THE OBLATENESS EFFECT ON THE EXTRATERRESTRIAL SOLAR RADIATION

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Abstract—Calculations of the daily solar radiation incident at the top of the Earth's atmosphere, with and without the effect of the oblateness, are presented in a figure illustrating the seasonal and latitudinal variation of the ratio of both insolations. It is shown that, in summer, the daily insolation of an oblate Earth is slightly increased in two regions symmetric with respect to the summer solstice. In winter, the flattening effect results in a somewhat more extensive polar region, the solar energy input being always reduced (in some cases by more than 2 per cent) when compared to a spherical one. In addition, we also numerically studied the mean daily solar radiation. It is found that the mean summer daily insolation is scarcely increased between the equator and the subsolar point, but decreased poleward of the above mentioned limit. In winter, however, the mean daily insolation is always reduced, the maximum loss of insolation attaining as much as 1 per cent in the 55–85° latitude interval. The partial gain of the mean summertime insolation being much smaller than the reduction during winter season evidently yields a mean annual daily insolation which is decreased, maximally by about 0.3 per cent, at all latitudes.

Resumé—L'insolation diurne au sommet de l'atmosphère de la Terre est calculée, d'une part, en assimilant la Terre à une sphère, d'autre part, en tenant compte de son aplatissement. Les résultats sont présentés dans une figure, illustrant les variations saisonnières et latitudinales du rapport des deux insolations. On montre qu'en été, l'aplatissement donne lieu à une insolation légèrement supérieure dans deux régions symétriques par rapport au solstice d'été. En hiver, l'effet de l'aplatissement implique une région polaire faiblement étendue, l'insolation diurne d'une Terre aplatie s'avérant toujours plus petite (dans un rapport parfois supérieur à 2 pour cent) comparée au cas d'une Terre sphérique. Nous avons également étudié numériquement l'insolation diurne moyenne. Il en résulte que l'insolation diurne moyenne en été augmente légèrement pour des latitudes comprises entre l'équateur et le point subsolaire, mais diminue pour des latitudes supérieures. En hiver, l'insolation diurne moyenne est toujours moindre, la réduction maximale atteignant une valeur approximative de 1 pour cent dans un intervalle de latitude allant de 55 à 85°. Le gain d'insolation diurne moyenne en été étant beacoup plus petit que la la perte en hiver, il s'ensuit que l'insolation diurne moyenne annuelle est réduite à toutes les latitudes (de l'ordre de 0.3 pour cent au maximum).

INTRODUCTION

In studies involving the distribution of solar radiation incident at the Earth's surface and its variability with latitude and season, the solar energy reaching the top of the atmosphere constitutes a very important input data. This amount of solar radiant flux is governed by two factors: on one hand, the solar flux as a function of the orbital distance and, on the other hand, the cosine of the solar zenith angle.

Considering more particularly the latter parameter, it should be emphasized that for a planet assumed to be spherical, the zenith distance may be expressed as a simple function of the latitude, the solar declination and the local hour angle of the Sun, the radius vector being normal to the horizon plane. For an oblate planet, however, this is not the case since in general there is an angle between the two directions mentioned above. This so-called angle of the vertical depends upon the flattening but vanishes at the equator and the poles.

Although the upper-boundary insolation of the Earth's atmosphere[1, 2] and the radiation incident on its surface [3–8] have been treated extensively in recent years, it must be pointed out that, to the best of our knowledge, the effect of the oblateness on the extraterrestrial solar radiation has never been discussed at least on a quantitative basis. This paper, therefore, is an attempt to investigate in detail the influence of the flattening on the radiation which would be incident in the absence of any atmosphere.

Our results are presented in the form of a contour map giving the seasonal and latitudinal variation of the ratio of the daily insolation with and without the effect of the oblateness. For the sake of completeness we have included a diagram showing the percentage difference of the mean (summer, winter and annual) daily insolations as a function of latitude.

