

Phase noise elimination in interferometric experiments by sawtooth phase modulation

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A technique has been developed for eliminating the phase sensitivity in interferometric measurements. It is based on sawtooth phase modulation of the signal and double phase sensitive detection. It overcomes the restrictions due to background transmission that have been recognized by us in a previously proposed method based on rms detection. This paper focuses on its use in time resolved stimulated Raman gain measurements. *Key words:* Phase noise, interferometry, sawtooth phase modulation, time resolved stimulated Raman gain.

I. Introduction

A common method of detecting weak signals is through their interference with a so-called local oscillator. This method is sensitive to the phase difference between the signal and the local oscillator, and, consequently, it also suffers from phase noise. Optical phase noise in particular is a nuisance. Such is the case, for example, in time resolved stimulated Raman gain (TRSRG)¹⁻⁵ measurements that make use of this interferometric detection scheme, the local oscillator field being one of the applied laser fields. The following discussions will mostly specify to this application.

Earlier,² we designed an audio phase modulation technique for eliminating the phase sensitivity in TRSRG measurements where basically the audio modulated signal passes through a broadband filter, and the rms output voltage is measured. It has the disadvantage of introducing a small background in the detected signal, arising from mainly laser noise within the detection bandwidth. This limits the dynamic range of the technique, and must also correctly be taken into account in the analysis of the signal.

An alternative handling of the TRSRG signal uses a sinusoidal audio phase modulation together with

phase sensitive lock-in detection.³ The recorded signal displays the amplitude and phase dependence of the TRSRG signal; in the concurrent signal analysis, the pure modulus is extracted from the recorded signal through a deconvolution procedure. This works well for the systematic (periodic) part of the phase variations, on condition that enough data points per period are sampled and that any random phase variations due to external disturbances, hereafter called phase noise, are sufficiently small.

The technique presented here overcomes all of the above disadvantages without any reduction in signal-to-noise ratio. It is based on an audio-frequency sawtooth phase modulation and corresponding lock-in phase sensitive detection of both the in- and out-of-phase signals. After vector summing, the signal modulus is directly recorded.

II. Outline of the Modulation Technique

The modulation detection technique presented here is applicable to any signal that, in the interferometric detection scheme, has the following dependence on a variable z ,

$$S(z) \propto A(z) \cos[\Delta\phi(z)], \quad (1)$$

in which $\Delta\phi(z)$ denotes the phase difference between the local oscillator and the signal of interest, and $A(z)$ the signal's amplitude.

The example to which the next discussions will specify, is the TRSRG spectroscopy. TRSRG is a pump-probe experiment, in which the pump pair consists of two laser fields E_L and E_S at the frequencies ω_L and ω_S , respectively, and the probe pair of a delayed replica of each of them, denoted as $E_{L'}$ and $E_{S'}$, respectively. The frequency difference $\omega_L - \omega_S$ is adjusted to the resonance frequency ω_R of the Raman mode, the relaxation of which is to be studied. The laser pump pair

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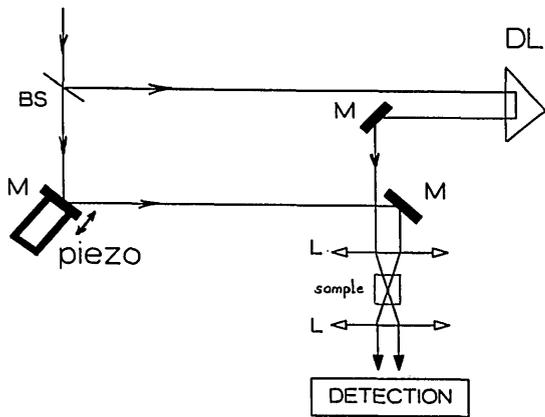


Fig. 1. Simplified diagram of the TRSRG setup, typical of the interferometric detection scheme; the signal is generated where both laser fields are overlapping. M: mirror, BS: beam splitter, DL: delay line; L: lens.

excites a third-order nonlinear polarization field in the sample $P_S^{(3)}(t_D)$ at the Stokes frequency ω_S ; one measures the decay of this polarization in time, by probing it (with the probe pair) at a variable delay t_D with respect to the pump pair. The essential features of the TRSRG setup that are typical of the interferometric detection scheme, are shown in Fig. 1.

