Electromagnetic instabilities in solar wind interaction with dusty cometary plasmas

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Abstract. Dusty plasmas contain charged dust grains which are much more massive than protons and carry high negative charges due to preferential capture of electrons. Fluctuations in the grain charges due to capture or liberation of additional electrons and protons translate as momentum losses or gains, which can lead to wave damping or growth. Many authors have addressed the pickup of ions of cometary origin by the solar wind, partly due to relative streaming between cometary and solar wind ions which excites low-frequency electromagnetic turbulence. In the present work we include effects due to charged dust in cometary environments. We have investigated several frequency regimes: nonresonant below the cometary charged dust gyrofrequency (new and interesting but highly unlikely!), nonresonant below the cometary watergroup gyrofrequency and resonant with the cometary watergroup ions. For most parameter ranges either existing instabilities can be enhanced, showing that charged dust facilitates the cometary ion pickup by the solar wind, or new instabilities or damping mechanisms have been shown to exist.

1. Introduction

Dusty plasmas are observed in many space environments such as nebulas, comets, planetary rings and magnetospheres, etc. The massive dust grains in these plasmas are usually highly charged, due to various processes as plasma currents, photoemission, secondary emission or field emission, and may significantly influence various physical processes [*Goertz*, 1989; *Mendis and Horanyi*, 1991; *Northrop*, 1992; *Mendis and Rosenberg*, 1994]. In such a dusty plasma the charges are not fixed as for electrons or ordinary ions, hence mass and momentum exchanges between electrons, protons and dust particles occur. The sticking or loosing of electrons or protons on and off dust grains changes the charge-to-mass ratio of dust particles, due to fluctuations in the plasma currents that flow to the dust grains. Charged massive dust grains affect electromagnetic modes also by substantially lowering the Alfvén speed [*Shukla*, 1992]. The linear and nonlinear evolution of various low-frequency electromagnetic modes in dusty plasmas have been discussed recently [*Shukla*, 1992; *Rao*, 1993], but with constant dust charges.

Low-frequency, long-wavelength, nonresonant electromagnetic fluctuations in multibeam plasmas were studied to explain the Alfvén wave turbulence in cometary environments upstream of the bow shock [*Lakhina and Verheest*, 1988; *Galeev et al.*, 1991; *Verheest and Lakhina*, 1991]. It is the aim of our paper to study such low-frequency and long-wavelength fluctuations in a multibeam dusty plasma by taking account of the momentum loss of electrons, protons and dust grains. Sink/source terms have been added in the momentum equations to make the analysis selfconsistent, elaborating on earlier ideas [*Verheest and Meuris*, 1995], and taking the dust dynamics into account [*Reddy et al.*, 1995]. For numerical estimates of dust grain sizes and charges we refer to the latter paper.

2. Methodology

We consider a homogeneous multibeam dusty plasma immersed in a uniform magnetic field $\mathbf{B}_0 = B_0\mathbf{e}_z$ and consisting of solar wind electrons and protons, and of electrons, protons and dust particles of cometary origin. For simplicity, we restrict ourselves to wave propagation parallel to $\mathbf{B}_0$ and to only one species of dust. A true distribution of dust grains in charge and mass cannot be dealt with at present, since all existing charging models only treat interactions between dust grains and electrons and protons, not between grains. Electromagnetic waves may be described by the following basic equations, taking into account charge fluctuations and the associated momentum loss of solar wind and cometary particles,

$$\left(\frac{\partial}{\partial t} + \mathbf{u}_\alpha \cdot \nabla\right)\mathbf{u}_\alpha = \frac{q_\alpha}{m_\alpha}(\mathbf{E} + \mathbf{u}_\alpha \times \mathbf{B}) - \sum_\beta \gamma_{\alpha\beta}(\widehat{\mathbf{u}}_\alpha - \widehat{\mathbf{u}}_\beta), \tag{1}$$

$$\nabla \times \mathbf{E} + \frac{\partial}{\partial t}\mathbf{B} = 0, \tag{2}$$

$$c^2\nabla \times \mathbf{B} = \frac{\partial}{\partial t}\mathbf{E} + \frac{1}{\varepsilon_0}\sum_\alpha n_\alpha q_\alpha \mathbf{u}_\alpha. \tag{3}$$

In these equations n_α, m_α, q_α and $\mathbf{u}_\alpha$ respectively refer to the number density, mass, charge and fluid velocity per species, labelled with subscript α, and $\mathbf{E}$ and $\mathbf{B}$ to the electric and magnetic fields.

