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On the scaling features of magnetic field fluctuations at non-MHD scales in turbulent space plasmas

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Abstract. In several different contexts space plasmas display intermittent turbulence at magneto-hydro-dynamic (MHD) scales, which manifests in anomalous scaling features of the structure functions of the magnetic field increments. Moving to smaller scales, i.e. below the ion-cyclotron and/or ion inertial length, these scaling features are still observed, even though its is not clear if these scaling features are still anomalous or not. Here, we investigate the nature of scaling properties of magnetic field increments at non-MHD scales for a period of fast solar wind to investigate the occurrence or not of multifractal features and collapsing of probability distribution functions (PDFs) using the novel Rank-Ordered Multifractal Analysis (ROMA) method, which is more sensitive than the traditional structure function approach. We find a strong evidence for the occurrence of a near mono-scaling behavior, which suggests that the observed turbulent regime at non-MHD scales mainly displays a mono-fractal nature of magnetic field increments. The results are discussed in terms of a non-compact fractal structure of the dissipation field.

1. Introduction

Turbulence is very common in the framework of space plasmas [1]. For instance, solar wind clearly displays the occurrence of turbulent fluctuations in several quantities, such as magnetic field and plasma parameters (velocity, plasma density, etc.). The turbulent nature of magnetized plasmas mainly manifests in power law spectral densities with power-law exponents near the one predicted by the Kolmogorov's (K41) and/or Iroshnikov-Kraichnan (IK) theory of turbulence. Furthermore, the observed turbulence at MHD scales is accompanied by anomalous scaling features, which are the counterpart for the occurrence of *intermittency*.

Intermittency, i.e. a non-homogeneous cascading process, is, indeed, a very well documented property of magnetic field fluctuations (increments) in turbulent space plasmas at MHD scales, and manifests in an anomalous scaling of the structure functions $S_p(\delta r)$ of magnetic field increments,

$$S_p(\delta r) = S_q(\delta r)^{\frac{p}{q}+\beta(p)} \tag{1}$$

where δr is a scale separation and $\beta(p)$ is a correction to scaling, which depends on the moment order p. This anomalous scaling is a manifestation of the multifractal nature of the turbulent cascading mechanism, which manifests in a dependence of the shape of the PDFs of magnetic field increments on the scale δr.

While the occurrence of anomalous scaling features at MHD scales in turbulent plasmas is a well documented phenomenon, the situation is less clear when one moves to non-MHD scales, i.e., typically at scales below the ion-gyroradius ρ_L and/or the ion inertial length η_i (typically, proton in solar wind). Indeed, at these scales there are controversial results indicating the occurrence of both anomalous and global scaling features, so that it is still not clear if the nature of fluctuations/increments at non-MHD scales is mono-fractal or multifractal [2, 3, 4, 5].

The multi- and/or mono-fractal character of fluctuations in the non-MHD/kinetic domain is very relevant to model the nature of the fluctuations at these scales. Indeed, because stochasticity plays a relevant role in the cascading mechanism (Landau's remark to K41 theory of turbulence), scale invariance should be not global but, only, local [6]. As a consequence, intermittency and inhomogeneity of the dissipation field is expected to grow indefinitely going to smaller and smaller scales [7]. Conversely, a mono-fractal character of magnetic field fluctuations/increments at non-MHD scales could be the consequence of a simple non-compact fractal structure of the dissipation field at these scales, as, for instance, it has been suggested in the case of fluid turbulence [8]. This would manifest in a saturation of the kurtosis/flattness of the magnetic field increments at these non-MHD scales.

Here, we show preliminary results on the scaling features of magnetic field increments at non-MHD scales for a period of fast solar wind conditions as observed by ESA-Cluster mission. The analysis is done using the novel method of Rank-Ordering-Multifractal-Analysis (ROMA) introduced by Chang and Wu [9].

