Physicalia Mag. 24 (2002) 2 pp 79-89

PLANETARY AND SOLAR EXOSPHERES

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The exosphere of a planet (or a star) is the very high altitude region of its atmosphere characterized by a vanishingly small density. Standard models of the exospheres were developed on the assumption of a complete absence of collisions between constituents above a critical level called the exobase. The collisionless models provide an extremely useful description of the exosphere, but they are somewhat artificial in the assumption of a discontinuous change in the behavior of the atmosphere at the level of the exobase, i.e., the atmosphere is assumed collision-dominated below the exobase and collisionless above the exobase. The gradual transition between the collision-dominated region at low altitude and the collisionless region at high altitude needs a rigorous kinetic theory treatment including the collisions.

1 Introduction

Kinetic problems of the atmospheres can be categorized in terms of the value of the Knudsen number, which is defined as the ratio of the mean free path λ of the particles and the density scale height H: $Kn = \lambda/H$. For $Kn \ll 1$, the system is dominated by collisions and hydrodynamic equations can be used to study the transport of the gas. For $Kn \gg 1$, the gas is collision-free and the particles can be assumed to move independently under the influence of the external forces. Kinetic models based on the Vlasov equation are then developed to describe the evolution of the velocity distribution function of the particles. Systems characterized by $Kn \simeq 1$ (i.e., in the transition region) must be studied with the kinetic collisional Boltzmann (for neutral particles) or Fokker-Planck (for charged particles) equations.

The collision-dominated region called barosphere (where $Kn \ll 1$) , the transition region ($Kn \simeq 1$), and the exosphere corresponding to the collisionless region ($Kn \gg 1$) are illustrated in Figure 1.

In the atmosphere of Earth and of other planets, as well as in planetary ionospheres or stellar coronae, there is always an altitude where the density is so low that Kn, the Knudsen number, becomes larger than unity. This level is called the exobase.

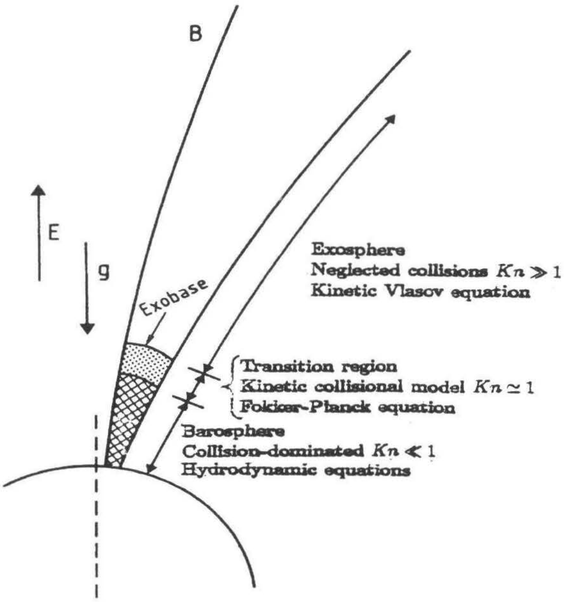

Figure 1: The regions of the barosphere (collision-dominated), the transition region and the exosphere (collisionless) of a planet or a star.

The region above the exobase is called the exosphere. Below the exobase the collision frequency is large enough to establish a nearly maxwellian and isotropic velocity distribution function (VDF) over a few collision times, so that the Chapman-Enskog expansions of the VDF, and the (magneto-)hydrodynamic equations tend to be applicable [1]. Between the collision-dominated and collisionless regions there is a transition region where none of these approximations are applicable. In the neutral atmosphere of the Earth, the exobase is situated at an altitude of 500 km. In the case of the polar wind, the transition region is located between 2000 km and 2500 km. In the solar wind the Knudsen number becomes equal to unity at about 6 (or 1.5) solar radii in the solar corona at equatorial (or polar) latitudes.

The principles of the exospheric models were developed in 1923 by Jeans [2] to study the escape of neutral atoms from planetary atmospheres. Later, similar models were adapted to the collisionless part of solar corona and of polar ionosphere ([3,4] for a review). These models were first based on Maxwellian VDFs and gave very interesting results. These exospheric models are able in particular to reproduce the observed properties of the slow solar wind when the appropriate electric field distribution is considered. Nevertheless it was difficult to reach the very high velocities observed in the fast solar wind with these models.