The form of the momentum exchanges have been inspired by previous work [*Verheest and Meuris*, 1995; *Reddy et al.*, 1995] and by what is usually written for ordinary collisional exchanges [*Booker*, 1984; *Pilipp et al.*, 1987]. For steady state equilibrium solutions the source/sink terms vanish because of constant grain charges, hence the hat over the variables on the r.h.s. of (1) to restrict them to deviations from the equilibrium values.

With the indices s, c and d we will refer to solar wind particles (electrons and protons), to cometary plasma particles (electrons and protons or cometary water group ions assumed to be singly charged) and average cometary dust grains. A second subscript e or i then refers to the corresponding electrons and ions. In the momentum equations for the different electrons and ions the sum over β is restricted to the interfacing with the dust, whereas for the dust all terms could be present. That means that only $\gamma_{\alpha d}$ and $\gamma_{d\beta}$ are nonzero, coefficients which are akin to collision frequencies, although they reflect the capture or liberation of plasma particles by the dust grains.

Furthermore, we will work in the cometary reference frame, in which all cometary material is essentially at rest in equilibrium, neglecting the small parallel outflow velocities of such particles ($\sim$ 1 km/s). Hence for their parallel equilibrium velocities we put $U_{ce} = U_{ci} = U_d = 0$. On the other hand, the beam velocity will be the projection of the solar wind velocity ($\sim$ 400 km/s) on the direction of the solar wind magnetic field, so that $U_{se} = U_{si} = U$.

We will split all variables according to $f = F + \delta f$, where capital letters represent the equilibrium quantities and δ the perturbations, assumed to vary as $\exp i(kz - \omega t)$. Linearizing the basic equations (1)–(3) yields after elimination a dispersion law. Intermediate steps in this derivation are analogous to those of our previous papers [*Verheest and Meuris*, 1995; *Reddy et al.*, 1995].

3. Nonresonant modes below the dust gyrofrequency

We first look at circularly polarized modes in the lowest possible frequency regime, which in the long-wavelength limit is defined such that for all species

$$\left|\frac{\gamma_{\alpha\beta}}{\Omega_\alpha}\right| \sim \left|\frac{\omega - kU_\alpha}{\Omega_\alpha}\right| \ll 1, \tag{4}$$

with $\Omega_\alpha = Q_\alpha B_0/m_\alpha$ being the different signed gyrofrequencies. We obtain the following linear dispersion relation [*Reddy et al.*, 1995]

$$\left(\omega - k\overline{U}\right)^2 - k^2(V_A^2 - W) + ik\Gamma = 0. \tag{5}$$

V_A is the global Alfvén velocity defined through

$$V_A^2 = \frac{B_0^2}{\mu_0(\rho_{se} + \rho_{si} + \rho_{ce} + \rho_{ci} + \rho_d)}, \tag{6}$$

where the different ρ refer to the equilibrium mass densities, $\overline{U}$ is the bulk mass velocity of the whole plasma, including the charged dust,

$$\overline{U} = \frac{\rho_{se} + \rho_{si}}{\rho_{se} + \rho_{si} + \rho_{ce} + \rho_{ci} + \rho_d}\, U, \tag{7}$$

and combined charging and streaming effects occur in

$$\Gamma = \frac{\rho_d(\gamma_{dse} + \gamma_{dsi}) - \rho_{se}\gamma_{sed} - \rho_{si}\gamma_{sid}}{\rho_{se} + \rho_{si} + \rho_{ce} + \rho_{ci} + \rho_d}\, U. \tag{8}$$

