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Ulysses electron distributions fitted with Kappa functions

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Abstract. We fit Kappa functions to 16,000 velocity distribution functions measured in the solar wind by the electron plasma instrument on board Ulysses. Statistically, the electron distributions are observed to have important high velocity tails in the fast solar wind but are closer to a Maxwellian in the slow wind. We also discuss how this result could support a recent kinetic model of the solar wind proposed by Maksimovic, Pierrard and Lemaire [1997].

Introduction

Up to now, the most often used model of the solar wind electron velocity distribution functions (VDFs) has been made of the sum of two Maxwellians: a core of "cold" electrons and a halo of "hot" ones [Feldman *et al.*, 1975]. However, Lorentzian or Kappa VDFs require one fewer parameter than the core/halo model [Vasyliunas, 1968], and there are moreover some possible, although limited, physical interpretations of the generation of Kappa VDFs [Hasegawa *et al.*, 1985; Christon *et al.*, 1991; Collier, 1993].

In the present paper, we fit electron VDFs observed in the solar wind with Kappa functions f^κ, defined in 3 dimensions, by:

$$f^\kappa(v) = \frac{n_\kappa}{(\pi\kappa v_\kappa^2)^{3/2}}\frac{\Gamma(\kappa+1)}{\Gamma(\kappa-1/2)}\left(1+\frac{v^2}{\kappa v_\kappa^2}\right)^{-(\kappa+1)} \qquad (1)$$

where $\Gamma(x)$ is the Gamma function and v_κ is an equivalent thermal speed, related to the equivalent temperature $T_\kappa = m\langle v^2\rangle/3k$ by $v_\kappa = (\frac{(2\kappa-3)}{\kappa}\frac{kT_\kappa}{m})^{1/2}$. The present attempt is not the first one to use Kappa functions to fit solar wind particle VDFs. VDFs of the solar wind protons, helium, oxygen and neon can be fitted quite well with Kappa functions [Bosqued *et al.*, 1977; Ogilvie *et al.*, 1980; Collier *et al.*, 1996].

In section 1 we present the observations, our fitting process and the results of our statistical data analysis. This analysis reveals a global anticorrelation between the solar wind bulk speed V and the parameter κ of the fitted functions f^κ, which quantifies the importance of the high velocity tails in the observed electron distributions.

In section 2, we discuss how this anticorrelation could support a recent kinetic model of the solar wind proposed by Maksimovic, Pierrard and Lemaire [1997].

1. Data analysis

Ulysses electron distributions

We use VDFs obtained with the solar wind electron plasma instrument on board the Ulysses spacecraft [Bame *et al.*, 1992]. This instrument measures three dimensional electron velocity distributions. Electrons with central energies in the range 0.86 to 814 eV are detected in many directions which cover the unit sphere comprehensively. We then average the observed 3-D electron VDFs over all directions.

Our data set contains 15,887 electron VDF spectra obtained during four periods (Table 1) which provide a good coverage of the large range of V, from $\approx$ 300 km/s near the ecliptic to $\approx$ 800 km/s at high latitudes.

The fitting process

We fit our spectra with 3-D Kappa functions. We first set $n_\kappa = n_c + n_h$ and $T_\kappa = T_{e\ tot}$: n_c and n_h are respectively the core and halo densities obtained from a fitting with two Maxwellians and $T_{e\ tot}$ is the total electron temperature obtained by integrating the observed electron distributions over the whole velocity range. The fitting with Kappa functions minimizes a χ^2 merit function with a single free parameter: κ.

Because the aim of our fitting is to accurately characterize the importance of the tails of the VDFs, we compute χ^2 in a $\log(f(v))$ space: in normal $f(v)$ space, the absolute phase space densities of the tails of the ob-

Table 1. The observations time periods

time interval	distance range in AU	latitude range
11 17, 90 to 11 26, 90	1.15 to 1.22	in ecliptic
01 15, 91 to 02 15, 91	1.74 to 2.08	in ecliptic
03 15, 91 to 04 15, 91	2.39 to 2.72	in ecliptic
05 30, 95 to 06 08, 95	1.60 to 1.66	60.3° to 64.9°

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served distributions are too low as compared to that of the thermal part of the distributions.

