Collisional Kinetic Theory of the Escape of Light Ions from the Polar Wind.

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SUMMARY

The departure from equilibrium in the ionized exosphere of the Earth's high altitude atmosphere is studied with the solution of the Boltzmann equation. A rigorous collisional formalism is outlined for a plane parallel model, and a methodology for the calculation of the escape fluxes of protons and He^+ ions is presented. Coulomb collisions between protons or He^+ with O^+ ions, the major constituent at these altitudes are treated rigorously with the Fokker-Planck collision operator in the Boltzmann equation. A half-range expansion of the distribution function is employed in place of the usual Legendre polynomial expansion, and permits an efficient method for fitting the boundary conditions of the distribution function.

1 INTRODUCTION

In the high altitude terrestrial ionosphere, the exobase level occurs where the average mean free path equals the density scale height. The escape of protons and alpha particles from this region of the topside ionosphere (approximately 1500 km) is referred to as the polar wind [1] in analogy with the supersonic expansion of the solar atmosphere known as the solar wind.

Previous kinetic theory treatments of the polar wind [2,3] involved the assumption that the ionosphere is collision dominated below the exobase and collisionless above. With this assumption, the ion velocity distribution function is given by the collisionless Boltzmann equation with the velocity distribution at the exobase as the boundary condition.

The present paper considers a detailed study of the effect of Coulomb collisions in the ionosphere in the vicinity of the exobase where the atmosphere undergoes a transition from collision dominated to collisionless. The objectives of the present paper are analogous to those of a previous paper which dealt with the neutral atmosphere [4]. The point of departure with the earlier paper is the use of half-range expansions of the velocity distribution function [5] in place of the usual Legendre polynomial expansion. Also, for this rarefied plasma, the Fokker-Planck operator instead of the hard sphere Boltzmann integral operator is employed.

The solution of the Boltzmann equation is obtained with a combination of a discrete ordinate method and a polynomial (or moment) method. The final objectives are the values of the escape fluxes of protons and alpha particles in comparison with the escape fluxes obtained with the collisionless approach.

2 PLANE PARALLEL MODEL

We consider a two-component atmosphere consisting of a major (heavy) constituent and a minor (light) constituent. The major constituent is bound to the planet and acts as a background gas in thermal equilibrium and a Maxwell-Boltzmann velocity distribution function. For the high altitude terrestrial ionosphere the major constituent is O^+ ions whereas the light species are either H^+ or He^+ ions. The requirement of quasi-neutrality in such an ionized plasma in a gravitational field leads to an outward directed electric field given by,

$$E = \frac{Mg(r)}{2e}\, r/r, \tag{1}$$

where M is the mass of O^+, g is the acceleration of gravity at geocentric radial distance r, and e is the electron charge. This polarization electric field, known as the Pannekoek Rosseland field [2], although not rigorously correct is adequate for the present effort. Owing to the presence of the electric field, positively charged particles are accelerated outwards and they escape regardless of their speed as long as they are directed outwards from the planet. With the assumption of a Maxwell-Boltzmann distribution at the exobase, the escape flux of protons is given by

$$F = n_c\left(\frac{kT}{2\pi m}\right)^{1/2}. \tag{2}$$

where n_c is the H^+ (or He^+) exobasic density. However, owing to the loss of particles, the distribution at the exobase is no longer a Maxwellian and the actual escape flux differs from that given by equation (2).

The objective of the present paper is to calculate the actual

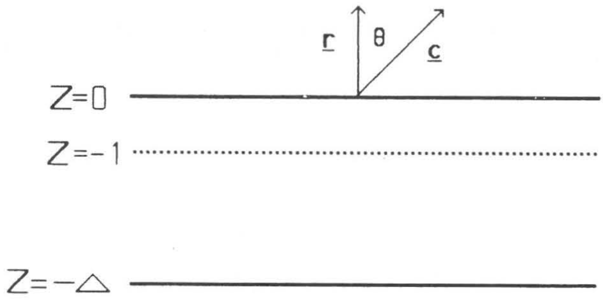

Figure 1: Plane Parallel Model of the Ion Exosphere.

nonequilbirium distribution function from the Boltzmann equation,

$$\frac{\partial f}{\partial t} + c \cdot \nabla_r f + \frac{1}{2m} e\, E \cdot \nabla_c f = C[f] \tag{3}$$

where C is the linear Fokker-Planck collision operator [8] for collisions between the minor ion species, of mass m, and the major background ion component taken to be at equilibrium.

