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Structural Aspects of Solitons in Dusty Plasmas

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Abstract

We can model a dusty plasma as a multispecies plasma, neglecting density and momentum losses and gains for the plasma electrons and ions, provided the dust charging is slow and hence the grain charges can be treated as constant.

It is well known that nonlinear waves in multispecies plasmas give rise to soliton structures. For electromagnetic waves propagating parallel to an ambient magnetic field a derivative nonlinear Schrödinger equation is obtained, while for perpendicular and oblique electrostatic and electromagnetic modes the situation is adequately described by a Korteweg–de Vries equation.

The localized solutions of these equations are used to analyze the width and height of the structures for a monosized dusty plasma. In reality however, different dust sizes coexist, and this affects the modes on the linear level. The influence on the nonlinear description is examined, and possible astrophysical applications as planetary rings, cometary environments and a dusty interstellar medium are discussed.

1. Introduction

A dusty plasma consists of charged dust grains embedded in an ambient plasma. Depending on the dust grain concentration, one has isolated screened dust grains (dust-in-plasma) or real collective dusty plasmas where the charged dust participates in the Debye screening. General reviews have been given by Goertz [1], Northrop [2] and Mendis and Rosenberg [3]. The interest in dusty plasmas is recently increased both from experimental as from theoretical point of view. From a mathematical point of view, the study of dusty plasmas is interesting and challenging. Indeed there are at least four reasons why dusty plasmas are essentially different from classical plasmas. First, the charge of a dust grain is variable, and is determined by the grain properties and the surrounding plasma. This changes the description of waves in the sense that plasma fluctuations related to a wave will induce grain charge fluctuations and a coupling between the wave and the charging mechanism occurs. Also, the charge-to-mass ratio is usually much lower for dust grains than for plasma particles and the dust characteristic frequencies (plasma frequency and gyrofrequency) are much smaller than those corresponding to the plasma particles. Furthermore the ion charge density is different from the electron charge density because charges are residing on the dust grains. These charges are immobilized and although the plasma including the dust is quasi-neutral, the collection of particles with a high mobility (electrons and ions) is not. Finally, the dust grains come in large range of grain sizes and for a realistic description we would need to use a mass distribution. When we use as a simplication a mono-sized dust population for the dusty plasma, an appropriate value must be taken to derive adequate results.

Besides the linear analysis of modes in dusty plasmas (see the reviews by Shukla [4] and Verheest [5]), the propagation of weakly nonlinear and dispersive waves in multispecies plasmas has been investigated by the use of the reductive perturbation method leading to a nonlinear evolution equation. It has been shown [6] that in the presence of an ambient magnetic field, we must make a distinction between the parallel [7–9] and quasi-parallel case [10–11], for which we come to a derivative nonlinear Schrödinger equation, and the oblique [6] (and perpendicular [12–13]) case which leads to a Korteweg–de Vries equation. Both nonlinear evolution equation are known to have soliton solutions. These solutions are characterized by a height and a width of the moving soliton structure.

In this paper we will look at the structural properties of stationary soliton-solutions for dust-acoustic modes (parallel, oblique and perpendicular). We have omitted the dust charge fluctuations keeping the grain charge as constant. This assumption is reasonable provided that the wave frequency is larger than the attachment frequencies. We will consider an electron–proton plasma with several negatively charged dust species. The special case of one (average) dust species is also considered.

2. Korteweg–de Vries equation

The standard form of the KdV equation looks like:

$$\phi_\tau + \alpha\phi\phi_\xi + \beta\phi_{\xi\xi\xi} = 0, \tag{1}$$

and the stationary soliton solution, vanishing at infinity for both time and space coordinates is:

$$\phi = \frac{3M}{\alpha}\,\mathrm{sech}^2\left[\sqrt{\frac{M}{4\beta}}\,(\xi - M\tau)\right], \tag{2}$$

provided that M and β have the same sign. The width and the height of the soliton structure is given by:

$$H = 3\,\frac{\varepsilon V_0}{\alpha}, \tag{3}$$

$$W^2 = \frac{4|\beta|}{\varepsilon V_0}. \tag{4}$$

The symbols ε and V_0 stand for a free (small) parameter (typically of the order of the ratio, e.g. wave magnetic field—ambient magnetic field) and the soliton velocity (equal to the Alfvén velocity or magnetosonic velocity for cold and warm plasma's respectively). As a general rule we see that the height of the solitons is essentially much smaller than their width, because of the presence of the $\varepsilon \ll 1$.

