THE SOLAR RADIATION INCIDENT AT THE TOP OF THE ATMOSPHERES OF URANUS AND NEPTUNE

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Abstract. The latitudinal and seasonal variation of the direct solar radiation incident at the top of the atmosphere of Uranus and Neptune has been recalculated by use of updated values for the period of axial rotation and the oblateness. Values for the solar radiation are given in Watt per square meter instead of the unit used in earlier papers (calories per square centimeter per planetary day). The solar radiation averaged over a season and a year as a function of planetocentric latitude has also been reviewed. In addition, attention is made to the ratio of the solar radiation incident on an oblate planet to that incident on a spherical planet.

1. Introduction

The solar radiation incident at the top of the atmosphere of outer planets (Jupiter, Saturn, Uranus, Neptune, and Pluto) has been calculated by several investigators (Vorob'yev and Monin, 1975; Levine *et al.*, 1977; Brinkman and McGregor, 1979; Van Hemelrijck, 1982a, b, c, 1985, 1987; Beebe *et al.*, 1986). In the first two of these papers, the seasonal variation of the solar energy outside the atmosphere as a function of latitude is calculated for the so-called Jovian or giant planets and the planets are assumed to be spherical. Brinkman and McGregor (1979) and Van Hemelrijck (1987) represented only the Saturnian upper-boundary insolation but included both the oblateness effect and the effect of the ring system. Corrections due to the flattening of the outer planets (excluding Pluto) were studied in detail by Van Hemelrijck (1982a), whereas an attempt has been made to compute the insolation at Pluto (Van Hemelrijck, 1982b, c, 1985). Very recently, calculations have been carried out by Beebe *et al.* (1986) defining the extent of variation in solar radiation incident at the top of the Jovian atmosphere for a selected set of planetocentric latitudes.

Although the orbital and planetary elements needed for the determination of the solar radiation reaching the top of the atmosphere of Jupiter and Saturn are known with sufficient accuracy, some of the parameters for the three most distant members of the solar system are more difficult to measure and are poorly determined. The rotation periods of Uranus and Neptune are not yet sufficiently known and their geometrical flattening is still questionable (Hughes, 1979; Davies *et al.*, 1980; Franklin *et al.*, 1980; Elliot *et al.*, 1980; Beebe, 1983; Belton and Terrile, 1984; French, 1984; Podolak *et al.*, 1985; Cruikshank, 1985; French *et al.*, 1985). For Pluto, measurements of the angle between the planet's spin axis and its orbit normal (or obliquity) vary greatly (Anderson and Fix, 1973; Golitsyn, 1979; *Handbook of the British Astronomical Association*, 1980). However, in a paper published by Dobrovolskis and

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Harris (1983), the authors investigated the history of Pluto's obliquity by numerical integration using analytic approximations of Williams and Benson (1971).

In this work, we recalculated the solar radiation at Uranus and Neptune using for the rotation period and the oblateness those values which seem to be in best accord with the observations and which are less or more accepted by the planetary science community.

In a first section we briefly summarize some expressions needed for the determination of the upper – boundary insolation. Then, taking into account their orbital and planetary data we calculate the daily insolation with (I_{DO}) and without (I_D) the effect of the oblateness. The results are presented in two contour maps showing the incident solar radiation in watts per square meter as a function of latitude and solar longitude and in two other ones giving the seasonal and latitudinal variation of the ratio of both insolations (I_{DO}/I_D). In addition, the latitudinal dependency of the mean annual [$(\bar{I}_{DO})_A$, $(\bar{I}_D)_A$], summer [$(\bar{I}_{DO})_S$, $(\bar{I}_D)_S$] and winter [$(\bar{I}_{DO})_W$, $(\bar{I}_D)_W$] daily insolations are included in two plots, whereas the percental differences 100 $(\bar{I}_{DO} - \bar{I}_D)/\bar{I}_D$ of the mean daily insolations are illustrated in two graphs.

In our calculations and for the northern hemisphere, the summer season is arbitrary defined as running from vernal equinox to autumnal equinox and spanning 180°; thus the planetocentric longitudes of the Sun equal to 180° and 360° respectively mark the beginning and the end of the winter period. In the southern hemisphere, the solar longitude intervals (0–180°) and (180–360°) divide the year into astronomical winter and summer, respectively.

2. Calculation of the Solar Radiation

The daily insolation for a spherical planet may be expressed as (see e.g., Ward, 1974; Vorob'yev and Monin, 1975; Levine *et al.*, 1977; Van Hemelrijck, 1982a, b, c, 1983, 1985, 1987):

$$I_D = [S_0 T(1 + e \cos W)^2/\pi a_\odot^2 (1 - e^2)^2] (h_0 \sin \phi' \sin \delta_\odot + \\ + \sin h_0 \cos \phi' \cos \delta_\odot), \tag{1}$$

where S_0 is the solar constant at the mean Sun-Earth distance of 1 AU taken at 1368 W m^{-2} (Wilson *et al.*, 1981; Wilson, 1982); T, the rotation period; e, the eccentricity; $a_\odot$, the semi-major axis; h_0, the local hour-angle at sunset or sunrise; ϕ', the planetocentric latitude, $\delta_\odot$ is the solar declination or subsolar latitude; and W is the true anomaly, which is given by

$$W = \lambda_\odot - \lambda_P; \tag{2}$$

where $\lambda_\odot$ and λ_P are, respectively, the planetocentric longitude of the Sun (called solar longitude in the Figures) and the planetocentric longitude of the planet's perihelion. Furthermore, h_0 and $\delta_\odot$ may be obtained from standard spherical trigonometric relationships and depend upon ϵ, called the obliquity or axial tilt of the planet.

