

SOLAR WIND INTERACTION WITH THE MAGNETOSPHERE

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The magnetopause is the interface region between the shocked solar wind in the magnetosheath and the hot and low-density plasma in the magnetosphere. Study of the fine structure and dynamics of the magnetopause current layer (MCL) is of fundamental importance in understanding mechanisms of transport of mass, momentum, energy, and waves from the solar wind to the magnetosphere. Many interesting plasma processes, which include solar wind impulsive penetration (Lemaire and Roth, 1991), tearing instability (Kuznetsova et al., 1994, 1995; Kuznetsova and Roth, 1995), Kelvin-Helmholtz instability (Miura, 1987) and microscopic plasma turbulence (Gary and Eastman, 1979), occur at this interface region.

The structure of the magnetopause was first studied by Ferraro (1952), who assumed that the Earth's magnetic field is confined by the ram pressure of an unmagnetized plasma impinging on the magnetosphere. Without the neutralizing particles trapped inside the magnetopause, the characteristic thickness of the current sheet is of the order of the electron skin depth c/ω_{pe} (≈ 1 km). A polarization electric field is present. It is due to charge separation whose thickness is of the order of the Debye length \mathcal{L}_D (≈ 1 m). With a population of trapped particles to provide partial or complete neutralization, the characteristic thickness is of the order of the ion gyroradius ρ^+ ($\rho^+ \approx 100$ km) (Parker, 1967). These early models of the magnetopause structure are described in extensive reviews by Willis (1971, 1975) and Roth (1980).

The simplest model of the MCL is described by a one dimensional tangential discontinuity (TD) of finite thickness within which the magnetic field rotates from an arbitrary interplanetary direction to the magnetospheric direction. The MHD conservation equations lead to the following so-called Ragnine-Hugoniot conditions for a tangential discontinuity (in Gaussian units) :

$$B_n = 0, u_n = 0, [\rho] \neq 0, [\mathbf{B}_t] \neq 0, [\mathbf{u}_t] \neq 0, [P + B_t^2/8\pi] = 0 \quad (1)$$

where \mathbf{B} , ρ , \mathbf{u} , and P are the magnetic field, plasma density, velocity, and pressure, respectively. In equation (1) the square brackets denote the difference between the values of any quantity on the two sides of the discon-

tinuity. It can be seen that across a TD the normal components B_n and u_n are equal to zero, while there are jumps in ρ and in the tangential components \mathbf{B}_t and \mathbf{u}_t . The pressure balance in the Ragnine-Hugoniot relations is a MHD jump condition. The kinetic theory tells us more about the pressure balance since, inside a TD of finite thickness, $P + B_t^2/8\pi$ remains everywhere a constant quantity.

Kinetic structure of the magnetopause

Vlasov equilibrium models of tangential discontinuities in collisionless plasmas have been described by, e.g., Grad (1961), Harris (1962), Nicholson (1963), Sestero (1964, 1966), Alpers (1969), Kan (1972), Roth (1976, 1978, 1979, 1980), Lemaire and Burlaga (1976), Channell (1976), Lee and Kan (1979); Roth et al. (1990, 1993, 1994), Kuznetsova et al. (1994), Kuznetsova and Roth (1995).

Table 1 summarizes the characteristics of most of these one-dimensional models.

A single plasma particle of species v (having electric charge q_v and mass m_v in a one dimensional TD parallel to the y - z plane is characterized by three constants of motion : the Hamiltonian ($H_v = m_v v^2/2 + q_v \phi$) and the y and z components of the canonical momentum ($P_{vy} = m_v v_y + q_v a_y/c$ and $P_{vz} = m_v v_z + q_v a_z/c$). The most generally used way to solve the time independent Vlasov equation is to introduce single-valued velocity distribution functions F_v in the (H_v, P_{vy}, P_{vz}) space. The partial number densities n_v and the y and z components of the partial current densities j_{vy} , j_{vz} can then be obtained after integrating the distribution functions $f_v(v_x, v_y, v_z, a_y, a_z, \phi) = F_v(H_v, P_{vy}, P_{vz})$ over velocity space (v_x, v_y, v_z) as functions of the electrostatic potential $\phi(x)$ and the y and z components of the vector potential $a_y(x)$ and $a_z(x)$. The charge density $\sigma = \sum q_v n_v$ and the y and z components of the total current density $J_y = \sum j_{vy}$, $J_z = \sum j_{vz}$ are then substituted into Maxwell's equations, leading to a set of coupled

