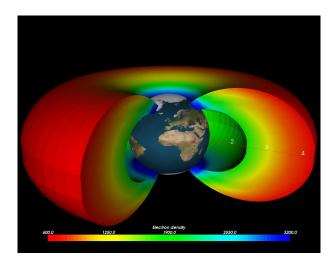


Figure 2. The number density of the electrons obtained during a quiet period with the 3D dynamic kinetic model of the plasmasphere.

H<sup>+</sup> ion velocity distributions (Pierrard & Barghouthi 2006). In the polar wind and auroral regions, Barghouthi et al. (2007) and Barghouthi et al. (2008) recently showed the velocity and altitude influence of the wave-particle interaction term on the ion distributions.

The kinetic models of the plasmaspheric region and of the polar regions have also been adapted to study the exospheres of Jupiter and Saturn and the escape flux from Io and Titan (Pierrard 2009). The models developed at BISA especially emphasize the effects of non-Maxwellian velocity distributions. The presence of suprathermal particles in the tail of the distribution, as often observed in space plasmas, leads to a significant increase of the escape flux.

# 3. Kinetic models of solar plasmas

#### 3.1. The solar wind

The solar wind is constituted by electrons, protons and some heavier ions escaping from the solar corona. This phenomenon is in some ways similar to the polar wind escape and their theoretical studies followed the same historical evolution (Lemaire & Pierrard 2001). The ion-exosphere model developed by Pierrard & Lemaire (1996) was adapted to the solar wind case by Maksimovic et al. (2001) who showed the importance of suprathermal electrons in the acceleration mechanism of the solar wind. The presence of suprathermal electrons increases the electrostatic potential difference between the solar corona and the interplanetary space and thus increases the solar wind velocity. This acceleration is especially large when it takes place at low radial distances in the coronal holes where the number density is lower than in other regions of the corona. Moreover, Pierrard et al. (2001a) showed that a spiral magnetic field decreases the temperature anisotropy.

The heavy ions were then added in the model since they give invaluable information on the physical mechanisms responsible for their heating in the corona and for the solar wind acceleration due to their different masses and charges (Pierrard et al. 2004). The last version of this solar wind model (Lamy & Pierrard 2003) is accessible on the space weather portal (www.spaceweather.eu) as well as on the CCMC (Community Coordinated Modeling Center) website (http://ccmc.gsfc.nasa.gov).

Pierrard et al. (1999) added the effects of Coulomb collisions and solved the Fokker–Planck equation using the spectral method of expansion of the solution in orthogonal polynomials. They showed that suprathermal electrons should exist already in the low corona. In a self-consistent model of the solar wind electrons, Pierrard et al. (2001b) showed the transformation of the velocity distribution function from the collision-dominated region in the low corona to the collisionless regions at higher radial distances in the solar wind.

The next step is to add the wave-particle interactions that can also play an important role in some space plasmas. Pierrard & Voitenko (2009) have studied the effects of kinetic Alfvén waves on the formation of a proton beam. Note that the wave-particle interaction term depends on the spectra of the waves to be considered. Such spectra being often unknown, different kinds of waves have been invoked to explain different characteristics of space plasmas.

### 3.2. The solar corona

A simple kinetic model of the solar corona has been developed at BISA (Pierrard & Lamy 2003) in order to study the temperatures of the solar ions in coronal regions where observations are only obtained by spectroscopy for some rare heavy ions like O<sup>5+</sup> and Mg<sup>9+</sup>. Such temperatures are important boundary conditions in the solar wind models. In the simple solar coronal model developed in BISA, the plasma is assumed to be in hydrostatic equilibrium. The hypothetical presence of suprathermal particles in the tail of the ion distribution leads to an increase of the heavier ion temperatures, due to the velocity filtration mechanism (Scudder 1992). Solar wind observations show indeed that the temperatures of the ions are more than proportional to their mass, as obtained by velocity filtration. Note that other processes reviewed in Cranmer (2002) have also been suggested to explain such high temperatures, like ion-cyclotron waves for instance.

#### 4. Conclusions

Kinetic models have been developed at BISA for different kinds of space plasmas ranging from planetary exospheres to stellar atmospheres. Depending on the approximations that are done (temporal variations, collisional regime, waveparticle interactions...), different degrees of sophistications are necessary in the methods used to solve the evolution equation. In addition to their importance to study the acceleration and heating mechanisms in space plasmas, the models have practical applications useful for space weather predictions. The kinetic solar wind model and the model for the position of the terrestrial plasmapause can be run on the space weather portal.

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