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# ON THE MOLECULAR SCATTERING IN THE TERRESTRIAL ATMOSPHERE: AN EMPIRICAL FORMULA FOR ITS CALCULATION IN THE HOMOSPHERE

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**Abstract**—A recent determination by D. R. Bates of the Rayleigh scattering cross section ($\sigma_{RS}$) for air from 0.2 to 1 $\mu$m leads to a simple empirical formula ($\lambda$ in $\mu$m)

$$\sigma_{RS} = 4.02 \times 10^{-28}/\lambda^{4+x}\ \text{cm}^2$$

where $x = 0.389\lambda + 0.09426/\lambda - 0.3228$ for the spectral region 0.2 $\mu$m $< \lambda <$ 0.55 $\mu$m; the accuracy is within $\pm 0.5\%$. From the visible at 0.55 $\mu$m to the infrared (i.r.) at 1 $\mu$m, the same accuracy can be obtained using a constant value, $x = 0.04$. The formula accounts for the degree of depolarization which varies with the wavelength according to the latest determination by Bates.

## 1. INTRODUCTION

In a recent paper with Meier and Anderson (Nicolet *et al.*, 1982) I discussed (Appendix 2) the problem of the numerical value of the molecular scattering cross section $\sigma_{RS}$ given by Fröhlich and Shaw (1980). The adopted formula was written as follows:

$$\sigma_{RS} = (3.93 \pm 0.05) \times 10^{-28}/\lambda^{4+x}\ \text{cm}^2 \qquad (1)$$

assuming a median value of about $1.03 \pm 0.01$ for the correction factor of King (1923) which corresponds to a possible error of about $\pm 1\%$ for the molecular scattering optical depth. But, for the direct application to the atmosphere (Meier *et al.*, 1982), we adopted the formula given by Elterman (1968). However, the series of publications by Young (1980, 1981a, b, 1982) on the value of King's correction factor, and also the observations of the direct and the scattered solar flux within the stratosphere (Herman and Mentall, 1982), have led us to reexamine the problem of the radiation field in the stratosphere (cf. Meier and Anderson, 1984), and to make a critical study of the experimental data relating to the depolarisation factors.

## 2. NEW DETERMINATION OF RAYLEIGH SCATTERING BY AIR

The detailed reexamination of the problem by Bates (1984) has brought about a clarification of the question. This study makes it possible first to find an accurate value for the molecular scattering cross-section, which can be used throughout the homosphere (troposphere, stratosphere and mesosphere) where the mean molecular mass remains constant. It makes it possible also to know the variation of King's correction factor with wavelength; Bates shows that the factor varies from about 1.05 in the visible to 1.08 in the ultraviolet (u.v.) at 0.2 $\mu$m. These values are well above the value given by Fröhlich and Shaw (1980).

Finally, Bates' detailed results, which he kindly made available to me prior to publication, can lead to the determination of a simple expression which permits the rapid calculation of the atmospheric optical thickness attributable to molecular scattering. The simplicity of a formula makes it possible to determine quickly, and as a function of wavelength, the importance of the effects due to the various components of atmospheric absorption. The most important of these arise from oxygen at wavelengths less than 240 nm, and from ozone throughout the u.v. and also in the visible (Chappuis bands). The absorption in these two cases must be compared with the effect of molecular diffusion in different spectral regions.

## 3. EMPIRICAL FORMULA

Between 0.55 $\mu$m in the visible and the shorter u.v. wavelengths, the data provided by Bates (1984) can be represented by the expression

$$\sigma_{RS} = 4.02 \times 10^{-28}/\lambda^{4+x} \qquad (2)$$

where

$$x = 0.389\lambda + 0.09426/\lambda - 0.3228 \qquad (3)$$

and $\lambda$ is expressed in $\mu$m. The value of $x$ increases from about 0.04 at 0.55 $\mu$m to 0.23 at 0.2 $\mu$m. In practice, the accuracy of the simple formula is better than $\pm 0.5\%$.

Hence it can be assumed that, for atmospheric applications, the molecular scattering optical depth is known to an accuracy of better than $\pm 1\%$; this leads to a determination of the optical thickness better than that resulting from oxygen and ozone. It should also result in a better understanding of the radiation field in the stratosphere.

Between the visible ($\lambda > 0.55\ \mu m$) and the i.r. regions, the required accuracy can be obtained by adopting the constant value: $x = 0.04$. Thus for $0.55 < \lambda < 1\ \mu m$, the simple expression

$$\sigma_{RS} = 4.02 \times 10^{-28}/\lambda^{4.04}\ \mathrm{cm}^2 \quad (4)$$

can be used for making comparisons with the absorption of the Chappuis bands of ozone.

The importance of the new determination of molecular scattering cross-section that has been made by Bates can be clearly seen in the study of the atmospheric transmissivity at 0.2 $\mu m$. At this wavelength, the $O_2$ absorption cross-section may reach a low value of about $6 \times 10^{-24}$ cm$^2$ (Nicolet, 1983), while that for ozone is near its minimum value, and hence molecular scattering must play an important role. The molecular scattering optical depth may be as great as 30% of the $O_2$ optical depth near 0.2 $\mu m$. Near 0.225 $\mu m$, its effect is greater than 50% of the $O_2$ optical depth when the theoretical absorption cross section of the $O_2$ Herzberg continuum is reduced by a factor of 2.

If it is permissible to consider the Herzberg continuum in the spectral region* of the (2-0) to (5-0) Schumann–Runge bands of $O_2$ (at wavelengths greater than 0.19 $\mu m$), where the rotational lines do not play any role in the atmospheric absorption, the contribution of molecular scattering in the absorption is of the order of 30% of that of molecular oxygen.

When new experimental values of the absorption cross-sections of $O_2$ and $O_3$ become available, it will be possible to determine with better accuracy the relative importance of the direct attenuated solar flux and of the multiply-scattered flux at various levels in the stratosphere and mesosphere.

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* Bates (private communication) has extended his calculations to cover this region: he finds that the Rayleigh scattering cross-section at 0.195 $\mu m$ is $4.123 \times 10^{-25}$ cm$^2$ and at 0.190 $\mu m$ is $4.735 \times 10^{-25}$ cm$^2$. The estimated accuracy is around 2%. Formula (2) with $x$ as in (3) reproduces his new results quite well.