Düsseldorf 29th August - 2nd September 1983, Edited by W. Bötticher. H. Wenk, E. Schulz-Gulde, 139-147, 1983

M/I?

BOUNDARY LAYERS IN SPACE PLASMAS : A KINETIC MODEL OF TANGENTIAL DISCONTINUITIES

by

M. Roth

Institute for Space Aeronomy

Avenue Circulaire, 3

B - 1180 Brussels, Belgium.

Abstract

A kinetic model of tangential discontinuity in a collisionless magnetized plasma with multiple particle species has been reviewed in this paper. It is a one-dimensional and stationary model whose boundary conditions on each side of the current sheet are specified by unlike densities, temperatures and average velocities for each plasma component. These average velocities must however be compatible with the asymptotic conditions of plasma uniformity. The model also allows the magnetic field to make an arbitrary rotation in the plane of the discontinuity. Vlasov equations for the particle species and Maxwell's equations for the fields are solved simultaneously. The theory is self-consistent in that the electric potential and the electric field are obtained from the charge-neutral approximation which is verified in most cases. In particular, it is shown that the electric field inside the current sheet is far from being identical with the convection electric field which is assumed to be a good approximation for the actual electric field in the MHD framework. The velocity distribution functions complying with Vlasov equation are linear combinations of shifted Maxwellians. Asymptotically their first moments are identical with those of the actual velocity distribution functions an experiment would measure on both sides of a current sheet. With such distributions all the moments of any order are analytically determined in terms of the electric potential and of the components of the vector potential. A numerical method is used to solve Maxwell's equations for the fields and their corresponding potentials. Therefore a full description of the microscopic structure of the current sheet can be made. The model is a powerful tool for studying the structure of tangential discontinuities which occur in collisionless space (or laboratory) plasmas.

1. Introduction

Space plasmas have a natural propensity to break up into distinct regions with characteristic densities, compositions, temperatures, magnetizations and average particle velocities (Fälthammar et al., 1978, Alfvèn, 1981, p. 40). The boundaries between these regions are often stable transition layers with very high observed lifetimes. These layers contain interpenetrated plasmas from the adjacent zones and most of them have electric current sheets which change the orientation and intensity of the local magnetic field. Their observed thickness is usually of the order of a few ion Larmor gyroradii (see for instance : Burlaga, 1971; Burlaga et al., 1977, for the solar wind discontinuities). Typical examples of such a cellular structure of space plasmas are the filamentary character of the solar wind and the containment of plasmas originating from planets and stars inside distinct compartments dividing the neighborhood of planetary magnetospheres.

Boundary layers resulting from a partition of magnetized plasmas can be classified into (Nishida, 1978, p. 17) : tangential discontinuities, contact discontinuities and shocks. For tangential and contact discontinuities there is no mass flow across the boundary layers which are then convected along with the flow. To distinguish between these two types of discontinuities one has to examine the magnetic field component along the normal to the boundary. If this component vanishes, then it is a tangential discontinuity for which the total plasma and field pressure across the discontinuity is conserved, while in the case of a finite normal component, it is a contact discontinuity for which the particles rapidly diffuse along field lines making the plasma pressure uniform. On the other hand, when the plasma flows across the boundary layers, these structures are propagated through the medium. The resulting waves are large amplitude, sharp structures or shock waves. These shocks include rotational discontinuities (intermediate shocks) as well as slow and fast shocks.

Equilibrium configurations of tangential discontinuities in collisionless plasmas have been discussed by a number of authors in the context of thermonuclear containment. Harris (1962) considered a pinch configuration in which an exactly charge-neutral layer (i.e., with no electric field) was confined between two oppositely directed magnetic fields produced by perpendicular flowing currents. On the other hand, Nicholson (1963) obtained a pinch configuration for an exactly charge-neutral layer confined between two reversing flowing diamagnetic currents perpendicular to a given external magnetic field. A superposition of Harris and Nicholson's models was made by Kan (1972). In Kan's model, reversing and field-aligned currents, perpendicular to each other, produced corresponding diamagnetic and opposite magnetic fields inside an exactly charge-neutral plasma with a slab configuration. Sestero (1964) provided a model for the microscopic description of hydrogen plasma sheaths using Vlasov equations for ions and electrons coupled with Maxwell's equations for the fields. The plasma sheaths he considered connected two different uniform states of a plasma in a magnetic field. In the charge-neutral approximation (i.e., with a small charge separation), solutions were obtained which scaled according to some representative electron Larmor radius, or ion Larmor radius, or piecewise, according to both. Sestero generalized this model by considering also cases involving shearing plasmas (Sestero, 1966). Inside these sheaths the magnitude of the magnetic field changed while the direction did not. However, Sestero did not include changes in composition, temperatures, anisotropies, etc... Although Kan's model included multidirectional currents (Kan, 1972), it was restricted to exactly charge-neutral layers.

These early models of tangential discontinuities were adjusted and generalized to describe the microscopic structure of current sheets in space plasmas. Thus, kinetic theories of tangential discontinuities were elaborated for the purpose of explaining the structure of the earth's plasmapause (Roth, 1976), of current sheets in the solar wind (Lemaire and Burlaga, 1976) and of the terrestrial magnetopause (Alpers, 1969; Roth, 1978, 1979, 1980; Lee and Kan, 1979). In this context the most improved model was elaborated by Roth in a series of papers (Roth, 1976, 1978, 1979, 1980). This model considers the structure of steady-state tangential discontinuities in a collisionless magnetized plasma with multiple particle species. It includes changes in magnetic field intensity and direction, plasma bulk velocity, composition, temperatures and anisotropies. It is not restricted to exactly charge-neutral layers. The role of collisions as prime mover for the dissipation is played by wave-particle interactions which determine the stability and thickness of the current layers (Roth, 1980).