Considering the fact that the velocity distribution functions of the particles are often observed to have suprathermal tails in space plasmas, we have developed a new exospheric model based on a Lorentzian VDF taking into account the effects of an enhanced population of high energy particles [5]. This model is presented in Section 2 and we show in Section 3 how the enhanced non-maxwellian tails in the electron VDF (simulated with Lorentzian VDFs) contribute to increase the electrostatic potential between the exobase and infinity and then to accelerate the solar wind and the polar wind at large distances. Since the space plasmas that we study are neither completely collisionless, especially in the transition region, collisional models have also been developed at the Belgian Institute for Space Aeronomy and are described in Section 4. Applications to the neutral atmosphere of the Earth and Mars, to the terrestrial polar wind and to the solar wind are presented in Section 5.

2 The Lorentzian collisionless model

2.1 Principles of the collisionless approach

In collisionless models of exospheres, the particles are assumed to move freely under the influence of the gravitational force for neutral particles. In the case of charged particles, electromagnetic forces (electric and Lorentz forces) have also to be considered. In the absence of collisions, the equation describing the evolution of the velocity distribution function $f(\mathbf{r}, \mathbf{v}, t)$ of the particles is reduced to the Vlasov equation:

$$\frac{\partial f(\mathbf{r}, \mathbf{v}, t)}{\partial t} + (\mathbf{v}.\nabla_{\mathbf{r}}) f(\mathbf{r}, \mathbf{v}, t) + (\mathbf{a}.\nabla_{\mathbf{v}}) f(\mathbf{r}, \mathbf{v}, t) = 0 \tag{1}$$

where $\mathbf{r}$ and $\mathbf{v}$ are respectively the position and velocity vectors of the particles, $\mathbf{a}$ is the acceleration due to external forces and t is the time.

We are first interested by steady state solutions of this equation. As a consequence of Liouville's theorem, any function of the constants of the motion of the particles is a solution of the Vlasov equation.

The first constant of the motion is the total energy. For neutral particles, the angular moment is the second constant of the motion. For charged particles in a magnetic field, the magnetic moment of the particles is the second invariant when the gyroradius of the particles is small compared to the length characterizing the inhomogeneities of the plasma. Once a velocity distribution function is assumed at the exobase, the VDF of the particles can be uniquely determined at any upper radial distance by Liouville's theorem. Depending on their velocity and pitch angle, the trajectories of the particles can be divided in four different classes: escaping, incoming, ballistic and trapped (or satellite) particles [6]. The different classes of particles are represented in the velocity plane in Figure 2 at the exobase (left) and at a larger radial distance (right) for the case of an attractive potential.

The moments of the VDF are then calculated separately for each class of particles. The number density is given by:

$$n(\mathbf{r}) \;=\; \int f(\mathbf{r}, \mathbf{v})\, d\mathbf{v}. \tag{2}$$

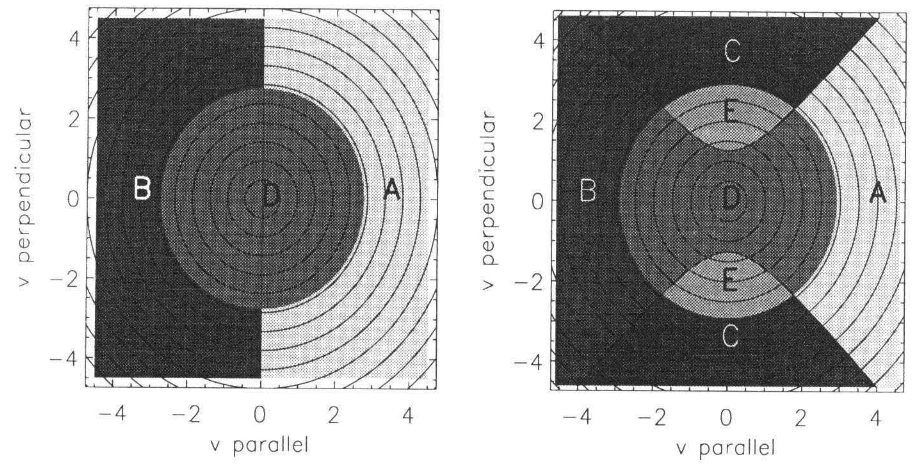

Figure 2: The different classes of particles in an attractive potential represented in the velocity plane at the exobase (left) and at a radial distance above the exobase (right): A Escaping particles, B Incoming reaching the exobase, C Incoming not reaching the exobase, D Ballistic particles, E Trapped particles.

the parallel particle flux:

$$F(\mathbf{r}) = \int v_{\parallel} f(\mathbf{r}, \mathbf{v})\, d\mathbf{v}. \tag{3}$$

the parallel and perpendicular momentum flux:

$$P_{\parallel}(\mathbf{r}) = m \int v_{\parallel}^2 f(\mathbf{r}, \mathbf{v})\, d\mathbf{v} \tag{4}$$

$$P_{\perp}(\mathbf{r}) = \frac{1}{2} m \int v_{\perp}^2 f(\mathbf{r}, \mathbf{v})\, d\mathbf{v}. \tag{5}$$

the parallel energy flux:

$$\epsilon(\mathbf{r}) = \frac{1}{2} m \int v^2 v_{\parallel} f(\mathbf{r}, \mathbf{v})\, d\mathbf{v}. \tag{6}$$

Note that in case of charged particles, the parallel and perpendicular directions refer to the direction of the magnetic field lines while for neutral particles, they refer to the radial direction.

