GEOPHYSICAL RESEARCH LETTERS, VOL. 11, NO. 6, PAGES 587-590, JUNE 1984

A PLAUSIBLE MODEL FOR THE SYNCHRONEITY OR THE PHASE SHIFT BETWEEN CLIMATIC TRANSITIONS

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Abstract. A class of climate models describing the dynamics of the sea ice-ocean surface temperature interactions is considered. The coupling mechanism between different spatial regions is analyzed, and the possibility of inhomogeneous phase shifted states is established. It is concluded that many major climatic transitions should occur in an asynchronous fashion. A new examination of the climatic record seems therefore to be necessary, in the light of this possibility.

Introduction

The evidence that the climatic system has undergone in the past massive changes, reminiscent of transitions between altogether different states, is compelling. Yet, little is known on the mechanisms which may be at the origin of the large scale behavior of the climatic system. A continuing debate is going on about the relative role of external factors and of internal processes, and the consensus established by the community of geophysicists at any given time determines, to a large extent, the way that climatic data will be calibrated and interpreted.

Our principal goal in the present letter is to draw attention on the need to reconsider the interpretation of paleoclimatic data, in the light of the possibility that climatic changes may well occur in a markedly asynchronous fashion. This possibility appears as the inevitable result of the dynamics of large classes of model systems whose relevance in climatology is well accepted. Its implications are considerable, since they might help to confirm or, on the contrary, rule out some of the scenarios of climatic change advanced so far.

Undoubtedly, the idea of synchroneity constitutes currently the basis of interpretation of climatic data. Still, examples of manifestly asynchronous behavior are available. For instance, in the so called Boreal period North America was still essentially in an ice age regime while Europe was warm (Lamb, 1977). In particular about 6500-6000 B.C. there was a major halt, or readvance, of the Laurentide ice sheet (contrary to its European counterpart) which produced the moraine systems. Even more striking is the coincidence in time of a period of cold climate in China and Japan with the height of the early medieval warm epoch in Europe, Greenland and North America east of the Rockies. An antiphase relationship in the opposite sense is apparent during Europe's cold phases around 600-200 B.C. and 600-700 A.D.

In order to assess the possibility of asynchronous behavior we focus our attention on climate models describing the interactions between sea ice extent and ocean surface temperature. Let η be the deviation of the sine of the latitude of the sea ice extent from a reference state, θ the excess mean ocean surface temperature. As suggested by Saltzman (1978) and Saltzman et al (1981, 1982) the following set of equations describes the essence of these interactions (we denote the time derivative by a dot) :

$$\dot{\theta} = - a\eta + b\theta - \eta^2\theta$$
$$\dot{\eta} = - \eta + \theta \qquad (1)$$

in which a and b are positive parameters. Both the initial variables and parameters have been suitably scaled to nondimensional quantities.

The first equation (1) represents the ocean surface heat balance whereas the second one stands for the mass balance of sea ice. The term $- a\eta$ describes the negative ("insulating") effect of sea ice on θ and the term $b\theta$ the positive feedback of θ on itself via the atmospheric CO_2 among others. The nonlinear term $- \eta^2\theta$ accounts for the nonlinear restoring mechanism that becomes effective when the system is displaced far from the reference state.

It has been shown (Saltzman et al 1981, 1982) that equations (1) predict an oscillatory behavior in time, with periods in the range of 10^3 - 10^5 years. In view of the periodicities observed in quaternary climatic changes, Saltzman's model appears therefore to be well suited to study the main qualitative features of these transitions. On the other hand, in all studies reported so far the assumption of a lumped oscillator, representing some sort of space averaged behavior of the climatic system, has been invariably adopted. Here we want to relax this assumption and study spatially distributed oscillators.

As a first step toward this goal we consider in the next section two identical Saltzman oscillators coupled through energy transfer and explore various possible modes of dynamical behavior. We find that, for reasonable parameter values, such a system admits perfectly entrained and phase locked solutions. Lumping appears therefore to be justified in this case.

The remaining part of this note is devoted to the coupling of two different Saltzman oscillators representing for instance the two hemispheres or even two regions of different spatial characteristics along a longitude zone. The system now admits solutions with entrainment but no phase locking, as well as solutions in which the two spatial regions evolve according to different periods. The implications of the results in the interpretation of the climatic record are briefly discussed in the final section.

