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SCALE TIMES AND SCALE LENGTHS ASSOCIATED WITH CHARGED PARTICLE FLUCTUATIONS IN THE LOWER IONOSPHERE

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Abstract—The concept of chemical fluctuations associated with electron attachment and detachment processes in the terrestrial *D*-region implies the existence of scale times and scale lengths over which these phenomena can be observed. An extension of Chapman's definition of scale times and scale lengths to fluctuating quantities leads to an eigenvalue problem which can be solved numerically, and for which simple analytical approximations are obtained. Two scale times associated with negative ion chemistry and with neutralization, respectively, differ by several orders of magnitude. One scale length indicates how the classical Debye length is modified by the presence of negative ions. Two other scale lengths are the spatial correspondence of the scale times. Whereas the scale length associated with negative ion chemistry ranges from 10 to 80 cm, the scale length corresponding to recombination processes ranges from 5 to 10 m.

1. INTRODUCTION

A quantitative analysis of any geophysical phenomenon requires an estimation of characteristic times and lengths over which the process is significant. The scale height associated with the atmospheric pressure was introduced by Chapman (1936), and a generalization for other atmospheric parameters (Chapman, 1961) clearly indicates the usefulness of scale times and scale lengths. This is particularly true when analytical expressions can be developed for the logarithmic derivative of a given quantity. Such a technique has been applied by Kockarts (1966) for a quantitative description of the neutral thermosphere above 120 km altitude.

Recently, Kockarts and Wisemberg (1981) and Wisemberg and Kockarts (1983) showed that fluctuations of the electron concentration can occur around an equilibrium value in the terrestrial *D*-region as a consequence of electron attachment to neutral particles followed by detachment or photodestruction processes. Such fluctuations should be characterized by scale times and scale lengths which are related to temporal and spatial variations of the physical processes.

The straightforward generalization of Chapman's definition given in Section two is applied to the continuity and momentum equations for the fluctuating quantities presented in Section three. The resulting eigenvalue problem is solved numerically in Sections four and five for the various scale times and scale lengths, respectively. Simple analytical approximations are presented simultaneously for practical applications to the lower ionosphere. The geophysical significance of each scale time and scale length is obvious from the parameters involved in the analytical approximations.

2. GENERALIZATION OF CHAPMAN'S DEFINITION

For a geophysical quantity y_i, Chapman (1961) defines a scale time or a scale length by $1/[\partial(ln y_i)/\partial x]$ depending on whether x represents the time or a spatial coordinate, respectively. Let us consider a set of quantities y_i representing scalar functions or cartesian components of vectors. This set forms a vector $\mathbf{y}$ for which it is eventually possible to write a relation such as

$$\frac{\partial \mathbf{y}}{\partial x} = A\mathbf{y}, \tag{1}$$

where A is a square matrix with dimensions equal to the number of quantities y_i, and x is a time or spatial coordinate. If matrix A has several non-zero eigenvalues $a(j)$, the solution of equation (1) is a linear combination of all corresponding eigenvectors $\mathbf{y}(j)$. The eigenvalues $a(j)$ are obtained by solving the secular equation

$$\det(A - aI) = 0, \tag{2}$$

where I is a unitary matrix. For each eigenvector $\mathbf{y}(j)$, equation (1) reduces to

$$\frac{\partial \mathbf{y}(j)}{\partial x} = a(j)\mathbf{y}(j). \tag{3}$$

Integration of equation (3) leads to

$$\mathbf{y}(j) = \mathbf{y}_0(j)\exp[a(j)x], \tag{4}$$

where $\mathbf{y}_0(j)$ is the eigenvector at a reference level.

The quantity $1/a(j)$ has the dimension of time or length depending on the dimension of x, respectively. The quantity $1/a(j)$ is, therefore, the *scale time* or *scale length* associated with the eigenvector $\mathbf{y}(j)$. If $1/a(j)$ is positive, we have a phenomenon growing with time or space, and for negative values the phenomenon decays by 1/e over a time (or space) interval $x = -1/a(j)$.

