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New model of magnetospheric current-voltage relationship

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Abstract. A new model based on the generalized Lorentzian velocity distribution function (VDF) is used to estimate the total current density J_{tot} along the magnetic field lines as a function of the field-aligned electric potential difference V between the ionosphere and the magnetosphere. As in the earlier kinetic models based on the Maxwellian VDF, J_{tot} is a nonlinear function of V, except for values of V between 0.1 and 10 kV, where the current-voltage relationship becomes linear. The effect of an enhanced population of suprathermal particles in the tail of the VDF is to increase the value of the applied potential for which the linear relationship is a valid approximation. The application of this current-voltage relationship to the auroral precipitation region and return current region is discussed.

Introduction

In the auroral regions, kilovolt magnetospheric particles are injected into the Earth's ionosphere, because of the field-aligned potential difference created by electrostatic interactions between the relatively cold ionospheric plasma and the hot plasma sheet particles [*Evans*, 1974; *Chiu et al.*, 1981]. *Knight* [1973] has been first to determine a current-voltage relationship between the field-aligned electric potential difference V and the field-aligned current density J_{tot}. The expressions used by Knight are similar to those published by *Lemaire and Scherer* [1970, 1971, 1973] in their kinetic models of the polar wind and of plasma sheet particle precipitation. According to these kinetic models, cold electrons and ions evaporate out of the topside ionosphere into the collisionless ion-exosphere, while hot plasma sheet electrons and ions spiral down the magnetic field lines and precipitate into the atmosphere. The partial currents J_i contributed by the escaping cold ionospheric electrons and ions and by the precipitated hot electrons and protons, are functions of V. However, in *Knight*'s [1973] model, the current carried by the ions was neglected. *Lemaire and Scherer* [1974] have shown that such an omission underestimates J_{tot} and leads to erroneous results for $V < 100\,\text{V}$.

The total field-aligned current density, $J_{\text{tot}} = \sum_i J_i$, is in general a nonlinear function of V [*Knight*, 1973; *Lemaire and Scherer*, 1974, 1983]. Nevertheless, there exists a range of potential ($100\,\text{V} < V < 10\,\text{kV}$) where the current-voltage relationship is linear. The extrapolation of this linear relationship is not a valid approximation in the return current region where $J_{\text{tot}} < 0$.

The kinetic models of *Knight* [1973] and *Lemaire and Scherer* [1974] rest on the assumption that the velocity distribution functions (VDF) are truncated Maxwellians. Most of the time, however, the VDF of space plasmas has a non-Maxwellian superthermal tail: this VDF, $f(\vec{v})$, decreases generally as a power law of the velocity v instead of exponentially [*Bame et al.*, 1967]. A useful function to model such plasmas VDFs is the generalized Lorentzian (or kappa) distribution [*Summers and Thorne*, 1991].

Kappa distributions have been used to analyze spacecraft data collected in the Earth's magnetospheric plasma sheet [*Vasyliunas*, 1968; *Lui and Krimigis*, 1981; *Christon et al.*, 1988] and in the solar wind [*Scudder*, 1992a, b]. Since many space plasmas VDFs can be better fitted by Kappa distributions than by Maxwellians or exponential functions, we developed a new kinetic model for the current-voltage relationship based on the kappa VDF.

First, we determine the expressions of the partial current density using the Kappa VDF and then compare them with those of the earlier model based on the Maxwellian VDF. We calculate the partial and total current densities in an auroral magnetic flux tube for typical temperature and number densities of the ionospheric and magnetospheric particles. We compare the results obtained for different values of the parameter κ, including values of κ deduced from observed auroral energy spectra. Finally, we discuss the current-voltage relationship in the limit of very low values of V which is relevant in the return current region.

Model Description

The current density parallel to magnetic field lines is defined by

$$J_{\|}(\vec{r}) = Ze \int v_{\|} f(\vec{r}, \vec{v})\, d^3\vec{v}\,, \tag{1}$$

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where $\vec{r}$ and $\vec{v}$ are, respectively, the position and the velocity of the particle, Ze is the electric charge of the particles, $v_{\|}$ is the velocity component parallel to the magnetic field direction, and $f(\vec{r},\vec{v})$ is the velocity distribution function, normalized so that $\int f(\vec{r},\vec{v})\, d^3\vec{v} = n(\vec{r})$ is the number density. The domain of the integration corresponds to the velocity space where $f(\vec{r},\vec{v})$ is not equal to zero.