Coupling of Two Identical Saltzman Oscillators

We first consider two coupled oscillators having identical characteristics. This will provide a simple description of the dynamics of an

Paper number 4L6041.
8276/84/004L-6041 $03.00

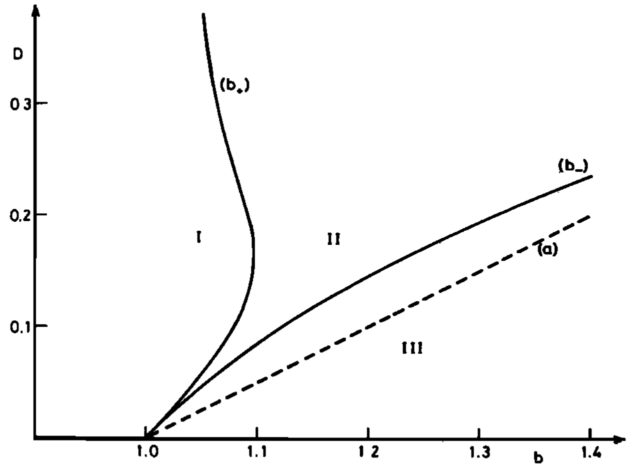

Fig. 1. Stability diagram of two coupled oscillators.
Curve (a) : locus of marginal stability for two identical oscillators (a = 6.4 from Saltzman et al 1981, 1982).
Curves (b_+) and (b_-) : loci of marginal stability for two different oscillators ($a_2 = a_1 + 1$).

extended zone along a given longitude belt. The coupling is expected to arise from energy transfer, and should therefore affect only the equations for the mean surface temperature. A reasonable assumption is that the strength of the coupling is proportional to the temperature inhomogeneities within the system. Introducing the labels 1 and 2 for the two longitude "boxes" we therefore write the following set of four coupled equations :

$$\dot{\theta}_1 = b\theta_1 - a\eta_1 - \eta_1^2\,\theta_1 + D(\theta_2 - \theta_1)$$
$$\dot{\eta}_1 = \theta_1 - \eta_1$$
$$\dot{\theta}_2 = b\theta_2 - a\eta_2 - \eta_2^2\,\theta_2 + D(\theta_1 - \theta_2)$$
$$\dot{\eta}_2 = \theta_2 - \eta_2 \qquad (2)$$

where D is the (positive) energy transfer coefficient.

Let us see how the presence of coupling affects the stability properties of the reference state. Linearizing eqs (2) around the steady state ($\theta_i = 0$, $\eta_i = 0$, $i = 1,2$) we find that linear stability will no longer be secured if $\mathrm{Re}\,\omega_\alpha \geq 0$, where $\mathrm{Re}\,\omega_\alpha$ is the real part of at least one of the roots of the characteristic equation (Cesari, 1962). This latter equation can be solved analytically near the bifurcation point b = 1 of the lumped oscillator and for small values of D. A straightforward algebra gives that the boundary between linear stability and instability corresponds to the following relation between parameters D and b :

$$D \cong \frac{1}{2}\,(b - 1) \qquad (3)$$

Curve (a) of Figure 1 represents this relation. The instability region is below the curve while on the curve itself the roots of the characteristic equation are purely imaginary. Curve (a) is therefore the locus of bifurcation of time periodic solutions (Hopf bifurcation).

The next question to be asked refers to the type of oscillatory regime that will prevail in the unstable region. We first observe that $\theta_1 = \theta_2 = \theta_0(t)$, $\eta_1 = \eta_2 = \eta_0(t)$ where θ_0 and η_0 are solutions of eqs (1), is also a solution of eqs (2) representing a perfectly synchronous behavior, whereby both boxes oscillate with the same period and phase. Let us study the stability of this regime. Linearizing around $\theta_0(t)$, $\eta_0(t)$ ($\theta_i = \theta_0(t) + \delta\theta_i$, $\eta_i = \eta_0(t) + \delta\eta_i$, $\delta\theta_i$, $\delta\eta_i \ll 1$, $i = 1,2$) and introducing the excess quantities $k_1 = \delta\theta_1 - \delta\theta_2$, $k_2 = \delta\eta_1 - \delta\eta_2$, we can easily obtain a closed set of two equations :

$$\dot{k}_1 = (b - \eta_0^2 - 2D)\,k_1 - (a + 2\eta_0\,\theta_0)\,k_2$$
$$\dot{k}_2 = k_1 - k_2 \qquad (4)$$

This linear system with periodic coefficients can be studied by Floquet's theory (Cesari, 1962). As it turns out, there is a domain of small D and of b close to 1 for which the trivial solution $k_1 = k_2 = 0$ is a stable solution. We conclude that in this range the perturbations $\delta\theta_i$ and $\delta\eta_i$ from the perfectly synchronous solution $(\theta_0(t), \eta_0(t))$ will themselves synchronize sooner or later. In other words, the system will behave for long times as a single lumped oscillator.