As a consequence, the matrix formulation (1) associated with the solutions of the secular equation (2) leads to a set of eigenvalues $a(j)$, which are the inverse values of scale times and scale lengths describing the behavior of the geophysical quantities y_i.

3. CONTINUITY, MOMENTUM AND POISSON'S EQUATIONS FOR FLUCTUATING QUANTITIES

Using signal flow graph theory, Wisemberg and Kockarts (1980) and Kockarts and Wisemberg (1981) showed that five equivalent processes are sufficient to reproduce the total negative, positive and electron concentrations in the terrestrial D-region. These reactions are as follows

$$\text{input} \rightarrow \mathrm{e} + X^+; \quad \gamma_e\,(\mathrm{cm^{-3}\,s^{-1}}) \tag{5}$$

$$\text{neutrals} + \mathrm{e} \rightarrow X^-; \quad L_e\,(\mathrm{s^{-1}}) \tag{6}$$

$$\text{neutrals or light} + X^- \rightarrow \mathrm{e}; \quad L_-\,(\mathrm{s^{-1}}) \tag{7}$$

$$\mathrm{e} + X^+ \rightarrow \text{neutrals}; \quad \alpha_e\,(\mathrm{cm^3\,s^{-1}}) \tag{8}$$

$$X^- + X^+ \rightarrow \alpha_-\,(\mathrm{cm^3\,s^{-1}}), \tag{9}$$

where γ_e is the external production rate, L_e is the effective attachment rate for electrons, L_- is the effective loss rate by collisional detachment, by photodissociation and by photodetachment for negative ions. The electron–ion and ion–ion recombination rates are α_e and α_-, respectively.

For such a model, the continuity equation for each type s of charged particles (s = e, +, −) can be written in a one-dimensional space as

$$\frac{\partial n_e}{\partial t} + \frac{\partial}{\partial z}(n_e v_e) = \gamma_e + n_- L_- - n_e L_e - \alpha_e n_e n_+ \tag{10}$$

$$\frac{\partial n_-}{\partial t} + \frac{\partial}{\partial z}(n_- v_-) = n_e L_e - n_- L_- - \alpha_- n_- n_+ \tag{11}$$

$$\frac{\partial n_+}{\partial t} + \frac{\partial}{\partial z}(n_+ v_+) = \gamma_e - \alpha_e n_e n_+ - \alpha_- n_+ n_-, \tag{12}$$

where t is time and z is altitude.

Neglecting viscosity, three momentum equations (s = e, +, −) can be written as

$$\frac{\partial v_s}{\partial t} + v_s\frac{\partial v_s}{\partial z} + (n_s m_s)^{-1}\frac{\partial p_s}{\partial z} + g - (q_s/m_s)E = -\nu_{sn}(v_s - v_n), \tag{13}$$

where q_s, m_s and p_s are the electrical charges, the masses and partial pressures of the charged species, respectively. The vertical component of the velocity is v_s (s = e, +, −) for each charged species. The collision frequencies between charged particles and neutral components are ν_{sn}, and v_n is the velocity of the neutral atmosphere. The acceleration of gravity is represented by g and the vertical component E of the electric field must satisfy Poisson's equation

$$\frac{\partial E}{\partial z} = \varepsilon_0^{-1}\sum_s q_s n_s, \tag{14}$$

where ε_0 is the permittivity of free space.

When concentrations n_s, velocities v_s and electric field E are solutions of equations (10)–(14), it is possible to imagine that small fluctuations occur around these solutions. Such fluctuations result from thermal agitation and from chemical reactions when the latter are considered as stochastic processes. For each n_s and v_s, we assume that $n_s = n_{s0} + n'_s$ and $v_s = v_{s0} + v'_s$, n_{s0} and v_{s0} being the non-fluctuating solutions of equations (10)–(13). Similarly, the electric field is $E = E_0 + E'$ where E_0 satisfies equation (14). The fluctuating quantities n'_s, v'_s and E' are always small, so that we can neglect second order fluctuations such as $n'_s v'_s$.