By reason of the considerable significance of such kinetic models for our understanding of the microscopic structure of current layers in space plasmas we will devote this paper to a review of the author's model. Section 2 will be devoted to the description of this model, with particular emphasis on the boundary conditions. Unidimensional plane current layers are considered and the determination of their microscopic structure is based on both Vlasov and Maxwell's equations for plasma and fields. In section 3, the moments of the velocity distribution functions are determined in terms of the electric potential and the components of the vector potential. The numerical method for solving Maxwell's equations is explained in section 4. Hence, by solving these equations for both the potentials and the fields we can also compute the various moments determined in section 3. This is done by means of a suitable numerical program (see numerical results in Roth, 1978, 1979, 1980). Therefore a complete description of the microscopic structure of the current sheet can be achieved. In particular, the electric field is found by assuming that the charge-neutrality approximation remains true. It is also shown that the actual electric field is far from being identical with the convection electric field. Finally, conclusions will be summarized in section 5.

2. Description of the model

In a cartesian coordinate system, the plane of a discontinuity is parallel to the (y, z) plane and all the variables are assumed to depend on the x coordinate, normal to the discontinuity. Since there is no mass flow across the transition and since the parallel conductivity is very large, the electric field is everywhere oriented along the x-axis. Furthermore, the normal component of the magnetic field (B_x) is assumed to vanish since this model applies to the description of tangential discontinuities.

In rationalized MKSA units, Maxwell's equations become for the one-dimensional geometry considered here :

$$\frac{d^2 a_y}{dx^2} = - \mu_0 \sum_{\nu=1}^{\mu} j_y^{(\nu)} \tag{1}$$

$$\frac{d^2 a_z}{dx^2} = - \mu_0 \sum_{\nu=1}^{\mu} j_z^{(\nu)} \tag{2}$$

$$\frac{d^2 \phi}{dx^2} = - \frac{e}{\epsilon_0} \sum_{\nu=1}^{\mu} Z^{(\nu)} n^{(\nu)} \tag{3}$$

where ϵ_0 and μ_0 are the vacuum permittivity and permeability, respectively; μ, the number of particle species; a_y and a_z, the non-vanishing components of the vector potential ($\underset{\sim}{a}$); ϕ, the electric potential; $j_y^{(\nu)}$ and $j_z^{(\nu)}$, the components of the current density ($\underset{\sim}{j}^{(\nu)}$) produced by the flow of particles of species ν (each particle carrying a charge $Z^{(\nu)}e$; e being the magnitude of the electronic charge); and $n^{(\nu)}$, the corresponding number density.

The magnetic and electric structures of a transition are determined by solving the system of differential equations (1) - (3). This can be achieved by numerical methods when the current and number densities are known functions of ϕ, a_y, and a_z. These functions are the first moments of the plasma velocity distribution functions determined from Vlasov equation. These moments will be determined in section 3. For the potential ϕ, we replace Poisson's equation (3) by the quasi-neutrality approximation

$$\sum_{\nu=1}^{\mu} Z^{(\nu)} n^{(\nu)} (a_y, a_z, \phi) = 0 \tag{4}$$

A self-consistent electric field is obtained whenever the charge density proportional to the Laplacian of ϕ is much smaller than the charge density associated with the positive (or negative) particles.

The velocity distribution functions must satisfy Vlasov equation whose most general solution is any function depending on the constants of the motion of a single particle. These constants are the kinetic energy (H) and the y- and z- components of the generalized momentum ($\underset{\sim}{p}$).

From Liouville's theorem, it follows that the velocity distribution functions satisfying Vlasov equation are any functions of H, p_y and p_z. By generalizing the method used by Sestero (1964, 1966), the following velocity distribution functions $F^{(\nu)}$ have been used by Roth (1978, 1979, 1980) :

$$F^{(\nu)}(H, p_y, p_z) = \sum_{i=1}^{2} g_i^{(\nu)}(p_y, p_z)\, \eta_i^{(\nu)}(H, p_y, p_z) \tag{5}$$

where H, p_y and p_z vary in the set defined by the inequalities :

$$-\infty < p_y < +\infty \tag{6}$$

$$-\infty < p_z < +\infty \tag{7}$$

$$H_0^{(\nu)} < H < +\infty \tag{8}$$

with

$$H_0^{(\nu)} = Z^{(\nu)} e \phi + (2 m^{(\nu)})^{-1} [(p_y - Z^{(\nu)} e a_y)^2 + (p_z - Z^{(\nu)} e a_z)^2] \tag{9}$$

Here $m^{(\nu)}$ is the mass of particles of species ν. In the (H, p_y, p_z) space, the set defined by the inequalities (6) - (8) corresponds to the interior of a paraboloid of revolution whose symmetry axis is parallel to the H- axis and whose vertex is located at the point ($Z^{(\nu)} e \phi$, $Z^{(\nu)} e a_y$, $Z^{(\nu)} e a_z$). In equation (5), $\eta_i^{(\nu)}$ are shifted Maxwellians in the presence of an electric field while $g_i^{(\nu)}$ (i = 1, 2) are discontinuous functions in the (p_y, p_z) plane taking non-negative constant values $c_i^{(\nu)}(k)$ in each quadrant E_k (k = 1, 2, 3, 4) dividing this plane into four parts from a finite origin $\underset{\sim}{p}_{0,i}^{(\nu)}$. For $Z^{(\nu)} > 0$, these quadrants are defined in the following way :

$$E_1 =]-\infty, p_{0y,i}^{(\nu)}] \times [p_{0z,i}^{(\nu)}, +\infty[$$

$$E_2 = [p_{0y,i}^{(\nu)}, +\infty[\times [p_{0z,i}^{(\nu)}, +\infty[$$

$$E_3 =]-\infty, p_{0y,i}^{(\nu)}] \times]-\infty, p_{0z,i}^{(\nu)}]$$

$$E_4 = [p_{0y,i}^{(\nu)}, +\infty[\times]-\infty, p_{0z,i}^{(\nu)}]$$

For $Z^{(\nu)} < 0$, quadrants E_1, E_4 and E_2, E_3 are permuted. Therefore the asymptotic parts of the quadrants E_k are related to the asymptotic values of the components of the vector potential located in the corresponding quadrants E_k' of the (a_y, a_z) plane. This can be easily seen from the definition of the generalized momentum. Across the transition, from x = $-\infty$ (i = 1) to x = $+\infty$ (i = 2), the point of the vector potential draws a curve in the (a_y, a_z) plane starting in the asymptotic part of a quadrant E_{k_1}' and ending in the asymptotic part of another quadrant E_{k_2}'. (Transition from an asymptotic configuration of $\underset{\sim}{a}$ towards another one actually determines the structure of the magnetic field $\underset{\sim}{B}$ and, in particular, the amount of rotation of this vector across the tangential discontinuity). Therefore if