For an oblate planet, characterized by a flattening factor $f = (r_e - r_p)/r_e$ where

r_e and r_p are respectively the equatorial and the polar radius, there is an angle v (the so-called angle of the vertical) between the radius vector and the normal to the horizon plane. This angle is also equal to the mathematical difference between the planetographic latitude (ϕ) and the planetocentric latitude (ϕ'). In terms of the latter, v can be written as

$$v = \tan^{-1}[(1-f)^{-2} \tan \phi'] - \phi'. \tag{3}$$

The daily insolation of an oblate planet may be expressed (see, e.g., Van Hemelrijck, 1982a, 1987) as

$$\begin{aligned} I_{DO} = {} & [S_0 T(1 + e \cos W)^2/\pi a_\odot^2 (1-e^2)^2] \times \\ & \times \{\cos v(h_{00} \sin \phi' \sin \delta_\odot + \sin h_{00} \cos \phi' \cos \delta_\odot) + \\ & + \sin v[-\tan \phi' (h_{00} \sin \phi' \sin \delta_\odot + \sin h_{00} \cos \phi' \cos \delta_\odot) + \\ & + h_{00} \sin \delta_\odot \sec \phi']\}, \end{aligned} \tag{4}$$

where h_{00}, the local hour-angle of the rising or setting Sun for an oblate planet is, in general, slightly different from h_0.

Finally, the mean summer, winter or annual daily solar radiations may be found by integrating relation (1) or (4) within the appropriate time limits, yielding the total amount of solar energy received over a season or a year, and by dividing the obtained result by the corresponding length of the summer (T_S) or winter (T_W) or tropical year (T_0). For the calculation of T_S or T_W we refer e.g. to Van Hemelrijck (1982c).

As already mentioned in the introduction, the period of rotation (T) and the planetary flattening (f) are not easy to determine especially in the case of the outer planets. Table I (Uranus) and Table II (Neptune) represent some values of the period of rotation reported in the literature over the last 15 y.

The Tables clearly demonstrate that the discrepancy between the various determinations of the rotation rates of Uranus and Neptune present a major problem. However, in the planetary science community there seems to be a consensus that the rotation period of Uranus is about 16 hr and that of Neptune is of the order of 18 hr (Podolak *et al.*, 1985) since those values seem to be in reasonable agreement with recent analysis of the observations.

In the Report of the IAU Working Group on "Cartographic Coordinates and Rotational Elements of the Planets and Satellites" (Davies *et al.*, 1980), the recommended value for the geometric oblateness or flattening (also called ellipticity) of Uranus was taken from Dollfus (1970) to be equal to 0.030; other measurements cited in the same paper are 0.033 (Elliot *et al.*, 1980), 0.022 (Franklin *et al.*, 1980) and 0.010 (Danielson *et al.*, 1972). According to Franklin *et al.* (1980) a correction has to be made to the value deduced by Danielson *et al.* (1972). After applying the correction the new value amounts to 0.020. Hildebrand *et al.* (1985) and Orton *et al.* (1986) used a value $f = 0.024$ which was given by Elliot *et al.* (1981), whereas the calculations made by Wagener *et al.* (1986) are based on an oblateness factor equal to 0.022 (French, 1984).

TABLE I

Rotation period of Uranus

Rotation period (hr)	Reference
24 (+3/−3)	Hayes and Belton (1977)
24	Slavsky and Smith (1981)
23.923 (+0.003/−0.003)	Smith and Slavsky (1979)
23 (+5/−2)	Trafton (1977)
16.6 (+0.5/−0.5)	Franklin *et al.* (1980)
16.4	O'Meara (1984)
16.2	O'Meara (1984)
16.16 (+0.33/−0.33)	Brown and Goody (1980)
16.0	O'Meara (1984)
16 (+1/−1)	French (1984)
15.57 (+0.80/−0.80)	Brown and Goody (1977)
15.5	Elliot *et al.* (1981)
15.0 (+4.0/−2.6)	Münch and Hippelein (1980)
15–17	Belton and Terrile (1984)
13.0 (+1.3/−1.3)	Trauger *et al.* (1978)
12.8 (+1.7/−1.7)	Elliot *et al.* (1980)
10.8	Newburn and Gulkis (1973)

TABLE II

Rotation period of Neptune

Rotation period (hr)	Reference
22 (+4/−4)	Hayes and Belton (1977)
19.583 (+0.005/−0.005)	Cruikshank (1978)
18.56	Belton *et al.* (1981)
18.44 (+0.01/−0.01)	Slavsky and Smith (1978)
18.432	Smith and Slavsky (1978)
18.29	Belton *et al.* (1981)
18.173 (+0.005/−0.005)	Cruikshank (1978)
17.83 (+0.1/−0.1)	Podolak *et al.* (1985)
17.73 (+0.1/−0.1)	Brown *et al.* (1981)
17.73	Belton *et al.* (1981)
17.7–18.6	Belton and Terrile (1984)
15.8	Newburn and Gulkis (1973)
15.4 (+3/−3)	Belton *et al.* (1980)
15 (about)	Kovalevsky and Link (1969)
15 (about)	Freeman and Lyngå (1970)
15 (about)	Hubbard *et al.* (1985)
11.2 (+1.8/−1.8)	Münch and Hippelein (1980)

In the IAU report, the flattening for Neptune (0.0259) came from analysis of occultation observations by Freeman and Lyngå (1970). Finally, Hildebrand *et al.* (1985), Wagener *et al.* (1986) and Orton *et al.* (1986) used an ellipticity $f = 0.021$ given by Elliot (1979).

In our earlier paper (Van Hemelrijck, 1982a) dealing with the oblateness effect on the solar radiation incident at the top of the atmosphere of the outer planets (exclud-

TABLE III

Elements of the planetary orbits of Uranus and Neptune

Parameters		Uranus	Neptune
Semi-major axis	$a_{\odot}$ (AU)	19.18	30.06
Eccentricity	e	0.04727	0.00859
Longitude of perihelion	λ_P (deg)	3.02	5.23
Obliquity	ϵ (deg)	97.86	29.56
Rotation period	T (Earth days)	0.66 (0.45)	0.75 (0.66)
Tropical year	T_0 (Earth days)	30684.80	60190.5
Oblateness	f	0.022–0.033 (0.05769)	0.0259 (0.02066)

ing Pluto) the planetary and orbital data for Uranus and Neptune were based on Vorob'yev and Monin (1975), Levine *et al.* (1977) and the *Handbook of the British Astronomical Association* (1981). In the present paper the same values are used except, of course, for the rotational period (T) and for the oblateness (f). For Uranus and Neptune we used $T = 0.66$ (Earth days, corresponding to 16 hr) and $T = 0.75$ (18 hr), respectively. For the oblateness of Uranus two values were taken – i.e., $f = 0.022$ and 0.033; for Neptune we used $f = 0.0259$.

