EVIDENCE OF CHAOTIC DYNAMICS OF BRAIN ACTIVITY
DURING THE SLEEP CYCLE

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Recent progress in nonlinear dynamics provides the means for the characterisation of the behavior of natural systems from time series. The analysis of electroencephalogram data from the human brain during the sleep cycle reveals the existence of chaotic attractors for sleep stages two and four. The onset of sleep is followed by increasing "coherence" towards deterministic dynamics involving a limited set of variables.

Recent progress in nonlinear dynamics have provided new methods for the study of multi-variable, complex systems [1,2]. These methods are specially valuable for the analysis of experimental data obtained from a single variable (one-dimensional) time series. Their chief merit is to discriminate between the random or deterministic nature of the dynamical system. For instance, they allow one to determine the minimum number of variables necessary for the description of the dynamical system. Furthermore, they give criteria for the existence of attractors, which characterize deterministic dynamics, as well as information about such quantitative properties as dimensionality.

Such an approach has been applied to laboratory experiments on hydrodynamic and chemical instabilities and to complex natural systems such as climatic variability [3] or the activity of the human brain [4].

The mammalian brain is certainly one of the most complex systems encountered in nature. It is made of billions of cells endowed with individual electrical activity and interconnected in a highly intricate network. The average electrical activity of a portion of this network may be recorded in the course of time

and is called the electroencephalogram (EEG) [5]. The EEG reflects the sum of elemental self-sustained neuronal activities of a relatively long period (of the order of 0.5–40 Hz). Recordings from the human brain show that to various stages of brain activity there correspond characteristic electrical wave forms. For example a resting and alert brain shows activity with an average frequency of about 10 c/s and amplitudes of the order of 10 μV (α waves). During a normal night's sleep α waves give way to other repeated cycles of activity, each marked by several stages [6].

In stage one, the individual drifts in and out of sleep. In stage two, the sleeper is disturbed by the slightest noise. In stage three a loud noise would be needed to arouse the sleeper. Finally, the deep sleep of stage four sets in. Afterwards the cycle is reversed back through stages three and two. After this stage the sleeper enters the phase of rapid eye movement sleep (REM) in which he dreams. This episode is followed by stage two and a new cycle begins. The sleep cycles continue through the night; however the periods of REM get longer and those of deep sleep shorter.

As sleep sets in the fast α waves gradually give way

to slower and higher amplitude waves. At deep sleep stage four the average frequency is of the order of 3–5 c/s and amplitudes are of several hundred μV (δ waves). During REM sleep intense bursts of high frequency activity appear.

In a recent paper the EEG of deep sleep stage four, considered as a time series, was analysed [4]. It was shown that a deterministic dynamics described by a minimum of five variables is sufficient for the description of the deep sleep. Moreover the existence of an attractor characterized by a dimensionality $d = 4.08$ has been established.

In the present paper we extend the study to the α waves, sleep stage two and also to REM sleep. Moreover with the help of Lyapunov exponents we show unambiguously that some stages of sleep are characterized by chaotic attractors.

It has been shown that from a unique time series $x_0(t)$, a set of variables describing the dynamics of the system could be defined. These variables are obtained by shifting the original time series by a fixed lag τ ($\tau = m \Delta t$ where m is an integer and Δt is the interval between successive samplings). These variables span a phase space which allows the drawing of the phase portrait of the system or, more precisely, its projection to a low dimensional subspace of the full phase space.

In the phase space the instantaneous state of the system is characterized by a point, a sequence of such states followed in time defines the phase space trajectory. If the dynamics of the system is reducible to a set of deterministic laws, the system reaches in time a state of permanent regime. This fact is reflected by the convergence of families of phase trajectories towards a subset of the phase space. This invariant subset is called an attractor.

With the help of the above cited procedures we have constructed the phase space portraits of EEG data sets provided by the sleep laboratory of the department of Psychiatry of the University of Brussels.

Figs. 1a–1d depict the phase portraits corresponding, respectively, to the awake state, sleep stage two, sleep stage four and REM activity of the human brain. A total of six sleep episodes taken from different individuals were analysed. The complexity of these graphs reflects the complexity of the brain activity. However, the most striking feature is the evolution of this complexity as the sleep cycle unfolds.

The phase portrait of the awake subject is densely filled and occupies a small portion of the phase space (fig. 1a). The representative point undergoes deviations from some mean position in practically all directions. At sleep stage two, already a tendency towards a privileged direction is seen, and a larger portion of the phase space is visited (fig. 1b). This tendency is amplified in sleep stage four and one sees preferential pathways, suggesting the existence of reproducible relationships between instantaneous values of the pertinent variables (fig. 1c). This phase portrait is the largest and exhibits maximum "coherence" which diminishes again when the REM sleep sets in (fig. 1d).

A universal attractor for different REM episodes of a single night and a given individual seems unlikely, as the REM episodes are associated with intense brain activity and generation of dreams. Our analysis of 3 consecutive REM periods of a single individual during a night's sleep showed that one episode was strikingly different from the two other REM periods, as depicted in fig. 1d. The phase space portrait showed a comet-like tail.