$$c_1^{(\nu)}(k_2) = 0 \tag{10}$$

$$c_2^{(\nu)}(k_1) = 0 \tag{11}$$

it is seen from Eq. (5) and the definition of $g_i^{(\nu)}$ that the velocity distribution function changes from $c_1^{(\nu)}(k_1)\,\eta_1^{(\nu)}$ at x = $-\infty$ to $c_2^{(\nu)}(k_2)\,\eta_2^{(\nu)}$ at x = $+\infty$. Note however that the same result is also achieved without any restriction on the values of $c_1^{(\nu)}(k_2)$ and $c_2^{(\nu)}(k_1)$ in the particular cases for which $\eta_2^{(\nu)} \to 0$ at x = $-\infty$ and $\eta_1^{(\nu)} \to 0$ at x = $+\infty$.

The asymptotic velocity distribution functions $c_i^{(\nu)}(k_i)\,\eta_i^{(\nu)}$ must have the same first order moments as the actual velocity distribution functions observed on each side of the tangential discontinuity. A simple description of these functions is given by shifted Maxwellians in the presence of an electric field :

$$\eta_i^{(\nu)}(H, p_y, p_z) = N_0 \left(\frac{m^{(\nu)}}{2\pi k T_i^{(\nu)}} \right)^{3/2} \exp\left(-\frac{H}{kT_i^{(\nu)}} \right) \times \exp\left\{ -\frac{1}{kT_i^{(\nu)}} \left[\frac{1}{2} m^{(\nu)} {V_i^{(\nu)}}^2 - \underset{\sim}{p} \cdot \underset{\sim}{V}_i^{(\nu)} \right] \right\} \tag{12}$$

where the lower indices i = 1 and i = 2 refer to quantities evaluated at x= $-\infty$ and x = $+\infty$, respectively. In Eq. (12), $T_i^{(\nu)}$ and $\underset{\sim}{V}_i^{(\nu)}$ are the observed average asymptotic temperatures and velocities of the particles of species ν, respectively; while N_0 is a constant which has the dimension of a number density. To simplify, asymptotic isotropic temperatures have been considered in this paper (However, asymptotic anisotropies have also been taken into account by Roth, 1980). It must be noticed that the velocity distribution functions defined by Eq. (5) are solutions of Vlasov equation in a weak way. Indeed, these solutions have mathematical discontinuities in the (p_y, p_z) plane since their derivatives are singular at the boundaries of quadrants E_k. However, as shown in section 3, any moment of the velocity distribution functions defined by Eq. (5) is continuous with respect to the potentials ϕ, a_y and

a_z. Furthermore, these moments strictly meet the full hierarchy of transport equations. It must also be noticed that the observed asymptotic densities $N_i^{(\nu)}$ and average velocities $\underset{\sim}{V}_i^{(\nu)}$ are not arbitrary. Indeed, the plasma at $x = \mp \infty$ is charge-neutral and homogeneous. This implies that

$$\sum_{\nu=1}^{\mu} Z^{(\nu)} N_i^{(\nu)} = 0 \tag{13}$$

$$V_{\perp,i}^{(\nu)} = C_{\perp,i} \tag{14}$$

$$\sum_{\nu=1}^{\mu} Z^{(\nu)} N_i^{(\nu)} V_{/\!/,i}^{(\nu)} = 0 \tag{15}$$

Equation (13) is the condition of charge neutrality at $x = \mp \infty$. In Eqs. (14) and (15), $V_{/\!/,i}^{(\nu)}$ and $V_{\perp,i}^{(\nu)}$ are respectively the parallel and perpendicular (with respect to the magnetic field direction) average velocities of the particles of species ν, at $x = \mp \infty$. Equation (14) means that the perpendicular components ($V_{\perp,i}^{(\nu)}$) are uniform and all are equal to the perpendicular component ($C_{\perp,i}$) of the asymptotic mass-velocity ($\underset{\sim}{C}_i$) defined by

$$\underset{\sim}{C}_i = \frac{\sum_{\nu=1}^{\mu} m^{(\nu)} N_i^{(\nu)} \underset{\sim}{V}_i^{(\nu)}}{\sum_{\nu=1}^{\mu} m^{(\nu)} N_i^{(\nu)}} \tag{16}$$

Indeed, since the plasma and fields become uniform asymptotically, the electric drift remains the only perpendicular drift. This also implies that the asymptotic electric field ($\underset{\sim}{E}_i$) is a convection electric field given by

$$\underset{\sim}{E}_i = - \underset{\sim}{C}_i \times \underset{\sim}{B}_i \tag{17}$$

Finally, equation (15) shows that the parallel electric current density vanishes at $x = \mp \infty$. Indeed, since the magnetic field becomes uniform at $x = \mp \infty$, the electric current becomes vanishingly small. From equation (14), it can be seen that, in frames of reference moving with the asymptotic mass-velocity of the plasma, the average peculiar velocity of the charged particles of species ν ($\underset{\sim}{u}_i^{(\nu)}$) is parallel to the asymptotic magnetic field, i.e.,

$$\underset{\sim}{u}_i^{(\nu)} = \underset{\sim}{V}_i^{(\nu)} - \underset{\sim}{C}_i = u_{/\!/,i}^{(\nu)} \underset{\sim}{e}_i \tag{18}$$

In this equation, $\underset{\sim}{e}_i$ is the unit vector parallel to the asymptotic magnetic field direction. From Eqs. (13), (15) and (18), it is easy to show that

$$\sum_{\nu=1}^{\mu} Z^{(\nu)} N_i^{(\nu)} u_{/\!/,i}^{(\nu)} = 0 \tag{19}$$

while, from Eqs. (18) and (16)

$$\sum_{\nu=1}^{\mu} m^{(\nu)} N_i^{(\nu)} u_{/\!/,i}^{(\nu)} = 0 \tag{20}$$

Equations (14), (19) and (20) are the conditions that the asymptotic average velocities $\underset{\sim}{V}_i^{(\nu)}$ must fulfill for the plasma and fields to remain uniform at $x = \mp \infty$.