Assuming that there are no incoming particles from interplanetary space, the flux escaping from the atmosphere is contributed only by the escaping particles i.e. the particles moving outwards with a speed greater than the escape speed. The ballistic and trapped particles do not contribute to the flux since their distributions are symmetric.

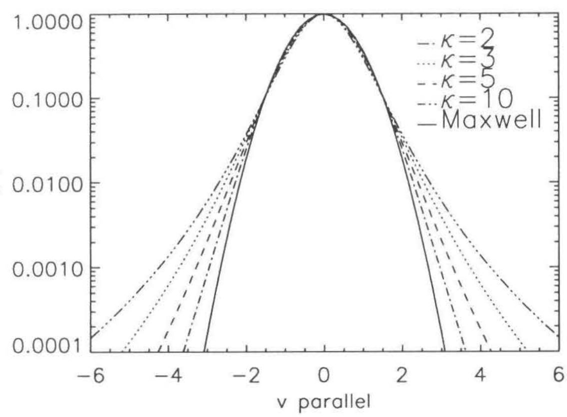

Figure 3: The Lorentzian velocity distribution function for different values of the kappa parameter.

2.2 The Lorentzian distributions

Standard earlier models were developed for Maxwellian distributions [7 for a review]. But in space plasmas, the VDFs of the particles are often observed to have non-maxwellian suprathermal tails. That is why we have generalized the maxwellian model to take into account the population of suprathermal particles by developing a new model based on the generalized Lorentzian VDF [5]. The Lorentzian (or Kappa) function is very convenient since it can fit distributions characterized by enhanced suprathermal tails as generally observed in the solar wind and in the Earth's magnetosphere. The slope of the tail is determined by the value of a kappa index; when $\kappa \to \infty$, one recovers the maxwellian VDF. Figure 3 presents different Lorentzian VDFs for different values of kappa.

A large number of solar wind electron VDFs observed with the electron spectrometer on Ulysses were fitted with Kappa functions. The parameter κ was found to globally range from 2 to 5 [8]. Adapted to study the solar wind [9], the Lorentzian model shows that electron suprathermal tails increase the electrostatic potential necessary to warrant the equality of outward fluxes of electrons and protons, so that no net current is transported by the solar wind. This increased electrostatic potential difference has for consequence to accelerate the solar wind at large distances for small kappa values of the electron VDF. Bulk velocities as large as those observed in the high speed solar wind can be obtained at 1 AU, without unreasonably large coronal temperature. Satisfactory agreement with the observations had already be obtained with earlier maxwellian models in the slow speed solar wind, but they were unable to reach velocities

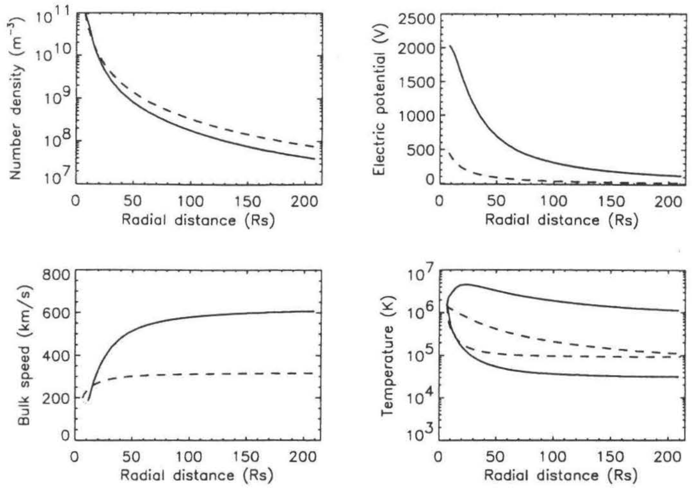

Figure 4: Number density, electric potentiel, bulk velocity and electron (upper lines) and proton (lower lines) temperatures in the solar wind as a function of the radial distance for a Maxwellian model (dashed line) and for a Lorentzian model with $\kappa = 2$ (solid line) for similar conditions at the exobase (6.6 R_s).

higher than 500 km/s at 1 AU as observed in the fast solar wind. A comparison of the density, bulk velocity temperature and potential difference profiles obtained for the solar wind with an exospheric model based on a Maxwellian and a Lorentzian function with $\kappa = 2$ is given in Figure 4. In both models, the exobase is chosen at 6.6 solar radii with a number density $n = 3 \times 10^{11}$ cm^{-3} and a temperature $T = 1.5 \times 10^6$ K.

