THE EFFECT OF ORBITAL ELEMENT VARIATIONS ON THE MEAN SEASONAL DAILY INSOLATION ON MARS

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Abstract. In this paper we briefly study changes in the mean seasonal insolations on the planet Mars caused by significant large-scale variations in the following orbital elements: the eccentricity (e), the obliquity (ϵ) and the longitude of perihelion (λ_p). Three orbital configurations have been investigated. In the first, the eccentricity equals successively 0, 0.075, and 0.15, whereas for the obliquity and the longitude of perihelion we took the present values which amount, respectively to 25° and 250°. In the second situation, $\epsilon = 15, 25$, and 35° for a circular orbit ($e = 0$) and with $\lambda_p = 250°$. In the last model we have set $e = 0.075$ and $\epsilon = 25°$ for $\lambda_p = -90, 0$, and 90°.

Although long-term periodic oscillations of e (first case) and λ_p (third case) produce, respectively, very small or no variations in the average yearly insolation, fluctuations of the above mentioned planetary data strongly effect the mean summer and winter daily insolations. Indeed, the calculations reveal that between the two extreme values of the orbital elements used, the seasonal insolations exhibit a change in amplitude of about 15 to 20% difference over the entire latitude interval.

Considering more particularly the second case it is found that the summertime insolation experiences a nearly similar variation as the mean annual daily insolation – i.e., a decrease of about 7% at the equator and a more than twofold increase at the poles. The corresponding mean winter daily insolation varies maximally by approximately 60% in the 60–80° latitude range.

1. Introduction

Some orbital elements of Mars are sensitive to significant long-term periodic variations: the obliquity (ϵ), the eccentricity (e) and the longitude of perihelion (λ_p). These oscillations are caused by gravitational perturbations from the Sun and the other planets.

The large-scale changes in the eccentricity and the longitude of perihelion of Mars were first obtained, from a secular perturbation analysis, by Brouwer and van Woerkom (1950). On the other hand, calculations of the obliquity variations are relatively new (Ward, 1973, 1974). More recently Ward (1979) and Ward *et al.* (1979) recalculated the present and past obliquity variations of Mars.

The eccentricity of Mars is currently 0.093 and ranges from a value near zero to a theoretical maximum of about 0.14 (Murray *et al.*, 1973; Ward, 1979). There are relatively rapid fluctuations on a time scale of about 9.5×10^4 yr, superimposed on a much longer quasi-periodic variation every 1.8 to 2.3×10^6 yr.

The time dependence of Mars' obliquity was, as previously mentioned, investigated by Ward (1973, 1974). He found a present obliquity of 25.1° (coinciding practically with the long-term average value of 25.2°), oscillating between a minimum and a maximum value of 14.9 and 35.5°, respectively. The oscillation has two superposed periods: one of 1.2×10^5 yr and another of 1.2×10^6 yr.

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A paper recently published by Ward (1979) updates the earlier calculations. The obliquity of Mars is now 25.2°. The maximum oscillation amplitude amounts to 13.6° and is centered on a long-term average value of 24.4°. This implies an obliquity ranging from a minimum of 10.8 to a maximum of 38.0°. The rapid oscillation is characterized by a dominant periodicity of 1.2×10^5 yr, whereas the much longer variation has a period modulation equal to about 1.3×10^6 yr. In a companion paper (Ward *et al.*, 1979) it has been demonstrated that intermittently even more extreme values are possible.

Finally the equinoxes, and as a consequence the longitude of perihelion (being the angle between the data of equinox and the perihelion passage), precess through 360° with a time period of approximately 5.1×10^4 yr (Ward, 1974).

Due to the fact that the solar radiation is dependent upon the above mentioned orbital elements, it is obvious that important periodic insolation variations on Mars are closely associated with changes in e, ϵ and λ_p.