2. Data Description and Methods

To investigate the scaling features of magnetic field increments at non-MHD scales, we have considered a period of fast solar wind observed by Cluster satellites and characterized by quasi-stationary conditions. In detail, the observations refers to January, 20th, 2007 from 12:00 UT to 14:00 UT when the Cluster satellites encountered a stream of fast solar wind characterized by a mean velocity $\bar{v} \sim 600$ km/s, a mean magnetic field $\bar{B} \sim 4$ nT, a proton density $n_p \sim 2$ cm^{-3}, a temperature $T_p \sim 30$ eV, and a plasma beta $\beta \sim 1.6$.

Figure 1 shows the three magnetic field components for the selected time interval as observed by the FGM and STAFF experiments on-board of Cluster mission.

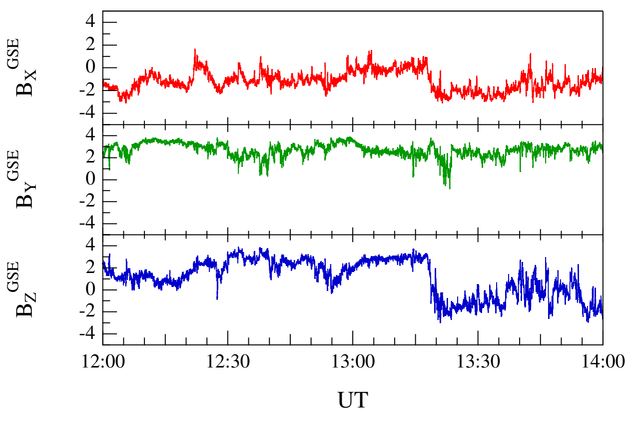

Figure 1. The magnetic field measurements relative to the period of fast solar wind observed by Cluster satellite.

The data used to investigate the scaling features refer to FGM and STAFF experiments on-board of Cluster. The merging is done by low- and high-pass filtering of respectively FGM and

STAFF data, with a cut-off frequency of 1 Hz using a finite response filter. In particular, data have been merged so to get the very high resolution of 450 samples per second. Magnetic field is measured in the GSE coordinate system.

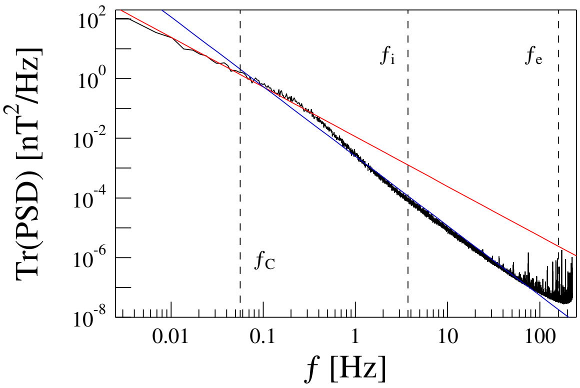

Figure 2. The trace of the PSD. The dashed vertical lines refer to the proton gyrofrequency f_C, and to the frequency corresponding to the proton and electron inertial lengths $f_k \sim \bar{v}/\rho_k$. Red and blue lines are power-law best fits.

Figure 2 shows the trace of the power spectral density (PSD) matrix of the magnetic field, which is defined as

$$\mathrm{Tr}(PSD) = \sum_i S_i(f) \tag{2}$$

where $S_i(f)$ is the power spectral density of the i-component of the magnetic field.

The PSD is characterized by two main spectral domains; a nearly $f^{-5/3}$ domain below the proton gyrofrequency f_C and a nearly $f^{-7/3}$ power-law domain at frequencies above f_C, which corresponds to a brown-to-dark spectrum in the non-MHD domain.

To analyze the scaling features of the magnetic field increments, $\delta b_i^\tau(t) = b_i(t+\tau) - b_i(t)$, we have applied the novel method of Rank-Ordered Multifractal Analysis (ROMA), introduced by Chang and Wu [9] to characterize the multifractal properties of a signal.