In the coronal holes, the exobase is still lower (typically 1.5 Rs). When the position of the exobase is very low, the total energy of the protons is a non monotonic function of the radial distance. We recently generalized the model to this particular case and showed that a low position of the exobase increases the bulk velocity of the solar wind at large radial distance. Values of 800 km/s occasionally observed in the high speed solar wind can be reached even assuming low exobase temperatures as indeed observed in the coronal holes of the solar corona [10].

The Lorentzian model was developed assuming a radial magnetic field. The spiral geometry of the magnetic field lines was taken into account in a more recent work [11]. It mainly influences the electron and proton temperatures whereas the interplanetary electric potential, the wind density and bulk speed are not significantly modified.

Our model was also applied to other space plasmas to study the influence of an enhanced population of high energy particles. Important consequences were deduced for the heating of the terrestrial plasmasphere [6,12]: with kappa distributions, the temperature of the particles increases as a function of the altitude contrary to the case of Maxwellian VDFs where the temperature is constant. A new current-voltage relationship was also calculated in relation with the field-aligned currents in the auroral region [13].

3 Collisional models

3.1 Principles of the kinetic collisional approach

The exospheric theory has served as model of the exosphere for comparison with satellite and ground-based measurements. Nevertheless the hypothesis made in exospheric models that the gas is completely collisionless is very restrictive. Collisions still have some effects above the exobase. A discontinuous change in the behavior of the atmosphere at the level of the exobase is not very realistic.

A comprehensive treatment of the exosphere and of the transition region between the collision-dominated region at low altitude and the collisionless region at high altitude has to proceed from the Boltzmann equation for neutral particles and Fokker-Planck equation for charged particles:

$$\frac{\partial f(\mathbf{r},\mathbf{v},t)}{\partial t} + (\mathbf{v}.\nabla_{\mathbf{r}})f(\mathbf{r},\mathbf{v},t) + (\mathbf{a}.\nabla_{\mathbf{v}})f(\mathbf{r},\mathbf{v},t) = \left(\frac{df}{dt}\right)_c \tag{7}$$

The term on the right hand side $(df/dt)_c$ represents the effects of the collisions. For neutral particles, it represents the Boltzmann collision operator and for charged particles, the Fokker-Planck collision operator appropriate when the scattering by cumulative small-angle deflections is predominant over large-angle deflections.

To solve this equation, we have developed a spectral method based on the expansion of the solution in orthogonal polynomials [14], assuming that the VDF of the background particles is a Maxwellian or a Lorentzian distribution. The solution allows to study the transformation of the VDF in the transition region from a nearly isotropic maxwellian VDF in the collision-dominated region at low altitude, to an anisotropic VDF at large radial distance.

3.2 Transition region

A collisional kinetic model based on the Boltzmann equation has been developed to study the escape of light neutral atoms (H and He) from the atmosphere of the Earth and of Mars [15]. For the Earth, the exobase is approximately at an altitude of 500 km in an atmosphere mainly composed of oxygen atoms. For Mars, it is situated at an altitude of 250 km in an atmosphere dominated by CO_2. Density, temperature profiles and departures from purely collisionless flux are calculated. The collisional approach based on the resolution of the Boltzmann equation shows that the actual thermal escape flux is reduced with respect to the exospheric flux, with an important

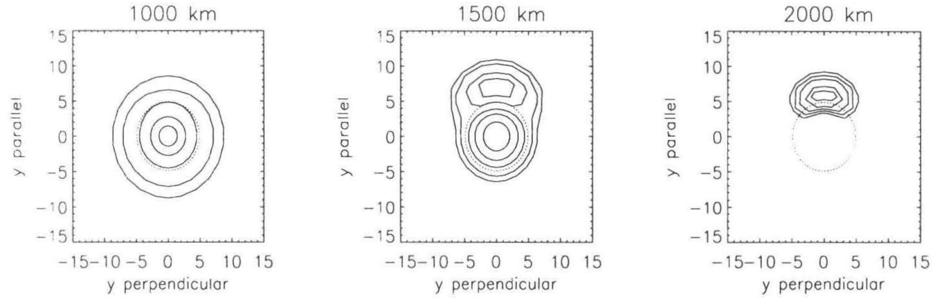

Figure 5: Isocontours of the proton velocity distribution function in the polar wind represented in the velocity plane from the collision-dominated region at low altitude to the collisionless region at high altitude.

reduction factor when the mass ratio between the escaping minor constituent and the background major constituent is weak. The actual VDF departs from a truncated Maxwellian.