We propose to solve the Boltzmann equation (equation (3)) for the portion of the atmosphere of thickness equal to several mean free paths in the vicinity of the exobase as shown in Figure 1. We employ a dimensionless altitude variable defined by

$$z = -\int_r^{r_{top}} \sigma_{eff} N(r')dr' \tag{4}$$

where σ_{eff}, defined explicitly later, is some constant cross section equal to the Coulomb cross section at some average energy, and r_{top} is some sufficiently large radial distance. With the assumption of a plane parallel atmosphere, linear trajectories and the introduction of dimensionless speed variable $y^2 = \frac{mc^2}{2kT}$,

we have the Boltzmann equation,

$$D[f] = C[f] \tag{5}$$

where

$$D[f] = y\mu \frac{\partial f}{\partial z} + a(r)\left(\mu \frac{\partial f}{\partial y} + \frac{1}{y}(1-\mu^2)\frac{\partial f}{\partial \mu}\right), \tag{6}$$

$a(r)$, defined explicitly later, is the field term, and $\mu = \cos(\theta)$ where θ is the angle between c and r. The collision operator on the RHS of equation (5) is given by

$$C[f] = L_0[f] + L_1[f], \tag{7}$$

were L_0 is the isotropic portion of the operator given by,

$$L_0[f] = \frac{\gamma}{y^2}\frac{\partial}{\partial y}\left(F(y)[2\rho y f + \frac{\partial f}{\partial y}]\right), \tag{8}$$

where $\rho = (m/M)^{1/2}$ is the mass ratio parameter. The anisotropic operator is given by

$$L_1[f] = \gamma P(y)\left(\frac{\partial}{\partial x}(1-x^2)\frac{\partial}{\partial x} + 2\frac{\partial}{\partial x}(1-x)\frac{\partial}{\partial x}\right)f. \tag{9}$$

where

$$\sigma_{eff} = \frac{c_0}{(2kT)^2}$$

$$a(r) = eEkT/Nc_0$$

$$x = 2\mu - 1$$

$$\gamma = \frac{mc_0}{M^2}\left(\frac{m}{2kT}\right)^2$$

$$P(y) = \frac{1}{y^3\rho}\frac{dg}{dy}$$

$$g(y) = \mathrm{erf}(\rho y)[\rho y + \frac{1}{2\rho y}] + \exp(-\rho^2 y^2)/\pi^{1/2}$$

$$F(y) = \mathrm{erf}(\rho y)/\rho y - 2\exp(-\rho^2 y^2)/\pi^{1/2}$$

where c_0 contains the plasma parameters and is given by Hinton [6]. We seek a solution to equation (5) subject to the two boundary conditions, one at the lower boundary where the atmosphere is collision dominated and a second boundary condition at the top boundary to account for the escape of ions. At the bottom boundary we take the distribution function to be of the form,

$$f(y,\mu,-\Delta) = f^M + \mu U(y), \tag{10}$$

where f^M is the Maxwellian and the additional term accounts for the finite diffusion of particles. At the upper boundary, we assume that there are no incoming particles from above, hence,

$$f(y,\mu,0) = 0 \qquad -1<\mu<0. \tag{11}$$

3 HALF RANGE SOLUTION METHOD

The Boltzmann equation is solved by splitting the distribution function into two portions, one for upward moving particles and one for downward moving particles, that is,

$$f(y,\mu,z) = H(\mu)f^+(y,\mu,z) + [1-H(\mu)]f^-(y,\mu,z), \tag{12}$$

where $H(\mu)$ is the Heaviside function. Further details of this half range moment method are given in a companion paper [7]. It is convenient to define two functions

$$F(y,\mu,z) = f^{+}(y,\mu,z) + f^{-}(y,\mu,z) \tag{13}$$

and

$$G(y,\mu,z) = f^{+}(y,\mu,z) - f^{-}(y,\mu,z), \tag{14}$$

and to employ the expansions,

$$F(y,\mu,z) = \sum_{\ell=0}^{\infty} F_{\ell}(y,z)P_{\ell}(2\mu-1) \tag{15}$$

$$G(y,\mu,z) = \sum_{\ell=0}^{\infty} G_{\ell}(y,z)P_{\ell}(2\mu-1) \tag{16}$$

where the coefficients in these expansions are to be obtained from the Boltzmann equation. The final desired quantity is the particle flux which is given by,

$$J(z) = \pi\left(\frac{2kT}{m}\right)^{2}\int_{0}^{\infty}[G_{0} + \frac{1}{3}G_{1}]y^{3}dy. \tag{17}$$

