# Collisionless model of the solar wind in a spiral magnetic field

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**Abstract.** We present a kinetic collisionless model of the solar wind generalized to take into account the spiral structure of the interplanetary magnetic field. This model, which also includes Kappa velocity distributions, calculates self-consistently the electric potential profile and derives the solar wind speed and the temperatures of the medium. We study how the inclusion of the spiral geometry changes the plasma parameters compared to the case of a radial magnetic field. Whereas the interplanetary electric potential, the wind density and bulk speed are not significantly changed, we show that the electron and proton temperatures are modified; in particular, we find a decrease of the proton temperature and of its anisotropy, and an increase of the electron temperature. We discuss these results and the validity of the model.

## Introduction

Four decades after the first in situ detection of the solar wind, its physics is still not correctly understood. An important theoretical difficulty is that the particle free paths are nowhere sufficiently small in the solar wind to justify a fluid description closed by using the classical Spitzer-Härm conductivity. Besides, sophisticated fluid modelizations have to make arbitrary assumptions on the energy transport in order to close the hierarchy of equations. At the other extreme, exospheric models [*Lemaire and Scherer*, 1971] assume the medium to be collisionless above a certain altitude, namely the exobase, which is also subject to criticism since the free paths are not sufficiently large to justify a fully collisionless description.

However, collisions between particles change their velocity directions and redistribute their energies, but they do not change significantly the mean electron and ion kinetic energy owing to the large ion-to-electron mass ratio. An important consequence is that the mean electron temperature profiles may be reliably calculated from exospheric models. This may not be true for the proton temperature, which may be affected by collisions with alpha particles and by wave particle interactions. The wind velocity and temperatures have been recently calculated numerically by *Maksimovic et al.* [1997a] with an exospheric model, and analytically by *Meyer–Vernet and Issautier* [1998], who have shown in particular that the mean electron temperature varies roughly midway between isothermal and adiabatic behavior near 1 AU. However, these calculations assume a radial magnetic field, which is not justified farther than about 1 AU, except at high latitudes.

The present work is a generalization which takes into account the spiral structure of the magnetic field. This effect had been included in some fluid models and simulations [*Brandt*, 1970; *Hundhausen*, 1972, *Chen et al.*, 1972; *Washimi and Sakurai*, 1993] but not in completely collisionless kinetic models. Section 2 discusses the streamlines and the magnetic field structure in the rotating and non rotating frames. Section 3 presents the basic assumptions of the model. In section 4, we compare the results to those described in *Maksimovic et al.* [1997a], which did not take the spiral structure into account.

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## Streamlines

Assuming (i) that the Sun rotates with an angular velocity $\Omega = 2.865 \times 10^{-6}$ rad/s, (ii) that in an inertial frame, the solar wind expands radially with a constant velocity beyond some distance $b$ and (iii) that the magnetic lines of force carried in the gas remain connected to the Sun for at least a few days, *Parker* [1958] showed that the lines of force of the interplanetary magnetic field have the general shape of Archimedes spirals.

In a frame of reference rotating with the Sun, the gas velocity in spherical coordinates is then given by

$$v_r = u_r \quad (1)$$

$$v_\theta = 0 \quad (2)$$

$$v_\phi = \Omega(r-b)\cos\lambda \quad (3)$$

where $u_r$ is the wind radial velocity, $r$ the radial distance and $\lambda$ the heliographic latitude. Writing

$$\frac{B_r}{B_\phi} = \frac{v_r}{v_\phi} \quad (4)$$

$$\frac{dr}{r\cos\lambda d\phi} = \frac{u_r}{\Omega(r-b)\cos\lambda} \quad (5)$$

the streamline with azimuth $\phi_0$ at $r = b$ is given by

$$\frac{r}{b} - 1 - \ln\left(\frac{r}{b}\right) = \frac{u_r}{b\Omega}(\phi - \phi_0) \tag{6}$$

In the frame of reference rotating with the Sun, the velocity of the particles is parallel to the spiral magnetic field lines beyond the radial distance $b$; this holds true if the plasma no longer corotates there. If we take the simple view that a stellar wind corotates out to the point where the flow speed equals the Alfvèn speed, the maximum distance of corotation is evaluated to be around 30 Rs [*Weber and Davis*, 1967]. In practice, $b$ can be taken smaller because in the corotating region, the distance to the Sun is so small that the Parker spiral roughly coincides with the radial direction.