The planetary data for the calculations are listed in Table III, values in parenthesis where those used in Vorob'yev and Monin (1975) (T), Levine *et al.* (1977) (T) and Van Hemelrijck (1982a) (T and f).

As mentioned earlier, the incident solar radiation is given in Watt per square meter; insolation values expressed in calories per square centimeter per planetary day as in the papers by Vorob'yev and Monin (1975) and Levine *et al.* (1977) may be obtained by multiplying the unit used by a factor of about 2.065.

3. Discussion of Calculations

3.1. DAILY SOLAR RADIATION

The daily solar radiation of Uranus (and also of Neptune) is presented in the form of a contour map giving the seasonal distribution in terms of the planetocentric longitude of the Sun taken to be 0° at the northern vernal equinox. Application of expression (4) with $f = 0.033$ leads to the isopleths illustrated in Figure 1. (The isopleths corresponding to $f = 0.022$ are only slightly different from Figure 1).

As already stated by Levine *et al.* (1977), the very large obliquity of Uranus results in a position reverse of both hemispheres: the northern hemisphere lies 'below' the ecliptic, the southern one 'above' it.

From the figure it can be seen that the incident solar radiation reaches its maximum at the poles around the summer solstices with values of about 2.5 W m^{-2}. The insolation at the north pole during summer solstice $(I_D)_{NP(ss)}$ is approximately equal to that of the south pole at its summer solstice $(I_D)_{SP(ss)}$ with $(I_D)_{NP(ss)} > (I_D)_{SP(ss)}$. This can easily be evaluated by computing the insolation at both poles; it follows (cf. Van Hemelrijck, 1982c, 1985) that

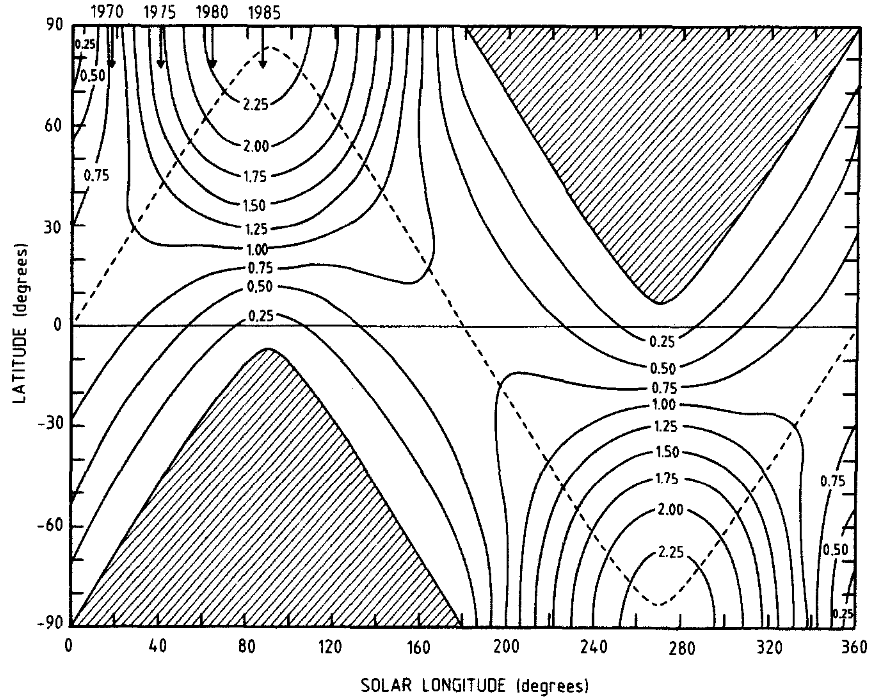

Fig. 1. Seasonal and latitudinal variation of the daily solar radiation at the top of the atmosphere of Uranus with a period of rotation equal to 16 hr and an oblateness factor of 0.033. Solar declination is represented by the dashed line. The areas of permanent darkness are shaded. Values of the solar radiation in watts per square meter are given on each curve.

$$(I_D)_{NP(ss)}/(I_D)_{SP(ss)} = [(1 + e \sin \lambda_p) / (1 - e \sin \lambda_p)]^2. \qquad (5)$$

Hence,

$$(I_D)_{NP(ss)} > (I_D)_{SP(ss)} \text{ if } 0 < \lambda_p < \pi.$$

For Uranus, this difference is of the order of 1%.

The equatorial summer solstice insolation, hereafter denoted as $(I_D)_{E(ss)}$ is related to the polar insolation at summer solstice $(I_D)_{P(ss)}$ by the well-known relationship

$$(I_D)_{P(ss)} / (I_D)_{E(ss)} = \pi \tan \epsilon, \qquad (6)$$

stating that the ratio of both insolations is larger than unity for $17.7° < \epsilon < 162.3°$ and that it is exclusively dependent upon the obliquity. For Uranus, application of expression (6) yields about 22.8.

Another point of interest regards the distribution of the daily solar radiation in the equatorial region. At $\epsilon \lesseqgtr 45°$ the solar radiation as a function of latitude has two

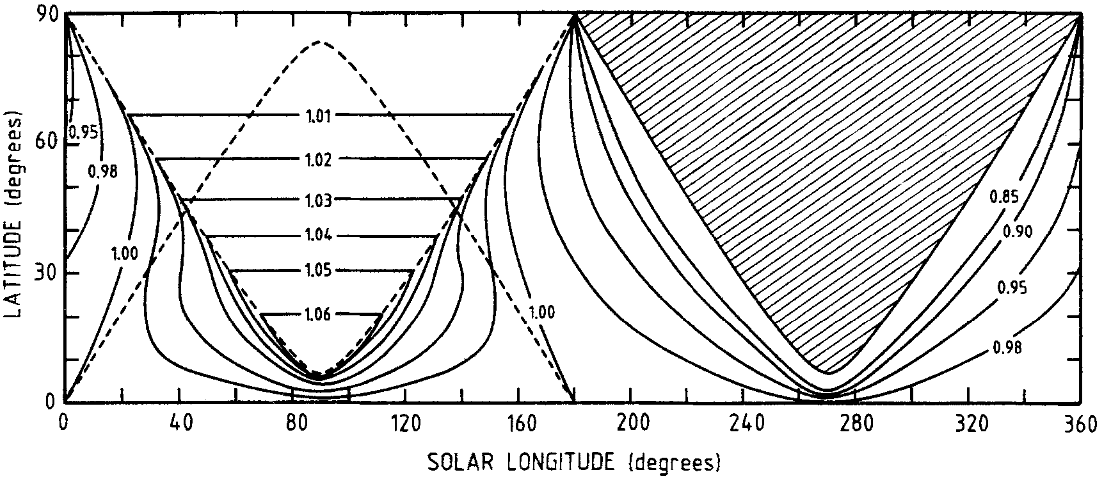

Fig. 2. Seasonal and latitudinal variation of the ratio of the daily solar radiation with and without the oblateness effect ($f = 0.033$) at the top of the atmosphere of Uranus. Solar declination and the region where the Sun does not set are represented by the dashed lines. The areas of permanent darkness are shaded. Values of the ratio of both insolations are given on each curve.