The effect of the time evolution of those three orbital parameters on the insolation on Mars has already been concisely discussed (Murray *et al.*, 1973) and briefly reviewed (Toon *et al.*, 1980). Moreover, Ward (1974) has presented in more detail the essential elements of a theory for insolation variations on Mars that results from changes in the eccentricity and obliquity. It should, however, be emphasized that in the previous works one deals mainly with the daily insolation and with the mean annual daily insolation. In this paper, we specially accentuate the impact of the periodic fluctuations of e, ϵ, and λ_p on the mean summer and mean winter daily insolation over the entire latitudinal interval.

In a first section, we briefly discuss the calculation of the mean (annual, summer, and winter) daily insolations. For more details see e.g. Ward (1974), Vorob'yev and Monin (1975), Levine *et al.* (1977), Van Hemelrijck and Vercheval (1981) and Van Hemelrijck (1982a, b, c, d, 1983a). Then, the influence of the oscillating orbital elements on the solar radiation averaged over a season are considered.

2. Mean Daily Insolation

In our calculations and for the northern hemisphere, the summer season is arbitrary defined as running from vernal equinox over summer solstice to autumnal equinox and spanning 180°; consequently, the planetocentric longitudes of the Sun $\lambda_\odot = 180°$ and $\lambda_\odot = 360°$ mark the beginning and the end of the winter period. In the southern hemisphere, the solar longitude intervals (0, 180°) and (180, 360°) divide the year into astronomical winter and summer, respectively.

The daily insolation can be expressed as

$$I_D = [S_0 T(1 + e\cos W)^2/\pi a_\odot^2 (1-e^2)^2](h_0 \sin\varphi \sin\delta_\odot + \sin h_0 \cos\varphi \cos\delta_\odot), \qquad (1)$$

where S_0 is the solar constant at the mean Sun–Earth distance of 1 AU taken at 1.96 cal cm^{-2}(min)$^{-1}$ or 2.82×10^3 cal cm^{-2}(day)$^{-1}$ (Wilson, 1982), T is the sidereal day, e is the eccentricity, W is the true anomaly, $a_\odot$ is the semi-major axis, h_0 is the local hour angle at sunset (or sunrise), φ is the planetocentric latitude and $\delta_\odot$ is the solar declination.

Furthermore, W, h_0, and $\delta_\odot$ can be calculated from the following well-known relationships

$$W = \lambda_\odot - \lambda_p, \tag{2}$$

$$\begin{aligned} h_0 &= \arc\cos(-\tan\delta_\odot \tan\varphi) \\ &= \arc\cos[-\tan\varphi \sin\epsilon \sin\lambda_\odot/(1-\sin^2\epsilon\sin^2\lambda_\odot)^{1/2}, \end{aligned} \tag{3}$$

and

$$\sin\delta_\odot = \sin\epsilon\sin\lambda_\odot, \tag{4}$$

where λ_p and ϵ, as mentioned earlier, are the longitude of perihelion (sometimes called argument of perihelion) and the obliquity.

The mean (summer, winter, or annual) daily insolations, hereafter denoted as $(\bar{I}_D)_S$ $(\bar{I}_D)_W$ and $(\bar{I}_D)_A$ respectively, may be found by integrating relation (1) within the appropriate time limits, yielding the total insolation over a season or a year, and by dividing the obtained result by the corresponding length of the summer (T_S) or winter (T_W) or tropical year (T_0). For the calculation of T_S or T_W we refer to Van Hemelrijck (1982d).

As an example, $(\bar{I}_D)_A$ may be written under the form (Vorob'yev and Monin, 1975; Van Hemelrijck, 1982d)

$$\begin{aligned} (\bar{I}_D)_A &= (1/T_0)\int_0^{T_0} I_D\,\mathrm{d}t \\ &= \{S_0 T\sin\varphi\sin\epsilon/[2\pi^2(1-e^2)^{1/2}a_\odot^2]\}\int_0^{2\pi}(h_0-\tan h_0)\sin\lambda_\odot\,\mathrm{d}\lambda_\odot, \end{aligned} \tag{5}$$