In the models of *Lemaire and Scherer* [1970, 1971, 1973], the VDF is assumed to be a truncated Maxwellian at the reference level r_0,

$$f(\vec{r_0},\vec{v}) = N_0 \left(\frac{m}{2\pi kT_0}\right)^{3/2} \exp\left(-\frac{mv^2}{2kT_0}\right) \tag{2}$$

where m is the mass of the particle. For ionospheric particles, the VDF is given by (2) outside the downward loss cone and is equal to zero inside the downward loss cone (i.e. for $\pi - \theta_m < \alpha < \pi$, where α is the pitch angle of the particles and θ_m is the loss cone angle). For the plasma sheet particles, the VDF is given by (2) outside the upward loss cone and is equal to zero inside the upward loss cone (i.e., for $0 < \alpha < \theta_m$). This means that we neglect the contribution of the ionospheric particles from the conjugate auroral region and consider that the precipitated plasma sheet particles with mirror points below the baropause are not reflected but are all scattered and lost by inelastic collisions. N_0 and T_0 are constants determined by the number density and the temperature at the altitude of the reference level r_0. The Boltzmann constant is k.

Lemaire and Scherer [1971] obtained analytical expressions of the integral equation (1), $J_i = J_i(V)$, for two extreme cases. When the total potential energy $U(r) = Z_i eV(r) + m_i\phi_g(r)$ (where m_i is the particle mass and ϕ_g is the gravitational potential) is positive and is a uniformly decreasing function of altitude, the current density is found to be

$$J_i(r_0) = Z_i e N_{0i} \left(\frac{kT_{0i}}{2\pi m_i}\right)^{1/2}. \tag{3}$$

For ions and electrons whose total potential energy $U(r) = Z_i eV + m_i\phi_g$ is negative and is a uniformly increasing function of altitude, the current density is given by

$$\begin{aligned} J_i(r_0) &= Z_i e N_{0i} \left(\frac{kT_{0i}}{2\pi m_i}\right)^{1/2} \left(1 - \frac{U(r_0)}{kT_{0i}}\right) \\ &\quad \exp\left(\frac{U(r_0)}{kT_{0i}}\right). \end{aligned} \tag{4}$$

In these equations, the parameters N_{0i} are determined by the actual number densities $n_i(r_0)$ at the reference level r_0. For equation (3) (when $U(r) > 0$), the normalization factor is given by

$$N_{0i} = 2n_i(r_0). \tag{5}$$

For equation (4) (when $U(r) < 0$), the normalization factor is given by

$$N_{0i} = \frac{n_i(r_0)}{\frac{1}{2}(\psi_i + \zeta_i) + (\psi_i - \zeta_i)\, K_2\left(\sqrt{\frac{-U(r_0)}{kT_{0i}}}\right)}, \tag{6}$$

where $\psi_i = 0$ and $\zeta_i = 1$ for the incoming magnetospheric particles which are confined in the downward loss cone; $\psi_i = 1$ and $\zeta_i = 0$ for the ionospheric particles which escape into the magnetotail or into the opposite hemisphere, and $K_2(x) = (2/\pi^{1/2}) \int_0^x dt \exp(-t^2)t^2 = \mathrm{erf}(x)/2 - \pi^{-1/2}x\exp(-x^2)$.