Since the dust grains typically are much more massive than ordinary protons, V_A could be much smaller than usual. In addition, W refers to the mass averaged relative kinetic energy in the parallel flows, given by

$$W = \frac{(\rho_{se} + \rho_{si})(\rho_{ce} + \rho_{ci} + \rho_d)}{(\rho_{se} + \rho_{si} + \rho_{ce} + \rho_{ci} + \rho_d)^2}\, U^2. \tag{9}$$

If the charging effects conserve momentum between any two species, as true collisional effects usually do, then

$$\rho_d\gamma_{dse} = \rho_{se}\gamma_{sed}, \qquad \rho_d\gamma_{dsi} = \rho_{si}\gamma_{sid} \tag{10}$$

and Γ vanishes exactly. For $W > V_A^2$, (5) yields as usual unstable waves

$$\omega = k\overline{U} + ik\sqrt{W - V_A^2}. \tag{11}$$

For $Zm_{ce}/m_d = Zm_{se}/m_d \ll m_{ce}/m_{ci} \leq m_{se}/m_{si} \ll 1$, equations (6), (7) and (9) are simplified to

$$\overline{U} = \frac{U}{1+\sigma}, \quad W = \frac{\sigma U^2}{(1+\sigma)^2}, \quad V_A^2 = \frac{V_{A,sw}^2}{1+\sigma}, \tag{12}$$

where $V_{A,sw}$ is the solar wind Alfvén velocity and $\sigma = (\rho_{ce} + \rho_{ci} + \rho_d)/(\rho_{se} + \rho_{si}) \simeq (\rho_{ci} + \rho_d)/\rho_{si}$ is the ratio of the mass densities of the cometary and the solar wind material.

Coming back to the conditions of validity (4) of our dispersion law, with Ω_d by far the smallest of all gyrofrequencies, we need for the moduli of the complex wave frequencies that

$$|\omega|^2 = |\mathrm{Re}\,\omega|^2 + |\mathrm{Im}\,\omega|^2 \ll \Omega_d^2. \tag{13}$$

This gives the allowable range of wave numbers as

$$k \ll \frac{|\Omega_d|}{V_{A,sw}}\sqrt{\frac{1+\sigma}{M^2-1}}, \tag{14}$$

with the Alfvén Mach number $M = |U|/V_{A,sw}$. The instability criterion reduces to

$$\frac{1}{M^2-1} < \sigma \simeq \frac{\rho_{ci}}{\rho_{si}}\left\{1 + \left(1 - \frac{N_{ce}}{N_{ci}}\right)\frac{m_d}{Z_d m_{ci}}\right\}. \tag{15}$$

In the absence of dust the cometary material is globally neutral, with $N_{ce} = N_{ci}$, and the inclusion of dust enhances the density ratio to obtain instability. At given Mach number the instability will be easier to excite.

The presence of the dust is an additional stimulus, as it augments the mass density of the slower species.

4. Nonresonant modes below the watergroup ion gyrofrequency

At frequencies below the watergroup ion gyrofrequency but well above that associated with the dust, the massive dust grains are treated as an immobile background, and we are deriving whistler-like modes [*Verheest and Meuris*, 1995]. Even though the dust is treated as stationary, we retain the explicit possibility of variable charges.