where the dependence of h_0 in terms of $\lambda_\odot$ is given by the second equality on the right-hand side of relation (3). It should be pointed out that, considering the complexity of the integrand, Equation (5) has generally to be integrated numerically. However, it is easy to show that at the poles, the mean annual daily insolation is given (Murray *et al.*, 1973; Ward, 1974; Vorob'yev and Monin, 1975; Van Hemelrijck, 1982d) by

$$(\bar{I}_D)_{A_p} = S_0 T\sin\epsilon/\pi(1-e^2)^{1/2}a_\odot^2. \tag{6}$$

Expressions (5) and (6) clearly state that the average yearly insolation is independent on the longitude of perihelion λ_p.

In the following sections we have studied three orbital situations. In the first configuration we treated $e = 0$, 0.075, and 0.15 for $\epsilon = 25°$ and $\lambda_p = 250°$. The data for e represent approximately the minimum and the maximum value of the orbital eccentricity of Mars during its early history and the midpoint between the two; the values for ϵ and λ_p correspond to the present orbital elements.

In the second model, we have analyzed the cases when $\epsilon = 15$, 25, and 35° for $e = 0$ (circular orbit) and $\lambda_p = 250°$. The values of ϵ represent the extreme reasonable values and the average one for the currently accepted periodic cycle.

In the last example, we put $\lambda_p = -90$ (or 270), 0 (or 180) and 90° with $e = 0.075$ and $\epsilon = 25°$. $\lambda_p = -90$ and 90° signify, respectively, that winter and summer solstices occur at the perihelion passage of the planet; $\lambda_p = 0°$ means that perihelion is at vernal equinox.

2.1. First Configuration ($e = 0$, 0.075 and 0.15, $\epsilon = 25°$, $\lambda_p = 250°$)

Figure 1 illustrates calculations of the mean summer daily insolation as a function of latitude for ϵ and λ_p, respectively, equal to 25° and 250° and for the three adopted values of the eccentricity. The non-coincidence at the equator of the two curves representing the northern and southern solar radiation distributions for $e = 0.075$ and 0.15 is owing to the arbitrary chosen definition of the summer season in both hemispheres.

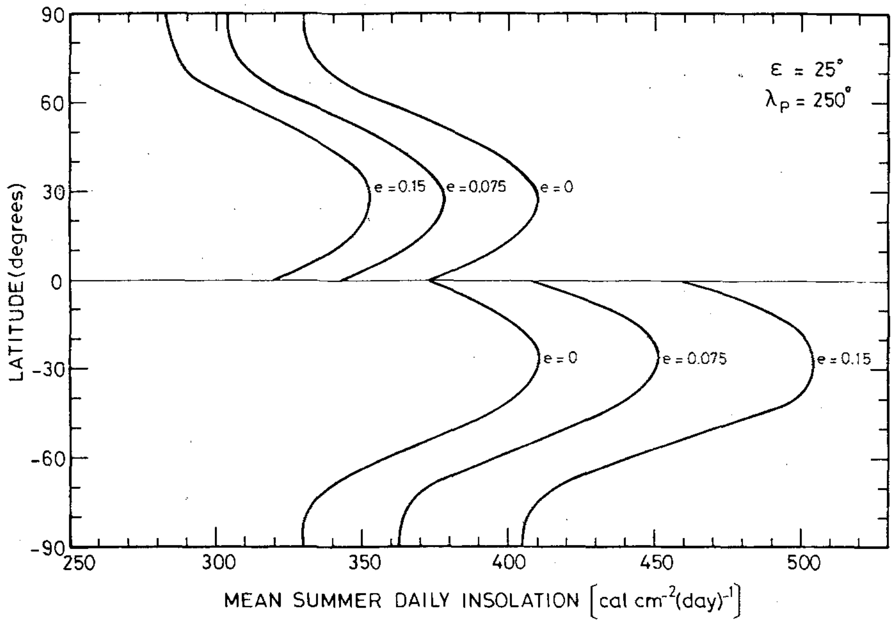

Fig. 1. Latitudinal variation of the mean summer daily insolation for various values of the eccentricity and with $\epsilon = 25°$ and $\lambda_p = 250°$.