Table 1 : Characteristics of kinetic TD models

Models	Properties
<i>Alpers</i> [1969]: A whole class of distribution functions are constructed by prescribing the magnetic field profile and a bulk velocity profile in the direction of the magnetic field. Magnetic shear is included ($B_y \neq 0$). Two plasma species (electrons and ions). Asymptotic isothermal plasma ($T = T^+(\pm\infty) = T^-(\pm\infty)$). No trapped populations.	Electrostatics Exact charge neutrality. Thickness $\geq \rho^+$
<i>Roth</i> [1976]: Magnetized plasma on both sides without trapped populations. Unidirectional magnetic field. Change in the plasma bulk velocity in the direction perpendicular to the field. Multi-species plasma with different densities and temperatures.	Electrostatics Charge neutral approximation. Non-zero normal electric field. Thickness ρ^- or ρ^+
<i>Lemaire and Burlaga</i> [1976]: Magnetized plasma on both sides without trapped populations. Magnetic shear is included ($B_y \neq 0$). No change in the plasma velocity across the plasma sheet. Multi-species plasma with different densities and temperatures.	Electrostatics Charge neutral approximation. Non-zero normal electric field. Thickness ρ^- or ρ^+
<i>Roth</i> [1978, 1979, 1980]: Magnetized plasma on both sides with or without trapped populations. Magnetic shear ($B_y \neq 0$). Shear in the plasma bulk velocity ($u_y \neq 0$, $u_z \neq 0$). One single formalism for trapped and untrapped populations. Multi-species plasma with different densities and temperatures. Asymptotic temperature anisotropies ($T_\perp \neq T_\parallel$).	Electrostatics Charge neutral approximation. Zero or non-zero normal electric field. Thickness $> \mathcal{L}_D$ (inner only); ρ^- or ρ^+
<i>Lee and Kan</i> [1979]: Magnetized plasma on both sides with or without trapped populations. Magnetic shear ($B_y \neq 0$). Shear in the plasma bulk velocity ($u_y \neq 0$, $u_z \neq 0$). Different formalisms for trapped and untrapped populations. Two plasma components (protons, electrons) with different densities and temperatures.	Electrostatics Charge neutral approximation or exact charge neutrality. Thickness $\geq \rho^+$
<i>Roth et al.</i> [1995]: In the previous model of <i>Roth</i> [1980] the thickness is included as a free parameter. Other characteristics of this generalized model are unchanged, except that temperature anisotropies are not considered in the velocity distribution functions.	Electrostatics Charge neutral approximation. Zero or non-zero normal electric field. Thickness $> \mathcal{L}_D$ (inner only); $\geq \rho^-$

second order differential equations for $\phi(x)$, $a_y(x)$ and $a_z(x)$

$$\frac{d^2\phi}{dx^2} = -4\pi\sigma(\phi, a_y, a_z) \tag{2}$$

$$\frac{d^2 a_{y,z}}{dx^2} = -4\frac{\pi}{c} J_{y,z}(\phi, a_y, a_z) \tag{3}$$

The differential equation for $\phi(x)$ is usually replaced by the quasi-neutrality condition

$$n(x) = \sum_{v=v_+} Z_v n_v = \sum_{v=v_-} n_v \tag{4}$$

where v_+ correspond to ion populations and v_- to electron populations, Z_v is the ion charge ($Z_v = 1$ for protons). In the general case, the number of ion and electron populations can be arbitrarily large. All particle populations can

Table 1 : Characteristics of kinetic TD models (cont'd)