In this model, the magnetic field at the high-altitude region B_M is assumed equal to zero. In a more general magnetic model like that considered by *Lemaire and Scherer* [1973], the altitude of the large-scale monotonic potential distribution is limited. The magnetic field intensity is a nonzero constant B_M at this high altitude and B_I at the ionosphere's reference level. In this case, the relation between the current density and the potential becomes

$$\begin{aligned} J_i(r_0) &= Z_i e N_{0i} \left(\frac{kT_{0i}}{2\pi m_i}\right)^{1/2} a^{-1} \exp\left(\frac{U(r_0)}{kT_{0i}}\right) \\ &\quad \cdot \left[1 + (a-1)\exp\left(\frac{a}{1-a}\frac{U(r_0)}{kT_{0i}}\right)\right], \end{aligned} \tag{7}$$

where $a = B_M/B_I$. The normalization factor is slightly different from (6). It is easy to show that (7) tends to (4) when $a \to 0$. For the magnetosphere-ionosphere system, the value of a turns out to be of the order of 0.02 [*Lu et al.*, 1991] or smaller (0.001 in articles of *Lemaire and Scherer* [1971, 1983]), and (7) gives results very similar to (4) for potential lower than 10 kV. For higher potential, the current given by (7) tends to an asymptotic value depending on a (cf. Figure 3 for $a = 0.02$).

We determine here the corresponding expressions in the case of a Kappa VDF defined by

$$f_\kappa(\vec{r_0},\vec{v}) = \frac{N_0}{2\pi(\kappa w^2)^{3/2}} A_\kappa \left(1 + \frac{v^2}{\kappa w^2}\right)^{-(\kappa+1)} \tag{8}$$

with $A_\kappa = \Gamma(\kappa+1)/[\Gamma(\kappa - 1/2)\Gamma(3/2)]$, where κ is the spectral index, w is the characteristic thermal speed of the distribution ($\overline{v} = w$), and $\Gamma(x)$ is the gamma function. At high velocities, the distribution obeys an inverse power law: $f_\kappa \sim (mv^2/2)^{-(\kappa+1)}$.

A comparison between the Maxwellian distribution and the generalized Lorentzian (kappa) is shown on Figure 1 for different values of the κ parameter. There are more suprathermal particles in the high-energy tail of the kappa VDF, but the difference becomes less significant as κ increases. When the spectral index $\kappa \to \infty$, the kappa distribution tends to a Maxwellian with a temperature T_0 related to w by $kT_0 = mw^2/2$.

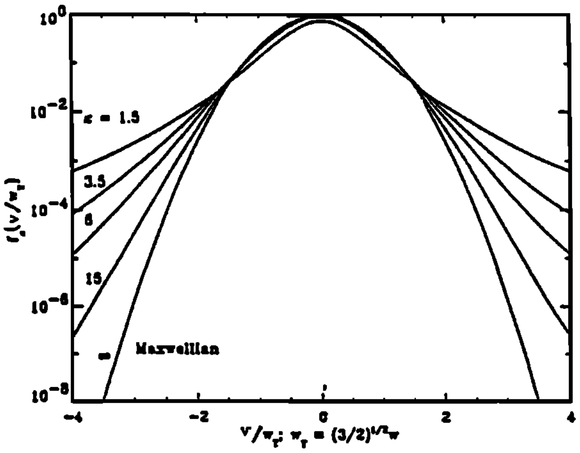

Figure 1. Comparison of generalized Lorentzian distributions for different spectral index κ with the corresponding Maxwellian distribution ($\kappa = \infty$) [*Scudder*, 1992a].

The current densities obtained for a kappa distribution function (integrating (1) with the same truncated pitch angle distributions) are

$$J_i(r_0) = \frac{1}{4} Z_i e N_{0i} \left(\frac{2kT_{0i}}{m_i}\right)^{1/2} \frac{A_\kappa \kappa^{-1/2}}{(\kappa - 1)}, \qquad (9)$$

when the total potential energy $U(r) = Z_i e V(r) + m_i \phi_g(r)$ is positive and uniformly decreasing with altitude, and

$$J_i(r_0) = \frac{1}{4} Z_i e N_{0i} \left(\frac{2kT_{0i}}{m_i}\right)^{1/2} \frac{A_\kappa \kappa^{-1/2}}{(\kappa - 1)} \left(1 - \frac{U(r_0)}{kT_{0i}}\right)\left(1 - \frac{U(r_0)}{\kappa(kT_{0i})}\right)^{-\kappa}, \qquad (10)$$

when $U(r)$ is negative and uniformly increasing with altitude. The current density given in (9) is independent on the parallel potential drop V while, according to (10), J_i is an increasing function of V.