From Figure 1 it is obvious that the solar radiation distribution curves are roughly parallel and that the peak insolations occur at approximately the same latitude. It can also be seen that the summer insolations of the northern and southern hemisphere become asymmetrical for $e \neq 0$. This insolation imbalance evidently vanishes for a circular orbit. The mean summer daily insolation for an eccentric orbit and at a given latitude is higher in the southern hemisphere than it is in the northern hemisphere, the ratio of both insolations being respectively equal to 1.43 and 1.19 for $e = 0.15$ and $e = 0.075$ at all latitudes. It is instructive to note that the value of those ratios corresponds to the ratio of the length of the northern summer to the length of the southern one for the two orbital configurations. Furthermore, Figure 1 clearly demonstrates that, in both hemispheres, the equatorial summertime insolation exceeds the solar radiation incident at the poles. The percentual difference amounts to about 13% and is independent on the eccentricity.

An analysis of Figure 1 also reveals that in the northern hemisphere $(\bar{I}_D)_S$ decreases with increasing eccentricity; in the southern hemisphere the steady increase of e is accompanied by a corresponding gain of $(\bar{I}_D)_S$. It is found that the variation of the mean summer daily insolation when e changes from 0 to 0.15 is of the order of 14% (northern hemisphere) and 23% (southern hemisphere) and that the rate of decrease or increase is latitudinal independent.

The distribution pattern of the mean winter daily insolation for the model under consideration is shown in Figure 2. Again, for an eccentric orbit, a hemispheric seasonal asymmetry exists in that their is significantly more insolation over the northern hemisphere than over the southern hemisphere. Similarly to the mean summertime insolation the ratio of both insolations amounts to 1.43 ($e = 0.15$) and 1.19 ($e = 0.075$) at all latitudes.

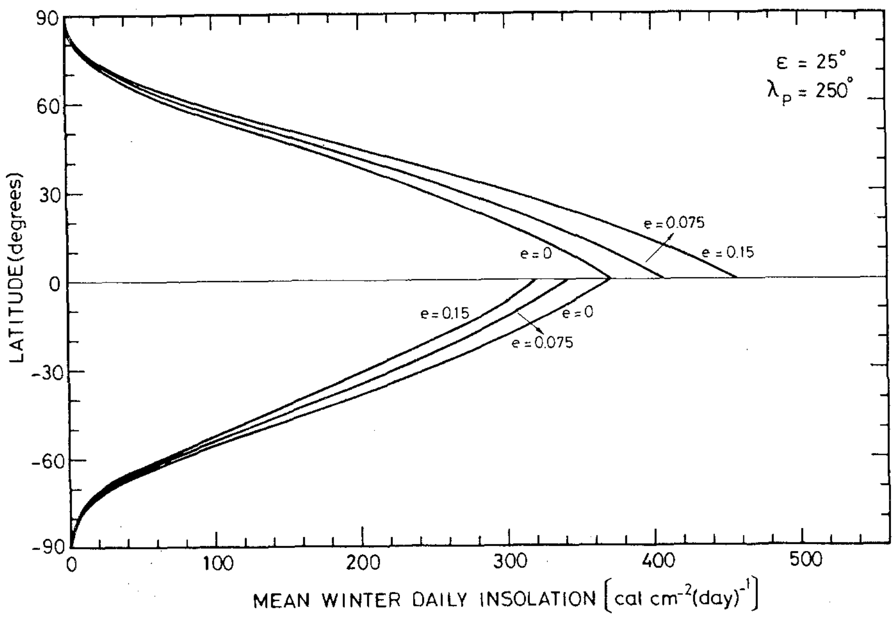

Fig. 2. Latitudinal variation of the mean winter daily insolation for various values of the eccentricity and with $\epsilon = 25°$ and $\lambda_p = 250°$.