Models	Properties
<i>Grad</i> [1961]: A <i>unique</i> and monotone B -field profile exists for the <i>thinnest</i> transition describing the exponential decrease of a field-free plasma into a unidirectional magnetic field region, if there are no "trapped" particles and if the asymptotic distributions are isotropic.	Electrostatics Charge separation effects in the case of particles of different masses are ignored. Thickness $c/\omega_p = \rho$
<i>Harris</i> [1962]: Plasma slab separating plasma-free regions of oppositely directed magnetic fields ($\mp \mathbf{B}_R$ along the z axis). The trapped populations of electrons ($-$) and protons ($+$) are described by Maxwellian distribution functions shifted along the v_y axis by the drift velocity $V_H^\mp = \pm 2cT/eB_R\mathcal{L}$ (\mathcal{L} =characteristic thickness).	Electrostatics Electric field vanishes in the reference system where $V_H^- = -V_H^+$. Thickness $\mathcal{L} > \mathcal{L}_D$
<i>Nicholson</i> [1963]: Plasma slab separating plasma-free regions of constant magnetic field, the field being in the same direction on the two sides of the slab. The trapped populations of electrons and protons have velocity distribution functions that differ from Maxwellians to the extent that a parameter a entering into the characteristic length differs from zero.	Electrostatics Exact charge neutrality. This condition fixes the parameter a and the thickness. Thickness ρ^+
<i>Sestero</i> [1964]: Magnetized plasma on both sides without trapped populations. Unidirectional magnetic field. No change in the plasma velocity across the plasma sheet. Two plasma components (electrons and ions). Asymptotic isothermal plasma ($T^+(\pm\infty) = T^-(\pm\infty)$).	Electrostatics Charge neutral approximation. Non-zero normal electric field. Thickness ρ^- or ρ^+
<i>Sestero</i> [1966]: Magnetized plasma on both sides without trapped populations. Unidirectional magnetic field. Change in the plasma bulk velocity in the direction perpendicular to the field. Two plasma components (electrons and ions). Asymptotic isothermal plasma ($T^+(\pm\infty) = T^-(\pm\infty)$). The maximum velocity shear is the thermal velocity of the particles carrying the current (ions in ion-dominated layers, electrons in electron-dominated layers).	Electrostatics Charge neutral approximation. Non-zero normal electric field. Thickness ρ^- or ρ^+

be subdivided into three groups associated with each of the two sides ("outer") of the transition and its "inner" region. For magnetopause modeling it is reasonable to introduce magnetosheath, magnetospheric, and trapped (i.e., inner MCL) populations.

The density of magnetosheath particles tends to zero on the magnetospheric side ($x \rightarrow +\infty$), while the density of magnetospheric particles tends to zero on the magnetos-

heath side ($x \rightarrow -\infty$). The inner (or trapped) populations are confined inside the current layer, their density having a maximum inside the MCL and tending to zero on both sides ($x \rightarrow \pm\infty$). The inner population is especially important in MCLs with large magnetic shear. The dependence of the distribution function on H_y is usually introduced in a Maxwellian form

$$F_v = s_v (m_v/2\pi T_v)^{3/2} \exp(-H_v/T_v) G_v(P_{vy}, P_{vz}) \quad (5)$$

that corresponds to Poisson distributions of the partial number densities in the electrostatic potential

$$n_v(\phi, a_y, a_z) = s_v \exp(-q_v \phi/T_v) g_v(a_y, a_z) \quad (6)$$

$$g(a_y, a_z) = \pi^{-1} \int \exp\{-(v_{vy}^2 + v_{vz}^2)\} G_v(P_{vy}, P_{vz}) dv_{vy} dv_{vz}, \\ v_{vy,uz}^2 = (m_v / 2T_v) v_{y,z}^2.$$

The functions $G_v(P_{vy}, P_{vz})$ represent cutoff factors in phase space to describe the fact that charged particles from one side cannot penetrate arbitrarily deeply into the other side of the current layer and that trapped particles are confined inside it. The form of $G(P_{vy}, P_{vz})$ determines the gradient scale D_v of the partial current density of the v 'th species

$$j_{vy,vz}(\phi, a_y, a_z) = cT_v (\partial n_v / \partial a_{y,z})$$

An isotropic Maxwellian distribution (with zero current velocity) corresponds to $G_v = 1$. The spatial variation of the density of such a population can only be controlled by the nonuniform electrostatic potential profile (that is, by the equilibrium electric field, E_x inside the layer).