The detected phase relaxation signal $S(t_D)$ can be written according to the form of Eq. (1)²⁻⁵; now, t_D plays the role of the variable z , and $\Delta\phi(t_D)$ is the phase difference between the carrier (E_S) and the nonlinear polarization field $P_S^{(3)}(t_D)$. With regard to the relaxation, one is interested mainly in the dependence of A on t_D . The dependence of $\Delta\phi$ on t_D is essentially limited to a trivial linear growth weighed with the driving frequency $\omega_L - \omega_S \approx \omega_R$ ^{3,4}:

$$\Delta\phi(t_D) = (\omega_L - \omega_S)t_D + \Delta\phi_0 + \Delta\phi_S(t_D); \quad (2)$$

herein, $\Delta\phi_0$ represents a constant phase offset, and $\Delta\phi_S(t_D)$ denotes any other phase terms that are intrinsic of the signal.⁶ In practice, the so-called signal phase $\Delta\phi(t_D)$ is also sensitive to external phase perturbations that alter the phase relationship between pump and probe replicas of the same laser beam. Therefore, in fact, the signal phase fluctuates.

From Eq. (1), it is clear that if one forces into $\Delta\phi(t_D)$ an additional term that is linearly dependent on time, e.g., a term Ωt , then phase sensitive lock-in detection at this frequency Ω yields the signal amplitude $A(t_D)$ if the lock-in reference phase matches the signal phase. This last condition must be circumvented because the signal phase is not constant in time. Therefore, both the in-phase and quadrature components must be sampled and processed in a vector summing circuit: one retains the pure signal amplitude, as desired.

In practice, the signal manipulation is performed with a vector lock-in amplifier. And the required linear phase sweep is approximated by a sawtooth phase modulation, at the frequency Ω , with a duty cycle,⁷ θ/π , that is almost equal to one, and a peak-to-peak (pp) amplitude corresponding to $\sim 2\pi$ phase difference.

The technical realization of this phase modulation is discussed after elucidating, in the next paragraph, the required characteristics of the sawtooth. The amplitude of the sawtooth waveform will always be expressed in terms of the corresponding phase difference induced upon the signal.

The appropriate shape of the sawtooth modulation function, $\Delta\phi_M(t)$, follows from the requirement that in a Fourier decomposition of Eq. (1), the signal phase should enter as a pure phase term in one of the Fourier components, e.g., the n^{th} harmonic. As a result, the n^{th} harmonic contribution to the signal, $S_n(t_D)$ ($n \neq 0$), looks like:

$$S_n(t_D) \propto A(t_D)a_n \cos[n\Omega t + \Delta\phi(t_D)],$$

with

$$a_n = \mp \sin(2n\pi\theta/\pi)(-1)^m/\pi \left[\frac{1 - \theta/\pi}{(m \pm n) \mp 2n\theta/\pi} + \frac{\theta/\pi}{m \mp 2n\theta/\pi} \right], \quad (3)$$

if the pp -amplitude Φ_{nm} of the sawtooth waveform with a duty cycle $\theta/\pi < 1$ equals:

$$\Phi_{nm} = 2\pi(m \mp n\theta/\pi) \quad (m \text{ integer}, m \neq 0). \quad (4)$$

Then vector lock-in amplifier detection at this n^{th} harmonic of the modulation frequency Ω immediately yields the signal amplitude $A(t_D)$ scaled with a_n . In the limit $\theta/\pi = 1$, a pp -amplitude equal to $2n\pi$ is required, yielding the unscaled signal amplitude $A(t_D)$ ($a_n = 1$). A sawtooth waveform with a duty cycle $\theta/\pi = 1/2$, i.e., a triangle, fails for our purposes.