peaks: a maximum one and a minimum one (the polar night being considered as a minimum). Uranus, and also Pluto, occupies a rather exceptional position in that the spin axis lies nearly in the orbital plane. In the polar regions the day and the night are approximately half an Uranian year long (about 42 Earth years). In the equatorial region summer and winter are, roughly speaking, repeated twice a year and the two seasons are substantially more temperate than in the polar areas.

Application of Equations (1) and (4) leads to the isopleths illustrated in Figure 2 where values of constant ratio distribution are given on each curve. As already stated by Van Hemelrijck (1982) there are, in summer, two obviously distinguished regions where $I_{DO} > I_D$. The first zone coincides with the area of permanent sunlight and the isocontours parallel the lines of constant latitude. The second is limited by the seasonal march of the Sun (or the area between the equator and the subsolar point). In general, the two above mentioned parts are linked by the isocontour $I_{DO}/I_D = 1$, coinciding remarkably well with the two branches of an hyperbola symmetric with respect to the solar longitude $\lambda_\odot = 90°$. In winter, the horizon plane is always tilted away from the Sun (Brinkman and McGregor, 1977; Van Hemelrijck, 1982a) causing both the cosine of the zenith angle and the length of the day to decrease. Thus the insolation is reduced. This findings are clearly demonstrated in Figure 2.

As a consequence of the very large obliquity of Uranus the solar radiation of the oblate planet is increased over practically the entire summer season. The obliquity and oblateness of Uranus cause in the vicinity of the equator a gain of insolation of approximately 7% ($f = 0.033$) and 4.5% ($f = 0.022$) decreasing systematically to about 1% at a planetocentric latitude of 70°.

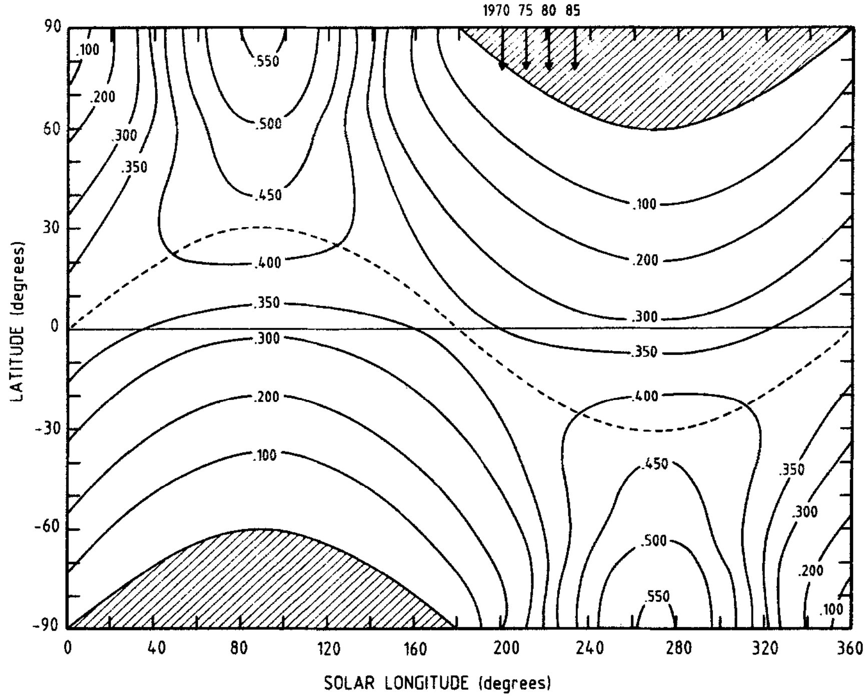

Fig. 3. Seasonal and latitudinal variation of the daily solar radiation at the top of the atmosphere of Neptune with a period of rotation equal to 18 hr and an oblateness factor of 0.0259. See Figure 1 for full explanation.

In winter, as noted earlier, the insolation decreases: the rate of change being extremely rapid near winter solstice, but less sensitive near the equinoxes. The effect of the flattening results also in a more extensive polar region; the maximum difference of the Arctic circles $I_D = 0$ and $I_{DO} = 0$ occurs at solar longitudes 225° and 315° with values of about 1.9° (f = 0.033) and 1.3° ($f = 0.022$). At winter solstice the differences are rather small (~ 0.5° and 0.3°, respectively).

The solar radiation incident on Neptune with an oblateness factor equal to 0.0259 and a rotational period of 18 hr is given in Figure 3. The maximum solar energy is attained at the poles near the summer solstices with a value of about 0.55 W m^{-2}. As for Uranus the solar energy at the north pole during summer solstice is greater than the corresponding insolation at the south pole but due to the much smaller eccentricity the difference is extremely small and reaches scarcely $3^0/_{00}$ (evidently the latest value is only valid if λ_p and e are constants as a function of time). Application of Equation (6) leads to a ratio $(I_D)_{P(ss)}$ / $(I_D)_{E(ss)}$ of 1.8. It can also be seen that there is no seasonal assymetry in the distribution of the solar radiation.

The obliquity of Neptune, being situated in the (0 – 45°) interval, it follows that in the equatorial region there exist only one maximum and only one minimum in the upper-boundary insolation.

