Stationary equilibria of self-gravitating quasineutral dusty plasmas

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Self-consistent stationary equilibrium states of a quasineutral, self-gravitating dusty plasma have been investigated by avoiding the usual ''Jeans swindle'' assumption. The analysis has been carried out for the Cartesian one-dimensional, cylindrical, as well as spherical symmetric cases. It is shown that allowed equilibria permit steady but inhomogeneous dust flows governed by a nonlinear differential equation, which has a singularity at the dust-acoustic speed. The qualitative nature of the admissible solutions of the latter equation have been discussed. The corresponding results for the case of a self-gravitating neutral fluid have also been pointed out. [DOI: 10.1063/1.1409960]

I. INTRODUCTION

Dust is an important component of astrophysical systems wherein self-gravitational effects are dominant. When the dust component is uncharged, the collective dynamics is governed entirely by gravitational interaction. However, dust grains surrounded by ionized media and radiative environments can become highly charged, and thus bring in important contributions arising from electromagnetic interactions. Since both interactions are long range, and the gravitational interaction is much weaker than the electromagnetic interaction, the latter can significantly influence the collective dynamics of self-gravitating extended systems. More specifically, the study of the so-called dusty plasmas, which consist of electrons, ions, and charged dust grains, has received wide attention in recent years.[1–6] Typically, the grains are in the range of a few (tens of) microns, while the charging can be up to a few thousand electrons. Thus, the physics of self-gravitating dusty plasmas is becoming increasingly relevant in determining the macroscopic behavior of extended systems in astrophysical scenarios.

Even for grain sizes in the micron range, the mass of an individual grain is typically about 10–12 orders of magnitude larger than the ion mass, and hence the mass of dusty plasmas is essentially contained in the dust component. The presence of such massive dust grains opens up a new, ultra-low-frequency regime for the existence of different types of collective modes in dusty plasmas, which do not exist in the usual electron-ion plasmas. In particular, there are three fundamental dusty plasma modes. First of all, there is the dust-acoustic wave,[7] which is associated with dust number density perturbations and which exists in the tenuous, low fugacity[8] regime. Second, the dust-Coulomb wave[8] is associated with dust charge perturbations and exists in the dense, high fugacity regime. Third, the dust-lattice wave[9] is associated with lattice vibrations when plasma crystals are formed. All three modes have been extensively studied in recent years.[10–23]

When the self-gravitational interaction due to the heavier dust component is included, dusty plasmas are found to exhibit macroscopic instabilities[24–26] of the Jeans type.[27] Physically, the Jeans instability of an extended massive system arises due to the purely attractive gravitational force, which is unlike the electromagnetic force. Recently, we have analyzed elsewhere[28] the occurrence of the Jeans instability in dusty plasmas over the entire range of dust fugacity.[8] It is found that low fugacity dusty plasmas are subjected to the Jeans instability associated with the dust-acoustic waves, while in the high fugacity range the instability is related to the dust-Coulomb modes. Furthermore, in the high fugacity regime, the instability occurs at much smaller scale sizes than in the case of the low fugacity regime.[28]

An outstanding and fundamental problem in the study of the Jeans instability of self-gravitating systems, whether neutral or (dusty) plasmas, is related to the question of the equilibrium state. By and large, this problem is generally circumvented by invoking what has been named the ''Jeans swindle.'' Stated briefly, the latter subterfuge enables one to study local perturbations of a system which is assumed to be homogeneous over several wavelengths of interest. Mathematically, this necessitates ignoring the equilibrium part of the gravitational Poisson equation, and thus considering only the perturbed part. However, the above-mentioned assumption cannot always be checked easily *a posteriori* for self-consistency since the equilibrium state cannot, in general, be determined, except in some simple models, while the system is always inhomogeneous over larger scale lengths. Notwithstanding this difficulty, the Jeans swindle has been extensively used in the literature to derive some physical insights into the collective dynamics of self-gravitating systems. An

[a]Deceased. This paper is dedicated by F.V. and V.M.Č. to the memory of N. N. Rao, whose untimely and unfortunate death deprived many of a stimulating physicist and true friend.

extensive discussion about the Jeans swindle and associated difficulties can be found in a recent book.[29]

We investigate in this paper the possible existence of self-consistent equilibria of a self-gravitating dusty plasma by avoiding the Jeans swindle. To this end, we allow for inhomogeneous but stationary flows, and derive, for the first time, a nonlinear differential equation which governs the self-consistent equilibrium flows and which is singular at the characteristic speed in dusty plasmas, namely, the dust-acoustic speed. The singularity is a manifestation of the equilibrium gravitational potential which is inhomogeneous. Several important and qualitative results about the possible equilibria have been obtained through an analysis of the governing equation. It is shown that, as a limiting case, the present analysis is applicable also for neutral self-gravitating fluids.