The well known analytical Harris distribution, modified by a superposed constant B_y magnetic field component (b), is

$$\mathbf{B} = B_R \tanh \frac{x}{L} \mathbf{e}_z + b \mathbf{e}_y \quad (7) \\ a_z = -bx, a_y = \mathcal{L} B_R \ln(\cosh(x/\mathcal{L}))$$

It is described by trapped particles (protons and electrons) with $G(P_{vy}, P_{vz}) = \exp(-m_v V_{Hv}^2 / 2T_v + V_{Hv} P_{vy} / T_v)$ corresponding to Maxwellian distribution functions shifted by the diamagnetic drift velocity ($V_{Hv} = -2cT_v/q_v B_R \mathcal{L}$) in the y direction :

$$F_v = n(x) (m_v/2\pi T_v)^{(3/2)} \exp\{-m_v[v_x^2 + (v_y - V_{Hv})^2 + v_z^2]/2T_v\} \\ (n(x) = n_0 \cosh^{-2}(x/\mathcal{L})).$$

Other forms of trapped distributions were introduced in the papers by Nicholson (1963) and Lee and Kan (1979).

The cutoff factors $G_v(P_{vy}, P_{vz})$ for magnetosheath and magnetospheric populations are usually chosen in the form of step functions (e.g., Sestero, 1964, 1966; Lemaire and Burlaga, 1976; Roth, 1976, 1978, 1979, 1980) or error functions (e.g., Alpers, 1966; Lee and Kan, 1979; Roth et al., 1995) because they lead to relatively simple analytical expressions for the moments, n_v , j_{vy} , and j_{vz} of the distribution functions. The choice of error functions allows one to introduce arbitrary gradient scales $D_v \geq \rho_v$ (ρ_v is the gyroradius of the v 'th species). Even for step-like cutoffs the characteristic thickness of the TD can not be less than one electron gyroradius $\rho_{v-} = \rho^-$ (in electron-dominated layers, where ions are isotropic, i.e. $G_{v+} = 1$, and the electric current is only carried by electrons), or one ion gyroradius $\rho_{v+} = \rho^+$ (in ion-dominated layers, where

the electric current is carried by ions). However, in symmetrical transitions of the Harris type (7), the minimum thickness can approach the Debye length. In the general case the characteristic thickness of the transition is determined by the gradient scales D_v of all populations collectively. Thin electron layers appear to be extremely unstable (Roth et al, 1993; Drake et al., 1994), so it is usual to only consider layers with characteristic thickness of a few ion gyroradii. The choice of functions $G_v(P_{vy}, P_{vz})$ is of course not unique.

In summary, the existing one-dimensional Vlasov models can be characterized by the following set of attributes :

- The number of different particle populations (magnetosheath, magnetospheric, and inner). For instance, the models by Harris (1962) and Nicholson (1963) include only inner (i.e., trapped) populations of electrons and protons, while Sestero (1964, 1966) and Alpers (1969) introduced only magnetosheath and magnetospheric particles without trapped populations. Both "inner" and "outer" populations were incorporated by Roth (1978, 1979, 1980), Lee and Kan (1979) and Roth et al. (1995). Multi-species plasma with different densities, ion charges and temperatures were considered by Lemaire and Burlaga (1976), Roth (1976, 1978, 1979, 1980) (including asymptotic temperature anisotropies), and Roth et al. (1995).

- Assumptions about the charge neutrality.

- The form of the cutoff functions $G_v(P_{vy}, P_{vz})$ and corresponding gradient scales D_v that control the thickness of the MCL.

- The degree of asymmetry in boundary conditions that can be described by the model (e.g., the velocity shear; the angle of magnetic field rotation, θ_0 ; density and temperature asymmetries). For instance, models by Sestero (1966) and Roth (1976), where velocity shear was taken into account imply unidirectional magnetic fields ($\theta_0 = 0$). The model by Alpers (1969) without inner populations can describe MCLs with velocity shear but small magnetic shear ($\theta_0 < 90^\circ$). The unified model by Lee and Kan (1979) can describe asymmetric MCLs with zero velocity shear and arbitrary magnetic shear (including $\theta_0 > 90^\circ$) as well as MCLs with finite velocity shear and small magnetic shear ($\theta_0 < 90^\circ$) but due to different formalisms for inner and outer populations their model is unable to describe MCLs with both velocity shear and large magnetic shear ($\theta_0 > 90^\circ$).