With these assumptions, the continuity equations for the fluctuating quantities are obtained from equations (10)–(12) as

$$\frac{\partial n'_e}{\partial t} + n_{e0}\frac{\partial v'_e}{\partial z} = -\mathscr{L}_e n'_e + L_- n'_- - \alpha_e n_{e0} n'_+ \tag{15}$$

$$\frac{\partial n'_-}{\partial t} + n_{-0}\frac{\partial v'_-}{\partial z} = L_e n'_e - \mathscr{L}_- n'_- - \alpha_- n_{-0} n'_+ \tag{16}$$

$$\frac{\partial n'_+}{\partial t} + n_{+0}\frac{\partial v'_+}{\partial z} = -\alpha_e n_{+0} n'_e - \alpha_- n_{+0} n'_- - \alpha_+ n_{+0} n'_+, \tag{17}$$

where

$$\alpha_+ = (\alpha_e n_{e0} + \alpha_- n_{-0})/n_{+0} = (\alpha_e + \lambda\alpha_-)/(\lambda + 1) \tag{18}$$

with $\lambda = n_{-0}/n_{e0}$ and

$$\mathscr{L}_s = L_s + \alpha_s n_{+0} \quad (19)$$

for $s = \mathrm{e}$ and $s = -$. It should be mentioned that equations (15)–(17) also imply that $v_{s0} = 0$ and that the external production γ_e does not fluctuate. The three momentum equations (13) lead in absence of any neutral movement ($v_n = 0$) to

$$\frac{\partial v'_s}{\partial t} + (V_s^2/n_{s0})\frac{\partial n'_s}{\partial z} - (q_s/m_s)E' = -\nu_{sn}v'_s, \quad (20)$$

where $V_s = (\mathrm{k}T/m_s)^{1/2}$ is the thermal speed of the charged species s (s = e, +, −) with temperature T and mass m_s, k being Boltzmann's constant. As a consequence of concentration and velocity fluctuations, the fluctuating part E' of the electric field must satisfy Poisson's equation

$$\frac{\partial E'}{\partial z} = \varepsilon_0^{-1}\sum_s q_s n'_s. \quad (21)$$

The generalization of Chapman's definition given in Section two can now be applied to equations (15)–(17), (20) and (21) for the deduction of scale times and scale lengths related to the fluctuating part of charged particle species.

4. SCALE TIMES

4.1. *General case*

Scale times can be obtained by ignoring all spatial derivatives in the continuity and momentum equations. In such a way, the continuity equations (15)–(17) and the momentum equations are decoupled and the time derivatives of the fluctuating part of the concentrations are given by

$$\frac{\partial n'_e}{\partial t} = -\mathscr{L}_e n'_e + L_- n'_- - \alpha_e n_{e0} n'_+ \quad (22)$$

$$\frac{\partial n'_-}{\partial t} = L_e n'_e - \mathscr{L}_- n'_- - \alpha_- n_{-0} n'_+ \quad (23)$$

$$\frac{\partial n'_+}{\partial t} = -\alpha_e n_{+0} n'_e - \alpha_- n_{+0} n'_- - \alpha_+ n_{+0} n'_+. \quad (24)$$

Using expressions (18) and (19), it appears that

$$\frac{\partial}{\partial t}(n'_e + n'_- - n'_+) = 0. \quad (25)$$

This expression implies that $n'_+ = n'_e + n'_-$. Eliminating n'_+ in the system (22)–(24), one obtains

$$\frac{\partial n'_e}{\partial t} = -[L_e + \alpha_e(\lambda+2)n_{e0}]n'_e + (L_- - \alpha_e n_{e0})n'_- \quad (26)$$

$$\frac{\partial n'_-}{\partial t} = (L_e - \lambda\alpha_- n_{e0})n'_e - [L_- + \alpha_-(2\lambda+1)n_{e0}]n'_-, \quad (27)$$