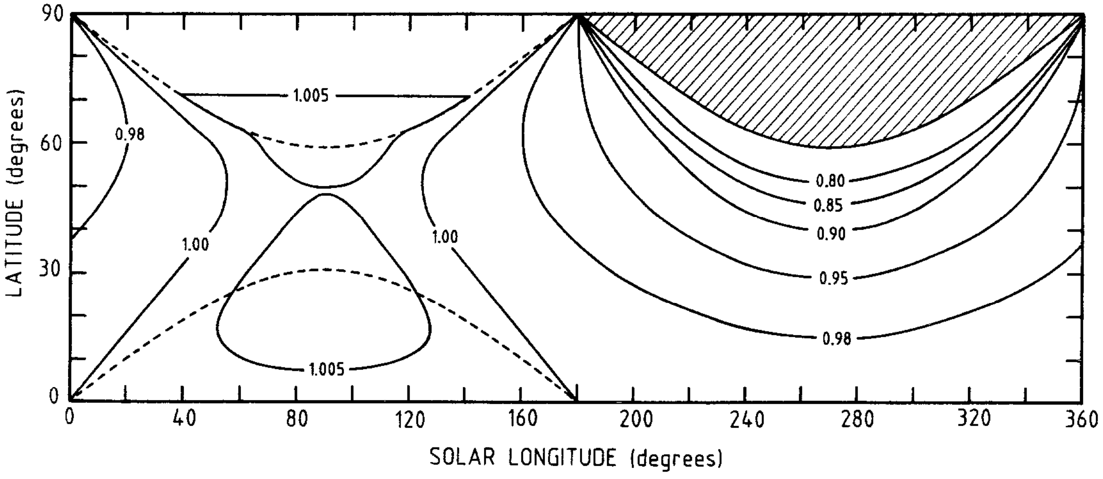

Fig. 4. Seasonal and latitudinal variation of the ratio of the daily solar radiation with and without the oblateness effect at the top of the atmosphere of Neptune. See Figure 2 for full explanation.

The solar radiation ratio pattern (I_{DO}/I_D) of Neptune is plotted in Figure 4. The maximum value (at a latitude of about 60° and at summer solstice) amounts to about 1.01 (1%). In winter, the solar radiation very slowly decreases with increasing latitude, this effect being ascribed to the small value of f. Another point about the curves is that the isocontours $I_{DO} = 0$ and $I_D = 0$ practically coincide. It should also be emphasized that the results presented in Figures 2 and 4 apply to either hemispheres.

Finally, we have indicated four epochs i.e. 1970, 1975, 1980 and 1985 in the upper part of the Figures 1 and 3. Values of the mean anomaly (M) related to the position of the planets in January of the above mentioned years were obtained from the *American Ephemeris and Nautical Almanac*. The corresponding true anomaly (W) was calculated from the well-known relationship:

$$W = M + [2e - (e^3/4)] \sin M + (5/4)\, e^2 \sin 2M + (13/12)\, e^3 \sin 3\, M. \quad (7)$$

In this expression we kept only terms up to the third degree in e, but this is sufficiently accurate for our computations.

According to Vorob'yev and Monin (1975) the planetocentric longitude of the perihelion may be written in terms of the argument of perihelion (ω) as

$$\lambda_p = \omega + \Lambda, \quad (8)$$

where Λ is the planetocentric longitude of the ascending node altered by 180° and may be expressed in the general form

$$\Lambda = f\,(i,\ \Omega,\ \tilde{\omega},\ \epsilon,\ \epsilon_0,\ \alpha_0,\ \delta_0), \quad (9)$$

with

$$\tilde{\omega} = \Omega + \omega; \quad (10)$$

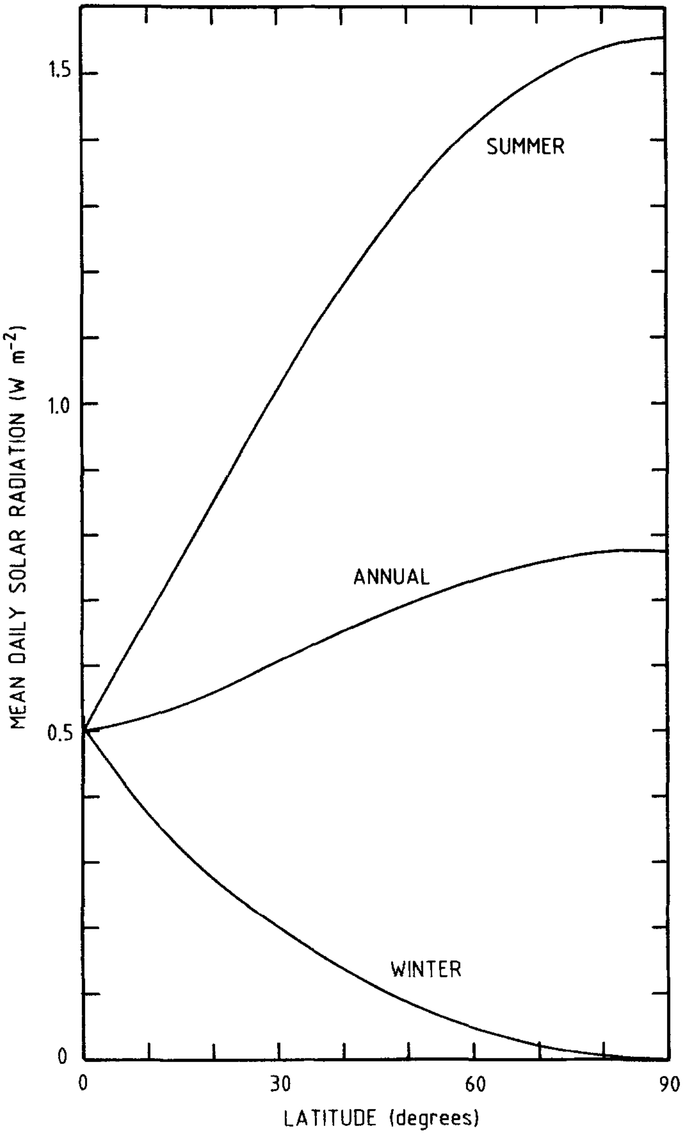

Fig. 5. Latitudinal variation of the mean daily solar radiations at the top of the atmosphere of Uranus.

where i, Ω, $\tilde{\omega}$, ϵ, ϵ_0, α_0, and δ_0 are, respectively, the inclination to the ecliptic, the mean longitude of the ascending node, the mean longitude of the perihelion, the obliquity, the angle between the Earth's spin axis an its orbit normal ($\epsilon_0 = 23.44°$) and the right ascension and declination of the north pole. i, Ω and $\tilde{\omega}$ are taken from the *American Ephemeris and Nautical Almanac*, whereas α_0 and δ_0 are the recommended values reported by Davies *et al.* (1983). Expressions (2), (7) and (8) allow the calculation of the planetocentric longitude of the Sun ($\lambda_\odot$) corresponding approximately to the beginning of the years 1970, 1975, 1980, and 1985.