Presently, let us see if attractors could be identified for the other stages of brain activity. In other words, is it possible to view the salient features of brain activity as the manifestation of deterministic dynamics or, rather, are the phase portraits of figs. 1a–1b the result of an irreducible stochastic element?

The existence of an attractor and the evaluation of its dimensionality may be achieved in the following manner [2]. We introduce vector notation: $x_i(t)$ stands for a point of phase space whose coordinates are $\{x_0(t_i), ..., x_0(t_i + (n-1)\tau)\}$. A "reference" point x_i from these data is chosen and all its distances $|x_i - x_j|$ from the $N-1$ remaining points are computed. This allows us to count the data points that are within a prescribed distance r from the point x_i in phase space. Repeating the process for all values of i, one arrives at the quantity

$$C(r) = \frac{1}{N^2} \sum_{\substack{i,j=1 \\ i \neq j}}^{N} \theta(r - |x_i - x_j|), \qquad (1)$$

where θ is the Heaviside function, $\theta(x) = 0$ if $x < 0$ and $\theta(x) = 1$ if $x > 0$. The non-vanishing of $C(r)$ measures the extent to which the presence of a data point x_i affects the position of the other points. $C(r)$ may
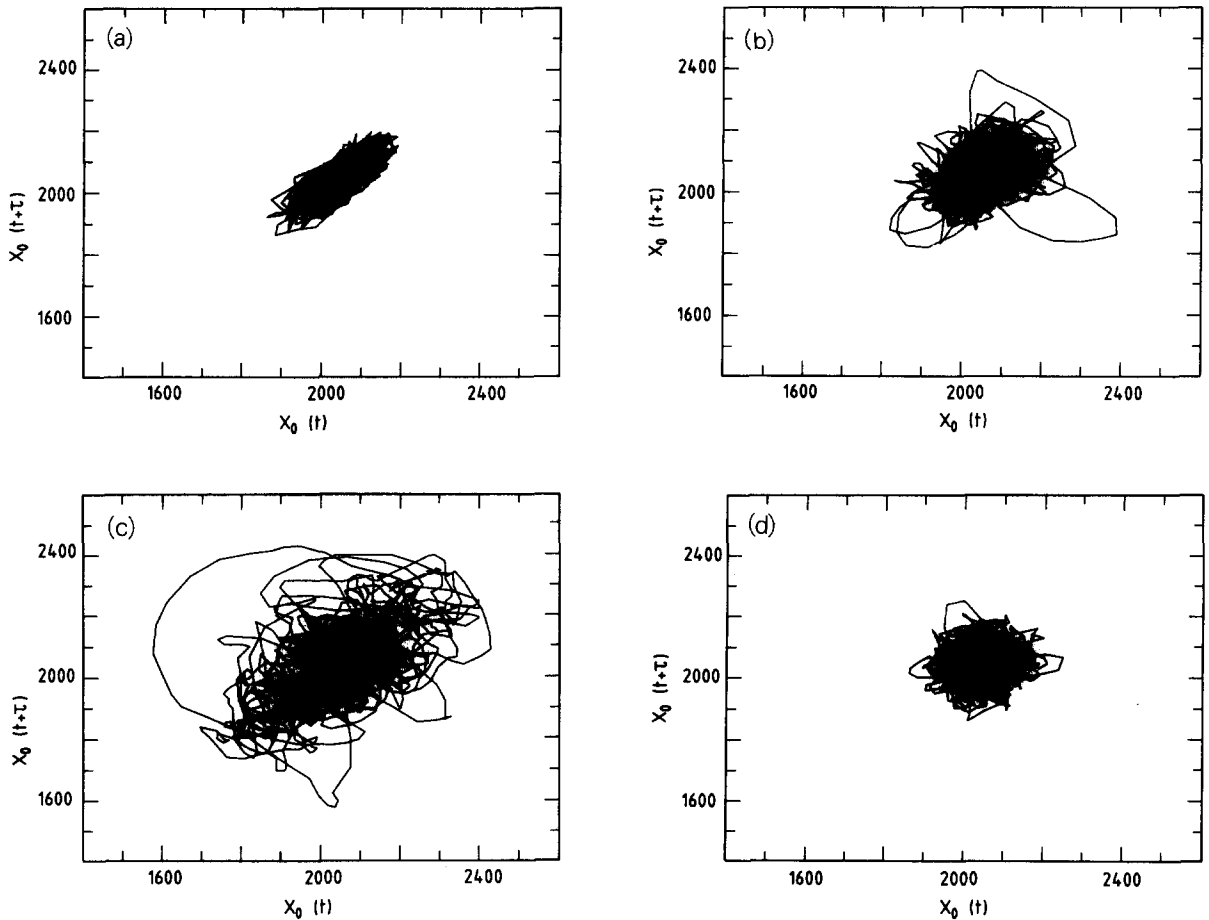
Fig. 1. Two-dimensional phase portraits derived from the EEG of (a) an awake subject, (b) sleep stage two, (c) sleep stage four, (d) REM sleep. The time series $x_0(t)$ is made of $N = 4000$ equidistant points. The central EEG derivation C4-A1 according to the Jasper system. Recorded with PDP 11-44, 100 Hz for 40 s. The value of the shift from 1a to 1d is $\tau = 10 \, \Delta t$.

thus be referred to as the integral *correlation function* of the attractor.