where $\lambda = n_{-0}/n_{e0}$. The secular equation defined by (2) for the system (26)–(27) is then given by

$$a^2 + a[L_e + L_- + \alpha_e(\lambda+2)n_{e0} + \alpha_-(2\lambda+1)n_{e0}] + L_e[\alpha_-(2\lambda+1) + \alpha_e]n_{e0} + L_-[\alpha_e(\lambda+2) + \lambda\alpha_-]n_{e0} + 2\alpha_e\alpha_-(\lambda+1)^2 n_{e0}^2 = 0. \quad (28)$$

The two solutions a_1 and a_2 of equation (28) are eigenvalues of the matrix associated with (26) and (27). Since they are both negative, the two time scales $\tau_1 = 1/|a_1|$ and $\tau_2 = 1/|a_2|$ correspond to time exponential decays of the fluctuations. Figure 1 shows the vertical distribution of the two time scales associated with chemical fluctuations. Although equation (28) can be solved analytically, the algebraic structure of the solutions does not show a simple physical interpretation. Therefore, two approximations are now developed in order to give a physical significance to the two time scales of Fig. 1.

4.2. *Recombination effects*

All numerical parameters used in the computation of Fig. 1 are taken from the equivalent model of the *D*-region established by Kockarts and Wisemberg (1981). In this model, negative ions are negligible above 75 km where $\lambda = n_{-0}/n_{e0}$ becomes smaller than 2.3×10^{-2}. Therefore, assuming $\lambda = \alpha_- = L_e = L_- = 0$, the secular equation (28) reduces to

$$a^2 + 2\alpha_e n_{e0} a = 0. \quad (29)$$

The solution $a = -2\alpha_e n_{e0}$ leads to a scale time

$$\tau_3 = (2\alpha_e n_{e0})^{-1}. \quad (30)$$

Below 60 km altitude, negative and positive ions progressively become the major charged species. With $\alpha_e = L_e = L_- = n_{e0} = 0$ and noting that $n_{-0} = \lambda n_{e0} \approx n_{+0}$, the secular equation (28) reduces to

$$a^2 + 2\alpha_- n_{-0} a = 0, \quad (31)$$

which leads to a scale time

$$\tau_4 = (2\alpha_- n_{-0})^{-1}. \quad (32)$$

The combined expression

$$\tau_n = (2\alpha_e n_{e0} + 2\alpha_- n_{-0})^{-1} \quad (33)$$

is not an exact solution of the secular equation (28). Numerical evaluation of (33) leads, however, to scale times which are almost identical to the exact profile τ_1 in Fig. 1. This scale time ranging from 100 s to more than 1 h corresponds, therefore, to fluctuations related