Since $\lim_{\kappa\to\infty} (1 + x/\kappa)^{-\kappa} = \exp(-x)$ and $\lim_{\kappa\to\infty} \kappa^{b-a}[\Gamma(\kappa + a)/\Gamma(\kappa + b)] = 1$ [*Abramowitz and Stegun*, 1968], one can verify that (9) and (10) tend, respectively, to (23) and (21) given by *Lemaire and Scherer* [1970] when $\kappa \to \infty$. The actual number density at the reference level $n_i(r_0) = \int f_\kappa(\vec{r_0}, \vec{v})\, d^3\vec{v}$ is related to N_{0i} by

$$N_{0i} = 2 n_i(r_0) \qquad (11)$$

in the former case, when $U(r) > 0$, and

$$N_{0i} = \frac{n_i(r_0)}{\psi_i + \frac{(\zeta_i - \psi_i)}{2}\beta\left(\left(1 - \frac{U(r_0)}{\kappa(kT_{0i})}\right)^{-1}\right)} \qquad (12)$$

in the latter case, when $U(r) < 0$. The classical incomplete beta function given in most standard mathematical libraries is $\beta(x) = \int_0^x A_\kappa t^{\kappa-3/2}(1-t)^{1/2}\, dt$, $(0 \leq x \leq 1)$.

Comparison of the Theoretical Results

Consider an auroral magnetic flux tube extending from the ionosphere up into the plasma sheet and containing both cold ionospheric plasma and hot magnetospheric electrons and protons. A quasi-stationary field-aligned electrostatic potential difference can develop between the low-altitude reference level, taken as 1000 km, where the magnetic field is B_I, and the high altitude equatorial region where the magnetic field intensity B_M is considered to be equal to zero. The temperature T_{0i} and number density $n_i(r_0)$ of the different particles species at the reference altitude 1000 km are given in Table 1.

Figure 2a shows the partial and total field-aligned current densities J_{he^-}, J_{p^+}, J_{ce^-}, J_{H^+}, and J_{tot} as a function of the applied potential difference V for a Maxwellian VDF. The results of Figure 2a are analogous to those of Figure 1 in the work of *Lemaire and Scherer* [1983] except that the latter give the current densities in a dayside cusp magnetic flux tube containing magnetosheathlike particles instead of in an auroral magnetic tube containing plasma sheet particles.

Figure 2b shows the same partial and total current densities as in Figure 2a but calculated for a kappa VDF with $\kappa = 5.5$ instead of a Maxwellian VDF; the same boundary conditions were used at the exobase altitude (1000 km) (see Table 1). From the comparison of Figures 2a and 2b, one sees that by enhancing the suprathermal tail population of particles (i.e., by decreasing the value of κ) all partial currents densities are also enhanced.

For $V < 10$ kV, the hot electron current density J_{he^-}, given by (10), can be approximated by a linear function of V:

$$J_{he^-} = \frac{e^2 N_{0he^-}}{(2\pi m_e k T_{0he^-})^{1/2}}\, V = K^{\mathrm{Maxw}}\, V. \qquad (13)$$

The approximate linearity of the current-voltage characteristic curves of the current density carried by the plasma sheet electrons has already been demonstrated earlier for a Maxwellian VDF [*Lundin and Sandahl*, 1978; *Fridman and Lemaire*, 1980; *Lyons*, 1981]. K^{Maxw} is a field-aligned conductance whose value can be determined from measurements of auroral electron spectra [*Lyons et al.*, 1979; *Weimer et al.*, 1985].

Table 1. Average Temperature (T_{0i}) and Number Densities (n_i) of Cold Ionospheric Electrons and Ions and of Warm Magnetospheric Electrons and Protons at 1000 km Altitude

i	ce^-	O^{+*}	H^+	he^-	p^+
T_{0i}, K	4500	1500	4000	10^7	5×10^7
n_i, cm^{-3}	2200	2000	200	0.1	0.1

*The mobility of the ions O^+ is considered too low to contribute significantly to the total current density.