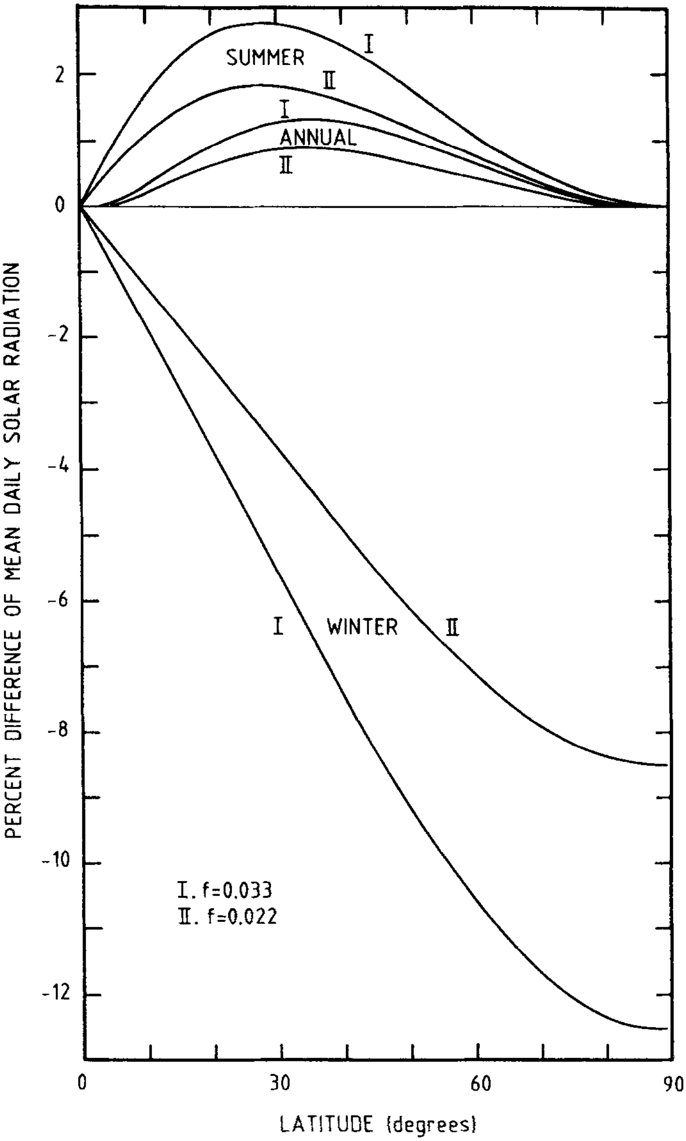

Fig. 6. Latitudinal variation of the percent difference of the mean daily solar radiations with and without the oblateness effect on Uranus.

3.2. MEAN DAILY SOLAR RADIATION

The mean (summer, winter and annual) daily solar radiations on Uranus are depicted in Figure 5 for $f = 0.033$ and for the northern hemisphere. Due to the insufficiency of the scale adopted for the ordinate the curves corresponding to an oblateness factor equal to 0.022 coincide with those of Figure 5. Values for the southern hemisphere are only scarcely different from the northern ones except for the mean annual daily solar radiation values which are symmetric with respect to the planet's equator. The percentual differences between an oblate and a spherical planet Uranus are illustrated in Figure 6.

From Figure 5 it is obvious that the equatorial daily solar radiations averaged over a season or a year are approximately equal and amount to about 0.5 W m^{-2}. At the pole, the mean summer daily insolation reaches its maximum value (1.55 W m^{-2}) and the yearly averaged solar radiation is about one-half of the summertime insolation. In winter, the Sun does not rise at the poles and, consequently, $(\bar{I}_{DO})_W = 0$.

The very large obliquity of Uranus results in an increased mean summertime insolation over the entire latitudinal region (Figure 6). The gain of insolation is of most importance between 25 and 30° with values of about 2.8% ($f = 0.033$) and 1,8% ($f = 0.022$). In winter, and at polar region latitudes, as much as 12.5% and 8.5%, respectively, of the mean winter daily insolation is lost through the oblateness effect.

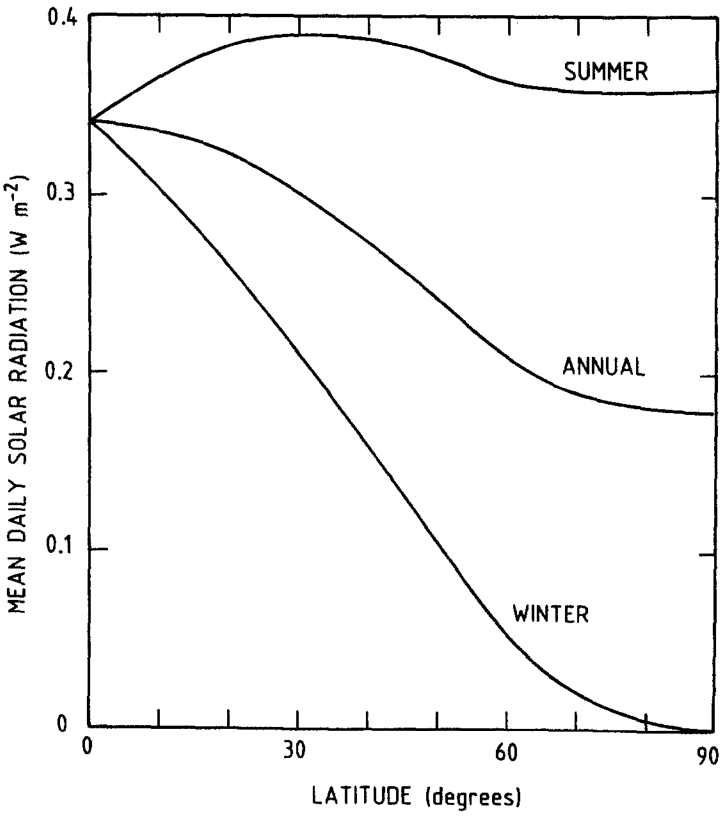

Fig. 7. Latitudinal variation of the mean daily solar radiations at the top of the atmosphere of Neptune.