The moment equations obtained from the Boltzmann equation are of the form

$$\sum_{\lambda=-1}^{1}\left(\alpha_{\lambda}(\ell)[y\frac{\partial G_{\ell-\lambda}}{\partial z} + a\frac{\partial G_{\ell-\lambda}}{\partial y}] + \frac{a}{y}\beta_{\lambda}(\ell)G_{\ell-\lambda}\right)$$
$$+\frac{a}{y}\sum_{\ell=\ell'}^{\infty}(-1)^{\ell-\ell'}G_{\ell} = \frac{1}{2\ell+1}L_{0}F_{\ell} + H_{\ell}(z,y), \tag{18}$$

where,

$$H_{\ell}(z,y) = P(y)\left(-\frac{\ell(\ell+1)}{2\ell+1}F_{\ell} + \sum_{\ell'=\ell+1}^{\infty}(-1)^{\ell'-\ell}(\ell'-\ell)(\ell'+\ell+1)F_{\ell'}\right.$$
$$\left. - \sum_{\ell'=0}^{\infty}(-1)^{\ell'+\ell}\ell'(\ell'+1)F_{\ell}\right) \tag{19}$$

Similarly we get the second set of equations coupled to equation

(18) as given by,

$$\sum_{\lambda=-1}^{1}\left(\alpha_\lambda(\ell)[y\frac{\partial F_{\ell-\lambda}}{\partial z}+ a\frac{\partial F_{\ell-\lambda}}{\partial y}]+\frac{a}{y}\beta_\lambda(\ell)F_{\ell-\lambda}\right)$$
$$+\frac{a}{y}\sum_{\ell=\ell'}^{\infty}(-1)^{\ell-\ell'}F_\ell = \frac{1}{2\ell+1}L_oG_\ell + J_\ell(z,y), \tag{20}$$

where,

$$J_\ell(z,y) = P(y)\left(-\frac{\ell(\ell+1)}{2\ell+1}G_\ell+\sum_{\ell'=\ell+1}^{\infty}(-1)^{\ell'-\ell}(\ell'-\ell)(\ell'+\ell+1)\,G_{\ell'}\right)$$

The quantities $\alpha_\lambda(\ell)$ and $\beta_\lambda(\ell)$ are the matrix elements of the operators in the drift term in the Boltzmann equation that involve μ. These are easily evaluated with the recursion relations and orthogonality properties of the Legendre polynomials.

4 DISCRETE ORDINATE METHOD OF SOLUTION

The moment equations (18) and (20) are to be solved subject to the boundary conditions, equations (10) and (11). These moment equations are partial differential equations in the altitude z and particle speed y. The numerical method of solution of these equations is the discrete ordinate (DO) method employed in previous papers [4,5] and described in detail elsewhere [8]. The method is based on the numerical evaluation of derivatives at a set of Gauss quadrature points as given by

$$\left(\frac{\partial G_{\ell-\lambda}}{\partial z}\right)_{z=z_i} = \sum_{j=1}^{N} D^{(z)}{}_{ij}G_{\ell-\lambda}(z_j),$$

where the matrix $D^{(z)}$ is the DO representation of the altitude derivative operator. For the altitude variable we employed a quadrature based on Legendre polynomials on the interval $[0,-\Delta]$

where the first and last quadrature points coincide with the interval boundaries. A set of speed quadrature points is employed for the speed variable y in the same way. With the application of this differentiation rule, the moment equations are reduced to a set of coupled algebraic equations for the distribution function as given by $F_\ell(z_i,y_j)$ and $G_\ell(z_i,y_j)$. These equations are solved with an iteration as discussed in the previous paper. The boundary conditions are imposed at each step in the iterative solution. With the converged values of G_0 and G_1, the particle flux is then calculated with equation (17). The use of the half range expansion and the quadrature procedure based on speed polynomials is expected to give very rapid convergence of the solution. Numerical results based on the formalism presented in this paper will be published elsewhere.

5 ACKNOWLEDGEMENTS

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