1. DAILY INSOLATION WITH AND WITHOUT THE OBLATENESS EFFECT

The instantaneous insolation I_0 at the upper-boundary of the Earth can be expressed as [9–11]

$$I_0 = S \cos \theta_z \tag{1}$$

with

$$S = S_0/r_s^2 \tag{2}$$

and

$$r_s = a_s(1 - e^2)/(1 + e \cos W) \tag{3}$$

where θ_z is the zenith angle of the incident radiation, S is the solar flux at an heliocentric distance r_s and S_0 is the

Table 1. Elements of the planetary orbit and dimensions of the Earth

a_s (AU)	e	λ_p (°)	ε (°)	a_e (km)	a_p (km)	f	T (Earth days)
1	0.01672	282.05	23.45	6378	6357	0.00329	1

solar constant at the mean Sun–Earth distance of 1 AU taken at 1368 W m^{-2} [12]. Furthermore, in expression (3), a_s, e and W are, respectively, the Earth's semi-major axis, the eccentricity and the true anomaly which is given by:

$$W = \lambda_s - \lambda_P \qquad (4)$$

where λ_s and λ_P are the planetocentric longitude of the Sun and the planetocentric longitude of the Earth's perihelion. The numerical values of the parameters used for the computations are listed in Table 1. In this table one can find also the obliquity ϵ, the equatorial radius a_e, the polar radius a_p, the flattening $f = (a_e - a_p)/a_e$ and the sidereal period of axial rotation T (sidereal day). Note that the elements of the planetary orbit and the dimensions of the Earth are taken from the Handbook of the British Astronomical Association [13] and from Vorob'yev and Monin [1].

For a spherical planet, the zenith angle θ_z may be expressed as:

$$\cos \theta_z = \sin \phi' \sin \delta_s + \cos \phi' \cos \delta_s \cos \omega_s \qquad (5)$$

where ϕ' is the geocentric latitude (which in this case, equals the geographic latitude ϕ), δ_s is the solar declination and ω_s is the hour angle of the Sun. Furthermore, δ_s can be calculated using the following expression:

$$\sin \delta_s = \sin \epsilon \sin \lambda_s \qquad (6)$$

The daily insolation H_0, being defined as the amount of incoming solar radiation at the top of the Earth's atmosphere over the planet's day, can now be obtained by integrating expression (1) over daytime assumed to be equal to the time that elapses between rising and setting of the Sun; we obtain:

$$H_0 = (ST/\pi)(\omega_{ss} \sin \phi' \sin \delta_s + \sin \omega_{ss} \cos \phi' \cos \delta_s) \qquad (7)$$

where ω_{ss} is the sunset hour angle and may be determined from expression (5) by the condition that at sunset $\cos \theta_z = 0$. It follows that:

$$\omega_{ss} = \text{arc cos} (- \tan \delta_s \tan \phi') \qquad (8)$$

if $|\phi'| < \pi/2 - |\delta_s|$.

In regions where the Sun does not rise ($\phi' < - \pi/2 + \delta_s$ or $\phi' > \pi/2 + \delta_s$) we have $\omega_{ss} = 0$; in regions where the Sun remains above the horizon all day ($\phi' > \pi/2 - \delta_s$ or $\phi' < - \pi/2 - \delta_s$) we may put $\omega_{ss} = \pi$.
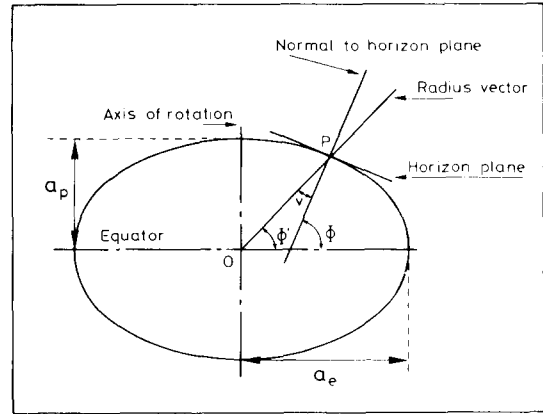

Fig. 1. Representation of some parameters involved with the oblate calculations.

In the case of an oblate planet (Fig. 1), there is an angle v, the so-called angle of the vertical, between the radius vector and the normal to the horizon plane; as already mentioned in the introduction, it vanishes at the equator and the poles while elsewhere $\phi > \phi'$ numerically. The angle v is dependent upon the geocentric latitude ϕ' and the flattening f by the relationship:

$$v = \arctan [(1 - f)^{-2} \tan \phi'] - \phi'. \qquad (9)$$

Defining $\theta_{z,0}$ as the zenith distance for an oblate planet, the following relation can easily be found by applying the formulas of spherical trigonometry:

$$\cos \theta_{z,0} = \cos v \cos \theta_z + \sin v(- \tan \phi' \cos \theta_z + \sin \delta_s \sec \phi'). \qquad (10)$$