II. FORMALISM

In order to keep the formulation of the problem mathematically tractable as well as physically transparent, and to be able to derive later *analytical* conclusions, we consider an unmagnetized dusty plasma having electrons, ions, and dust grains, and use the standard model[7] of dusty plasmas, which is sufficient for our purpose. Since the mass content of dusty plasmas is essentially contained in the dust component, it is appropriate to neglect the mass of the electrons and ions (compared to the dust grain mass), and assume the latter to be in thermal equilibrium at the respective temperatures (T_e and T_i) in the presence of a self-consistent electrostatic potential. Thus, the electron (n_e) and ion (n_i) number densities are given by the respective Boltzmann distributions,

$$n_e = N_e \exp\left(\frac{e\phi}{\kappa T_e}\right),\qquad n_i = N_i \exp\left(-\frac{e\phi}{\kappa T_i}\right), \tag{1}$$

where N_e and N_i are true integration constants. The dust fluid is governed by the fluid equations

$$\frac{\partial n_d}{\partial t} + \nabla\cdot(n_d \mathbf{u}_d) = 0,\qquad \frac{\partial \mathbf{u}_d}{\partial t} + (\mathbf{u}_d\cdot\nabla)\mathbf{u}_d + \frac{v_{Td}^2}{n_d}\nabla n_d = -\frac{q_d}{m_d}\nabla\phi - \nabla\psi. \tag{2}$$

The system of equations is closed by the Poisson equations for the electrostatic (ϕ) as well as gravitational (ψ) potentials,

$$\nabla^2\phi = -\frac{1}{\varepsilon_0}(q_d n_d + e n_i - e n_e), \tag{3}$$

$$\nabla^2\psi = 4\pi G m_d n_d. \tag{4}$$

In these basic equations n_d, $\mathbf{u}_d$, m_d, q_d, and v_{Td} denote the dust number density, the flow velocity, the mass, the charge, and the thermal speed, respectively, and G is the gravitational constant.

Equations (1)–(4) constitute a complete set of governing equations for describing electrostatic collective processes in self-gravitating dusty plasmas. We now look for self-consistent stationary equilibria, governed by

$$n_{e0} = N_e \exp\left(\frac{e\phi_0}{\kappa T_e}\right), \tag{5}$$

$$n_{i0} = N_i \exp\left(-\frac{e\phi_0}{\kappa T_i}\right), \tag{6}$$

$$\nabla\cdot(n_{d0}\mathbf{u}_{d0}) = 0, \tag{7}$$

$$(\mathbf{u}_{d0}\cdot\nabla)\mathbf{u}_{d0} + \frac{v_{Td}^2}{n_{d0}}\nabla n_{d0} = -\frac{q_{d0}}{m_d}\nabla\phi_0 - \nabla\psi_0, \tag{8}$$

$$\nabla^2\phi_0 = -\frac{1}{\varepsilon_0}(q_{d0}n_{d0} + e n_{i0} - e n_{e0}), \tag{9}$$

$$\nabla^2\psi_0 = 4\pi G m_d n_{d0}, \tag{10}$$

where the subscript "0" denotes the respective equilibrium quantities. As remarked earlier, we have dropped the electron and ion mass contributions in (4) and (10) since the mass content of dusty plasmas arises almost entirely due to the dust component. In addition, the Boltzmann assumption is equivalent to treating the electrons and ions as massless.