A generalized one-dimensional kinetic multi-species model of MCLs was developed recently by Roth et al. (1995). In this model all particle populations (from both outer regions and from inside the layer) are described using a unique formalism for the velocity distribution functions. Most of the previous models can be retrieved as special

cases. The model also describes current layers with velocity shear and large angles of magnetic field rotation.

As illustrated by figure 1, such a model with a large number of free parameters and different gradient scales can in principle illustrate many observable features of the MCL, including its multiscale fine structure.

Discussion

A number of problems associated with the one-dimensional, time-independent Vlasov approach should be kept in mind :

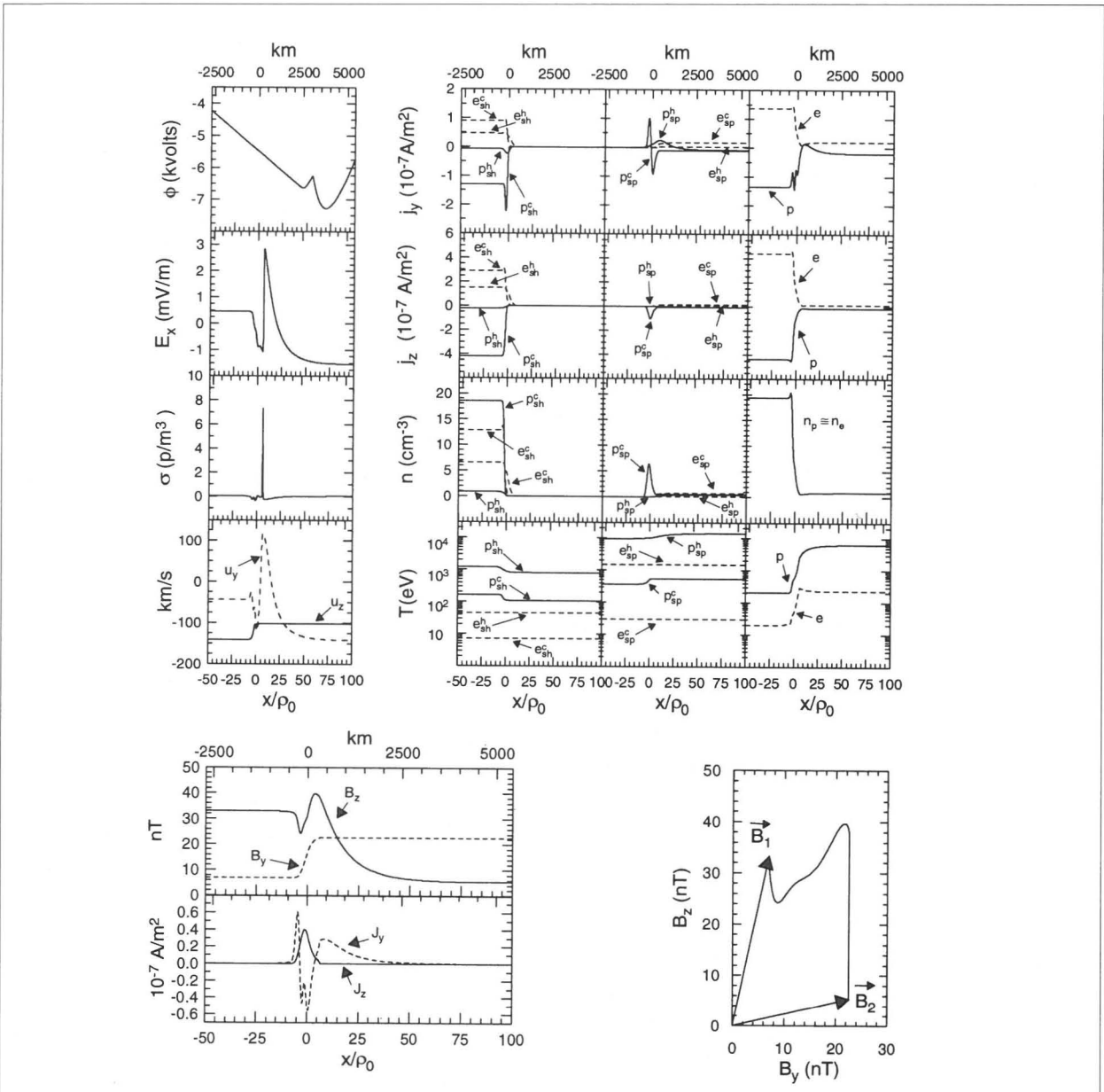


Figure 1 : Simulation of an ISEE magnetopause crossing. Top panels : (left) electric structure and plasma flow; (right) plasma structure. Labels **e** and **p** refer to electron and proton populations. Subscript **sh** identifies particles originating in the magnetosheath, while **sp** refers to particles from the magnetosphere. Superscripts **c** and **h** refer to cold and hot populations. Bottom panels : (left) magnetic field and total current density; (right) magnetic field hodogram.

- Vlasov theories of plane TD's yield non-unique solutions. On a macroscopic scale, any pressure profile $p(x)$ and magnetic field $B(x)$ related by $p + B^2/8\pi = C_{\text{const}}$ define an allowable equilibrium solution. On a microscopic scale, this non-uniqueness shows up in the arbitrariness with which particle velocity distribution functions can be chosen. Only consideration of particle accessibility (i.e., tracing the origin of the populations) can remove this non-uniqueness; the plane TD models themselves are inadequate to solve the problem of particle accessibility, both to the current layer itself, and more specifically to different phase space regions (Whipple et al, 1984). The accessibility question is, of course, also related to the temporal behavior of the sheet (see Morse, 1965).

- The large number of free parameters obscures the relation between boundary conditions and the internal structure of the layer.

- One-dimensional current layers with magnetic shear are thermodynamical nonequilibrium systems (Kuznetsova et al., 1994) that have an excess of free energy and are potentially unstable with respect to the excitation of large scale electromagnetic perturbations, resulting in the destruction of magnetic surfaces. Therefore, MCLs most likely are in a state of turbulence rather than in a state of one-dimensional Vlasov equilibrium.