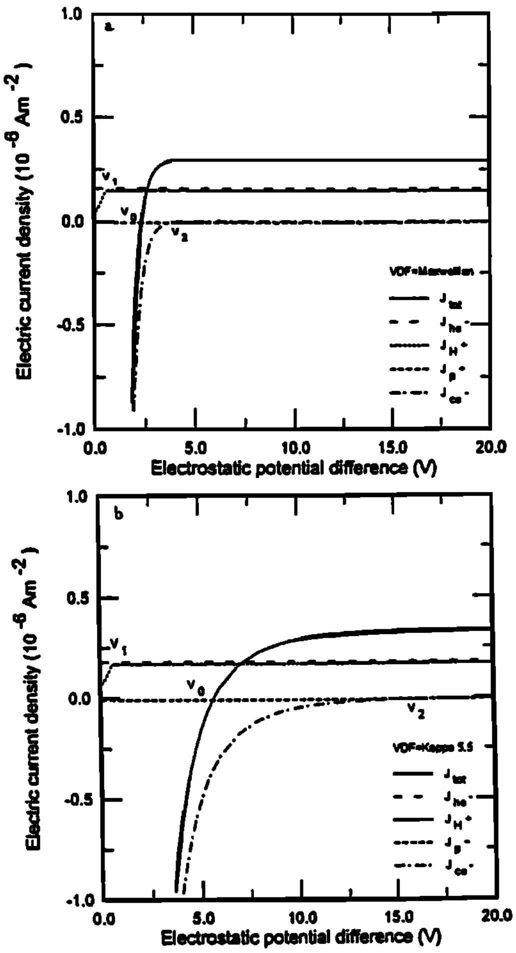

Figure 2. With (a) a Maxwellian VDF model and (b) a kappa VDF model with $\kappa = 5.5$ for each distribution, partial and total field-aligned currents in an auroral magnetic flux tube as a function of the applied field-aligned potential difference V between the ionosphere and the magnetosphere. Plasma densities and temperatures at 1000 km are summarized in Table 1.

We confirm here that this ohmiclike behavior remains true when kappa VDFs are used instead of Maxwellians. However, in this case, the conductance $K = J_{\mathrm{he}^-}/V$ is larger than K^{Maxw}:

$$K^{\mathrm{Kappa}} = \frac{\Gamma(\kappa+1)}{\Gamma(\kappa-1/2)\kappa^{1/2}(\kappa-1)} K^{\mathrm{Maxw}}. \tag{14}$$

It can be verified that $K^{\mathrm{Kappa}} \to K^{\mathrm{Maxw}}$ when $\kappa \to \infty$.

The flux of the plasma sheet protons, injected downward, is negligibly small in both models since their thermal velocity is much smaller than that of the electrons, unless they have a field-aligned bulk velocity.

The contribution of the ionospheric hydrogen current density J_{H^+} is constant (equation (3)) except for $V < V_1 = 0.65\,\mathrm{V}$. V_1 is independent on κ. The importance of the ionospheric ions in the total current depends on their density $n_{\mathrm{H}^+}(r_0)$ relative to the hot magnetospheric plasma density $n_{\mathrm{he}^-}(r_0)$ at the reference altitude.

Table 2. Variations of V_0, the Potential for Which $J_{\mathrm{tot}} = 0$, as a Function of κ

Values of κ	V_0, V
Maxwellian ($\kappa = \infty$)	2.32
$\kappa = 100$	2.42
$\kappa = 10$	3.63
$\kappa = 5.5$	5.66
$\kappa = 3$	17.65
$\kappa = 2$	159.04

The partial current density J_{ce^-} carried by the upward flowing cold electrons accounts for the nonlinearity of J_{tot} as a function of V. There always exists a positive value V_0 of the electrostatic potential for which $J_{\mathrm{tot}} = 0$. When $V < V_0$, the downward current carried by the escaping ionospheric electrons J_{ce^-} becomes dominant. A field-aligned potential V_0 of only a few volts can inhibit these cold ionospheric electrons from escaping to infinity. Table 2 shows how V_0 increases when κ decreases.