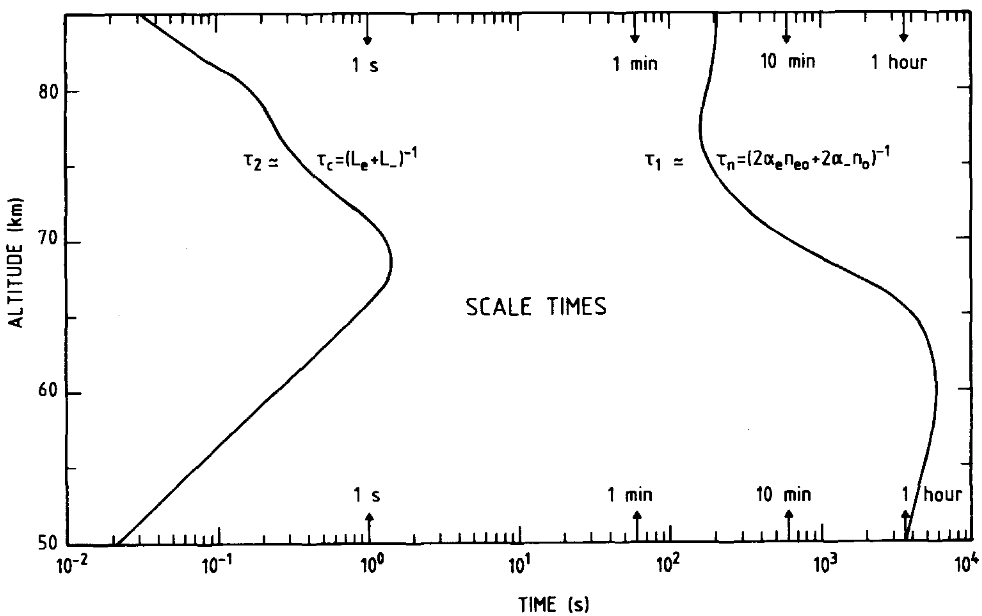

FIG. 1. VERTICAL DISTRIBUTION OF SCALE TIMES.
τ_c is associated with negative ion chemistry. τ_n is associated with recombination processes. Approximate expressions are indicated for each scale time and no significant difference can be seen on the figure.

to recombination processes between negatively charged species (electrons and negative ions) and positive ions.

4.3. *Electron attachment and detachment effects*

Kockarts and Wisemberg (1981) and Wisemberg and Kockarts (1983) have shown that the ratio $\lambda = n_{-0}/n_{e0}$ is mainly controlled by electron attachment and detachment processes and that recombination processes play a negligible role. Assuming, therefore, that $\alpha_e = \alpha_- = 0$ and using the relation $\lambda = L_e/L_-$, the secular equation (28) can be written as

$$a^2+(L_e+L_-)a = 0. \quad (34)$$

The scale time τ_c corresponding to the eigenvalue $a = -(L_e+L_-)$ is simply

$$\tau_c = (L_e+L_-)^{-1}. \quad (35)$$

Numerical evaluation of τ_c leads to scale times which differ by less than 0.3% from the exact values given by curve τ_2 in Fig. 1. The maximum in curve τ_2 occurs exactly at the height where $\lambda = 1$. The scale time τ_2, for which an excellent approximation is given by τ_c, corresponds to electron attachment and detachment processes which control the negative ion distribution in the D-region. Above 70 km altitude the negative ion loss rate L_- is the leading term in τ_c, whereas below 70 km altitude, the electron loss rate L_e is dominant.

5. SCALE LENGTHS

5.1. *General case*

Scale lengths can be obtained by ignoring all time derivatives in the continuity and momentum equations for the fluctuating quantities. Contrary to the case of scale times, these equations are now strongly coupled and Poisson's equation is used as a closure equation.

From the continuity equations (15)–(17) in which $\partial/\partial t$ is set to zero, it can be shown that

$$\frac{\partial}{\partial z}[v'_e+\lambda v'_- -(\lambda+1)v'_+] = 0, \quad (36)$$

where the relation $\alpha_+ = (\alpha_e+\lambda\alpha_-)/(\lambda+1)$ has been used. Equation (36) leads to the following relation between the fluctuating parts of the velocities:

$$v'_+ = (\lambda+1)^{-1}(v'_e+\lambda v'_-). \quad (37)$$

Using (37) to eliminate v'_+, the momentum, continuity and Poisson's equations from Section three can be written as

$$\frac{\partial n'_e}{\partial z} = -(\nu_{en}n_{e0}/V_e^2)v'_e-(q_e n_{e0}/kT)E' \quad (38)$$

$$\frac{\partial n'_-}{\partial z} = -(\nu_{-n}n_{-0}/V_-^2)v'_- - (q_e n_{-0}/kT)E' \quad (39)$$

$$\frac{\partial n'_+}{\partial z} = -(\nu_{+n}n_{e0}/V_+^2)v'_e - (\nu_{+n}n_{-0}/V_+^2)v'_- + (q_e n_{+0}/kT)E' \quad (40)$$

$$\frac{\partial v'_e}{\partial z} = -(\mathscr{L}_e/n_{e0})n'_e + (L_-/n_{e0})n'_- - \alpha_e n'_+ \quad (41)$$

$$\frac{\partial v'_-}{\partial z} = (L_e/n_{-0})n'_e - (\mathscr{L}_-/n_{-0})n'_- - \alpha_- n'_+ \quad (42)$$

$$\frac{\partial E'}{\partial z} = -(q_e/\varepsilon_0)(n'_e + n'_- - n'_+), \quad (43)$$

where q_e is the absolute value of the electron charge.