One shows that for small r

$$C(r) = r^d . \qquad (2)$$

The dimensionality d of the attractor is therefore given by the slope of log $C(r)$ versus log r.

With the help of relation (1) a dimensionality d is computed by considering successively higher values of the dimensionality n of the phase space. If the d versus n dependence is saturated beyond some relatively small n, the system represented by the time series should possess an attractor. The saturation value d is

regarded as the dimensionality of the attractor represented by the time series. The values of n beyond which saturation is observed provide the minimum number of variables necessary to model the behavior represented by the attractor.

Fig. 2 shows saturation curves computed for the awake state and several stages of the sleep cycle. They are compared with the behavior obtained from a random process, such as a gaussian white noise (crosses). There is no saturation for the awake state. However, we are far from the behavior of random noise. A satisfactory saturation exists for sleep stage two. In this case for two individuals we find, respectively, $d =$
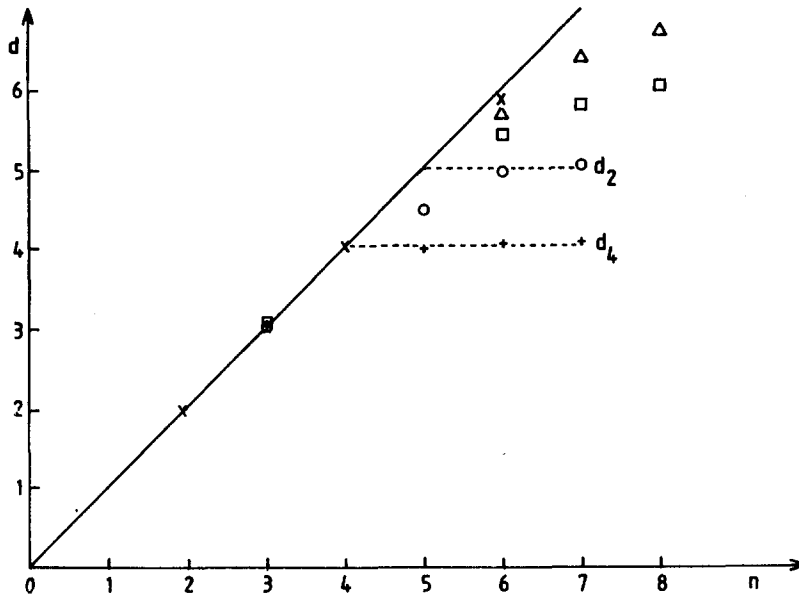
Fig. 2. The dependence of dimensionality d on the number of phase space variables n for a white noise signal (X), the EEG attractor of an awake subject (△), sleep-stage two (o), sleep stage four (+) and REM sleep (□), for the same number of data points as in fig. 1.

5.03 ± 0.07 and $d = 4.99 \pm 0.11$. Saturation curves were already found for sleep stage four [4]. The analysis of EEG data of stage four of three individuals showed $d = 4.05 \pm 0.5, d = 4.08 \pm 0.05$ and $d = 4.37 \pm 0.1$. Again the saturation was poor for the REM state.

In view of the uncertainties involved in the numerical procedure for the evaluation of the dimensionality d of the attractor it is important to determine wether $d = 4.05$ and $d = 5.03$ represent genuine strange attractors rather than quasi-periodic motions.

If we are in the presence of chaotic motion, the trajectories in phase space have the tendency to diverge. One can estimate the speed of this divergence. The average of these individual measures over a large number of data is defined as a Lyapunov exponent. A positive Lyapunov exponent indicate unambiguously the presence of a chaotic attractor.

Recently algorithms have been developed which allow for the evaluation of the largest positive Lyapunov exponent from the knowledge of a time series describing a dynamical system. Using the Fortran code described by Wolf et al. [7] we have evaluated the largest positive Lyapunov exponents for stage two and stage four of deep sleep. For stage two

we find a positive value of λ_2 between 0.4 and 0.8. The inverse of this quantity gives the limits of predictability of the long-term behavior of the system. For stage four we find also a positive number, $0.3 < \lambda_4 < 0.6$.

The results reported here and in a previous paper [4] establish the presence of chaotic attractors during sleep stage two and sleep stage four. This in turn implies the existence of deterministic dynamics which may be described by a limited number of variables. The fact that the underlying attractor is a fractal and its dimensionality decreases as the deep sleep sets in, implies that the dynamics becomes more coherent during the deep sleep.

The fact that we could not find an attractor for the awake state or for the REM state may reflect two entirely different situations.

(i) Saturation occurs for a larger number of variables and therefore the analysis must be performed with a much longer time series.

(ii) The dynamics of the system exhibits intrinsic stochastic behaviour.

These points will be elucidated in a future paper. We also intend to extend our analysis to a large number of subjects.

155

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