The daily insolation of an oblate planet $H_{0,0}$ can now be obtained by integrating expression (1), within the appropriate time limits where $\cos \theta_z$ has to be replaced by relation (10) yielding:

$$H_{0,0} = (ST/\pi) \{\cos v (\omega_{ss,0} \sin \phi' \sin \delta_s$$
$$+ \sin \omega_{ss,0} \cos \phi' \cos \delta_s)$$
$$+ \sin v [- \tan \phi' (\omega_{ss,0} \sin \phi' \sin \delta_s$$
$$+ \sin \omega_{ss,0} \cos \phi' \cos \delta_s) + \omega_{ss,0} \sin \delta_s \sec \phi']\} \qquad (11)$$

where $\omega_{ss,0}$, the local hour angle at sunset for an oblate planet, is generally slightly different from ω_{ss}. As for a spherical planet, $\omega_{ss,0}$ may be derived from relation (10) by putting $\cos \theta_{z,0} = 0$. After some rearrangements, and as expected, the expression of $\omega_{ss,0}$ in terms of δ_s and ϕ is found to be similar to formula (8) and can be expres-

sed as:

$$\omega_{ss,0} = \text{arc cos} \, (-\tan \delta_s \tan \phi). \qquad (12)$$

Taking into account expressions (7) and (11), the ratio $H_{0,0}/H_0$ at the top of the Earth can now easily be determined as a function of geocentric latitude ϕ' and solar longitude λ_s.

2. DISCUSSION OF THE RATIO DISTRIBUTION OF THE DAILY INSOLATIONS

In a previous work (14), dealing with the oblateness effect on the solar radiation incident at the top of the atmospheres of the outer planets Jupiter, Saturn, Uranus and Neptune, we studied qualitatively some characteristic features of the ratio distribution $H_{0,0}/H_0$ in the northern hemisphere both for summer ($0° < \lambda_s < 180°$) and winter ($180° < \lambda_s < 360°$) period. It should, however, be emphasized that the results presented are also valid for the southern hemisphere and that they are evidently applicable to the Earth. The following are the major conclusions that were reached:

(1) In summer and in the region of permanent sunlight ($\omega_{ss} = \omega_{ss,0} = \pi$) the isocontours $H_{0,0}/H_0$ parallel the lines of constant geocentric latitude ϕ' and the daily solar radiation $H_{0,0}$ is always greater than H_0. The maximum value of the ratio $H_{0,0}/H_0$, occurring at $\phi' = \pi/2 - \epsilon$, can be expressed by the following relationship:

$$(H_{0,0}/H_0)_{max} = \sin \{\arctan[(1-f)^{-2} \cotan \epsilon]\}/\cos \epsilon. \qquad (13)$$

(2) In summer and in the equatorial region limited by the seasonal march of the Sun, the solar radiation of an oblate planet $H_{0,0}$ is increased with respect to the insolation of a spherical one H_0.

(3) For latitudes between the subsolar point and the region where the Sun remains above the horizon all day, the ratio $\cos \theta_{z,0}/\cos \theta_z$ is decreasing (with $\cos \theta_{z,0} < \cos \theta_z$), whereas $\omega_{ss,0}/\omega_{ss}$ is increasing (with the condition that $\omega_{ss,0} > \omega_{ss}$). Whether or how the regions

mentioned in (1) and (2) are linked depends on the relative effect of those two ratios, both being function of the flattening and the obliquity, and can only be determined by computation of the expression $H_{0,0}/H_0$.

(4) In winter, the effect of the flattening results in a more extensive polar region, the insolation is always reduced ($H_{0,0} < H_0$) and the curves of constant ratio $H_{0,0}/H_0$ roughly parallel the boundary of the polar night except in the neighborhood of the equinoxes.

Application of expressions (7) and (11) leads to the isocontour map illustrated in Fig. 2., where values of constant ratio distribution $H_{0,0}/H_0$ are given on each curve. Solar declination (lower part) is indicated by the dotted-dashed line and the region where the Sun does not set (upper part) is represented by the dashed line. The area of permanent darkness is shaded. Finally, the two regions of enhanced solar radiation ($H_{0,0} > H_0$) are dotted.