Furthermore, it is obvious that the percent differences of the mean annual daily insolations are governed by the two opposite summer and winter effects. For Uranus this results in an increase of insolation taking a maximum value approaching 1.3% and 0.9% at latitudes near 30-40°.

The mean daily solar radiations incident on Neptune are given in Figure 7, their percent differences are plotted in Figure 8. At the equator, the mean daily insolations are the same and amount to about 0.34 W m^{-2}. The mean summer daily insolation reaches a value of 0.36 W m^{-2} at the pole and a peak value of 0.39 W m^{-2} in the 25–40° latitudinal interval. Finally, at the poles, the daily solar radiation averaged over a Neptunian year is equal to 50% of the summertime insolation.

When comparing Figure 5 with Figure 7 a striking difference is noticed: the steady increase of the mean annual daily insolation of Uranus as a function of latitude –

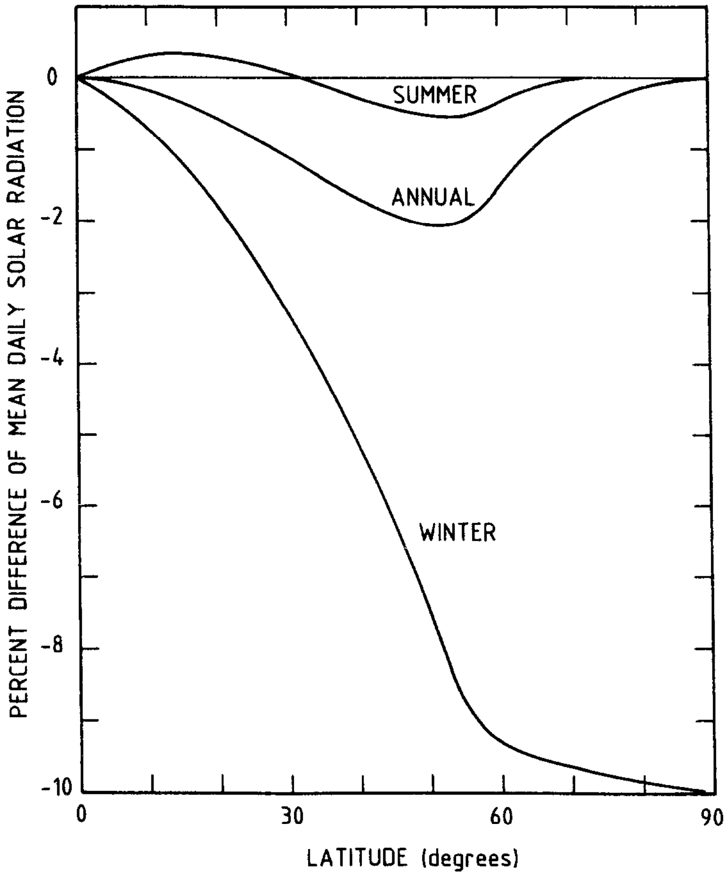

Fig. 8. Latitudinal variation of the percent difference of the mean daily solar radiations with and without the oblateness effect on Neptune.

the latitudinal decrease of the corresponding insolation for Neptune. This is explained by the fact that, for the outer planets, there exists a critical obliquity ($\epsilon \sim 54°$) (Ward, 1974; Vorob'yev and Monin, 1975; Toon *et al.*, 1980) past which the poles receive more annual average energy than the equator (see also French *et al.*, 1983). This situation is not only realised by Uranus but also by Pluto with a present pole position of about 118.5 ° (Harris and Ward, 1982; Dobrovolskis and Harris, 1983).

From Figure 8 it is clear that the mean summer daily insolation is increased between the equator and about the subsolar point, but decreased poleward of the latter limit. The maximum values are +0.35% and −0.55% at about 15 and 55 °. In winter, the horizon plane is always tilted away from the Sun causing the mean winter daiiy insolation to be reduced (maximally by 10% at polar region latitudes); the mean annual daily insolation is decreased by approximately 2% in the latitude interval 45−55 °.

Finally, it should be pointed out that Figure 6 and 8 seem to be somewhat misleading in that they might suggest that the percent difference of the mean winter daily solar radiation reaches its maximum value at the pole. This is, of course, not the case because at $\phi' = 90°$ both $(\bar{I}_D)_W$ and $(\bar{I}_{DO})_W$ equal zero. The curve in the 89-90° latitudinal interval has been omitted due to the fact that in this relatively small region the above mentioned curve roughly coincides with the ordinate at the right side of the two figures.

In conclusion, this paper constitutes an updating of the results obtained by Vorob'yev and Monin (1975), Levine *et al.* (1977) and Van Hemelrijck (1982a) for the planets Uranus and Neptune. Solar radiation values are expressed in the commonly adopted International System of Units.

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References

Anderson, L. E. and Fix, J. D.: 1973, *Icarus* **20**, 279.

Beebe, R. F.: 1983, *Rev. Geophys. Space Phys.* **21**, 143.

Beebe, R. F., Suggs, R. M., and Little, T.: 1986, *Icarus* **66**, 359.

Belton, M. J. S. and Terrile, R.: 1984, 'Rotational properties of Uranus and Neptune', in J. T. Bergstrahl (ed.), *Uranus and Neptune*, NASA CP-2330, pp. 327–347.

Belton, M. J. S., Wallace, L., Hayes, S. H., and Price, M. J.: 1980, *Icarus* **42**, 71.

Belton, M. J. S., Wallace, L., and Howard, S.: 1981, *Icarus* **46**, 263.

Brinkman, A. W. and McGregor, J.: 1979, *Icarus* **38**, 479.

Brown, R. H., Cruikshank, D. P., and Tokunaga, A. T.: 1981, *Icarus* **47**, 159.

Brown, R. A. and Goody, R. M.: 1977, *Astrophys. J.* **217**, 680.

Brown, R. A. and Goody, R. M.: 1980, *Astrophys. J.* **235**, 1066.

Cruikshank, D. P.: 1978, *Astrophys. J.* **220**, L57.

Cruikshank, D. P.: 1985, *Icarus* **64**, 107.