As the eccentricity increases, mean wintertime insolation increases in the northern hemisphere; in the southern hemisphere the opposite effect is found. The increase (23%) and decrease (14%) equal those obtained during the summer season.

Finally, as previously stated by Murray *et al.* (1973) and Ward (1973, 1974), the annual average insolation at the geometric poles of Mars varies by only about 1% between the two extremes of the eccentricity. This finding can easily be verified by application of expression (6). Moreover, relation (5) indicates that this maximum difference remains constant over the entire latitude interval.

Although the resulting alteration in the mean annual daily insolation is only of the order of 1% as the eccentricity varies from 0 to 0.15, it follows that the mean seasonal daily insolations are quite sensitive to changes in e, exhibiting a 1.8 to 2.3 $\times$ 10^6 yr periodicity in amplitude with maximum percent differences of about 23%.

2.2. SECOND CONFIGURATION ($\epsilon = 15, 25,$ AND $35°$, $e = 0$, $\lambda_p = 250°$)

The mean summer, winter and annual daily insolations as a function of latitude, for various values of the obliquity and where $e = 0$ and $\lambda_p = 250°$ are respectively given in Figures 3, 4, and 5. Furthermore, Figure 6 represents the percentage differences as a function of latitude for the two extreme values of the obliquity. It should be pointed out that for a circular orbit the solar energy distribution curves are perfectly symmetric with respect to the equator.

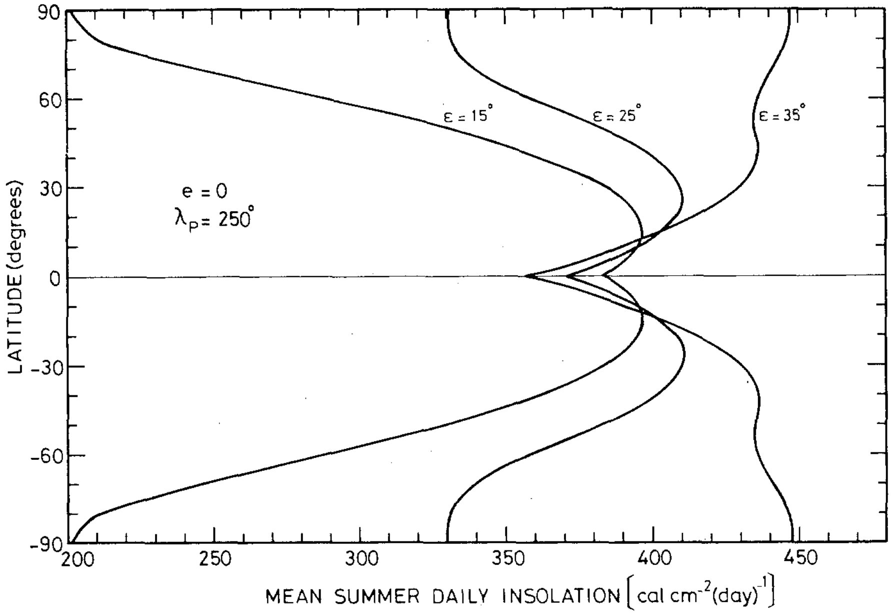

Fig. 3. Latitudinal variation of the mean summer daily insolation for various values of the obliquity, for a circular obit and with $\lambda_p = 250°$.