The system of equations (38)–(43) can be written in a form similar to equation (1), where the six components of vector **y** are n'_e, n'_-, n'_+, v'_e, v'_- and E', respectively. The secular equation (2) is given by

$$a^6 + p_4 a^4 + p_2 a^2 + p_0 = 0, \quad (44)$$

where

$$p_4 = -(\nu_{en}/V_e^2)\mathscr{L}_e - (\nu_{-n}/V_-^2)\mathscr{L}_- - (\nu_{+n}/V_+^2)\alpha_+ n_{+0} - (2q_e^2 n_{+0})/(\varepsilon_0 kT), \quad (45)$$

$$\begin{aligned} p_2 = {} & (\nu_{en}\nu_{-n})/(V_e^2 V_-^2) \\ & \times[\alpha_e\alpha_- n_{+0}^2 + L_-\alpha_e n_{+0} + L_e\alpha_- n_{+0}] \\ & + (\nu_{en}\nu_{+n})/(V_e^2 V_+^2)[L_e\alpha_+ n_{+0} + \alpha_e\alpha_- n_{-0}n_{+0}] \\ & + (\nu_{-n}\nu_{+n})/(V_-^2 V_+^2)[L_-\alpha_+ n_{+0} + \alpha_e\alpha_- n_{e0}n_{+0}] \\ & + (q_e^2 n_{e0})/(\varepsilon_0 kT)\{(\nu_{en}/V_e^2)[(2\lambda+1) \\ & \times L_e + 2(\lambda+1)\alpha_e n_{+0} + \lambda L_-] + (\nu_{-n}/V_-^2) \\ & \times[(\lambda+2)L_- + 2(\lambda+1)\alpha_- n_{+0} + L_e] \\ & + 2(\nu_{+n}/V_+^2)(\lambda+1)\alpha_+ n_{+0}\} \end{aligned} \quad (46)$$

and

$$\begin{aligned} p_0 = {} & -(q_e^2 n_{e0})/(\varepsilon_0 kT)\{\nu_{-n}\nu_{en}(\lambda+1)/(V_-^2 V_e^2) \\ & + \nu_{+n}\nu_{en}\lambda/(V_+^2 V_e^2) + \nu_{-n}\nu_{+n}/(V_-^2 V_+^2)\} \\ & \times\{2\alpha_e\alpha_- n_{+0}^2 + \alpha_e n_{e0}[(\lambda+2)L_- + L_e] \\ & + \alpha_- n_{e0}[(2\lambda+1)L_e + \lambda L_-]\}. \end{aligned} \quad (47)$$

The secular equation (44) of degree six is easily reduced to a cubic equation for which the three solutions are real. The system (38)–(43) has, therefore, six real eigenvalues which lead to three pairs of scale lengths. The positive value of each pair of scale lengths corresponds to decreasing altitudes, and the negative values are related to increasing altitudes such that both in upward and downward directions, fluctuations are a decaying process.

As a consequence of the algebraic complexity of the secular equation, it is useless to write down exact analytical solutions. We have, therefore, computed the eigenvalues corresponding to the system (38)–(43) by using the numerical method of Kaufman (1974, 1975). The absolute values of the corresponding scale lengths are shown in Fig. 2, for which the same equivalent ionospheric model as in Fig. 1 has been used.