ROMA is based on the construction of invariants in 2$^{\mathrm{nd}}$ order critical points displaying multi-scaling properties as a results of competing renormalization group (RG) fixed points. Its central idea is to analyze the singularity features of signal increments by grouping them according to their re-scaled sizes. In particular, the ROMA method consists in ranking the domain of scaled increments into separate intervals and in determining the associate scaling exponent s that satisfies the equation

$$\tau^s P(\delta x; \tau) = P_s(\tau^{-s}\delta x), \tag{3}$$

where δx are the signal increments at the timescale τ, and $P(\delta x; \tau)$ is the corresponding probability density function (PDF) of the signal increments. As a result, it is possible to construct an invariant shape for the PDFs and, simultaneously to get the scaling features of fluctuations/increments characterized by a different amplitude in terms of a rank-ordered multifractal (ROMA) spectrum $s(Y)$, where

$$Y = \tau^{-s(Y)}\delta x(\tau). \tag{4}$$

The ROMA spectrum $s(Y)$ can be found by solving the following functional equation for a range-limited structure function,

$$S'(\mid \delta x \mid; \tau) = \int_{\tau^s Y_i}^{\tau^s Y_{i+1}} \mid \delta x(\tau) \mid^q P(\mid \delta x \mid; \tau) d \mid \delta x \mid \simeq \tau^{qs}, \tag{5}$$

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and its knowledge allows the construction of a scale invariant PDF $\mathcal{I}(Y)$ defined according to the following transformation:

$$P(\delta x;\tau) \;\rightarrow\; \mathcal{I}(Y) = \tau^{s(Y)} P(\delta x;\tau). \tag{6}$$

The ROMA spectrum $s(Y)$ is, thus, the real quantity able to characterize the different scaling features of fluctuations/increments of different amplitude Y. This is a peculiar feature of ROMA method, which differently from other methods is not based on average behavior. In other words, ROMA is less affected by scaling exponents of the most probable fluctuation amplitude than the partition function based methods and, thus, is much more efficient in characterizing multifractal signals. In this framework, ROMA method is able to characterize the different universality classes of rank-ordered amplitude fluctuations/increments implying thus the possibility to envisioning the different physical processes.

3. Analysis and Results

To investigate the scaling features of magnetic field fluctuations/increments in the non-MHD domain (i.e. at timescales τ shorter than the proton gyroperiod T_C) we have confined our analysis to temporal scales $\tau < 1$ s. Indeed, this timescale of 1s separates two scaling domains as clearly shown in Figure 3 where we report the 2^{nd}-order structure function, $S_2(\tau)$, of the B_Z component.

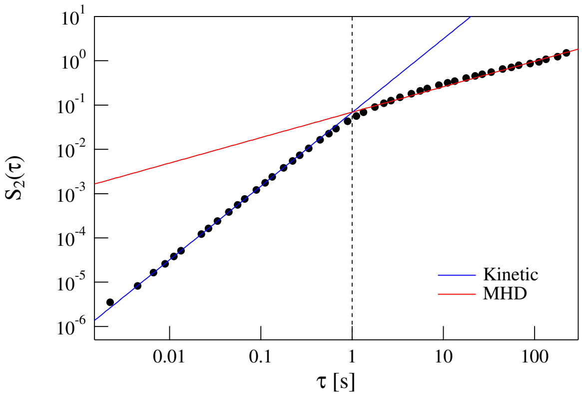

Figure 3. The 2^{nd}-order structure function, $S_2(\tau)$, of the B_Z component. The dashed vertical line indicates the typical scale separation between the MHD domain and the kinetic one. Red and blue lines are power-law best fits.

The evaluation of the ROMA spectrum $s(Y)$ has been done solving Eq. 4 for different values of the rank-ordered scaled variable Y at different moment order q as described in Ref. [9, 10, 11].

Figures 4 and 5 show the obtained ROMA spectra, $s(Y)$, for the kinetic ($\tau < 1$ s) domain and MHD ($\tau > 1$ s) one, respectively.