Numerical solutions confirm entirely this analytical prediction and show that the same conclusions hold for much larger values of b and D. Figure 2 describes a typical result : starting from phase shifted initial conditions we find that the system evolves to a limit cycle solution in which the phase difference between the two boxes remains equal to zero at all times.

Coupling of Two Climatic Oscillators Characterized by Different Parameter Values

We now consider the case in which the two coupled oscillators have different parameter values. This should provide a simple model of the dynamical behavior of the two hemispheres coupled through meridional energy transfer or even of two zones of the same hemisphere (for instance, two longitude zones) subjected to different environmental conditions. Actually, in the first problem it would be more realistic to apply Saltzman's model to the southern hemisphere only, and couple it to the oscillatory model of continental ice extent developped by Källén et al. (1979), which is more suitable for describing the northern hemi-

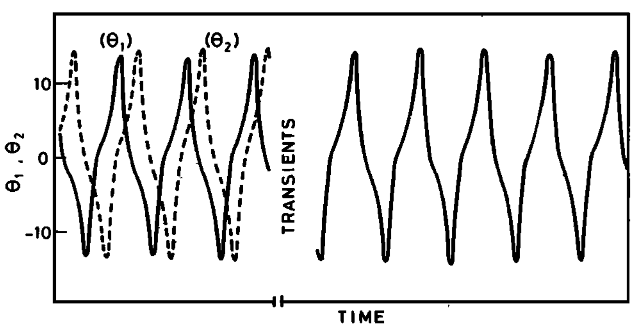

Fig. 2. Time evolution of θ_1, θ_2 for two coupled identical oscillators. Parameters used : a = 6.4, b = 4, from Saltzman et al, 1981, and D = 0.1.

sphere. In the present note however we limit ourselves to coupled Saltzman oscillators which are more tractable analytically and will thus allow us to sort out some general qualitative trends.

Since parameter a describing the insulating effect of sea-ice determines the intrinsic period of each oscillator near the bifurcation point (Nicolis, 1984), it is adopted as the principal parameter differentiating the two oscillators. Keeping, for simplicity, the same value of b and D throughout, the balance equations become therefore :

$$\dot{\theta}_1 = b\theta_1 - a_1\eta_1 - \eta_1^2\theta_1 + D\,(\theta_2 - \theta_1)$$
$$\dot{\eta}_1 = \theta_1 - \eta_1$$
$$\dot{\theta}_2 = b\theta_2 - a_2\eta_2 - \eta_2^2\theta_2 + D(\theta_1 - \theta_2)$$
$$\dot{\eta}_2 = \theta_2 - \eta_2 \qquad (5)$$

As previously we want to see first how the stability of the steady state is affected by the presence of the coupling. An answer to this question can again be found analytically for small D and for b near the bifurcation point (b = 1) of the lumped oscillator. Expanding the characteristic equation in powers of b-1 and D we find that the condition of marginal stability, Re $\omega_\alpha = 0$, is now satisfied along two curves in the space of the parameters D and b-1 :

$$D_+ \cong b - 1 + \frac{2(b-1)}{a_2 - a_1}$$
$$D_- \cong b - 1 - \frac{2(b-1)}{a_2 - a_1} \qquad (6)$$

where we adopted the convention $a_2 > a_1$. Each of the above two conditions corresponds to a regime in which the imaginary part of the critical characteristic root ω_α, Im ω_α, is given by

$$\gamma_+^2 = a_1 - 1 + O((b-1)^2)$$
$$\gamma_-^2 = a_2 - 1 + O((b-1)^2) \qquad (7)$$

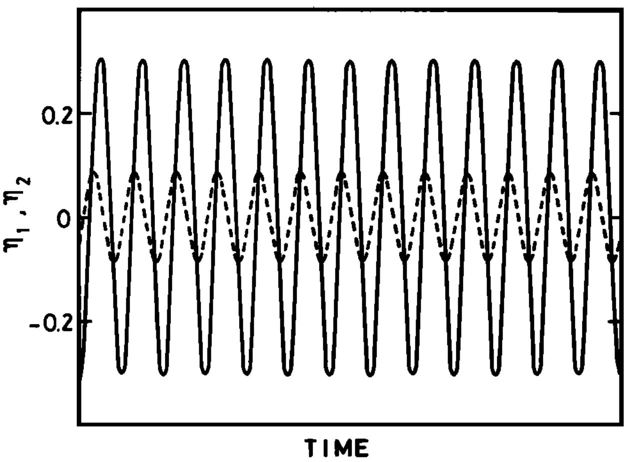

Fig. 3. Time evolution of η_1, η_2 for two different oscillators. Parameters corresponding to region II of Fig. 1 : $a_1 = 6.4$, $a_2 = a_1 + 1$, b = 1.1, D = 0.1.