For left- or right-circularly polarized, parallel electromagnetic modes the dispersion law is then [*Verheest and Meuris*, 1995],

$$\omega^2 = c^2k^2 + \sum_{\alpha\neq d} \frac{\omega_{p\alpha}^2(\omega - kU_\alpha)}{\omega - kU_\alpha + i\gamma_{\alpha d} \pm \Omega_\alpha}. \quad (16)$$

Here $\omega_{p\alpha}$ are the plasma frequencies, defined through $\omega_{p\alpha}^2 = N_\alpha e^2/\varepsilon_0 m_\alpha$. At low frequencies and long wavelengths, under the same conditions (4) except for the dust, we obtain

$$\left(\omega - k\overline{U}\right)^2 + \omega\left(i\overline{\gamma} \mp \frac{N_d Z_d e B_0}{\rho_{se} + \rho_{si} + \rho_{ce} + \rho_{ci}}\right) - k^2(V_A^2 - W) + ik\Gamma = 0. \quad (17)$$

Now V_A, $\overline{U}$ and W refer to the plasma species only, excluding the charged dust considered as an immobile background. In addition, $\overline{\gamma}$ is an average momentum loss or gain coefficient given by

$$\overline{\gamma} = \sum_{\alpha\neq d} \rho_\alpha \gamma_{\alpha d} \bigg/ (\rho_{se} + \rho_{si} + \rho_{ce} + \rho_{ci}), \quad (18)$$

and Γ has to be redefined as

$$\Gamma = -\frac{\rho_{se}\gamma_{sed} + \rho_{si}\gamma_{sid}}{\rho_{se} + \rho_{si} + \rho_{ce} + \rho_{ci}} U. \quad (19)$$

If there were no dust ($N_d = 0$), then the charging effects in $\overline{\gamma}$ and Γ would also disappear, and one would be led to nonresonant Alfvén modes below the cometary water group ion gyrofrequency [see e.g. *Lakhina and Verheest*, 1988; *Galeev et al.*, 1991]. However, even when there is dust, the charging is likely to be slow, so that the main influence of the dust is the presence of an additional term in (17), characterized by

$$A = \frac{N_d Z_d e B_0}{\rho_{se} + \rho_{si} + \rho_{ce} + \rho_{ci}} \simeq \frac{\rho_{ci}\Omega_{ci}}{\rho_{si} + \rho_{ci}}\left(1 - \frac{N_{ce}}{N_{ci}}\right), \quad (20)$$

which could become rather important if enough electrons are depleted onto the dust grains. With the foregoing in mind we will solve (17) under the assumptions that the charging and related momentum loss effects are small, and that the dominant part of the Alfvén modes is given by the streaming between the cometary and the solar wind plasma plus the mere presence of the dust. That would lead to ω_0 obeying

$$\left(\omega_0 - k\overline{U}\right)^2 \mp \omega_0 A - k^2(V_A^2 - W) = 0. \quad (21)$$

If this yields complex solutions, then unstable modes occur even in the absence of dust charging or decharging. However, if from (21) we would find real solutions and stable modes, then there will be a small damping or growth due to the charging/decharging effects:

$$\mathrm{Im}\,\omega = -\frac{\omega_0\overline{\gamma} + k\Gamma}{2(\omega_0 - k\overline{U}) \mp A}. \quad (22)$$

So even at what would correspond to stability in the absence of variable dust charge effects, we find that damping or growth becomes possible.

5. Resonant watergroup ion modes

We now approximate the dispersion law (16) under the conditions (4), except that the dust terms do not play a role and that the low-frequency conditions do not hold for the watergroup of cometary ions. Under these approximations we get instead of (17) that

$$\left(\omega - k\overline{U}\right)^2 + \omega(i\overline{\gamma} \mp A) - k^2(V_A^2 - W) + ik\Gamma - \frac{\omega(\omega + i\gamma_{cid})^2\rho_{ci}}{(\omega + i\gamma_{cid} \pm \Omega_{ci})(\rho_{si} + \rho_{ci})} = 0. \quad (23)$$

We will assume that $|\gamma_{cid}| \ll |\omega|$. The resonance condition means $|\omega| \sim \Omega_{ci}$, and it is the left-hand polarized mode in the comet frame which can go unstable. Upon transforming to a solar wind frame one finds as usual that the mode polarization changes and that right-hand polarization is obtained. If we restrict ourselves to the dominant terms when ω is close to Ω_{ci}, we can approximate the main solution of (23) as

$$\omega = \Omega_{ci} - i\gamma_{cid} + \frac{\Omega_{ci}^3\rho_{ci}}{\left[(\Omega_{ci} - k\overline{U})^2 + \Omega_{ci}A - k^2(V_A^2 - W)\right](\rho_{si} + \rho_{ci})}. \quad (24)$$