While the ROMA spectrum $s(Y)$ in the MHD inertial domain shows a large variability ($\Delta s(Y)/\langle s(Y)\rangle \sim 70$ %), the situation is less pronounced at non-MHD scales (kinetic domain) where the variability is very small ($\Delta s(Y)/\langle s(Y)\rangle \sim 15$ %). The results from ROMA confirm the multifractal nature of the magnetic field increments in the MHD domain and simultaneously indicate that at non-MHD domain the scaling features of the magnetic field fluctuations are compatible with the quasi mono-fractal nature, being the spectrum nearly flat. Furthermore, we observe that the scaling features of the different magnetic field components do not differ from each other in a significative manner, so that we can assume that scaling features at non-MHD kinetic scales are essentially isotropic.

The observed average scaling exponent $\langle s(Y)\rangle_{Y\leq 20}$ for the kinetic domain is $[0.84 \pm 0.05]$, a value which is quite well in agreement with other observations in different contexts at similar scales [4, 5].

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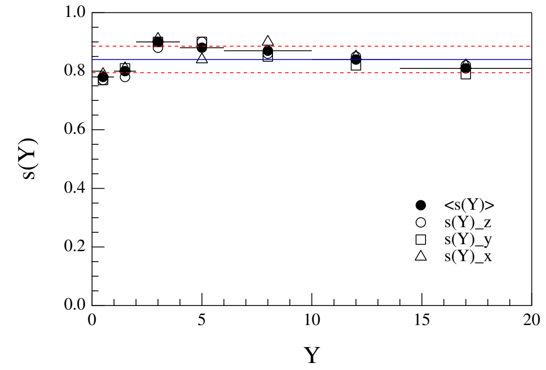

Figure 4. ROMA spectrum $s(Y)$ associated with the non-MHD/kinetic domain ($\tau < 1$ s) for the three magnetic field components. Horizontal lines indicates the mean value (blue line) and the values corresponding to $\pm 1\sigma$ (dashed red lines).

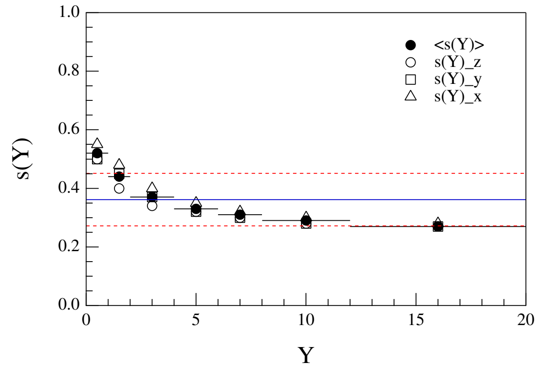

Figure 5. ROMA spectrum $s(Y)$ associated with the MHD domain ($\tau > 1$ s) for the three magnetic field components. Horizontal lines indicates the mean value (blue line) and the values corresponding to $\pm 1\sigma$ (dashed red lines).

As a result of ROMA method we have found a strong indication that the magnetic field fluctuations/increments at non-MHD kinetic scales are essentially mono-fractal. As a consequence of that, we should be able to construct an invariant PDF by simply collapsing all the PDFs at the different timescales τ onto a single invariant shape applying the following one-exponent scale transformation reported in Eqs. 4 and 6.

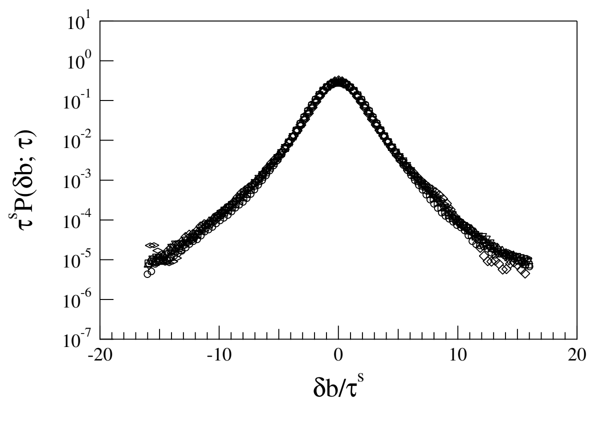

Figure 6. One-exponent collapsing of PDFs of the increments of the magnetic field B_Z component relative to the non-MHD domain ($\tau \in [4, 141]$ ms).