Figure 1 shows four typical Ulysses electron VDFs together with both the classical core/halo and the Kappa fits: the Kappa functions fit much better the high velocity tails than do the sums of Maxwellians.

The results

For each of our spectra, the solar wind bulk speed is plotted as a function of the fitted parameter κ (Figure 2). Two distinct populations appear on this scatter plot. This is because the solar wind is essentially a two-state phenomenon: the high speed streams, emanating from coronal holes, are associated with lower κ values and the low speed streams, emanating from equatorial regions, are associated with higher and more scattered values of κ. In order to quantify this result, we split our data set: events with $V < 550$ km/s and events with $V > 550$ km/s. For each of these two populations we compute the median value $\langle\kappa\rangle$ of the fitted parameter κ and that of the variances of the fits $\langle\sigma_{fit}\rangle$ (Table 2). $\langle\kappa\rangle$ is definitively smaller for the fast solar wind population, whose VDFs thus contain more important tails.

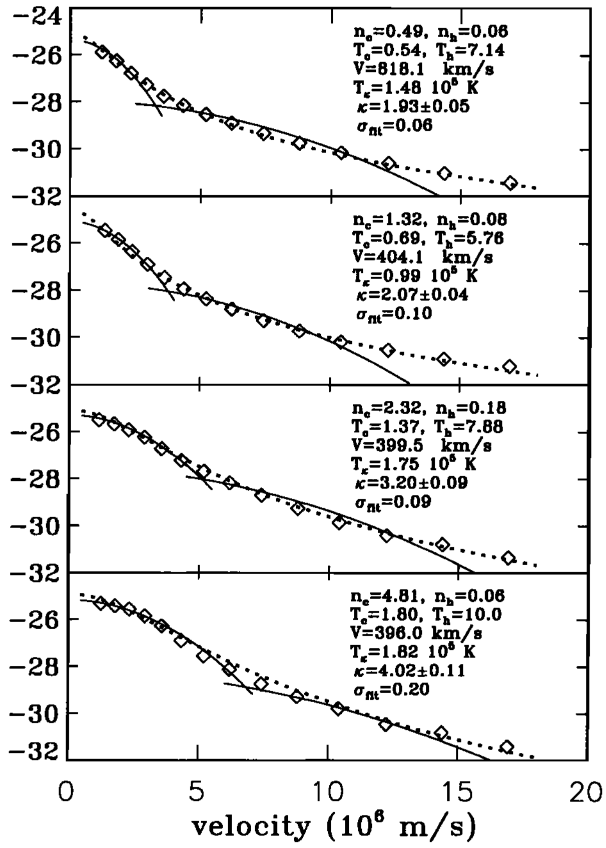

Figure 1. Four typical Ulysses electron distributions. The diamonds represent averages of the observed electron distributions over all spatial angles, the full lines the classical core/halo fits and the dotted lines the Kappa fits. Core/halo densities (n_c and n_h) are in cm^{-3} and the core/halo temperatures in units of 10^5 K. The Kappa fitting is processed with $n_\kappa = n_c + n_h$ and $T_\kappa = T_{e\ tot}$.

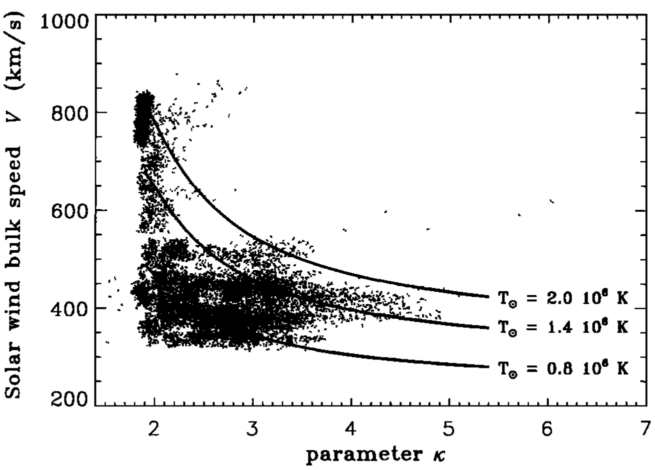

Figure 2. For each spectrum, the solar wind bulk speed V is plotted as a function of the fitted parameter κ. There are two distinct populations: high speed solar wind streams with lower values of κ and low speed streams with larger values of κ. Continuous lines: the expansion bulk speed at 1 AU from the MPL model, as a function of the parameter κ for the electrons and for three values of the initial coronal temperature $T_\odot$. The MPL bulk speed increases with decreasing κ.