It is worth making a comparison between the present method and the commonly applied sine function phase modulation.³ As is the case for the triangle-like modulation, the analogous Fourier decomposition procedure applied to a sine modulation proves that it is impossible to force the signal phase into a pure phase of only one Fourier component. Instead, the amplitude of the n^{th} harmonic contribution to the signal remains proportional to the sine or cosine of the signal phase:

$$S_n(t_D) \propto A(t_D)a_n \cos[\Delta\phi(t_D)] \cos(n\Omega t) \quad (n \text{ even}),$$

or

$$S_n(t_D) \propto A(t_D)a_n \sin[\Delta\phi(t_D)] \sin(n\Omega t) \quad (n \text{ odd}),$$

with

$$a_n = 2J_n(\Phi/2) \quad \text{for } n \neq 0,$$

and

$$a_0 = J_0(\Phi/2); \quad (5)$$

J_n denotes the Bessel function of integer order n , and Φ is the pp -amplitude of the sinusoidal modulation function.

Equations (2) and (5) elucidate the origin of the fringes that are commonly observed in TRSRG (see Fig. 2 in Ref. 3): their periodicity reflects the frequency of the probed Raman mode.

At the same time, we see that the ratio of signal strength in the sawtooth modulation scheme to that in the sine modulation scheme, at the fundamental fre-

frequency Ω , is at best (if $\theta/\pi = 1$) $1/1.16$ or 86%. Detection at higher harmonics always results in a smaller optimal signal amplitude; in case of sawtooth modulation this also requires a higher pp -amplitude of the sawtooth. The results of Eqs. (3) and (4) concerning the dependence of the signal amplitude on the indices n and m are illustrated in Fig. 2. For each choice of the harmonic n , the maximum signal amplitude is obtained if the duty cycle of the sawtooth waveform satisfies $2n\theta/\pi = m$, and amounts to θ/π ; the corresponding required pp -amplitude equals $2\pi m/2$. The higher the value of n , the steeper is the dependence of the signal amplitude on the exact value of θ/π .

The required phase modulation has been implemented as follows: A mirror in one of the optical paths of the TRSRG setup is mounted on a piezoelectric element (Physik Instrumente P170) driven by a high voltage sawtooth waveform at a frequency Ω of ~ 500 Hz (see Fig. 1). This frequency is dictated by the AM transmission bandwidth of the DRAKE radio that passes the TRSRG signal.⁸ For a suitable piezoelectric element, the applied voltage causes an expansion or contraction of the piezoelectric element, hence a corresponding mirror movement, that is linearly proportional to it. The 2π phase modulation depth, corresponding with an optical path length change of 600 nm which is about our laser beam wavelength, requires a pp -voltage amplitude of ~ 120 V. A block diagram of the home-built⁹ sawtooth generator/amplifier (SGA) is shown in Fig. 3. The SGA generates a sawtooth output at a frequency tunable between ~ 475 and 575 Hz, and amplifies it to a maximum pp -amplitude of ~ 270 V (slightly dependent on frequency) with a variable negative dc offset voltage up to ~ -70 V. The duty cycle tends to 1. At the same time, it is able to amplify any external ac input by a factor of ~ 57 .

As an alternative realization of a similar phase modulation, we mention the use of an electrooptical modulator. This device also can induce a phase change of the transmitted optical electric field that is linearly proportional to the voltage applied to it.^{2,4}

III. Performances of the Sawtooth Phase Modulation Technique

With the use of the sawtooth modulation and vector lock-in detection technique, the S/N ratio is unaltered in comparison with that for the previously applied technique based on sinusoidal modulation and corresponding phase-sensitive detection; the absolute signal amplitude still amounts to $\sim 90\%$. The main advantages of the new technique stem from the elimination of the signal phase from the detected signal. If the residual phase sensitivity of the TRSRG signal is estimated from the ratio of the residual uncertainty in the signal amplitude to the mean of this amplitude, $(\Delta A)_{pp}/\langle A \rangle$, it is reduced to $\sim 5\%$. As a result, the experimental alignment is less time consuming and more accurate. With regard to the subsequent data processing, a fringe deconvolution and consequently the requirement for a minimum of ~ 7 data points per fringe, are avoided. It turns out that the