It follows that the steady-state magnetic field resulting from a spherically symmetric outflow of gas from a rotating star is given in spherical coordinates by:

$$B_r = B_0\left(\frac{b}{r}\right)^2 \tag{7}$$

$$B_\theta = 0 \tag{8}$$

$$B_\phi = B_0\frac{\Omega}{u_r}(r-b)\left(\frac{b}{r}\right)^2\cos\lambda \tag{9}$$

where $B_0$ is the radial component of the magnetic field at the radial distance $b$. The streamlines (direction of the velocity vector of the particles) are radial in the inertial frame and spiral in the frame rotating with the Sun (see Eq. 6). A magnetic field line of the Parker spiral makes an angle of 45 degrees with the radius vector at 1 AU when $u_r = 440$ km/s.

## Model

We consider the motion of electrons and protons in a frame of reference rotating with the Sun, where the streamlines coincide with the lines of force of the spiral magnetic field, and there is no perpendicular electric field ($\mathbf{E} = \mathbf{v}\times\mathbf{B} = \mathbf{0}$). In this frame, the motion can be decomposed in a motion of gyration around the magnetic field lines and a motion parallel to the magnetic field lines, so that in the guiding-center approximation, we have two invariants of motion:

1. the total energy $E$ of the particles:

$$\frac{mv^2}{2} + ZeV(r) + m\phi_g(r) - \frac{m}{2}\Omega^2 r^2\cos^2\lambda = \text{cst} \tag{10}$$

where $m$ is the mass of the particles, $Ze$ their electric charge, $V(r)$ the electric potential and $\phi_g$ is the gravitational potential. The last term represents the centrifugal energy from the rotation of the frame, assuming that in the inertial frame the particles have radial velocities. Note that when this approximation does not hold true, i.e. at small distances, this term plays a negligible role.

2. the magnetic moment $\mu$:

$$\mu = \frac{mv_\perp^2}{2B} = \text{cst} \tag{11}$$

where $B$ is the average interplanetary magnetic field.

The conservation of $\mu$ implies that the particle pitch angles are smaller than $\theta_m$ given by

$$\sin^2\theta_m(r) = \frac{\sin^2\theta(r)}{\sin^2\theta(r_0)} = \frac{B(r)}{B(r_0)}\left[1 + \frac{R(r)}{v^2}\right] \tag{12}$$

with

$$\begin{aligned} R(r) = {} & 2\Bigg(\phi_g(r) + \frac{Ze}{m}V(r) - \frac{\Omega^2 r^2}{2}\cos^2\lambda \\ & -\phi_g(r_0) - \frac{Ze}{m}V(r_0) + \frac{\Omega^2 r_0^2}{2}\cos^2\lambda\Bigg) \end{aligned} \tag{13}$$

In the present paper, the spiral structure of the average interplanetary magnetic field is taken into account (from Eqs. (7), (8), (9)) as [*Parker*, 1958]:

$$B(r) = B(r_0)\left(\frac{r_0}{r}\right)^2\left(1 + \frac{\Omega^2(r-r_0)^2\cos^2\lambda}{u_r^2}\right)^{1/2} \tag{14}$$

where the exobase $r_0$ has been substituted to $b$, which is of no consequence since, as already noted, the non radial term is negligible at these distances.

While the protons have Maxwellian velocity distribution function, we assume that the electron velocity distribution $f(\mathbf{r},\mathbf{v})$ is a generalized Lorentzian of index $\kappa$ at the exobase level $r_0$. This kind of distribution is a generalization of the Maxwellian taking into account a possible suprathermal tail, currently measured in the solar wind by *Maksimovic et al.* [1997b] and in the solar corona [*Esser and Edgar*, 2000]; it reduces to the Maxwellian distribution when $\kappa$ tends to infinity.