Figure 6 shows the PDFs' collapsing for the B_Z-component magnetic field increments in the range $\tau \in [4, 141]$ ms, using the average scaling exponent $s \sim 0.84$. PDFs' collapsing is very good supporting the mono-fractal (mono-scaling) nature of the observed fluctuations/increments. Similar results are found for the other two components X and Y.

4. Discussion and Conclusions

The results obtained by the application of ROMA method to a case study of solar wind magnetic field fluctuations support the existence of a different character of the scaling features of fluctuations/increments at MHD and non-MHD (kinetic) scales. In particular, while in the MHD domain, and more specifically in the inertial range, these fluctuations are multifractal as expected in the case of a non-homogeneous dissipation pattern in turbulent media, in the kinetic domain we have found a strong indication of a nearly global scale invariance, i.e. a mono-fractal nature. This result on the mono-fractal character of magnetic field fluctuations/increments at non-MHD scales agrees with the previous findings in solar wind by Kiyani et al. [4], and also with other results from different space plasma context, such as, for instance, magnetic field fluctuations during magnetic reconnection [5] and geomagnetic tail current disruption events [3]. Furthermore, a comparison of the observed scaling exponents in the non-MHD domain indicates that there is a certain degree of universality in the scaling features, being $s \sim [0.8 \pm 0.1]$.

A possible physical scenario of the mono-fractal nature of magnetic field fluctuations/increments at non-MHD scales may be that the observed global scale-invariance is the counterpart of the fractal nature the dissipative structures at these scales. In this framework, the multifractal character of fluctuations at MHD scales is lost implying a saturation of the intermittent character. A similar scenario seems to occur in the case of fluido-dynamic turbulence (see e.g., [8]).

In turbulent space plasmas dissipation is generally expected to occur where currents flow. Thus, a reasonable candidate of the dissipative structure is a filamentary, not-space filling, current web at scales below the ion (proton) inertial length. In other words, the fluctuation zoo associated with the turbulent magnetic field may generate multiscale coherent structures, which at these scales would imply a filamentation of the current, i.e. a fractal structure of the the dissipative field at the finest scales [12, 13].

Following Milovanov et al. [14] we can try to infer the effective fractal dimension d_s of this filamentary current web from the spectral exponent and/or the Hurst exponent, $\mathcal{H}$, of the fluctuations at non-MHD scales. In our specific case we get for the effective dimension d_s,

$$d_s = \langle s(Y) \rangle + 1 \sim 1.84, \tag{7}$$

where we have assumed $\langle s(Y) \rangle \simeq \mathcal{H}$. The effective dimension d_s is below the embedding one $E = 3$, confirming the non compact nature of the current structure. This resultt is also confirmed by the well-known *Alexander-Orbach conjecture* [15],

$$D = \frac{2d_s}{2 + \delta}. \tag{8}$$

where d_s is the fractal dimension of a percolating network, *connectivity index* $\tilde{\delta}$ is the connectivity index and *fracton dimension* D (where $D = 4/3$ for percolation clusters). Using the above relation we obtain for the connectivity index $\tilde{\delta} \sim 3/4$, which again confirms that we are dealing with a non compact structure of the dissipation field (indeed, for compact sets the connectivity index $\tilde{\delta} = 0$) .

In conclusion, in this preliminary work we have clearly found a further confirmation of the occurrence of the quasi mono-fractal nature of the magnetic field fluctuations/increments at non-MHD scales, using the ROMA technique. A possible physical scenario for this mono-scaling nature and the related global scale-invariance was also suggested. In detail, the proposed scenario relates the observed mono-fractality to a complex topology (filamentary fractal web) of dissipation structure (perhaps the current web) instead of a special kind of wave-turbulence, not displaying multi-fractal properties in the cascading process. Recent, non-MHD simulations and observations support the existence of a complex topology of structures at these scale and the formation of a filamentary current web [16, 17, 18, 19].

The next step to confirm this scenario will involve a comparison between ad-hoc simulations on this domain and the analysis of high resolution data coming from multi-satellite missions (ESA-CLUSTER, NASA-MMS) and/or proposed mission (e.g., ESA-THOR).

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