There is another simple way to provide evidence of high velocity tails in the observed VDFs. This is to calculate a "width factor" ξ, defined as $\xi = \overline{v^4}/(\overline{v^2})^2$, where:

$$\overline{v^n} = \frac{\int_0^{+\infty} v^n f(v) v^2 dv}{\int_0^{+\infty} f(v) v^2 dv} \tag{2}$$

For a Maxwellian $\xi = 5/3$ and for a Kappa function $\xi = \frac{5}{3}\frac{\kappa - 3/2}{\kappa - 5/2}$. The more important the high velocity tails of a given VDF are, the smaller is κ and the larger is the value of ξ. One can thus quantify the importance of the suprathermal tails, without any fitting, by only calculating ξ from (2) and the observed VDFs. The median value of ξ is 3.82 for the high speed streams and only 2.82 for the low speed ones. This is in agreement with the global anticorrelation shown in Figure 2. We can thus definitively reach the following conclusion: the electron VDFs have well defined high velocity tails in the fast solar wind and are closer to a Maxwellian in the slow solar wind.

Table 2. Summary of the statistical analysis

	$V > 550$ km/s	$V < 550$ km/s
number of events	3,494	12,393
$\langle\kappa\rangle$	1.90 ±0.08	2.71 ±0.56
$\langle\sigma_{fit}\rangle$	0.16	0.11
$\langle\xi\rangle$	3.82	2.82

2. Discussion

Scudder [1992a,b] proposed a new mechanism to explain the high coronal temperature. This mechanism, "the velocity filtration effect", is based on the assumption that the VDFs of the ions and the electrons in the chromosphere, and therefore in the corona, are non-Maxwellians, for instance Kappa functions. Following Scudder's work, Maksimovic, Pierrard and Lemaire [1997] (MPL) proposed a new kinetic model of the solar wind based on Kappa VDFs for the electrons and protons escaping from the corona. We will now discuss how the observed global anticorrelation between V and κ can support the MPL model.

A kinetic model of the solar wind with Kappa distributions in the corona

The MPL model is based on the kinetic/exospheric approach of the solar wind by Lemaire and Scherer [1971a,b] for Maxwellian distributions in the corona. In this approach, two distinct regions are considered: first a collision-dominated barosphere where the particles are assumed to be in hydrostatic/hydrodynamic equilibrium, and second, a collisionless exosphere. The barosphere and the exosphere are separated by an exobase where the Coulomb mean free path equals the local density scale height (see Figure 1 and Table 1 in MPL [1997]). Starting at the exobase with a given VDF, the distribution function can be determined at any radial location in the exosphere by using Liouville's theorem. Various moments, such as the flux of particles and their pressures or temperatures, can then be calculated analytically by integrating the exosphere distribution function.

Using Kappa distribution functions for the particles at the exobase, the main result obtained with the MPL model is the fact that the high speed solar wind streams can be explained if the electron VDFs at the base of these streams have important high velocity tails, a suggestion previously made by Fairfield and Scudder [1985] and Scudder [1992b]. To illustrate this, we plot in Figure 2 the bulk speed V at 1 AU obtained with the MPL model as a function of the parameter κ for the electrons and for three values of the initial coronal temperature $T_{\odot}$ at the exobase. The model predicts that V increases with decreasing κ. This is explained by the fact that, for a given value of $T_{\odot}$, as in planetary atmospheres [Jeans, 1923; Brandt and Chamberlain, 1960], when the width of the VDF increases in the corona (i.e when κ decreases), the number of particles with a velocity larger than the local escaping velocity also increases, and so does V. High speed solar wind streams can thus be explained by the MPL model without any additional heating energy source in the corona.

Can we compare the MPL model with the observations ?

In Figure 2, the three $V(\kappa)$ curves, derived from the MPL model, fit rather well the scatter plot resulting from our Kappa fittings. Therefore, the present study provides some support to the MPL model. Nevertheless, there is an important property of the model, which is a consequence of Liouville's theorem: if the VDF is a Kappa function f^{κ} in the corona, it remains a Kappa function with the same value of κ at any heliocentric distance. This property should be only true if there are not too many collisisons in the medium.