Equation (8) can be rewritten as

$$\nabla\left(\tfrac{1}{2}u_{d0}^2 + \frac{q_{d0}}{m_d}\phi_0 + v_{Td}^2 \ln n_{d0} + \psi_0\right) = \mathbf{u}_{d0}\times(\nabla\times\mathbf{u}_{d0}). \tag{11}$$

We now consider one-dimensional geometries that can be either Cartesian, cylindrical, or spherical. The flow is assumed to be along the x axis or in the radial direction and function of the corresponding spatial coordinate, as the case may be. Such flows are irrotational ($\nabla\times\mathbf{u}_{d0}=0$), so that the right-hand side of (11) vanishes, leading to the conserved quantity

$$\tfrac{1}{2}m_d u_{d0}^2 + q_{d0}\phi_0 + m_d v_{Td}^2 \ln n_{d0} + m_d\psi_0 = C_1, \tag{12}$$

where C_1 is an integration constant. Equation (12) is the Bernoulli equation representing the conservation of the total energy which consists of the kinetic, electrostatic, thermal, and gravitational components.

We now proceed to derive the equation governing the self-consistent equilibria. For this purpose, it is illustrative as well as useful to concentrate on the flow speed u_{d0} in favor of the other variables (namely, n_{d0}, ψ_0, and ϕ_0). Taking the Laplacian of (12) leads to

$$\tfrac{1}{2}\nabla^2 u_{d0}^2 + \frac{q_{d0}}{m_d}\nabla^2\phi_0 + v_{Td}^2\nabla^2 \ln n_{d0} + \omega_{\mathrm{J}d}^2 = 0, \tag{13}$$

where $\omega_{\mathrm{J}d}^2 \equiv 4\pi G m_d n_{d0}$ is the square of the Jeans frequency for the self-gravitating dusty plasma. To eliminate n_{d0} in (13), we use (7) which yields

$$n_{d0} = \frac{C_2}{r^\nu u_{d0}}, \tag{14}$$

where C_2 is a constant, and $\nu=0$, 1, or 2 for a one-dimensional Cartesian, cylindrically symmetric, or spherically symmetric geometry, respectively.

It is tempting to use the electrostatic Poisson equation (9) to substitute for $\nabla^2\phi_0$ in (13). However, this does not lead to a closed form governing equation for the flow speeds, since (9) together with (5) and (6) is highly nonlinear and cannot be solved for ϕ_0 in terms of n_{d0} and hence u_{d0} [after using (14)]. In order to proceed with the analytic formulation of the problem, we now make use of the charge quasineutrality condition. While this is somewhat restrictive and limits the possible number of self-consistent equilibria, it is a reasonably good approximation for the electrostatic potential, particularly when the plasma inhomogeneities are sufficiently weak and smooth, as discussed in the following. Furthermore, it allows the derivation of a governing equation for the equilibrium flow speed. Thus, instead of the electrostatic Poisson equation (9), we use the charge quasineutrality condition, namely,

$$q_{d0}n_{d0}+en_{i0}-en_{e0}=0, \tag{15}$$

where n_{e0} and n_{i0} are given by (5) and (6), respectively, while n_{d0} is given in terms of u_{d0} by (14). Since

$$\nabla^2\equiv\frac{\partial^2}{\partial r^2}+\frac{\nu}{r}\frac{\partial}{\partial r}, \tag{16}$$

where $\nu=0$, 1, or 2, and $r\rightarrow x$ (the Cartesian one-dimensional coordinate) for $\nu=0$, we use (5), (6) and (16) in (15) to obtain

$$q_{d0}\nabla^2 n_{d0}=\frac{\varepsilon_0}{\lambda_{\rm D}^2}\nabla^2\phi_0+\frac{e^3}{\kappa^2}\left(\frac{n_{e0}}{T_e^2}-\frac{n_{i0}}{T_i^2}\right)\left(\frac{\partial\phi_0}{\partial r}\right)^2, \tag{17}$$

where we have defined the effective plasma Debye length $\lambda_{\rm D}$ through $\lambda_{\rm D}^{-2}\equiv\lambda_{\rm De}^{-2}+\lambda_{\rm Di}^{-2}$, and $\lambda_{{\rm D}j}=(\varepsilon_0\kappa T_j/n_{j0}e^2)^{1/2}$ is the electron or ion Debye length for $j=e$ or $j=i$. Equation (17) is nonlinear in ϕ_0, and relates the latter to n_{d0} and, hence, to u_{d0} through (14). In order to obtain a closed form equation for the dust flow speed, we neglect the nonlinear term in (17) and obtain

$$\nabla^2 n_{d0}\simeq\frac{\varepsilon_0}{\lambda_{\rm D}^2 q_{d0}}\nabla^2\phi_0. \tag{18}$$