From Fig. 2 it can be seen that in the region of permanent sunlight the incoming solar radiation $H_{0,0}$ is increased when compared to H_0. Furthermore, it follows from expression (13) that in the region considered $(H_{0,0}/H_0)_{max} = 1.00104$ (~ 0.1 per cent). This extremely small gain of insolation is mainly due to the even small value of the Earth's flattening. For comparison, it is instructive to note that $(H_{0,0}/H_0)_{max}$, is equal to 1.00033, 1.037, 1.124 and 1.010 for Jupiter, Saturn, Uranus and Neptune, respectively. The effect of parallelism between the isocontours $H_{0,0}/H_0$ and the lines of constant geocentric latitude ϕ' in the above mentioned zone and already pointed out previously is not illustrated in Fig. 2. The reason for this non-representation lies in the fact that the numerical value of $H_{0,0}/H_0$ is negligible small (1.0008 and 1.0002 at $\phi' = 70$ and 80°, respectively).

It can mathematically be proved that, in summer, in the region bounded by the equator and the solar declination curve, both the length of the day and the cosine of the zenith distance are enhanced by the effect of the flattening. Hence, it follows that in this particular region, the solar energy input at the top of the atmosphere of the
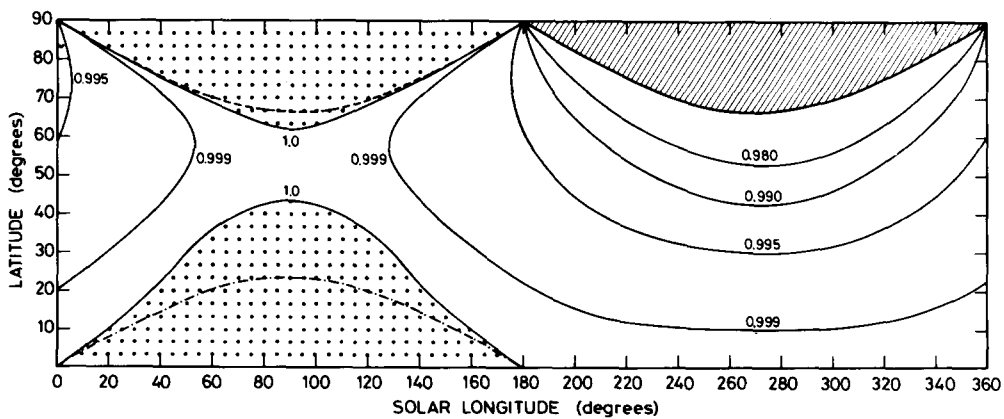

Fig. 2. Seasonal and latitudinal variation of the ratio $H_{0,0}/H_0$ of the daily insolation with ($H_{0,0}$) and without (H_0) the oblateness effect at the top of the atmosphere of the Earth. Solar declination (lower part) is indicated by the dotted-dashed line and the region where the Sun does not set (upper part) is represented by the dashed line. The area of permanent darkness is shaded. The two regions of enhanced solar radiation ($H_{0,0} > H_0$) are dotted. Values of constant ratio distribution $H_{0,0}/H_0$ are given on each curve.

Earth assumed as an oblate planet is increased with respect to the insolation of a spherical Earth.

In conclusion, there exists two obviously distinguished zones where the upper-boundary insolation $H_{0,0}$ is greater than H_0. For the sake of clearness it has to be emphasized that the enhancement is significantly small

Figure 2 also reveals that the two zones where $H_{0,0} > H_0$ are not linked. Indeed, in the mid-latitude interval 45–60°, it is obvious that the effect of the oblateness causes the solar energy outside the Earth's atmosphere to be reduced, the loss of insolation depending markedly upon the solar longitude. For example, if, at $\phi' = 60°$, λ_s increases from 0 to approximately 50°, the ratio $H_{0,0}/H_0$ increases from 0.995 (~ 0.5 per cent) to about 0.999 (~ 0.1 per cent). The non-linkage of the above mentioned areas is mainly ascribed to the explicitely small value of the Earth's flattening (0.00329).

As for all planets [14], the polar region is extended. It should, however, be emphasized that the Arctic Circles $H_{0,0} = 0$ and $H_0 = 0$ practically coincide, the maximum difference attaining scarcely 0.14° at a solar longitude of 270°. This value is considerably lower than those for the other planets varying from about 0.4 (Jupiter) to approximately 5° (Saturn). It is clear that this finding is also ascribed to the small value of f.

Figure 2 reveals that in winter, as stated earlier, the insolation of an oblate Earth is always reduced when compared to a spherical one. The solar radiation very slowly decreases in passing from equator latitudes to midlatitudes; it is found that at winter solstice and if ϕ' increases from 10 to about 50°, the ratio $H_{0,0}/H_0$ decreases from 0.999 (~ 0.1 per cent) to approximately 0.980 (~ 2 per cent). At higher latitudes, especially near the region where the Sun does not rise, $H_{0,0}/H_0$ drops very rapidly to zero. Although the loss of insolation is not very striking it follows from Fig. 2 that for parts of the winter the incident solar radiation is decreased by more than 2 per cent.