To test for collisional effects, we will now quantify the role of Coulomb collisions on the solar wind electron VDFs. We define the Knudsen number Kn as the ratio λ_e/H, where λ_e is the Spitzer's [1962] 90° cumulative deflection Coulomb mean free path for electrons scattering off solar wind ions and other electrons, and $H = R/2$ is the scale height corresponding to inverse square density radial variation. Since the value of λ_e calculated for electron Kappa VDFs does not significantly depend on the value of κ [Pierrard, 1997], it is not unreasonable to take for λ_e the expression calculated for a Maxwellian: $\lambda_e(\mathrm{m}) = 9.25\ 10^7\ Te^2(\mathrm{K})/n_e(\mathrm{m}^{-3})$, with $Te = T_{e\ tot}$ and $n_e = n_c + n_h$. A value of $Kn < 1$ characterizes a solar wind in which the Coulomb interactions are important in determining the transport properties of the plasma. On Figure 3, the Kn of each spectrum is plotted as a function of the fitted κ. The median value of Kn is $\langle Kn \rangle = 7.33 \gg 1$: this confirms that Coulomb collisions can be neglected, to a first approximation, in the MPL model. In Figure 3, the scatter plot does not show any clear correlation between Kn and κ. To quantify this lack of correlation, we divide our data set into two classes: events with Kn larger than $\langle Kn \rangle$ of the whole data set (class I), and events with Kn lower than $\langle Kn \rangle$ (class II). For each of the two classes we compute the median values of the fitted index κ: $\langle \kappa \rangle = 2.7$ for class I and $\langle \kappa \rangle = 2.3$ for class II. The medians of κ are nearly equal for the two classes and thus, there is no clear observational evidence that the fitted parameter κ is related to some Coulomb collisional effects. How-

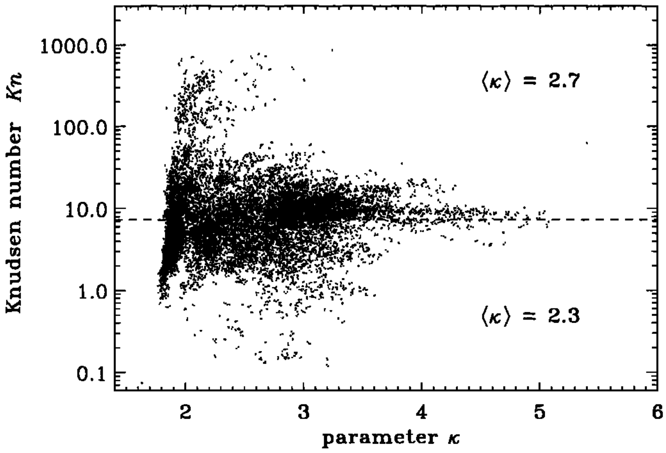

Figure 3. The Knudsen number Kn of each individual spectrum as a function of the fitted κ. The dashed line represents the median $\langle Kn \rangle$. There is no clear correlation between κ and Kn.

ever, a more comprehensive observational procedure is needed to provide better support to this statement. In addition, this statement should also be confirmed theoretically, with a model taking into account the effect of, even rare, collisions between particles in the solar wind.

Anyway, if the collisions played any role in the solar wind transport, it should be, in principle, by driving the electron VDFs closer to Maxwellians, i.e., by increasing κ between the corona and the interplanetary medium. In the present analysis, most electron VDFs are fitted with values of κ lower than five; this is in disagreement with a recent study by Ko *et al.* [1996] who find that the electron VDFs in the lower corona can be modelized by Kappa functions, but only with values of κ greater than five.

3. Conclusion

We have analyzed velocity distribution functions observed on board Ulysses, by fitting them with Kappa functions. Our main result is that the electron distributions contain important high velocity tails in the fast solar wind, while they are closer to a Maxwellian in the slow solar wind. If we now assume, first, that the particle velocity distributions in the corona can be modelized with a Kappa function, and second, that collisional effects do not basically modify the VDFs during their transport in the interplanetary medium, then our statistical analysis can support the kinetic model of the solar wind proposed recently by Maksimovic, Pierrard and Lemaire [1997].

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