This approximation is essentially the same as requiring $e\phi_0\ll\kappa T_e,\kappa T_i$ and then using (5) and (6) in (15). Furthermore, the assumption of charge quasineutrality can now be checked for consistency by using (18) in the electrostatic Poisson equation (9). Clearly, it follows from (9) and (18) that the assumption of quasineutrality holds well when the plasma inhomogeneity scale length L satisfies the condition $L^2\gg\lambda_{\rm D}^2$. For realistic dusty plasmas of practical interest, this is not very restrictive.

Finally, we use (14), (16), and (18) in (13) to obtain

$$\begin{aligned}&u_{d0}(u_{d0}^2-c_{da}^2-v_{Td}^2)\frac{\partial^2 u_{d0}}{\partial r^2}+(u_{d0}^2+2c_{da}^2+v_{Td}^2)\left(\frac{\partial u_{d0}}{\partial r}\right)^2\\&+\frac{\nu u_{d0}}{r}(u_{d0}^2+c_{da}^2-v_{Td}^2)\frac{\partial u_{d0}}{\partial r}\\&+\left(\omega_{\rm J\mathit{d}}^2+\frac{\nu c_{da}^2}{r^2}-\frac{\nu(\nu-1)v_{Td}^2}{r^2}\right)u_{d0}^2=0,\end{aligned} \tag{19}$$

where $c_{da}\equiv\lambda_{\rm D}\omega_{pd}$ is the dust-acoustic speed,[7] $\omega_{pd}=(n_{d0}q_{d0}^2/\varepsilon_0 m_d)^{1/2}$ is the dust plasma frequency and, as mentioned earlier, ν takes values 0, 1, and 2 for the Cartesian one-dimensional $(r\rightarrow x)$, cylindrically symmetric and spherically symmetric cases, respectively. It should be kept in mind for the subsequent discussions that both c_{da} and $\omega_{\rm J\mathit{d}}$ depend on n_{d0}, and hence indirectly on u_{d0}. However, since $c_{da}^2u_{d0}\sim r^{-\nu}$ and $\omega_{\rm J\mathit{d}}^2u_{d0}\sim r^{-\nu}$, the r-dependent coefficients in (19) become more intricate when written out in full than they appear now, but we prefer to let this equation stand in its present, more compact form.

Equation (19) is the self-consistent governing equation for determining the steady state equilibrium dust flow speed in a self-gravitating unmagnetized dusty plasma. The corresponding number density profiles are governed by (14). Equation (19) is a nonlinear equation and has, interestingly, a local singularity at the total dust-acoustic speed c_d given through $c_d^2\equiv c_{da}^2+v_{Td}^2$. This seems to be natural, and is to be anticipated since the latter speed is the fundamental characteristic speed in an unmagnetized dusty plasma. Equations (14) and (19) thus completely determine self-consistently the stationary equilibrium profiles of a quasineutral, self-gravitating dusty plasma.

Before discussing the qualitative nature of the solutions admissible for (19), we briefly address the case of a self-gravitating neutral fluid. The equilibrium equations for the latter are

$$\begin{aligned}&\boldsymbol{\nabla}\cdot(n_0\mathbf{u}_0)=0,\\&(\mathbf{u}_0\cdot\boldsymbol{\nabla})\mathbf{u}_0+\frac{c_s^2}{n_0}\boldsymbol{\nabla}n_0=-\boldsymbol{\nabla}\psi_0,\\&\nabla^2\psi_0=4\pi Gmn_0,\end{aligned} \tag{20}$$

where n_0, $\mathbf{u}_0$, m, and c_s denote, respectively, the neutral fluid number density, flow velocity, mass, and acoustic speed. Equation (20) admits the Bernoulli equation

$$\tfrac{1}{2}u_0^2+c_s^2\ln n_0+\psi_0=C_3, \tag{21}$$

where C_3 is another integration constant. Unlike the dusty plasma case discussed previously, (20) can be *exactly* reduced to a single equation given by