The instability or growth clearly is due to the dust charging or decharging process. Without dust and in the absence of ring current effects the modes would be stable. Equation (24) is valid except when k obeys

$$\left(\Omega_{ci} - k\overline{U}\right)^2 + \Omega_{ci}A - k^2(V_A^2 - W) = 0, \quad (25)$$

following a reasoning close to what Lee and Ip [1987] gave for the pickup of interstellar material by the solar wind. In that case we find from the dominant terms that

$$(\omega - \Omega_{ci})^2\left[2\Omega_{ci} - 2k\overline{U} + A\right] + i(\omega - \Omega_{ci})\left[(2\Omega_{ci} - 2k\overline{U} + A)\gamma_{cid} + \Omega_{ci}\overline{\gamma} + k\Gamma\right] - (\Omega_{ci}\overline{\gamma} + k\Gamma)\gamma_{cid} - \frac{\Omega_{ci}^3\rho_{ci}}{\rho_{si} + \rho_{ci}} = 0. \quad (26)$$

This yields purely damped or unstable modes, or else modes with a real frequency shift away from Ω_{ci}. In the total absence of dust we could find unstable waves from

$$(\omega - \Omega_{ci})^2(\Omega_{ci} - k\overline{U}) = \frac{\Omega_{ci}^3 \rho_{ci}}{2(\rho_{si} + \rho_{ci})}, \qquad (27)$$

which would only be possible if also $\Omega_{ci} < k\overline{U}$. Outside that one needs perpendicular ring current effects to render the modes unstable [*Lakhina and Verheest*, 1988].

6. Conclusions

Many authors have addressed the pickup of ions of cometary origin by the solar wind, which for the parallel part is due to relative streaming between cometary and solar wind ions which excites low-frequency electromagnetic turbulence. In the present work we have looked again at those instabilities by including effects due to the presence of charged dust in cometary environments, and including fluctuations in the dust charges and related effects. Without the fluctuations the charged dust would be tantamount to an additional heavy ion component, the treatment of which has been given (although not for the extreme charge-to-mass ratios envisaged here) by many authors [see *e.g. Brinca and Tsurutani*, 1988].

We have investigated several frequency regimes. For nonresonant modes with frequencies below the cometary charged dust gyrofrequency we found that the presence of the dust would enhance the possibility of the instability being excited. The main mechanism is the modifications of V_A and W by the inclusion of heavy dust material. Dust grain fluctuations as such have no direct influence. We hasten to add, however, that this new and interesting case is highly unlikely, as we would be working at extremely low frequencies, which for the time being have not the slightest chance of being observed.

Next come modes which can still be classed as nonresonant, but have frequencies below the cometary water group ion gyrofrequency. This generalizes previous work of ours [*Lakhina and Verheest*, 1988; *Verheest and Lakhina*, 1991] to include charged dust. We have assumed that at these and higher frequencies the dust dynamics can safely be ignored, and the main effect remaining is an additional source of wave damping or growth, except when the modes were stable in the absence of dust. The new mechanism which becomes possible is the momentum exchange between dust and plasma due to variable dust charges. This latter possibility, however, would lead to rather slow growth or decay.

Finally, we have looked at modes whose frequencies resonate with the cometary watergroup ion gyrofrequency. This is the preferred mechanism to excite Alfvénic turbulence in cometary environments, and we find that the dust charge fluctuations influence the instability leading to such turbulence. As a conclusion, it would seem that for most parameter ranges either existing instabilities are enhanced, showing that charged dust facilitates the cometary ion pickup by the solar wind, or new instabilities have been shown to exist.

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