We deduce from Liouville's theorem the particle distribution at any further radial distance using the conservation of the invariants of the motion. For these distributions, we calculate the number density, the flux and the parallel and perpendicular components of momentum flux by:

$$n(\mathbf{r}) = \int f(\mathbf{r},\mathbf{v})d\mathbf{v} \tag{15}$$

$$F_\parallel(\mathbf{r}) = \int v_\parallel f(\mathbf{r},\mathbf{v})d\mathbf{v} \tag{16}$$

$$P_\parallel(\mathbf{r}) = m\int v_\parallel^2 f(\mathbf{r},\mathbf{v})d\mathbf{v} \tag{17}$$

$$P_\perp(\mathbf{r}) = \frac{m}{2}\int v_\perp^2 f(\mathbf{r},\mathbf{v})d\mathbf{v}. \tag{18}$$

The bulk velocity, the parallel, perpendicular and total temperature are then obtained by

$$u_\parallel(\mathbf{r}) = \frac{F_\parallel(\mathbf{r})}{n(\mathbf{r})} \tag{19}$$

$$T_\parallel(\mathbf{r}) = \frac{P_\parallel(\mathbf{r}) - mu_\parallel(\mathbf{r})F_\parallel(\mathbf{r})}{kn(\mathbf{r})} \tag{20}$$

$$T_\perp(\mathbf{r}) = \frac{P_\perp(\mathbf{r})}{kn(\mathbf{r})} \tag{21}$$

$$T(\mathbf{r}) = \frac{T_{\|}(\mathbf{r}) + 2T_{\perp}(\mathbf{r})}{3}. \qquad (22)$$

and $u_r(r) = \sqrt{u_{\|}(r)^2 - \Omega^2 r^2 \cos^2\lambda}$ is the radial bulk speed.

The particle and energy fluxes, which are odd moments, are parallel to the local direction of the magnetic field. The radial dependence of these vectors is simply obtained by projecting them on the radial direction.

For a generalized Lorentzian velocity distribution function, the analytic expressions found in this case are similar to those obtained in *Pierrard and Lemaire* [1996], except that $R(\mathbf{r})$ is given by (13) and that $\eta = B(r)/B(r_0)$ is now given for a spiral magnetic field by Eq. (14). We will not repeat here these analytic expressions, but insist on the differences with the radial case. It is important to note that as in the radial case, the electric potential is calculated self-consistently at all altitudes above the exobase by imposing both neutrality and egality of the proton and electron fluxes, i.e., zero net electric current [*Lemaire and Scherer*, 1972].

## Importance of the spiral magnetic field

Figure 1 shows the radial profiles of the density, electric potential, bulk speed, and temperatures, with a spiral magnetic field (dashed line) compared to the radial case (solid line), with a Kappa distribution of $\kappa = 10$ corresponding to a small suprathermal tail. Similar conditions are assumed at the exobase level ($r_0 = 6.6$ Rs) in both cases: $n_e(r_0) = n_p(r_0) = 3 \times 10^{10}$ m$^{-3}$, $T_e(r_0) = T_p(r_0) = 1.5 \times 10^6$ K. The bulk velocity $u_r$ defining the spiral is set self-consistently equal to the terminal speed derived in the model. In this example, we have considered the solar equatorial plane, i.e. $\lambda = 0$, where the difference between the radial and spiral case is the most important.

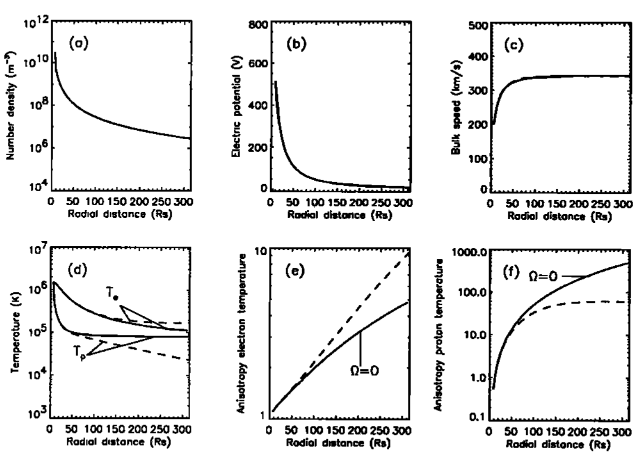


**Figure 1.** Comparison of different moments of a generalized lorentzian velocity distribution function when the spiral structure is neglected (solid lines) and when it is taken into account (dashed lines). Panel (d) shows that the electron temperature is higher in the spiral case than in the radial case whereas it is the opposite for the proton temperature.