$$\begin{aligned}&u_0(u_0^2-c_s^2)\frac{\partial^2 u_0}{\partial r^2}+(u_0^2+c_s^2)\left(\frac{\partial u_0}{\partial r}\right)^2+\frac{\nu u_0}{r}(u_0^2-c_s^2)\frac{\partial u_0}{\partial r}\\&+\left(\omega_{\rm J}^2-\frac{\nu(\nu-1)c_s^2}{r^2}\right)u_0^2=0,\end{aligned} \tag{22}$$

where $\omega_{\rm J}=(4\pi Gmn_0)^{1/2}$ is the Jeans frequency for a self-gravitating neutral fluid, and the definition of ν remains the same as earlier [cf. (19)]. Equation (22) governs the self-consistent (irrotational) flow speed of a self-gravitating neutral fluid, while the corresponding number density is given through $r^{\nu} n_0 u_0 = C_4$, where C_4 is a constant. Again, $\omega_J^2 u_0 \sim r^{-\nu}$, and similar remarks apply when discussing (22).

It should be pointed out that (22) follows *exactly* from the corresponding equation for dusty plasmas, namely (19), if in the latter we "switch off" the dust grain charge by letting $q_d \to 0$, which is equivalent to taking the limit $c_{da} \to 0$. This essentially corresponds to considering the case $c_{da}^2 \ll v_{Td}^2$. In the latter limit, the (uncharged) dust thermal speed is the same as the acoustic speed, that is, $v_{Td} \to c_s$, and hence (19) *identically* reduces to (22). It is, therefore, anticipated that many of the conclusions derived from (19) for dusty plasmas hold well also for the neutral fluid governed by (22). While stationary states with radial flows have been treated in neutral fluids,[30] there is no mention of the resonances and singular layers that we discuss in Sec. III.

In order to highlight the contributions arising from the electromagnetic force (because of the charged components of the dusty plasmas) as well as compare with the neutral fluids which are governed only by the gravitational interaction, it is illustrative to explicitly consider the opposite case when the dust thermal speed is much smaller than the dust-acoustic speed, that is, $v_{Td}^2 \ll c_{da}^2$. For typical dusty plasmas of current interest, this is generally a very good approximation. Accordingly, (19) reduces to

$$u_{d0}(u_{d0}^2 - c_{da}^2)\frac{\partial^2 u_{d0}}{\partial r^2} + (u_{d0}^2 + 2c_{da}^2)\left(\frac{\partial u_{d0}}{\partial r}\right)^2 + \frac{\nu u_{d0}}{r}(u_{d0}^2 + c_{da}^2)\frac{\partial u_{d0}}{\partial r} + \left(\omega_{Jd}^2 + \frac{\nu c_{da}^2}{r^2}\right)u_{d0}^2 = 0, \tag{23}$$

which is similar to (22) for the neutral fluids. Unlike in the case of the latter where the equilibrium is maintained due to the fluid thermal pressure, in the case of dusty plasmas the electromagnetic forces contribute (additional) balancing forces which are accompanied by self-consistent flows.

The most essential feature of the governing (22) and (23) which is *common* to all geometries ($\nu = 0$, 1, or 2) mentioned earlier is the existence of the *singularity* at the characteristic speed, namely, the dust-acoustic speed c_{da} in the case of dusty plasmas, and the acoustic speed c_s in the case of neutral fluids. This singularity is a consequence of the inhomogeneous equilibrium self-gravitational potential, which manifests itself in the governing equations through the Jeans frequency. Compared to the Cartesian one-dimensional case ($\nu = 0$), the cylindrically ($\nu = 1$) and spherically ($\nu = 2$) symmetric cases bring in additional terms, but the essential structure of the equations remains the same. In fact, as anticipated, the limit $r \to \infty$ is mathematically identical to taking $\nu = 0$.

Equations (22) and (23) are thus nonlinear singular differential equations whose exact analytic solutions are not yet available. In the absence of such solutions, these equations have to be carefully analyzed using appropriate (singular) perturbation techniques, or solved by suitable numerical methods, both of which are beyond the scope of the present work. However, certain qualitative results about the possible equilibria can indeed be derived from the general structure of the governing equations (22) and (23), and will be given in Sec. III.