From Eqs. (5) and (12) the asymptotic number densities are of the form :

$$N_i^{(\nu)} = c_i^{(\nu)}(k_i)\, N_0 \exp\left\{ - \frac{Z^{(\nu)}\, e(\phi_i - \underset{\sim}{a}_i \cdot \underset{\sim}{V}_i^{(\nu)})}{k\, T_i^{(\nu)}} \right\} \tag{21}$$

where the exponential term is the asymptotic Boltzmann factor. In this equation, $\underset{\sim}{a}_i$ and ϕ_i are the asymptotic vector potentials and the asymptotic electric potentials, respectively. The Boltzmann factor becomes homogeneous at $x = \mp \infty$, since the asymptotic electric field vanishes in each frame of reference moving with the velocity $\underset{\sim}{V}_i^{(\nu)}$. From Eq. (17), it can be deduced that

$$\phi_i = \underset{\sim}{a}_i \cdot \underset{\sim}{C}_i + \phi_{0,i} \tag{22}$$

where $\phi_{0,i}$ ($i = 1,2$) are the asymptotic values of the electric potential in frames of reference moving with $\underset{\sim}{C}_i$.

From Eqs. (21), (22) and (18), we can then calculate the asymptotic number densities. We find

$$N_i^{(\nu)} = c_i^{(\nu)}(k_i)\, N_0 \exp\left\{ - \frac{Z^{(\nu)}\, e\, \phi_{0,i}}{kT_i^{(\nu)}} + X_i^{(\nu)} \right\} \tag{23}$$

where

$$X_i^{(\nu)} = \frac{Z^{(\nu)}\, e}{k\, T_i^{(\nu)}}\, u_{/\!/,i}^{(\nu)}\, a_{/\!/,i} \tag{24}$$

In equation (24), $u_{/\!/,i}^{(\nu)}$ are the average peculiar speeds of the charged particles of species ν at $x = \mp \infty$ in a direction parallel to the magnetic field, while $a_{/\!/,i}$ are the parallel components of the asymptotic vector potentials (also with respect to the magnetic field direction). As the Boltzmann factor is homogeneous at $x = \mp \infty$, it is clear from Eqs. (23) and (24) that this must also be the case for $a_{/\!/}$.

Note however that the integration of Eqs. (1) - (3) requires only initial values at $x = -\infty$. Therefore, the direction of the magnetic field at $x = +\infty$ is not known a priori (This is not the case for the intensity B which can be deduced from a pressure balance condition). To avoid an iteration process we consider the still general cases for which $X_2^{(\nu)} = 0$. These cases can be classified into two distinct classes. The first class includes the transitions for which the magnetic field remains parallel to a given direction ($a_{/\!/} = 0$ in the whole transition) while the second includes the transitions for which the average velocities of all particle species at $x = +\infty$ are identical to the corresponding mass-velocity of the plasma ($u_{/\!/,2}^{(\nu)} = 0$ for $\nu = 1,\ldots \mu$).

If we normalize the electric potential in such a way that

$$\phi_{0,1} = 0 \tag{25}$$

we can see, by putting i = 1 in Eq. (23) that

$$c_1^{(\nu)}(k_1) = \frac{N_1^{(\nu)}}{N_0} \exp(-X_1^{(\nu)}) \tag{26}$$

In eq. (26), N_0 is the total number density of electrons on side 2, i.e., if there is j(1) electron species,

$$N_0 = \sum_{a=1}^{j(1)} N_2^{(a)} \tag{27}$$

If we assume that $\nu = 1$ corresponds to an electron species whose number density is non-vanishing at x = + ∞ ($N_2^{(1)} \neq 0$), it can be deduced, by putting $\nu = 1$ and i = 2 in Eq. (23) that

$$\phi_{0,2} = -\frac{k\,T_2^{(1)}}{Z^{(1)}\,e} \ln\left(\frac{N_2^{(1)}}{c_2^{(1)}(k_2)\,N_0} \right) \tag{28}$$

Taking account of Eqs. (22) and (25), this constant $\phi_{0,2}$ is seen to be the electric potential difference between x = + ∞ and x = − ∞, in a frame of reference moving with the plasma mass-velocity. From Eq. (28), it is also seen that the parameter $c_2^{(1)}(k_2)$ can be chosen as an arbitrary positive number regulating the electric potential jump across the transition layer. The other $c_2^{(\nu)}(k_2)$ for $\nu > 1$ can now be deduced from Eqs. (23), (28) and (24). We find, for $\nu = 2,\ldots,\mu$

$$c_2^{(\nu)}(k_2) = \frac{N_2^{(\nu)}}{N_0} \left(\frac{N_2^{(1)}}{c_2^{(1)}(k_2)\,N_0} \right)^{-\frac{Z^{(\nu)}\,T_2^{(1)}}{Z^{(1)}\,T_2^{(\nu)}}} \tag{29}$$

The role of the constants $c_i^{(\nu)}(k_3)$ and $c_i^{(\nu)}(k_4)$ related to quadrants k_3 and k_4 is to allow the point of the vector potential $\underline{a}$ to draw a curve within the (a_y, a_z) plane whose asymptotic limits turn out to be in predetermined quadrants : E'_{k_1} (corresponding to x = − ∞) and E'_{k_2} (corresponding to x = + ∞). Generally, this will be achieved if the kinetic plasma pressure associated with the asymptotic parts of quadrants E_{k_3} and E_{k_4} is larger than the total pressure (kinetic + magnetic) associated with the asymptotic parts of quadrant E_{k_1}. This pressure unbalance can be made possible by a suitable choice of the parameters $c_i^{(\nu)}(k_3)$ and $c_i^{(\nu)}(k_4)$. Therefore, we should be able to simulate arbitrary rotations of the magnetic field in the (y, z) plane.