Another point about the curves is that they roughly parallel the limit of the polar night except, of course, in the vicinity of the equinoxes. Finally, it is also suitable to remark that in summer, respectively in winter, the curves of constant ratio $H_{0,0}/H_0$ are perfectly symmetric with respect to the summer and winter solstices.

3. DISCUSSION OF THE MEAN DAILY INSOLATIONS

In the preceeding sections we investigated the oblateness effect on the extraterrestrial daily insolation. Here we discuss the influence of the flattening on the mean (summer, winter and annual) daily insolations.

In Fig. 3 this influence is plotted in terms of the percentage difference [100 $(\bar{H}_{0,0} - \bar{H}_0)/\bar{H}_0$] as a function of geocentric latitude, the bars over symbols signifying seasonal or annual averages.

Concerning more particularly the mean summertime insolation (the summer season being arbitrary defined as the period running from vernal equinox over summer solstice to autumnal equinox and spanning 180°), the influence of the oblateness, although very small, is obviously evident from Fig. 3. Indeed, in the latitude interval 50–60° it can be seen that about 0.15 per cent of
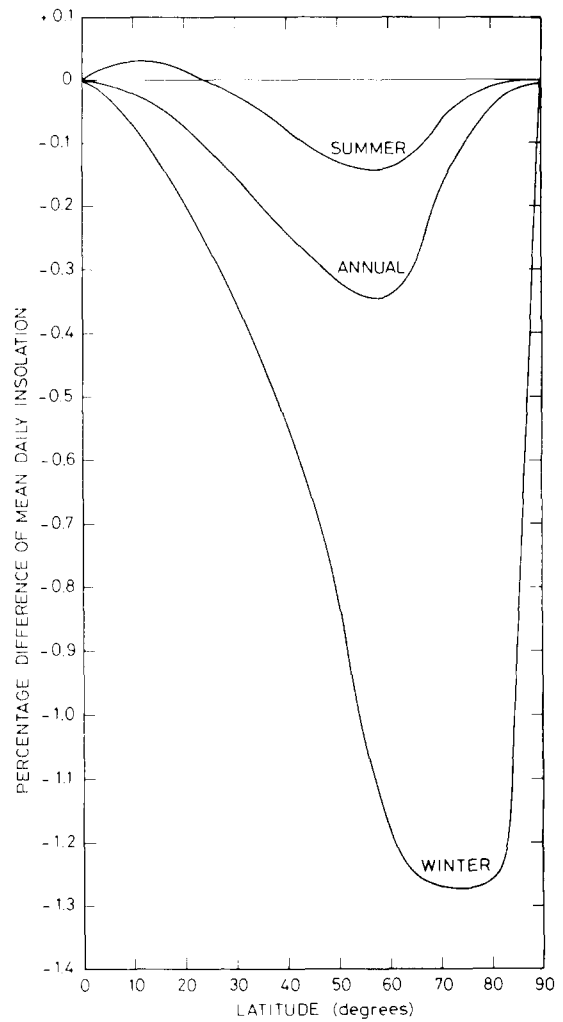

Fig. 3. Latitudinal variation of the percentage difference [100 $(\bar{H}_{0,0} - \bar{H}_0)/\bar{H}_0$] of the mean (summer, winter, annual) daily insolations. The bars over symbols signify seasonal and annual averages.

the mean summer daily insolation is lost through the flattening effect. Outside this interval, the effect is of decreasing significance. Another interesting phenomenon is that for latitudes between the equator and the subsolar point [14–15], the mean daily summer insolation of an oblate Earth is increased. However, owing to the small flattening, the rise of insolation is practically negligible, reaching only a maximum value of about 0.03 per cent at a latitude of approximately 10°.

During winter season ($180° < \lambda_s < 360°$), as already stated previously, the daily insolation $H_{0,0}$ is always reduced. Consequently, $\bar{H}_{0,0} < \bar{H}_0$ at any latitude. Figure 3 reveals that in winter the loss of insolation is of most importance between 55 and 85° where a percentage difference as much as 1 per cent has been found with a maximum value of about 1.3 per cent near 70°.