A reasonable application of these one-dimensional Vlasov models is to adopt them as an initial unperturbed state and then consider the temporal and spatial evolution of the system caused by superposed perturbations. In Kuznetsova and Roth (1995), the stochastic percolation model by Galeev et al. (1986), based on the symmetrical charge-neutral Harris equilibrium, was generalized for MCLs with asymmetrical \mathbf{B} field profiles. It was demonstrated that the asymmetry factor, $\kappa_B = |(B_2 - B_1)/B_2|$, strongly modifies the dependence of the marginal MCL thickness (below which the MCL is subjected to percolation) on the angle of magnetic field rotation θ_0 . In realistic asymmetrical cases ($\kappa_B > 0.3$), the marginal thickness should be thinner for $\theta_0 < 90^\circ$ (northward IMF) than for $\theta_0 > 90^\circ$ (southward IMF). For southward IMF ($\theta_0 > 90^\circ$) the marginal thickness depends only slightly on κ_B and on plasma β in the magnetosheath. For northward IMF the marginal thickness is likely to be thinner for larger values of β in the magnetosheath.

If the MCL thickness is much larger than the marginal one, a large domain of stable magnetic surfaces should exist within it, which should prevent particles diffusion across the layer. Note that microscopic plasma turbulence (e.g., lower hybrid drift instability that could provide diffusion when the magnetosheath and magnetospheric fields are parallel) is strongly stabilized by the magnetic shear when θ_0 is substantial. In this case a one-dimensional slab TD could be considered as a good model for the MCL.

When the thickness is close to the marginal value, the MCL can be modeled as a tangential discontinuity "spoiled" (in the first approximation) by embedded percolated magnetic filaments. In this case the MCL is likely to have a pore-like fractal structure, where percolated magnetic filaments (common for both sides of the MCL) are surrounded by closed magnetospheric and open magnetosheath field lines.

When the MCL thickness is much less than the marginal one, a large number of tearing modes are allowed to grow together. This large-scale magnetic turbulence leads to a diffusive broadening of the current layer.

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