5.2. *Generalized Debye length and negative ion chemistry*

We assume, as in Section 4.3, that the neutralization frequencies $\alpha_e n_{+0}$ and $\alpha_- n_{+0}$ are negligible compared to the electron loss rate L_e and to the negative ion loss rate L_-. The secular equation (44) is considerably simplified since coefficient p_0 is zero with $\alpha_e = \alpha_- = 0$. One obtains the following biquadratic equation

$$\begin{aligned} & a^4 - [(\nu_{en}/V_e^2)L_e + (\nu_{-n}/V_-^2)L_- + (2q_e^2 n_{+0})/(\varepsilon_0 kT)]a^2 \\ & + 2q_e^2 n_{e0}(\lambda+1)[(\nu_{en}/V_e^2) \\ & \times L_e + (\nu_{-n}/V_-^2)L_-]/(\varepsilon_0 kT) = 0, \end{aligned} \quad (48)$$

where the relation $\lambda = L_e/L_-$ (Kockarts and Wisemberg, 1981) has been used in the independent term. Equation (48) has two pairs of solutions with opposite signs, and the corresponding scale lengths z_1 and z_2 are the inverses of the absolute values of these solutions, i.e.

$$z_1 = [2q_e^2 n_{e0}(\lambda+1)/(\varepsilon_0 kT)]^{-1/2} \quad (49)$$

and

$$z_2 = [(\nu_{en}/V_e^2)L_e + (\nu_{-n}/V_-^2)L_-]^{-1/2}. \quad (50)$$

When $\lambda = 0$, i.e. no negative ions, scale length z_1 reduces to $\sqrt{2}\lambda_D$ where λ_D is the classical Debye length (see Yeh and Liu, 1972) for a collection of charged particles. With negative ions, expression (49) is, therefore, a generalization of the Debye length taking into account the extra shielding resulting from the presence of negative ions. Scale length z_1 is furthermore an excellent approximation of the numerical solution H_1 in Fig. 2. The agreement is better than 1%. It should be noted that the analysis of neutral and non neutral ambipolar diffusion made by Hill (1978) does not consider the strong reduction of the Debye length resulting from the presence of negative ions.

Scale length z_2 reflects the stochastic aspect of negative ion chemistry. It is the spatial counterpart of scale time τ_c discussed in Section 4.3. Approximation (50) cannot be distinguished from the numerical solution H_2 in Fig. 2.