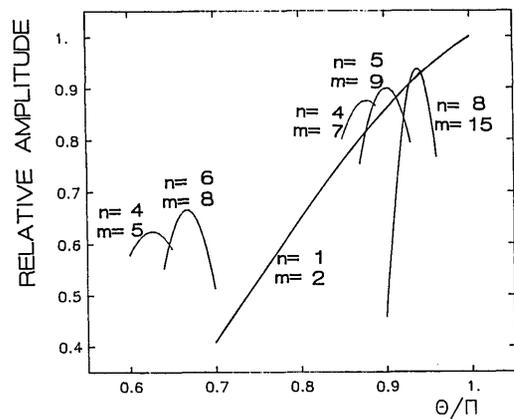


Fig. 2. Relative signal amplitude around its optimum value in the assumption of sawtooth phase modulation, of duty cycle θ/π and pp -amplitude Φ_{nm} , and detection at the n^{th} harmonic of the modulation frequency, for different choices of n and m . The optimum duty cycle satisfies $2n\theta/\pi = m$, requires a pp -modulation depth $\Phi_{nm} = 2\pi m/2$ and yields a relative signal amplitude equal to θ/π .

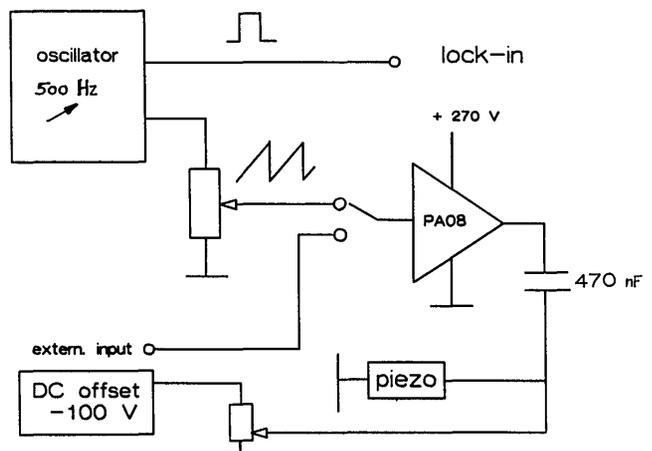


Fig. 3. Block diagram of the sawtooth voltage generator/amplifier. PA08: amplifier; piezo: PI P-170 (capacitance ≈ 15 nF).

number of data points may be reduced by a factor of ~ 5 .

The effective maximum scan velocity is the same for both techniques. According to Eq. (2), a delay scan at velocity v_s causes a frequency shift of the signal with respect to $\Omega/(2\pi)$, f_s , that is equal to $v_s\omega_R/(2\pi c)$. Therefore, the maximum value of v_s is determined by the product $f_s\tau$ in which τ is the time constant of the low-pass filter in the output of the lock-in amplifier.

A detailed comparison of performances between the technique proposed here and the previously mentioned rms detection method has been presented in Ref. 5. It should be remembered from Ref. 5 that the success of the present technique is based on the fact that the phase fluctuations are sufficiently slow in comparison with the response time of the detection system. On the contrary, the rms method does not suffer from any phase fluctuations, even fast ones, but is less attractive because a background due to noise is present in the detected signal.^{2,4,5}

IV. Conclusions

In this paper, we presented a technique for eliminating phase noise in interferometric detection schemes, based on sawtooth phase modulation and corresponding phase sensitive detection. The present discussion has been focused on TRSRG experiments. The optical phase modulation is generated by varying one of the optical path lengths with a piezoelectric element that is driven by a high voltage sawtooth waveform, and the detection is performed in a vector lock-in amplifier. The technique satisfies the need of making the measurements insensitive to phase variations, without any degradation in S/N ratio or dynamic range. Therefore, it is an improvement over previous applied detection methods.^{2,4,5,8} In addition, the phase information inherent to the signal is not lost; it can be retrieved from the dual output of the lock-in amplifier.

The wider applicability of the technique to any interferometric experiment has been proven by us in its use in laser field autocorrelation measurements.

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