In both cases, the density decreases as $r^{-2}$ at large distances. The electric potential difference between the exobase level and infinity is not significantly affected by the spiral structure of the magnetic field. This is because this potential is determined by the balance of the outward fluxes for the electrons and protons at the exobase, i.e., zero net electric current. This also holds for the radial bulk speed. Significant effects would appear only for stars rotating much faster than the Sun.

On the contrary, the spiral shape affects significantly the particle temperatures. The total electron temperatures are shown by the upper curves on Figure 1, in solid line for the radial case and in dashed line for the spiral case. The proton temperature corresponds to the two lowest curves. The proton temperature decreases faster when the spiral shape is taken into account, while the contrary holds for the electron temperature; significant differences appear at radial distances larger than 100 Rs. The faster decrease of the proton temperature is mainly due to the parallel component: with a radial magnetic field (i.e. $B \propto r^{-2}$), the parallel proton temperature is roughly constant at large distances because the magnetic field decreases in proportion to the density; with a spiral geometry, the magnetic field (defined in Eq. 14) decreases less than the density (varying as $r^{-2}$), thereby producing an adiabatic cooling of the parallel component, given by $T_{p\|} \propto r^{-2}$ from the double adiabatic equations (CGL). For the same reason, the spiral geometric decreases the proton anisotropy, as shown on the Figure. In contrast, the spiral shape increases the electron temperature with respect to the radial case, at radial distances larger than 1 AU. This is because the density of the electrons having enough energy to escape from the electrostatic potential varies as the inverse of the section of the flux tubes, which now decreases as $1/r$ at large distances, thus more slowly than the total density; hence the contribution of the escaping electron to the temperature increases with distance, so that the generic radial variation derived for the electron temperature by *Meyer–Vernet and Issautier* [1998] is modified. This larger contribution of the energetic electrons to the temperature occurs in the parallel direction, thus increasing the electron anisotropy with respect to the radial case.

The very large anisotropy of proton temperature obtained in the exospheric models was the main disagreement with observation. One can see in Figure 1 that the spiral structure of the magnetic field reduces the proton anisotropy, in agreement with the results of *Chen et al.* [1972], and as predicted by *Griffel and Davis* [1969]. This geometrical reduction is nevertheless still too small to bring the proton temperature anisotropy into the range of observed values at 1 AU, which is around $T_{p\|}/T_{p\perp} \sim 2-5$ [*Marsch et al.*, 1982 ; *Goldstein et al.*, 1996]. The excessive temperature anisotropies are the consequence of neglecting collisions and wave-particle interactions [*Philips and Gosling,* 1990; *Tam and Chang,* 1999]. *Griffel and Davis* [1969] pointed out that each particle has to make on average 2 or 3 collisions between 0.1 to 1 AU to be in agreement with the observations. As noted by *Lemaire and Scherer* [1972], pitch angle scattering by Coulomb collisions reduces the temperature anisotropies without changing the average energies and mean temperatures of the particles. In addition, the right hand magnetosonic instability, the heating of the perpendicular component by ion cyclotron waves and other wave particle interactions [*Parker*, 1958b; *Treumann and Baumjohann*, 1997] might also explain why $T_{p\parallel}$ is not very much larger than $T_{p\perp}$ in the solar wind. Finally, it is important to note that these results do not depend significantly on the value of the index $\kappa$, i.e. on the contribution of suprathermal particles in the corona.

## Conclusion

We have considered the effects of the spiral structure of the interplanetary magnetic field on the solar wind bulk speed and temperatures given by an exospheric model which generalizes the Lorentzian one developed by *Pierrard and Lemaire* [1996; 1998]; we use a reference frame rotating with the Sun, where the streamlines coincide with the lines of force of the spiral magnetic field for a non corotating solar wind.

The number density and the radial bulk velocity are not significantly affected by the spiral magnetic field, as expected since these parameters are mainly determined by the conditions at the exobase. In contrast, the temperatures of the particles are modified. The excessive proton temperature anisotropy obtained in radial exospheric models is reduced, albeit not sufficiently to agree with the observations. These excessive temperature anisotropies are the consequence of neglecting Coulomb collisions and wave particle interactions. Since pitch angle scattering reduces the temperature anisotropies without changing the average energies of the particles, the present model is expected to be more adequate to derive the profiles of the electron and ion total temperatures than their anisotropies. This is especially true of the electron temperature since the proton one may be affected by collisions with alpha particles and by wave particle interactions.

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