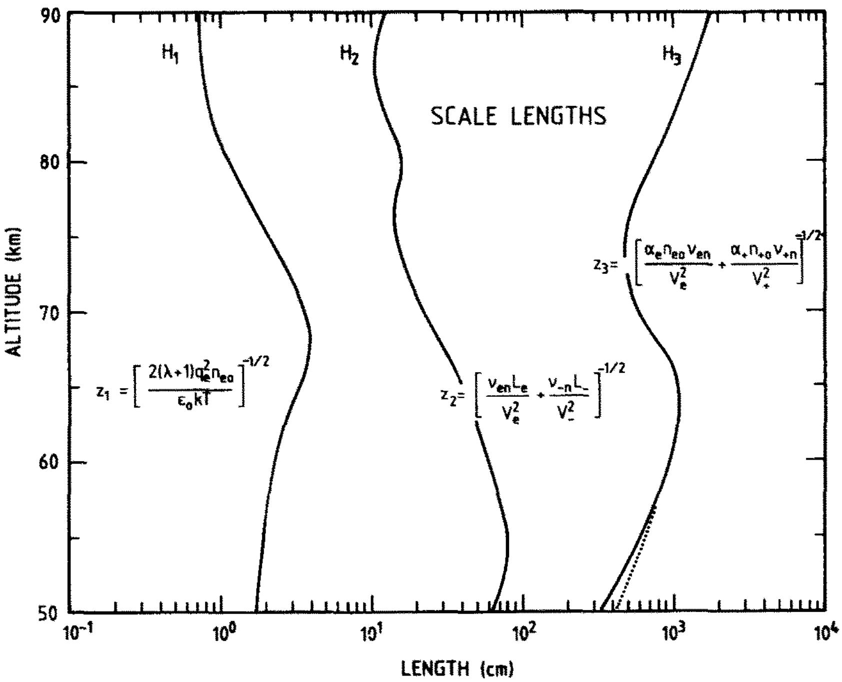

FIG. 2. VERTICAL DISTRIBUTION OF SCALE LENGTHS H_1, H_2 AND H_3.
H_1 is the modified Debye length. H_2 is the scale length associated with negative ion chemistry and H_3 corresponds to recombination processes. Approximate expressions are indicated and a significant difference appears only below 60 km for H_3, where the approximation is given by the points.

5.3. *Recombination effect*

We could not find an analytical approximation for scale length H_3 in Fig. 2 by making simplifying assumptions in the secular equation (43). Noting, however, that a simple solution for the recombination scale length is found when negative ions are completely ignored, we adopt for H_3 the approximation

$$z_3 = [(\alpha_e n_{e0} \nu_{en}/V_e^2) + (\alpha_+ n_{+0} \nu_{+n}/V_+^2)]^{-1/2}, \quad (51)$$

where the fictitious coefficient α_+ is given by equation (18). Approximation (51) is excellent except below 60 km altitude; the points in Fig. 2 show the departure from the exact solution.

6. CONCLUSION

A systematic analysis of the continuity and momentum equations for electron and ion fluctuations in the terrestrial D-region leads to the deduction of two scale times and three scale lengths for which simple analytical approximations are developed. These approximations can be used for any equivalent model in which effective recombination coefficients and electron and negative ion loss rate are available (Kockarts and Wisemberg, 1981).

The first scale time τ_n given by equation (33) is associated with recombination processes between electrons, negative ions and positive ions. Its value ranges from approximately 200 s in the upper D-region to more than 1 h in the lower D region where the ionospheric plasma is essentially composed of positive and negative ions. The second scale time τ_c given by equation (35) reflects the fast attachment and detachment processes for the electron–negative ion chemistry. The maximum value of τ_c of the order of 1.5 s is reached at the height where $\lambda = n_{-0}/n_{e0} = 1$. The shape of the curve τ_c is easily understood by remembering that the effective electron loss rate L_e is a decreasing function with increasing altitude, whereas the negative ion loss rate L_- increases with increasing height (Wisemberg and Kockarts, 1983).

The presence of negative ions in the D-region leads to a generalization of the classical Debye length. An increase of the negative ion abundance leads to a decrease of this scale length, given by equation (49). In the D-region below 80 km altitude, this scale length ranges from 1 to 4 cm. The second scale length given by equation (50) is directly associated with chemical fluctuations induced by electron attachment and detachment phenomena. Its value ranges from 10 to 80 cm in the *D*-region. Finally, the largest scale length is related to recombination processes, and expression (51) leads to values ranging from 5 to 10 m. In a discussion of nonthermal scattering from electrons, Harper and Gordon (1980) mention the necessity for scattering scale lengths of the order of 6 m in the lower *D*-region. Fluctuations resulting from recombination in the presence of negative ions induce scale lengths of such an order of magnitude.

Hill and Bowhill (1977) have analyzed the response of the *D*-region to an initial perturbation without production or loss terms in the continuity equation. Our scale lengths essentially result from internal fluctuations induced by photochemical processes. Introduction of such processes would considerably increase the complexity of the analysis made by Hill and Bowhill (1977). Whereas the modified Debye length z_1 reflects an additional shielding by negative ions, scale lengths z_2 and z_3 can be seen as characteristic distances over which the charged species cannot be considered as independent. Over the scale length z_2, negative ions and electrons are coupled by attachment and detachment processes. Over the scale length z_3, all negative species (electrons and ions) and positive ions are coupled by neutralization processes.

In summary, the various scale times and scale lengths presented in this paper give physical information on time and spatial extent of small fluctuations occurring constantly for all charged species in the terrestrial *D*-region.

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