The partial gain of the mean summertime insolation equatorward of the subsolar point being considerably lower than the loss of insolation over the same period in winter evidently results in a mean annual daily insolation which is reduced over the entire latitude interval as

illustrated in Fig. 3. It can be seen that the daily insolation averaged over a one year period is decreased by about 0.3 per cent at latitudes from 45 to 65°.

4. CONCLUDING REMARKS

In the present paper we have investigated the influence of the flattening on the solar energy input at the top of the Earth's atmosphere. As a result of this study one can draw the conclusion that the effect of the oblateness causes non-negligible, although relatively small, variations in both the planetary-wide distribution and the intensity of the extraterrestrial daily solar radiation.

In summer, the daily insolation is slightly increased in two obviously distinguished regions: the first, near the poles, coinciding practically with the area of permanent sunlight, the second, at lower latitudes, covering the region between the equator and the seasonal march of the Sun. The maximum increase of the incoming solar radiation, occuring at a geocentric latitude $(\pi/2 - \epsilon)$, is approximately equal to 0.1 per cent. Outside the two zones mentioned above, the direct solar radiation $H_{0,0}$ is decreased when compared to H_0. For example, in the neighborhood of the equinoxes the loss of insolation ranges from 0.1 to 0.5 per cent (except at equator latitudes), whereas elsewhere the oblateness effect on the extraterrestrial solar radiation is negligible small.

In winter, the influence of the flattening causes the polar region to enlarge over an extremely small distance of about 15 km at winter solstice. The insolation is always reduced, the rate of decrease depending to a large extent on the geocentric latitude. For comparison, at $\lambda_s = 270°$, the loss of solar energy amounts to about 0.1, 0.5, 1.0 and 2.0 per cent, respectively at latitudes of approximately 10, 30, 45 and 50°. Moreover, in the relatively small area limited by the isocontour $H_{0,0}/H_0 = 0.980$ and the region of permanent darkness the effect of the flattening plays a more significant role and the decrease of incident solar radiation is correspondingly greater. Furthermore, it is particularly evident from Fig. 2 that the curves of constant ratio $H_{0,0}/H_0$ roughly parallel the Arctic Circle bounding the polar region in which there are days without sunrise.

Finally, we also have studied the latitudinal variation of the percentage difference of the mean daily insolation. It is found that for latitudes between the equator and the subsolar point, the mean summer daily insolation of an oblate Earth is increased when compared to a spherical one, the maximum rise being extremely small (~ 0.03 per cent). At higher latitudes, there is a loss of insolation which is of most importance at midlatitude regions (~ 0.15 per cent).

In winter, the horizon plane is always tilled away from the Sun causing both the cosine of the zenith angle and the length of the day to be reduced. Consequently, the daily insolation as well as the mean daily insolation are reduced, the latter decreasing maximally by about 1.3 per cent at high latitudes.

Despite of the partial gain of the mean summertime insolation near the equator, the effect of the flattening can clearly be seen to reduce the mean daily insolation over the entire year.

In conclusion, as interest in applications of solar energy increases considerably over the last years, very accurate values of the energy input at the top of the Earth's atmosphere are absolutely needed. We, therefore, believe that the effect of the oblateness, although very small, has to be taken into account in studies related to theoretical models for the calculation of solar global insolation.

NOMENCLATURE

a_e equatorial radius
a_p polar radius
a_s semi-major axis
e eccentricity
f flattening
H_0 extraterrestrial daily insolation of a spherical Earth
$H_{0,0}$ extraterrestrial daily insolation of an oblate Earth
$\bar{H}_0$ extraterrestrial mean daily insolation of a spherical Earth
$\bar{H}_{0,0}$ extraterrestrial mean daily insolation of an oblate Earth
I_0 extraterrestrial instantaneous insolation
r_s heliocentric distance
T sidereal period of axial rotation
v angle of the vertical
S solar flux at an heliocentric distance r_s
S_0 solar constant (1368 W m^{-2})
W true anomaly

Greek symbols
δ_s solar declination
ϵ obliquity
θ_z zenith angle for a spherical Earth
$\theta_{z,0}$ zenith angle for an oblate Earth
λ_s planetocentric solar longitude
λ_P planetocentric longitude of the Earth's perihelion
ϕ geographic latitude
ϕ' geocentric latitude
ω_s hour angle of the Sun
ω_{ss} sunset hour angle (spherical Earth)
$\omega_{ss,0}$ sunset hour angle (oblate Earth)

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