Finally, for each velocity distribution function, 4 additional parameters ($p_{0y,1}$, $p_{0z,1}$) and ($p_{0y,2}$, $p_{0z,2}$) define two sets of quadrants whose origin is located at $\underline{p}_{0,1}$ and $\underline{p}_{0,2}$, respectively. Their role is to overlap or separate the contributions of the asymptotic plasma distributions of the form $c_i^{(\nu)}(k_i)\,\eta_i^{(\nu)}$.

If

$$\sum_{i=1}^{2} c_i^{(\nu)}(k_j) = \text{constant} \quad ; \quad j = 1, 2, 3, 4 \tag{30}$$

and

$$\eta_1^{(\nu)} = \eta_2^{(\nu)} \tag{31}$$

$$\underline{p}_{0,1}^{(\nu)} = \underline{p}_{0,2}^{(\nu)} \tag{32}$$

the corresponding ν - velocity distribution function ($F^{(\nu)}$) is a shifted Maxwellian everywhere within the transition and it is obvious that the temperature ($\Theta^{(\nu)}$) and the average velocity ($\langle \underline{v}^{(\nu)} \rangle$) remain uniform from x = − ∞ to x = + ∞. If the electron (/ion) species remain shifted Maxwellians from x = − ∞ to x = + ∞, only the ions (/electrons) can be accelerated inside the transition, on a characteristic scale length of the order of a few ion (/electron) Larmor gyroradii. As Sestero (1964) we call these kinds of transition ion(/electron) layers, respectively. On the other hand, if Eqs. (30) - (32) are not fulfilled together, then one obtains transitions which are variously scaled. Namely, the scale length is an electron Larmor radius near the middle of the sheath and an ion Larmor radius further out towards the two ends.

3. Moments of the velocity distribution functions

In this section, the results are related to any velocity distribution function. Therefore, the upper indices will be dropped, unless otherwise stated. All the variables can be made dimensionless by introducing four basic units for length (λ_x), velocity (λ_v), time (λ_t) and mass (λ_m), defined by

$$\lambda_x = \left(\frac{m_e}{e^2\,\mu_0\,N_0} \right)^{1/2} \tag{33}$$

$$\lambda_v = \left(\frac{k\,T_2^{(a)}}{m_e} \right)^{1/2} \tag{34}$$

$$\lambda_t = \frac{m_e}{e\lambda_B} \tag{35}$$

$$\lambda_m = m_e \tag{36}$$

In these equations (33) - (36), m_e is the electron mass and it is assumed that the upper index (a) pertains to an electron species. Corresponding to this electron species, the unit of length (λ_x) is the skin depth, the unit of velocity (λ_v) is the thermal velocity, the unit of time (λ_t) is proportional to the gyroperiod in a magnetic field whose magnitude is λ_B, the magnetic field unit, and, finally, the electron mass is the unit of mass (λ_m). With this system of units, it is easy to demonstrate that relations between dimensional physical variables and the corresponding dimensionless ones are unchanged provided that, in the dimensionless system used, the magnitude of the electronic charge (e), the permeability of vacuum (μ_0) and the Boltzmann's constant (k) be all together equal to unity, i.e. (identifying dimensionless quantities with a star index)

$$e^* = \mu_0^* = k^* = 1 \tag{37}$$

From Poisson's equation (3), it can also be deduced that

$$\epsilon_0^* = \frac{k\, T_2^{(\alpha)}}{m_e\, c^2} \tag{38}$$

where c is the speed of light in vacuum. Therefore, the dimensionless form of Eq. (3) is

$$\frac{d^2\phi^*}{dx^{*2}} = - \frac{m_e\, c^2}{k\, T_2^{(\alpha)}} \sum_{\nu=1}^{\mu} Z^{(\nu)}\, n^{*(\nu)} \tag{39}$$

and the charge density can be ignored provided that

$$\frac{k\, T_2^{(\alpha)}}{m_e\, c^2} \left| \frac{d^2\phi^*}{dx^{*2}} \right| << 1 \tag{40}$$

Eq. (40) is a necessary and sufficient condition for the quasi-neutrality approximation to hold.

Let us take, for $\nu = 1,\ldots,\mu$ and $i = 1,2$

$$\alpha_i^{(\nu)} = \frac{k\, T_2^{(\alpha)}}{k\, T_i^{(\nu)}} = \frac{1}{[k\, T_i^{(\nu)}]^*} \tag{41}$$

i.e., the inverse of the asymptotic thermal energies.

In the remaining part of this paper we shall only use dimensionless quantities and shall therefore leave out the star index.

Moments of any order Q_{rst} are defined by

$$Q_{rst} = n < v_x^r\, v_y^s\, v_z^t > \tag{42}$$

where n is the number density, v_x, v_y and v_z, the components of the particle velocity. The symbolization $< v_x^r\, v_y^s\, v_z^t >$ represents the average value of the variable quantity $v_x^r\, v_y^s\, v_z^t$ over the entire distribution of velocities.