Current-Voltage Characteristics for Different Values of κ

Figure 3 shows J_{tot} as a function of V for different values of κ. Logarithmic scales are used for J_{tot} and for V ranging between 1 and 10^5 V. One can see that the

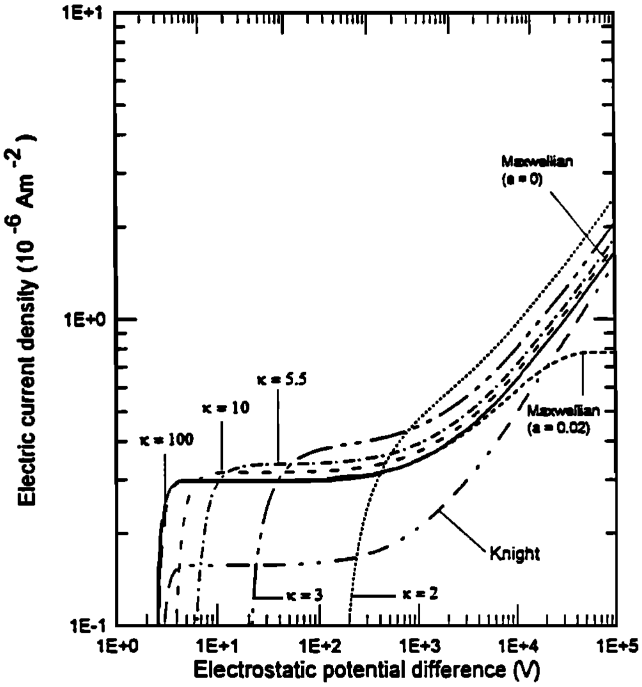

Figure 3. Influence of the κ parameter on the total current density (J_{tot}) in an auroral magnetic flux tube with characteristics summarized in Table 1. Note the nonlinear trend except in a limited range of the potential difference V. Note also the similarity between the results obtained with a Maxwellian VDF (earlier models) and those obtained with a Lorentzian VDF when the parameter κ is large. The curve called Knight takes into account the magnetospheric and ionospheric electrons but not the contributions of the ion currents.

curve marked Knight, neglecting the ions contributions (J_{p^+} and J_{H^+}), underevaluates J_{tot}. When $\kappa = 100$, the current-voltage curve is almost identical to that obtained in the case of a Maxwellian VDF.

In the range from 0.04 to 10 kV, many authors [*Yeh and Hill*, 1981; *Burch et al.*, 1983; *Marklund and Blomberg*, 1991; *Reiff et al.*, 1988] verified that $J_{he^-} \sim J_{tot}$ is almost a linear function of V. This linearity is still applicable for kappa functions, but the range of V for which the near linearity of the current-voltage applies shrinks when κ decreases from $\kappa = \infty$ to $\kappa = 2$.

The linear relationship fails to be applicable in the range below 100 V, that is, in the plasmasphere, plasma through, polar wind, and return current regions. Indeed, below the critical value V_0, J_{tot} drops rapidly and becomes negative. For a Maxwellian ($\kappa = \infty$), the negative return current can only subsist up to $V_0 = 2.32$ V. However, for $\kappa = 3$, it can subsist up to $V_0 = 17.6$ V and even up to $V_0 = 159$ V for $\kappa = 2$ (see Table 2). Negative values of J_{tot} are expected in the return current regions on both sides of the auroral region where the precipitated plasma sheet electrons produce a positive (upward) field-aligned current.

When κ decreases, $|\ J_{ce^-}\ |$ and $|\ J_{tot}\ |$ increase. Indeed, when κ is smaller, there are relatively more superthermal electrons in the tail of the kappa VDF, and the number of electrons which are able to escape is then larger.

Determination of Kappa Index From Observations

Christon et al. [1988] and *Williams et al.* [1988] have fitted measured plasma sheet ion and electron differential energy spectra with kappa functions. For the majority of cases studied, they found that a kappa VDF with values of κ between 4 and 7 provides a better overall description of the energy spectra than the Maxwellian VDF. The value of κ stays roughly constant, even when the temperature varies with time or with position in the plasma sheet transition regions. However, the value of κ differs from one species to the other.

In our model, we assumed for simplicity that all particle populations have the same kappa index. To be able to relax this assumption, it would be necessary to have reliable observations of the energy spectra for all ions species and electron populations considered in this study. The required observations should range from well below the peak of the spectrum to far up into the tail of the VDF (cf. Figure 1). This implies that detailed and high-resolution energy spectra are measured simultaneously, at the same location in space, and in the appropriate ranges of energies for each electron and ion population.