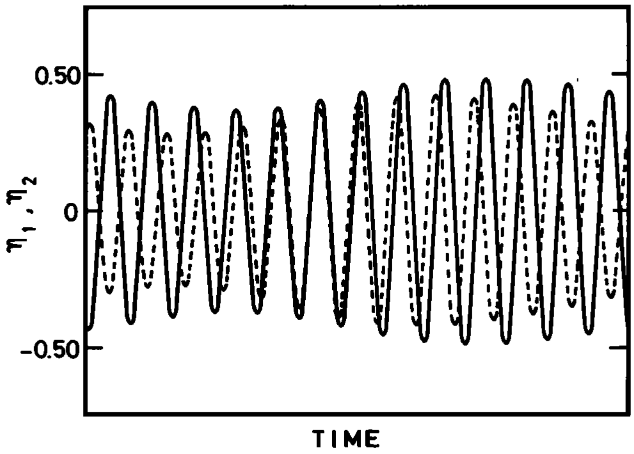

Fig. 4. Time evolution of η_1, η_2 for two different oscillators. Parameters corresponding to region III of Fig. 1 : $a_1 = 6.4$, $a_2 = a_1 + 1$, b = 1.1, D = 0.06.

Equations (6) and (7) allow us to identify the important parameters of the problem.

The interpretation of these relations is quite clear : In the absence of coupling (D = 0) one has two independent oscillators whose periods are determined by a_1 and a_2, and which reach simultaneously the bifurcation point b = 1. In the presence of the coupling ($D \neq 0$) one can no longer speak of individual oscillators. Nevertheless, the overall system exhibits two characteristic frequencies γ_+, γ_- which are slightly shifted from the values of the frequencies of the individual oscillators.

Curves (b_+), (b_-) of Figure 1 describe the stability boundaries of the system. They are obtained by solving numerically the characteristic equation for higher values of (b-1) and D, since the analytic expression eq. (6) is limited to small values of these parameters. As one moves in parameter space, say clockwise, the following phenomena will take place. On the left of curve(b_+) (region I), the steady state is stable. On crossing such a curve one enters in a region (II), in which one mode only (corresponding to γ_+) is unstable. Further crossing of curve(b_-) excites the second mode γ_- as well, and one thus enters in a region (III) of two competing unstable modes.

From the above one would expect that the system would show, respectively, damped oscillations, simple periodic and complex periodic or even non-periodic behavior. This trend is confirmed entirely by numerical solutions. Figure 3 describes the result of a numerical integration of eqs. (5) in region II. We observe that the variables in the two boxes oscillate with the same period but with a phase lag. This means that the regions described by the two oscillators will experience the same cyclic climatic change, but that they will not reach simultaneously the peaks or other characteristic stages of the various episodes. Clearly, the lumping approximation is inadequate in this case.

In Figure 4 we report results observed in region III of the stability diagram. The coupling of the two unstable modes gives now rise to an erratic non-periodic behavior, in which the two "boxes" evolve with different periods as well as different phases. As a result the amplitude of the successive oscillations becomes also erratic. In addition, it appears that a fast oscillation, corresponding to the period of the individual oscillators, or a combination thereof, is modulated by a much slower mode. We arrive therefore at the conclusion that a weak coupling between two oscillators can introduce a new, long time scale in the system.

The emergence of different periodicities characterizing the two spatial compartments in the results of Figure 4 implies that the two regions concerned will no longer be subjected to the same cycle of climatic variations. This provides a new, and hitherto unexplored mechanism of internal differentiation within the climatic system. Besides, it suggests that long scale phenomena like glacial maxima in the two hemispheres, whose dating involves necessarily considerable uncertainties could also be intrinsically asynchronous events.

Concluding Remarks

We have sketched a simple mechanism describing different modes of behavior of the climatic system, based on the coupling between "elementary" oscillators. We have seen that, depending on the parameter values, such different regimes as perfect phase locking or an erratic space and time variation of the variables can be observed. Most of our analysis was motivated by climatic changes in the 10^3 - 10^5 year range, but it is clear that the main ideas can be applied equally well to shorter term events. A better knowledge of the parameters would allow us to specify more sharply the specific domain within which a particular system is expected to operate.

There is no doubt that a detailed explanation of the lack of synchroneity should also take into account the specificity of the regions considered (orography etc) as well as atmospheric circulation patterns. Nevertheless, we believe that our results establish that the possibility of asynchronous behavior is deeply rooted in the dynamics of the climatic system, and that the coupling between local oscillators is a general mechanism for generating this behavior.

Acknowledgements. This work is supported, in part, by the EEC under contract n°STI-004-J-C (CD).

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(Received January 17, 1984);
accepted March 15, 1984.)