Figure 3 clearly illustrates the sensitivity of the summertime insolation to changes in the obliquity (contrast this diagram with Figure 1). From the figure it is obvious that the mean summer daily insolation at the poles is strongly dependent on the obliquity, whereas the equatorial solar energy is only a weak function of ϵ. Indeed, over the total obliquity interval, the polar insolation ranges from about 200 ($\epsilon = 15°$) to 450 cal cm^{-2}(planetary day)$^{-1}$ ($\epsilon = 35°$), the equatorial energy deposition varies between approximately 355 ($\epsilon = 35°$) and 385 cal cm^{-2} (planetary day)$^{-1}$ ($\epsilon = 15°$). In other words, the solar radiation incident at the poles exhibits a more than twofold increase in going from the minimum to the maximum value of ϵ, whereas the corresponding equatorial decrease is of the order of 7% (see also Figure 6). Furthermore, it can also be seen from Figures 3 and 6 that above a latitude near 15° the summertime insolation increases with increasing obliquity; below the above mentioned latitude the opposite effect is found. Another characteristic feature is that, for $\epsilon = 35°$, the summer polar energy exceeds the equatorial one and that at latitudes greater than approximately 40° the summer average insolation does not vary substantially.

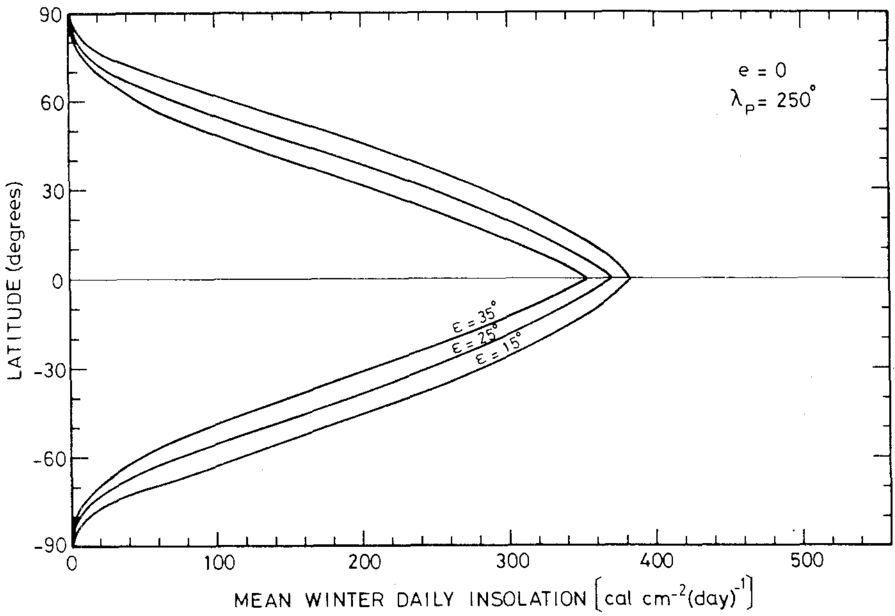

Fig. 4. Latitudinal variation of the mean winter daily insolation for various values of the obliquity, for a circular orbit and with $\lambda_p = 250°$.

The mean winter daily insolation is shown in Figure 4. The curves are nearly parallel and, over the entire latitude range, the increase of the obliquity is accompanied by a corresponding decrease of the solar radiation received in winter. Figure 4 and especially Figure 6 reveal that the solar energy distribution curves in winter, at least at high latitudes, are less sensitive to variations in the obliquity than the summer ones. At equatorial and mid-latitudes, the percentage differences are higher in winter than in summer (except at the equator where they are equal). Maximum percentage differences of the order of 60% are attained in the 60–80° latitude interval.

Figure 5 presents plots of the annual average solar radiation for the three obliquities. Similar calculations were made for the first time by Ward (1974) (see also Toon *et al.*, 1980). He argued that the polar insolation virtually doubles if ϵ increases from the minimum to the maximum value, but that conversely the equatorial energy deposition