2.1. *Dust-acoustic modes*

Electrostatic dust-acoustic modes are described by a KdV-equation as shown by Verheest [14]. The dispersion law of these modes is given by:

$$\frac{\sum_{\rm d} \omega_{\rm pd}^2}{V^2} = \frac{\omega_{\rm pe}^2}{c_{\rm se}^2} + \frac{\omega_{\rm pi}^2}{c_{\rm si}^2}, \tag{5}$$

and the width and height of the soliton solution in the electric potential are given by:

$$W = \sqrt{\frac{2}{\varepsilon}}\, \lambda_{\rm eff}, \tag{6}$$

$$H = \frac{3\varepsilon}{2\lambda_{\rm eff}^2} \left[\frac{\omega_{\rm pi}^2 q_{\rm i}}{m_{\rm i} c_{\rm si}^4} + \frac{3}{V^4} \sum_{\rm d} \frac{\omega_{\rm pd}^2 q_{\rm d}}{m_{\rm d}} \right]^{-1}, \tag{7}$$

with $\lambda_{\rm eff}$ the effective Debye length, which is defined as:

$$\lambda_{\rm eff}^{-1} = \lambda_{\rm De}^{-2} + \lambda_{\rm Di}^{-2}. \tag{8}$$

2.2. *Oblique electromagnetic modes*

For oblique electromagnetic modes propagating at an angle θ to the ambient magnetic field, we follow the outline of Verheest and Meuris [6] where it was shown that the wave related quantities obey a KdV equation. For a general cold multi-species plasma we recover:

$$H = \frac{2\varepsilon B_0}{\sin\theta}, \tag{9}$$

$$W^2 = \frac{2V^6}{\varepsilon c^4 B_0^6 \varepsilon_0^2} \left| \frac{1}{2} \sum_\alpha \sum_\beta N_\alpha m_\alpha N_\beta m_\beta \left(\frac{m_\alpha}{q_\alpha} - \frac{m_\beta}{q_\beta} \right)^2 - \cot^2\theta \left(\frac{\sum_\alpha N_\alpha m_\alpha^2}{q_\alpha} \right)^2 \right|, \tag{10}$$

for truly oblique modes for which $\varepsilon \leqslant \theta \leqslant (\pi/2)$. From (10), it is clear that there might exist a critical angle $\vartheta_{\rm cr}$ for which the width of the soliton structure vanishes, although the physical mechanism of this phenomena is not yet clear, this might indicate some kind of shock structure, which cannot adequately be described by a KdV without the dispersive term.

- For a dusty plasma with several dusty plasma components, the width of the soliton obeys:

$$W^2 = \frac{2V^2}{\varepsilon c^4 B_0^6 \varepsilon_0^2} \left| N_{\rm i} m_{\rm i} \left(\sum_d \frac{N_{\rm d} m_{\rm d}^3}{q_{\rm d}^2} + \frac{N_{\rm i} m_{\rm e} m_{\rm i}^2}{q_{\rm i}^2} \right) - \cot^2\theta \left(\sum_{\rm d} \frac{N_{\rm d} m_{\rm d}^2}{q_{\rm d}} + \frac{N_{\rm i} m_{\rm i}^2}{q_{\rm i}} \right)^2 \right|. \tag{11}$$

For spherical dust grains with size a, the mass and the equilibrium charge of the grains are given respectively by:

$$m_{\rm d} \sim a^3, \quad q_{\rm d} \sim a, \tag{12}$$

while the dust distribution is mostly given by a power law:

$$n_{\rm d}\, da = a^{-\beta} da, \quad \text{for } a \in [a_{\min}, a_{\max}]. \tag{13}$$

This kind of distribution is widely accepted in space plasmas. We find values of $\beta = 4.6$ for the F-ring of Saturn [14], while for the G-ring values of $\beta = 7$ or $\beta = 6$ we obtained [15–16]. For cometary environments we recall a value of $\beta = 3.4$ [17]. With this information in mind we can see that the first summation in (11) goes like $n_{\rm d} a^7$, while the second summation goes like $n_{\rm d} a^5$. This makes it possible to calculate an appropriate mean value of the dust grain size. For instance if $\beta < 5$, we must look at the upper part of the size spectrum, while for $\beta > 7$ the smaller grain sizes dominate.