It is found (Roth, 1980)

$$Q_{rst} = 0 \tag{43}$$

if r is odd

$$Q_{rst} = \sum_{i=1}^{2} \sum_{k=1}^{4} M_{rst}\,(k,i) \tag{44}$$

if r is even, with

$$M_{rst}(k,i) = c_i(k)\; \xi_{rst}\; \Delta_i(r)\; L_{y;k,i}(s)\; L_{z;k,i}(t) \tag{45}$$

where

$$\xi_{rst} = \frac{r!}{2^{r/2}\, \pi(r/2)!}\; Z^{s+t}\; m^{-\left(\frac{r+s+t}{2}\right)} \tag{46}$$

$$\Delta_i(r) = \alpha_i^{-r/2}\; \exp\,(-\varphi_i) \tag{47}$$

$$L_{y;k,i}(s) = \sum_{j=0}^{s} \left\{ \binom{s}{j} \frac{\epsilon_{y,k}^j}{(\frac{1}{2}\Lambda_i)^{j/2}}\; \kappa(s-j,\, D_{y,i}) \times R_j\left[-\left(\tfrac{1}{2}\Lambda_i\right)^{1/2} \epsilon_{y,k}\, U_{y,i}\right]\right\} \tag{48}$$

$$L_{z;k,i}(t) = \sum_{j=0}^{t} \left\{ \binom{t}{j} \frac{\epsilon_{z,k}^j}{(\frac{1}{2}\Lambda_i)^{j/2}}\; \kappa(t-j,\, D_{z,i}) \times R_j\left[-\left(\tfrac{1}{2}\Lambda_i\right)^{1/2} \epsilon_{z,k}\, U_{z,i}\right]\right\} \tag{49}$$

with

$$\varphi_i = \alpha_i\, Z\, (\phi - \mathbf{a} \cdot \mathbf{V}_i) \tag{50}$$

$$\mathbf{U}_i = \mathbf{h} + \mathbf{D}_i - \mathbf{\Omega}_i \tag{51}$$

$$\mathbf{\Omega}_i = Z^{-1}\, m^{-1/2}\, \mathbf{P}_{0,i} \tag{52}$$

$$\Lambda_i = \alpha_i\, Z^2 \tag{53}$$

$$\mathbf{h} = m^{-1/2}\, \mathbf{a} \tag{54}$$

$$\mathbf{D}_i = m^{1/2}\, Z^{-1}\, \mathbf{V}_i \tag{55}$$

The vectors pertaining to Eqs. (50) - (55) are two-dimensional vectors in the (y, z) plane. The usual binomial coefficients occur in the sums defined by Eqs. (48) and (49). In these sums, $\boldsymbol{\epsilon}_k$ (k = 1,...,4) are two-dimensional vectors whose definition is the following

$$\left.\begin{aligned} \boldsymbol{\epsilon}_1 &= (-1,\, +1) \\ \boldsymbol{\epsilon}_2 &= (+1,\, +1) \\ \boldsymbol{\epsilon}_3 &= (-1,\, -1) \\ \boldsymbol{\epsilon}_4 &= (+1,\, -1) \end{aligned}\right\} \tag{56}$$

Also, in Eqs. (48) and (49), expressions like $\kappa(i,y)$ and $R_i(y)$ are functions defined for real "y" and non-negative integral "i" as follows :

$$\kappa(i,y) = y^i \tag{57}$$

for i > 0

$$\kappa(i,y) = 1 \tag{58}$$

for $i = 0$

$$R_i(y) = \int_y^{+\infty} x^i \exp(-x^2)\, dx \tag{59}$$

In particular,

$$\kappa(0,0) = 1 \tag{60}$$

$$R_0(y) = \frac{\pi^{1/2}}{2} \text{ erfc } (y) \tag{61}$$

$$R_1(y) = \frac{1}{2} \exp(-y^2) \tag{62}$$

where erfc(y) is the complementary error function :

$$\text{erfc}(y) = \frac{2}{\pi^{1/2}} \int_y^{+\infty} \exp(-x^2)\, dx \tag{63}$$

4. <u>The numerical methods of solving Maxwell's equations</u>

We are now able to determine the second members of Eqs. (1) - (2) and the first member of Eq. (4) in terms of ϕ, a_y and a_z. Indeed, we have for each particle species (leaving out the species index)

$$n = Q_{000} \tag{64}$$

$$j_y = Z\, Q_{010} \tag{65}$$

$$j_z = Z\, Q_{001} \tag{66}$$

Let us define (with $k = 1$)

$$\underset{\sim}{A}_i = \left(\frac{Z^2}{2m\, k\, T_i} \right)^{1/2} (\underset{\sim}{a} + \underset{\sim}{\chi}_i) \tag{67}$$

with

$$\underset{\sim}{\chi}_i = Z^{-1} (m\, \underset{\sim}{V}_i - \underset{\sim}{p}_{0,i}) \tag{68}$$

Let us also define

$$\phi_i' = \phi - \underset{\sim}{a} \cdot \underset{\sim}{V}_i \tag{69}$$

i.e., the electric potential in a frame of reference moving with the average velocity $\underset{\sim}{V}_i$. Then, from Eqs. (50), (41) and (69), it can be seen that

$$\Psi_i = \frac{Z}{kT_i} \phi_i' \tag{70}$$

Now, from the formulae established in the previous section, it can be deduced that

$$Q_{000}^{(\nu)} = \sum_{i=1}^{2} \sum_{j=1}^{4} n_{ij}^{(\nu)} \tag{71}$$

$$Q_{010}^{(\nu)} = \sum_{i=1}^{2} \sum_{j=1}^{4} f_{ij,y}^{(\nu)} \tag{72}$$

$$Q_{001}^{(\nu)} = \sum_{i=1}^{2} \sum_{j=1}^{4} f_{ij,z}^{(\nu)} \tag{73}$$

with

$$n_{ij}^{(\nu)} = \frac{1}{4} K_i^{(\nu)} G_{ij}^{(\nu)} \tag{74}$$

$$f_{ij,y}^{(\nu)} = \frac{1}{4} K_i^{(\nu)} c_i^{(\nu)}(j) \text{ erfc } (-\epsilon_{z,j} A_{z,i}^{(\nu)}) \times$$
$$[V_{y,i}^{(\nu)} \text{ erfc } (-\epsilon_{y,j} A_{y,i}^{(\nu)}) + \epsilon_{y,j} \text{ sign } Z^{(\nu)} (2/\pi)^{1/2} \theta_i^{(\nu)} \times$$
$$\exp(-A_{y,i}^{(\nu)2})] \tag{75}$$