From such ideal spectral measurements, one would then be able to determine by standard parameter-fitting procedures the estimates of the temperature (T_{0i}), density (n_{0i}), and kappa index (κ) for each population of particles ($H^+, ce^-, he^-, p^+, \ldots,$). Usually, only the first two parameters are adjusted under the assumption that the VDF is a Maxwellian. However, there is now more experimental evidence that a Maxwellian VDF is not the best zero-order approximation in most collisionless space plasmas and that kappa functions should be preferred in future data analysis.

An interesting alternative model would be obtained by taking Maxwellians for the ionospheric distributions of particles and kappas for the magnetospheric distributions with a different index κ determined by satellite's observations. In this case, only hot electron and proton current densities would be enhanced. However, the value of V_0 would hardly change. The large increase of the crossover potential V_0 on Figure 3 is due to the cold electron kappa VDF.

Return Current Intensity

Figure 4 shows the negative current densities obtained for small values of V and for different values of κ. J_{tot} is very sensitive to small changes of V. When V increases from zero to more positive values, the total potential barrier $(-m_e\phi_g + eV)$ that the ionospheric electrons must overcome in order to escape and contribute to the net field-aligned current increases. In other words, the escape speed of an electron is increasing from 11 km/s to much larger values given by $(-2\phi_g + 2eV/m_e)^{1/2}$.

The effect of a higher magnetospheric plasma density is to increase the hot electron current density. However, the field-aligned current density remains negative without a reversal sign of the field-aligned potential difference. To compensate for the cold electron downward current density (which is around $-100\,\mu$A/m^2 for

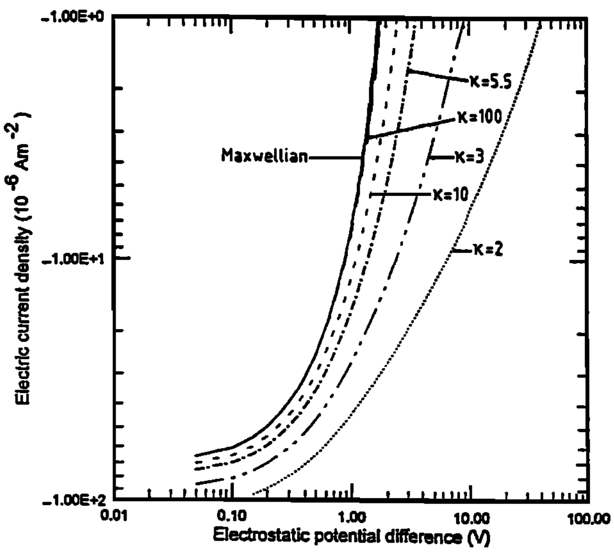

Figure 4. Negative current densities obtained for small but positive values of V and for different values of κ.

$V = 0$), it would be necessary to have very large density of magnetospheric particles lacking in consistency with the auroral particle data.

It should be emphasized that according the different kinetic models, negative currents are obtained without reversing the sign of V. In some applications and models of ionosphere-magnetosphere coupling [*Blomberg and Marklund*, 1991], the linear relationship (equation (13)) is extrapolated outside its range of validity. For instance, the same value of the conductance K is used in the return current region where $J_{\text{tot}} < 0$ as in the auroral precipitation region where $J_{\text{tot}} > 0$. This abusive extrapolation of the relationship $J_{\text{tot}} = KV$ would imply that $V < 0$ in the return current region. However, large-scale reversed field-aligned potential difference would lead to unreasonably large escape fluxes of the cold ionospheric electrons (cf. Figure 4). When $V = 0$, almost all cold electrons would blow out of the topside ionosphere because their thermal speed ($= 3500\,\text{km/s}$ for $T_0 = 3000\,\text{K}$) is much larger than $11\,\text{km/s}$ (i.e., the gravitational escape velocity for neutral particles). This would produce a downward field-aligned current density of more than $100\,\mu\text{A/m}^2$ and unreasonably large transverse magnetic field perturbations.