- For a three component plasma consisting of dust grains electrons and protons, we recover:

$$W^2 = \frac{2V^6}{\varepsilon c^4 B_0^6 \varepsilon_0^2} \left| N_{\rm i} m_{\rm i} \left(\frac{N_{\rm d} m_{\rm d}^3}{q_{\rm d}^2} + \frac{N_{\rm i} m_{\rm e} m_{\rm i}^2}{q_{\rm i}^2} \right) - \cot^2\theta \left(\frac{N_{\rm d} m_{\rm d}^2}{q_{\rm d}} + \frac{N_{\rm i} m_{\rm i}^2}{q_{\rm i}} \right)^2 \right|. \tag{14}$$

When $N_{\rm d} m_{\rm d}^2/q_{\rm d} \gg N_{\rm i} m_{\rm i}^2/q_{\rm i}$, as is usually the case in most dusty plasma environments, this expression becomes:

$$W^2 = \frac{2c^2}{\varepsilon \omega_{\rm pd}^2} \left(1 + \frac{N_{\rm i} m_{\rm i}}{N_{\rm d} m_{\rm d}} \right)^{-3} \left| \frac{N_{\rm i} m_{\rm i}}{N_{\rm d} m_{\rm d}} - \cot^2\theta \right|. \tag{15}$$

When most of the mass is in the dust grains and hence $N_{\rm i} m_{\rm i} \ll N_{\rm d} m_{\rm d}$, we recover:

$$W^2 = \frac{2c^2}{\varepsilon \omega_{\rm pd}^2} \left| \frac{N_{\rm i} m_{\rm i}}{N_{\rm d} m_{\rm d}} - \cot^2\theta \right|, \tag{16}$$

$$\sin^2\theta_{\rm cr} = 1 - \frac{N_{\rm i} m_{\rm i}}{N_{\rm d} m_{\rm d}},$$

which results in a critical angle very close to $\pi/2$. On the other hand, when most of the mass is found in the ions ($N_{\rm i} m_{\rm i} \gg N_{\rm d} m_{\rm d}$):

$$W^2 = \frac{2c^2}{\varepsilon \omega_{\rm pd}^2} \left(\frac{N_{\rm d} m_{\rm d}}{N_{\rm i} m_{\rm i}} \right)^3 \left| \frac{N_{\rm i} m_{\rm i}}{N_{\rm d} m_{\rm d}} - \cot^2\theta \right|,$$

and the critical angle is close to zero.

3. Derivative nonlinear Schrödinger equation

The standard form for the DNLS equation looks like:

$$\alpha\phi_\tau + \frac{1}{4\mu_0} (\phi |\phi|^2)_\xi \pm i\beta\phi_{\xi\xi} = 0, \tag{17}$$

with a 1/cosh-like soliton solution. It was shown [7] that the height and width of the soliton structure for parallel electromagnetic waves obey:

$$H^2 = \varepsilon \times \frac{8\mu_0 V \sum_\alpha N_\alpha m_\alpha (V - U_\alpha)}{\sqrt{2} \pm 1}, \tag{18}$$

$$W = \frac{1}{\varepsilon} \times \frac{1}{2B_0 V} \sum_\alpha \frac{N_\alpha m_\alpha^2}{q_\alpha} (V - U_\alpha)^3 \div \sum_\alpha N_\alpha m_\alpha (V - U_\alpha) V, \tag{19}$$

with the $\pm$ sign for sub-alfvenic and super-alfvenic solitions respectively. For a non-drifting plasma the width is given by:

$$H^2 = \frac{8}{\sqrt{2} \pm 1} \varepsilon B_0^2, \tag{20}$$

$$W = \frac{1}{\varepsilon} \times \frac{V}{2B_0} \sum_\alpha \frac{N_\alpha m_\alpha^2}{q_\alpha} \Big/ \sum_\alpha N_\alpha m_\alpha. \tag{21}$$

- When $N_{\rm d} m_{\rm d}^2/q_{\rm d} \gg N_{\rm i} m_{\rm i}^2/q_{\rm i}$ and the different species are non-drifting, we come to:

$$W = \frac{V_{\rm A}}{2\varepsilon \Omega_{\rm d}} \left(1 + \frac{N_{\rm i} m_{\rm i}}{N_{\rm d} m_{\rm d}} \right)^{-1}. \tag{22}$$

4. Conclusions

The localized solutions of the nonlinear evolution equations describing plasma waves in a multi-species plasma are analyzed. The width and the height of the structures for a monosized dusty plasma were calculated, and the appropriate expressions were given. In reality however, different dust sizes coexist and this means that the appropriate mean dust size must be taken into account.

The width of the dust-acoustic solitons, without streaming effects described by a KdV equation, is proportional to the effective Debye length, and therefore dust effects play only an indirect role. Oblique electromagnetic modes (including perpendicular magnetosonic modes) are also described by a KdV equation, at least for propagation at an angle different from the critical angle θ_{cr}. This angle is very close to zero when the mass is mostly in the ions, while $\theta_{cr} \approx \pi/2$ when the mass is mostly in the dust grains. For propagation at the critical angle, the width of the soliton vanishes. For both oblique as perpendicular electromagnetic solitons, the maximum amplitude is independent of the plasma environment, and only determined by the ambient magnetic field.

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