Danielson, R. E., Tomasko, M. G., and Savage, B. D.: 1972, *Astrophys. J.* **178**, 1972.

Davies, M. E., Abalakin, V. K., Lieske, J. H., Seidelmann, P. K., Sinclair, A. T., Sinzi, A. M., Smith, B. A., and Tjuflin, Y. S.: 1980, *Cel. Mech.* **22,** 205.

Davies, M. E., Abalakin, V. K., Lieske, J. H., Seidelmann, P. K., Sinclair, A. T., Sinzi, A. M., Smith, B. A., and Tjuflin, Y. S.: 1983, *Cel. Mech.* **29,** 309.

Dobrovolskis, A. R. and Harris, A. W.: 1983, *Icarus* **55,** 231.

Dollfus, A.: 1970, *Icarus* **12,** 101.

Elliot, J. L.: 1979, *Ann. Rev. Astron. Astrophys.* **17,** 445.

Elliot, J. L., Dunham, E., Mink, D. J., and Churms, J.: 1980, *Astrophys. J.* **236,** 1026.

Elliot, J. L., French, R. G., Frogel, J. A., Elias, J. H., Mink, D. J., and Liller, W.: 1981, *Astron. J.* **86,** 444.

Franklin, F. A., Avis, C. C., Colombo, G., and Shapiro, I. I.: 1980, *Astrophys. J.* **236,** 1031.

Freeman, K. C. and Lyngå, G.: 1970, *Astrophys. J.* **160,** 767.

French, R. G.: 1984, 'Oblateness of Uranus and Neptune', in J. T. Bergstrahl (ed.), *Uranus and Neptune,* NASA CP-2330, pp. 349–355.

French, R. G., Elliot, J. L., Dunham, E. W., Allen, D. A., Elias, J. H., Frogel, J. A., and Liller, W.: 1983, *Icarus* **53,** 399.

French, R. G., Melroy, P. A., Baron, R. L., Dunham, E. W., Meech, K. J., Mink, D. J., Elliot, J. L., Allen, D. A., Ashley, M. C. B., Freeman, K. C., Erickson, E. F., Goguen, J., and Hammel, H. B.: 1985, *Astron. J.* **90,** 2624.

Golitsyn, G. S.: 1979, *Icarus* **38,** 333.

Handbook of the British Astronomical Association, 1980, pp. 100–101, Sunfield and Day Ltd., Eastbourne, East Sussex.

Harris, A. W. and Ward, W. R.: 1982, *Ann. Rev. Earth Planet. Sci.* **10,** 61.

Hayes, S. H. and Belton, M. J. S.: 1977, *Icarus* **32,** 383.

Hildebrand, R. H., Loewenstein, R. F., Harper, D. A., Orton, G. S., Keene, J., and Whitcomb, S. E.: 1985, *Icarus* **64,** 64.

Hubbard, W. B., Frecker, J. E., Gehrels, J.-A., Gehrels, T., Hunten, D. M., Lebofsky, L. A., Smith, B. A., Tholen, D. J., Vilas, F., Zellner, B., Avey, H. P., Mottram, K., Murphy, T., Varnes, B., Carter, B., Nielsen, A., Page, A. A., Flu, H. H., Wu, H. H., Kennedy, H. D., Waterworth, M. D., and Reitsema, H. J.L 1986, *Astron. J.* **90,** 655.

Hughes, D. W.: 1979, *Nature (London)* **279,** 582.

Kovalevsky, J. and Link, F.: 1969, *Astron. Astrophys.* **2,** 398.

Levine, J. S., Kraemer, D. R., and Kuhn, W. R.: 1977, *Icarus* **31,** 136.

Münch, G. and Hippelein, H.: 1980, *Astron. Astrophys.* **81,** 189.

Newburn, R. L., Jr. and Gulkis, S.: 1973, *Space Sci. Rev.* **3,** 179.

O'Meara, S. J.: 1984, *IAU Circ.* No. 3912.

Orton, G. S., Griffin, M. J., Ade, P. A. R., Nolt, I. G., Radostitz, J. V., Robson, E. I., and Gear, W. K.: 1986, *Icarus* **67,** 289.

Podolak, M., Young, R., and Reynolds, P. T.: 1985, *Icarus* **63,** 266.

Slavsky, D. and Smith, H. J.: 1978, *Astrophys. J.* **226,** L49.

Slavsky, D. and Smith, H. J.: 1981, *Bull. Amer. Astron. Soc.* **13,** 733.

Smith, H. J. and Slavsky, D.: 1978, 'The Rotation Period of Neptune', presented at the Tenth Annual DPS/AAS Meeting, Pasadena, November 1, 1978.

Smith, H. J. and Slavsky, D.: 1979, *Bull. Amer. Astron. Soc.* **11,** 568.

Toon, O. B., Pollack, J. B., Ward, W., Burns, J. A., and Bilski, K.: 1980, *Icarus* **44,** 552.

Trafton, L.: 1977, *Icarus* **32,** 402.

Trauger, J. T., Roesler, F. L., and Münch, G.: 1978, *Astrophys. J.* **219,** 1079.

Van Hemelrijck, E.: 1982a, *Icarus* **51,** 39.

Van Hemelrijck, E.: 1982b, *Icarus* **52,** 560.

Van Hemelrijck, E.: 1982c, *Bull. Acad. R. Belg., Cl. Sci.* **68,** 675.

Van Hemelrijck, E.: 1983, *The Moon and the Planets* **28,** 125.

Van Hemelrijck, E.: 1985, *Earth, Moon, and Planets* **33,** 163.

Van Hemelrijck, E.: 1987, *Earth, Moon, and Planets* **38,** 217.

Vorob'yev, V. I. and Monin, A. S.: 1975, *Atm. Ocean. Phys.* **11,** 557.

Wagener, R., Caldwell, J., and Fricke, K.-H.: 1986, *Icarus* **67,** 281.

Ward, W. R.: 1974, *J. Geophys. Res.* **79**, 3375.
Williams, J. G. and Benson, G. S.: 1971, *Astron. J.* **76**, 167.
Willson, R. C.: 1982, *J. Geophys. Res.* **87**, 4319.
Willson, R. D., Gulkis, S., Janssen, M., Hudson, H. S., and Chapman, G. A.: 1981, *Science* **211**, 700.