and

$$f_{ij,z}^{(\nu)} = \frac{1}{4} K_i^{(\nu)} c_i^{(\nu)}(j) \text{ erfc } (-\epsilon_{y,j} A_{y,i}^{(\nu)}) \times$$
$$[V_{z,i}^{(\nu)} \text{ erfc } (-\epsilon_{z,j} A_{z,i}^{(\nu)}) + \epsilon_{z,j} \text{ sign } Z^{(\nu)} (2/\pi)^{1/2} \theta_i^{(\nu)} \times$$
$$\exp(-A_{z,i}^{(\nu)2})] \tag{76}$$

where $\theta_i^{(\nu)}$ is the thermal velocity

$$\theta_i^{(\nu)} = \left(\frac{k\, T_i^{(\nu)}}{m^{(\nu)}} \right)^{1/2} \tag{77}$$

and

$$K_i^{(\nu)} = \exp\left(- \frac{Z^{(\nu)} \phi_i'^{(\nu)}}{k\, T_i^{(\nu)}} \right) \tag{78}$$

$$G_{ij}^{(\nu)} = c_i^{(\nu)}(j) \text{ erfc } (-\epsilon_{y,j} A_{y,i}^{(\nu)}) \text{ erfc } (-\epsilon_{z,j} A_{z,i}^{(\nu)}) \tag{79}$$

$$\left.\begin{aligned} \text{sign } Z &= +1 \quad \text{for } Z > 0 \\ &= -1 \quad \text{for } Z < 0 \end{aligned}\right\} \tag{80}$$

Equations (1) - (3) can be written

$$\frac{dB_z}{dx} = -\mu_0 \sum_{\nu=1}^{\mu} Z^{(\nu)} Q_{010}^{(\nu)} \tag{81}$$

$$\frac{dB_y}{dx} = \mu_0 \sum_{\nu=1}^{\mu} Z^{(\nu)} Q_{001}^{(\nu)} \tag{82}$$

$$\frac{d^2\phi}{dx^2} = -\frac{1}{\epsilon_0} \sum_{\nu=1}^{\mu} Z^{(\nu)} Q_{000}^{(\nu)} \tag{83}$$

As shown by Eq. (40), the quasi-neutrality approximation is expected to hold in most cases. Therefore, the electric potential is more suitably determined from the plasma neutrality equation :

$$\sum_{\nu=1}^{\mu} Z^{(\nu)} Q_{000}^{(\nu)} = 0 \tag{4}$$

The electric field E can be obtained explicitly in terms of ϕ, a_y and a_z, by taking the first derivative of Eq. (4) with respect to x (Roth, 1980). It is found that

$$E = - \frac{\sum_{\nu=1}^{\mu} Z^{(\nu)2} \sum_{i=1}^{2} \sum_{j=1}^{4} \frac{1}{kT_i^{(\nu)}} (\underline{\ell}_{ij}^{(\nu)} \times \underline{B})_x}{\sum_{\nu=1}^{\mu} Z^{(\nu)2} \sum_{i=1}^{2} \sum_{j=1}^{4} \frac{1}{kT_i^{(\nu)}} n_{ij}^{(\nu)}} \tag{84}$$

This result shows that the convection electric field (E_c) differs generally from the actual electric field (E). Indeed,

$$E_c = - (\underline{C} \times \underline{B})_x$$

$$= - \frac{\sum_{\nu=1}^{\mu} m^{(\nu)} \sum_{i=1}^{2} \sum_{j=1}^{4} (\underline{\ell}_{ij}^{(\nu)} \times \underline{B})_x}{\sum_{\nu=1}^{\mu} m^{(\nu)} \sum_{i=1}^{2} \sum_{j=1}^{4} n_{ij}^{(\nu)}} \tag{85}$$

For the electric field (E) be equal to the convection electric field (E_c), it is sufficient that

$$\frac{Z^{(\nu)2}}{m^{(\nu)} \, k \, T_i^{(\nu)}} = \text{constant} \tag{86}$$

for $\nu = 1, \ldots \mu$ and $i = 1,2$.

For example, in the case of a hydrogen plasma whose ion temperature is much smaller than the electron temperature (cold plasma approximation), the condition given by Eq. (86) could be met. However, in the case of the earth's magnetopause, where the H^+ temperature is about 10 times as large as the electron temperature (Eastman, 1979), Eq. (86) could not be satisfied.

The charge density (q) can also be obtained by taking the second derivative of Eq. (4) with respect to x. The result can be found in Roth (1980).

The differential equations (81) and (82), together with the field equations

$$B_y = - \frac{da_z}{dx} \tag{87}$$

$$B_z = \frac{da_y}{dx} \tag{88}$$

form a system of four differential equations of the first order for a_y, a_z, B_y and B_z. This system of differential equations, coupled with Eq. (4) whose solution is obtained by Newton's method of successive approximation, is integrated numerically by using a Hamin's predictor-corrector scheme (Ralston and Wilf, 1965). In practice, one starts integrating with initial values of $|a_{y,1}|$ and $|a_{z,1}|$ large enough for the asymptotic values of Q_{rst} to be reached. Simultaneously with the computation of ϕ, a_y, a_z, B_y and B_z, the moments of the velocity distribution function are computed from Eqs. (43), (44) and (45) - (59). This gives the complete description of the structure of the current sheet. Indeed, any physical parameter describing this structure can be expressed in terms of Q_{rst}.

A numerical program is available at the Institute for Space Aeronomy (Brussels). It takes account of a maximum of 12 particle species. This program can be applied to the description of current sheets separating the "cells" of space plasmas and numerical applications can be found in papers by Roth (1978, 1979, 1980).