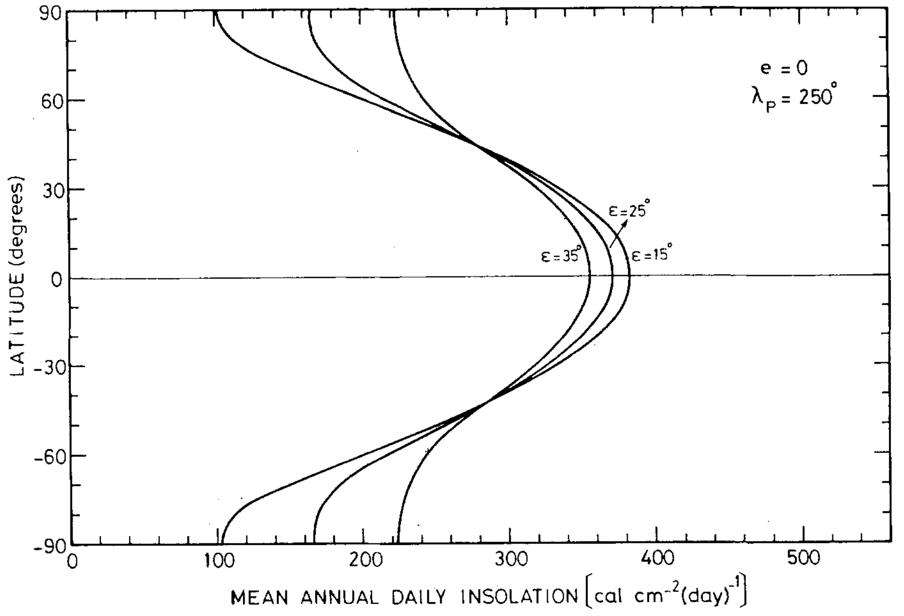

Fig. 5. Latitudinal variation of the mean annual daily insolation for various values of the obliquity, for a circular orbit and with $\lambda_p = 250°$.

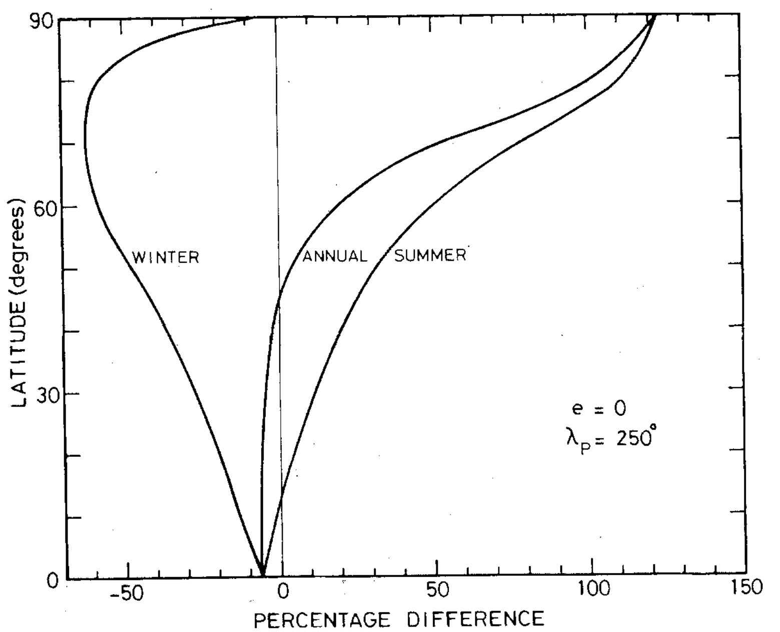

Fig. 6. Latitudinal variation of the percentage difference $100 \times [\bar{I}_{D(\epsilon=35)} - \bar{I}_{D(\epsilon=15)}]/\bar{I}_{D(\epsilon=15)}$ of the mean (summer, winter and annual) daily insolations for a circular orbit and with $\lambda_p = 250°$. The bars over symbols signify seasonal and annual averages.

declines. This is obviously demonstrated in Figures 5 and 6 where it can also be seen that the increase of ϵ causes the equatorial insolation to reduce by about 7% and that the critical latitude past which the yearly insolation at high obliquities exceeds that at lower obliquities is situated near approximately 40°.