A large-scale reversed potential difference would also lead to downward parallel electric field $E_{\parallel}$ which would be opposite to the ambipolar electric field required to maintain the quasi-neutrality of the ionospheric and magnetospheric plasma. A downward directed parallel electric field, $E_{\parallel} = -[dp_e/dr]/en_e$ (where e is the electron charge and n_e is the electron density), would necessarily imply an electron pressure p_e increasing with altitude. Since this is physically unrealistic, it can be concluded that negative field-aligned potential difference between the ionosphere and magnetosphere must be excluded in the return current region as well as elsewhere. In other words, to drive a negative current, it suffices to reduce the positive value of the field-aligned potential difference below the value of V_0 given in Table 2.

Note that we only consider here a large-scale potential distribution between the ionosphere and the magnetosphere, though localized structures with small-scale size may well be imbedded in the larger-scale regions. Small-scale downward electric fields have been observed at high altitudes [*Burch et al.*, 1983]. Since they are confined well above the ionosphere, they do not lead to large runaway outflows of ionospheric electrons.

Conclusions

Field-aligned currents (FAC) can be driven upward or downward along magnetospheric field lines by a quasi-stationary (dc) potential created by the temperature difference between the cold ionospheric and the warmer magnetospheric plasma. In earlier kinetic models [*Lemaire and Scherer*, 1970, 1971, 1973; *Knight*, 1973], truncated Maxwellian velocity distributions (VDF) have been assumed to determine the total FAC in an auroral magnetic flux tube. However, there is more and more evidence that the VDF of collisionless space plasma is closer to a kappa (or Lorentzian) function than to a Maxwellian [*Christon et al.*, 1988]. The spectra of the superthermal particles decrease as power law of the energy E with a slope which can be related to the value of the kappa index.

Since the particles with largest velocity contribute most to the escape or precipitated flux, we determined the FAC for a kappa VDF. The comparison between the results obtained with Maxwellian and kappa VDFs shows the following:

1. In both cases, one obtains nonlinear current-voltage characteristic curves.
2. When the index κ increases, the total FAC intensities decrease (increase) at large (small) values of V.
3. To the limit $\kappa \rightarrow \infty$, the current-voltage relationship for Maxwellian VDF is recovered.
4. For values of V between 0.1 and 10 kV, corresponding to field-aligned potentials observed in auroral arcs or strong plasma sheet precipitation, the total current is positive (upward) and can be approximated by a linear current-voltage relationship.
5. The slope of the current-voltage linear relationship corresponds to a conductance K, whose value depends on the index κ of the kappa VDF. When κ increases indefinitely, the value of K^{Kappa} decreases toward the Maxwellian values K^{Maxw}.
6. The current carried upward by the ionospheric hydrogen ions $J_{\text{H+}}$ is constant but not always negligible, although it is ignored in some studies of ionosphere-magnetosphere coupling [*Knight*, 1973; *Fridman and Lemaire*, 1980].
7. For $V > 10\,\text{kV}$, the current density mainly transported by the plasma sheet electrons increases exponentially with V for the Maxwellian VDF as well as for the kappa VDF, when the magnetic field intensity in the magnetosphere B_M is assumed to be equal to zero. In a more general magnetic model where B_M is a nonzero constant ($a \neq 0$), the current is limited.
8. For $V < 100\,\text{V}$, the current-voltage curves cease to be well approximated by a linear relationship like (13). The total current drops rapidly due to the negative current transported downward by the escaping cold electrons of the ionosphere.
9. The value of the thermoelectric charge separation potential V_0 for which $J_{\text{tot}} = 0$, increases when κ decreases, in models where the kappa index is identical for each particle distribution.
10. To obtain a negative total FAC density in a return current region, it is not necessary to reverse the sign of V. Negative (downward) current can be arbitrarily large when $V < V_0$ (see Figure 1). For $V = 0$, all the ionospheric electrons would be blown out of the

gravitational potential due to their large thermal velocity.

11. To determine appropriate values of κ for each particle population contributing to the total FAC, high-resolution energy spectra must be measured over an energy range from below the peak flux to far up into the tail of the VDF. This should serve as a general recommendation for designing particle spectrometers to be flown in future missions in Earth's magnetosphere.

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