5. Conclusions

The kinetic model that has been developed in this paper can be used to determine numerically the internal structure of a tangential discontinuity in a collisionless plasma. On each side of this plasma transition layer, the first moments of the theoretical velocity distribution functions are identified with the corresponding moments of the actual distribution functions. This model is not restricted to a hydrogen plasma but the formalism is developed to take account of a plasma with multiple particle species. It also includes the presence of an electric field, normal to the plane of the discontinuity. This electric field is self-consistent since it derives from the distribution of the electric potential which allows the plasma to be quasi-neutral. In most cases, the weak charge density sustaining this electric field does not however violate the quasi-neutrality condition.

Although discontinuous in the plane of the generalized momenta, the theoretical velocity distribution functions are not only solutions of Vlasov equation but also solutions of the transport equations. They form a set of linear combinations of shifted Maxwellians which moreover are just one class of functions amongst a number of others depending only on the constants of motion (in the Hamiltionian formalism). From a mathematical point of view, these distribution functions are very simple. Nevertheless, they include all the parameters describing the asymptotic characteristics of the plasma, i.e., the average velocity, number density and temperature of each particle species. Moments of arbitrary order have been determined analytically in terms of the electric potential (ϕ) and of the components of the vector potential (a_y and a_z).

Since 1978, a numerical program for the description of the internal structure of tangential discontinuities has been available at the Institute for Space Aeronomy (Brussels). This program is based on the theory developed in this paper and solves Maxwell's equations for the potentials and the magnetic field. Furthermore, this program computes the moments of the velocity distribution functions (up to 12 particle species), the electric field and the charge separation, thus leading to a complete description of the internal structure of the layer. It can be used daily and the results are given on a graphical format for every physical variable calculated from moments up to the third order. Numerical examples obtained with this program have already been published elsewhere (Roth, 1978, 1979, 1980). They gave a description of the internal structure of the terrestrial magnetopause.

As shown in this paper, our model is based on the kinetic description of plasma without resorting to the MHD theory. In particular, it has been shown that this latter theory is unable to describe the structure of thin current sheets, since the actual electric field is generally not equal to the convection electric field which is assumed to be a good approximation of the actual electric field in the MHD theory. Such a result was already pointed out by Eastman (1979, p. 97-101) and Roth (1982) for scale lengths and plasma parameters that are often observed near the earth's magnetopause and adjacent plasma boundary layer. Consequently, electric and gradient drifts in sharp boundary layers can be very different from what is usually assumed from the classical MHD approximation.

Finally, the many parameters used in this model make it very general. By a suitable choice of these parameters, it is expected to "mimic" the observed structure of a large class of tangential discontinuities. Furthermore, when the model resorts to moments of higher order, like the energy flow, the results that can be obtained precede the observations (Roth, 1980). More generally, the model and its associated numerical program are a powerful tool for studying the structure of tangential discontinuities which are inherent in collisionless space (or even laboratory) plasmas. This includes some of the boundary layers resulting from the cellular structure of space plasmas, from the planetary and pulsar magnetopauses to the cometary tails, without forgetting the micro-scale structure of stellar winds.

References

Alfvèn, H., Cosmic plasma, D. Reidel Publishing Company, Holland, Dordrecht, 164 p., 1981.

Alpers, W., Steady state charge neutral models of the magnetopause, Astrophys. Space Sci., 5, 425-437, 1969.

Burlaga, L.F., Hydromagnetic waves and discontinuities in the solar wind, Space Sci. Rev., 12, 600-657, 1971.

Burlaga, L.F., J.F. Lemaire and J.M. Turner, Interplanetary current sheets at 1 AU, J. Geophys. Res., 82, 3191-3200, 1977.

Eastman, T.E., The plasma boundary layer and magnetopause layer of the earth's magnetosphere, Ph. D. Thesis, Los Alamos Scientific Laboratory, LA-7842T, 246 p., 1979.

Fälthammar, C.-G., S.-I. Akasofu, and H. Alfvèn, The significance of magnetospheric research for progress in astrophysics, Nature, 275, 185-188, 1978.

Harris, E.G., On a plasma sheath separating regions of oppositely directed magnetic field, Nuovo Cimento, 23, 115, 1962.

Kan, J.R., Equilibrium configurations of Vlasov plasmas carrying a current component along an external magnetic field, J. Plasma Phys., 445-449, 1972.

Lee, L.C., and J.R. Kan, A unified kinetic model of the tangential magnetopause structure, J. Geophys. Res., 84, 6417-6426, 1979.

Lemaire, J., and L.F. Burlaga, Diamagnetic boundary layers : a kinetic theory, Astrophys. Space Sci., 45, 303-325, 1976.

Nicholson, R.B., Solution of the Vlasov equations for a plasma in a uniform magnetic field, Phys. Fluids, 6, 1581-1586, 1963.

Nishida, A., Geomagnetic diagnosis of the magnetosphere, Springer-Verlag, New York, 256 p., 1978.

Ralston, A., and H.S. Wilf (Eds.), Méthodes mathématiques pour calculateurs arithmétiques, Dunod, Paris, 482 p., 1965.

Roth, M., The plasmapause as a plasma sheath : a minimum thickness, J. Atmos. Terr. Phys., 38, 1065-1070, 1976.

Roth, M., Structure of tangential discontinuities at the magnetopause : the nose of the magnetopause, J. Atmos. Terr. Phys., 40, 323-329, 1978.

Roth, M., A microscopic description of interpenetrated plasma regions, in Magnetospheric Boundary Layers, edited by B. Battrick and J. Mort, ESTEC, Noordwijk, The Netherlands, ESA SP-148, pp. 295-309, 1979.

Roth, M., La structure interne de la magnétopause, Ph. D. Thesis, Brussels, 333 p., Aeronomica Acta A, 221, 1980.

Roth, M., On the relevance of the MHD approach to study the Kelvin-Helmholtz instability of the terrestrial magnetopause, Bull. Classe Sc., Académie Royale de Belgique, 68, 443-453, 1982.

Sestero, A., Structure of plasma sheaths, Phys. Fluids, 7, 44-51, 1964.

Sestero, A., Vlasov equation study of plasma motion across magnetic fields, Phys. Fluids, 9, 2006-2013, 1966.