III. RESULTS AND CONCLUSIONS

The results are summarized as follows.

(1) The governing equations (22) and (23) have a trivial solution corresponding to zero flow speed. In this case, the equilibrium gravitational force is balanced by the thermal and/or the electrostatic forces, as the case may be. This is consistent with the momentum balance equations like (8) and (20). On the other hand, the presence of the Jeans frequency term in (22) and (23) precludes any equilibria with constant flow speed (even for the Cartesian case when $\nu = 0$). Thus, in general, the equilibrium flow speed has to be inhomogeneous and, hence, so also the number density [cf. (14) and its equivalent for the neutral fluid case]. This is again consistent with the corresponding momentum balance (8) and (20). For the special case of inhomogeneous equilibria with zero flow speed, explicit profiles can be analytically derived in some special cases.[29]

(2) In view of (14) and its equivalent for the neutral fluids, the sign of the equilibrium flow speed is fixed once the constant of integration is determined by specifying the equilibrium flow speed and the number density at some spatial coordinate. Thus, the equilibrium flow speed cannot change sign with respect to the spatial coordinate.

(3) The detailed structure of the singular layer where the flow speed approaches the respective characteristic speeds for cold dusty plasmas and neutral fluids (c_{da} and c_s, respectively) depends on the geometry of the configuration ($\nu = 0$, 1, or 2). However, certain generic features of the singular layer can be easily deduced for the Cartesian one-dimensional case. Thus, for $\nu = 0$ and $r \equiv x$, (23) becomes

$$u_{d0}(u_{d0}^2 - c_{da}^2)\frac{\partial^2 u_{d0}}{\partial x^2} + (u_{d0}^2 + 2c_{da}^2)\left(\frac{\partial u_{d0}}{\partial x}\right)^2 + \omega_{Jd}^2 u_{d0}^2 = 0, \tag{24}$$

and (22) then similarly is

$$u_0(u_0^2 - c_s^2)\frac{\partial^2 u_0}{\partial x^2} + (u_0^2 + c_s^2)\left(\frac{\partial u_0}{\partial x}\right)^2 + \omega_J^2 u_0^2 = 0. \tag{25}$$

In both cases, the last two terms are positive definite. Since the sign of the flow speed is fixed (either positive or negative), we assume it to be positive for convenience. Thus, for non-zero flow speeds, it follows from (24) and (25) that the following condition should be satisfied:

$$(U^2 - C^2)\frac{\partial^2 U}{\partial x^2} < 0, \tag{26}$$

where $U \equiv (u_{d0}, u_0)$ and $C \equiv (c_{da}, c_s)$ for cold dusty plasmas and neutral fluids, respectively. Thus, in the singular layer when $U^2 - C^2 \to \pm 0$, it follows that $\partial^2 U/\partial x^2 \to \mp \infty$. This implies that the equilibrium flow speeds have steep profiles in the singular layer.

More specifically for cold dusty plasmas, governed by (23), the location r_c of the singular layer is determined from the condition $u_{d0}^2(r_c) = c_{da}^2(r_c)$, or more explicitly

$$u_{d0}^3(r_c)=c_{da}^2(r_0)u_{d0}(r_0)\left(\frac{r_0}{r_c}\right)^{\nu}, \tag{27}$$

where r_0 is some reference spatial position at which the boundary values are specified.

To summarize, we have investigated the existence of self-consistent stationary equilibrium states of self-gravitating dusty plasmas as well as neutral fluids by avoiding the usual Jeans swindle assumption. It is shown that, in general, stationary equilibria are possible when the medium is inhomogeneous with nonzero flow speeds. Starting with the respective basic equations, we have shown that the possible equilibria are governed by a nonlinear differential equation for the flow speed. The latter governing equation has a singularity at the respective characteristic speed of the medium, namely, the dust-acoustic speed in the case of dusty plasmas, and the acoustic speed in the case of neutral fluids. The singularity is a manifestation of the equilibrium (inhomogeneous) gravitational potential, and is common to all geometries (Cartesian, cylindrical, and spherical). Certain general, qualitative results about the admissible equilibria have been derived by analyzing the structure of the governing nonlinear equation for the equilibrium flow speed.

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