For charged particles, the assumption that the gas is collision-dominated, as implicitly assumed in fluid models, is difficult to justify in plasmas like the solar wind, since the mean free path of the particles becomes larger than the scale height above 7 solar radii. Furthermore, particles with velocities larger than the mean thermal velocity become collisionless even at lower altitudes. Indeed, the Coulomb cross section is inversely proportional to the fourth power of the relative velocity between the colliding particles, and thus the level of the exobase for a given particle decreases with its energy. Therefore, no unique exobase corresponding to the mean thermal velocity of particles should be used and the gradual transition has to be studied by solving the Fokker-Planck equation.

We have developed a collisional model to study the transition region of the polar wind, constituted by light ions escaping from the terrestrial atmosphere in high latitude regions [16]. Numerical solutions of the Fokker-Planck equation show a VDF changing from a nearly isotropic maxwellian in the low altitude collision-dominated region, to a doubly peaked function at high altitudes. The change of the H^+ ion VDF in the polar wind is illustrated in Figure 5. A tridimensional view of a doubly peaked function appearing in the transition region is presented on Figure 6. Similar doubly peaked VDF have been obtained by Direct-Monte-Carlo-Simulations of the polar wind flow [17]. This kinetic solution is drastically different from the VDF associated with the solution of five or eight moments hydrodynamic equations. Indeed in these hydrodynamic models, the VDF is always considered to be a displaced (bi-)maxwellian at a zero order approximation.

A self-consistent collisional model for the solar wind electrons was developed [18]. The aim was to determine whether electron VDFs with suprathermal tails already

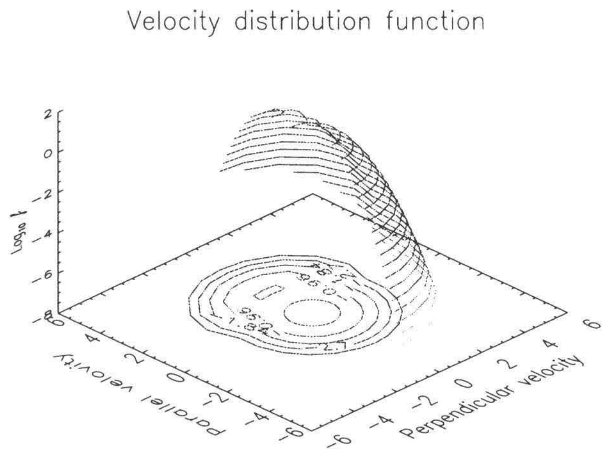

Figure 6: 3-D view of the polar wind protons velocity distribution function in the transition region.

exist in the low corona and how prominent these tails are there. The Fokker-Planck equation was solved for test electrons scattered by the background solar wind plasma. As upper boundary condition, we used velocity distribution functions observed at 1 AU by the 3DP instrument onboard of the WIND spacecraft [14, 19]. The density and temperature distributions obtained with the Lorentzian exospheric solar wind model were used to simulate the corresponding distributions of the background or target particles.

Our results indicate that suprathermal tails must exist in the low corona when they are observed at 1 AU. But they are much less important close to the Sun than those observed at larger distances. Deeper into the solar corona, the relative number density of these halo electrons forming the non-maxwellian tails becomes negligibly small compared to that of the low energy core electrons.

4 Conclusions

We have presented kinetic models developed to describe the exospheric part of planetary and solar atmospheres. The collisionless approach is outlined using examples of applications in the solar wind and in the terrestrial plasmasphere. Collisionless models have proven to be useful in the interpretation of observations of the solar corona and planetary exospheres. In particular, they allowed to understand how the solar wind ions are accelerated by the polarization electric field created by the tendency of the thermal electrons to escape out of the gravitational potential well faster than the heavier ions. But a more comprehensive treatment is applied in the transition region by using collisional models. They can not be solved analytically like purely exospheric ones, but show the transformation of the velocity distribution function of the particles from the collision-dominated region to the collisionless region.

Acknowledgments

This research work is supported by the Belgian FNRS. I thank Prof. J. Lemaire for many helpful discussions and suggestions concerning the development of kinetic models.

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