2.3. THIRD CONFIGURATION ($\lambda_p = -90, 0,$ AND 90°, $e = 0.075$, $\epsilon = 25°$)

The effect of the argument of perihelion fluctuation on the mean summer and winter daily insolations is respectively given in Figures 7 and 8. It has to be pointed out that the precession of the perihelion position causes no variation in the average yearly insolation (Toon *et al.*, 1980).

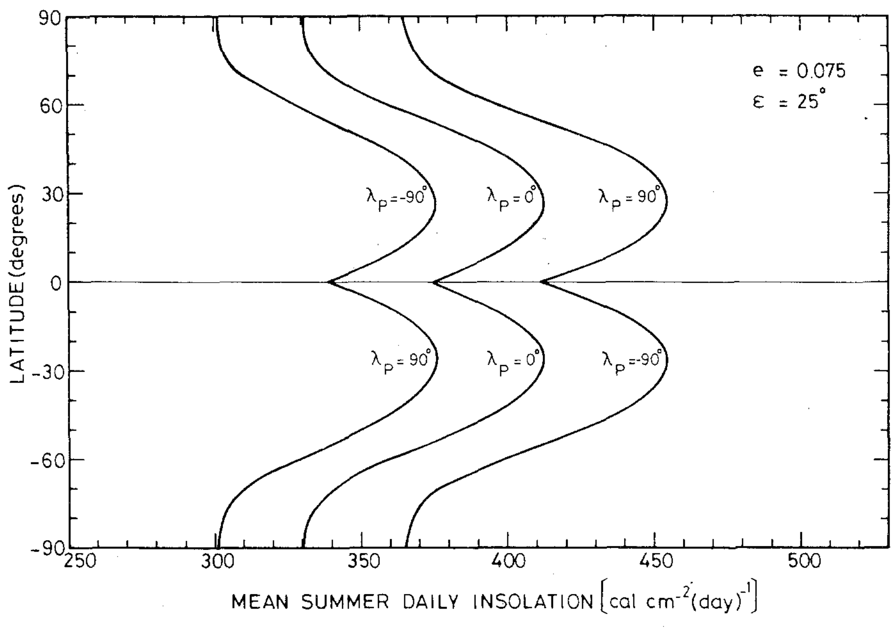

Fig. 7. Latitudinal variation of the mean summer daily insolation for various values of the longitude of perihelion and with $e = 0.075$ and $\epsilon = 25°$.

As expected, there is complete symmetry between the two hemispheres for $\lambda_p = 0°$ (or 180°). On the other hand, the northern summer distribution curve for $\lambda_p = -90°$ (or 90°) and the southern summer one for $\lambda_p = 90°$ (or $-90°$) are perfectly symmetric with respect to the equator. This is easily explained by observing that the length of the northern summer for $\lambda_p = -90°$ (90°) equals the length of the southern summer for $\lambda_p = 90°$ ($-90°$). These findings are evidently valid for the wintertime insolations (Figure 8).

Comparison of the curves represented in Figure 7 reveals that the insolation distributions for $\lambda_p = -90$ and 90° practically parallel the mean summer daily insolation for

$\lambda_p = 0°$. The alteration of $(\bar{I}_D)_S$ in the northern hemisphere with the 5.1×10^4 yr precessional cycle can easily be derived from Figure 7: it increases in the 0–90° longitude of perihelion interval, reduces if λ_p ranges from 90 over 180 (or 0) to 270° (or −90) and enhances again if λ_p varies from 270 to 360°. The maximum percentage difference $100 \times [(\bar{I}_D)_{S(90)} - (\bar{I}_D)_{S(-90)}]/(\bar{I}_D)_{S(-90)}$ amounts to about 21% and remains constant over the entire latitude range. It is worthwhile to notice that this constant difference equals the ratio of the lengths of the northern summer for $\lambda_p = -90$ and 90° respectively. Figure 7 also clearly indicates that in the southern hemisphere the behavior of the curves, when λ_p passes from 0 to 360°, is completely reversed compared to that in the northern summer hemisphere. The maximum percentage difference, computed in the same way as above, yields −17%.

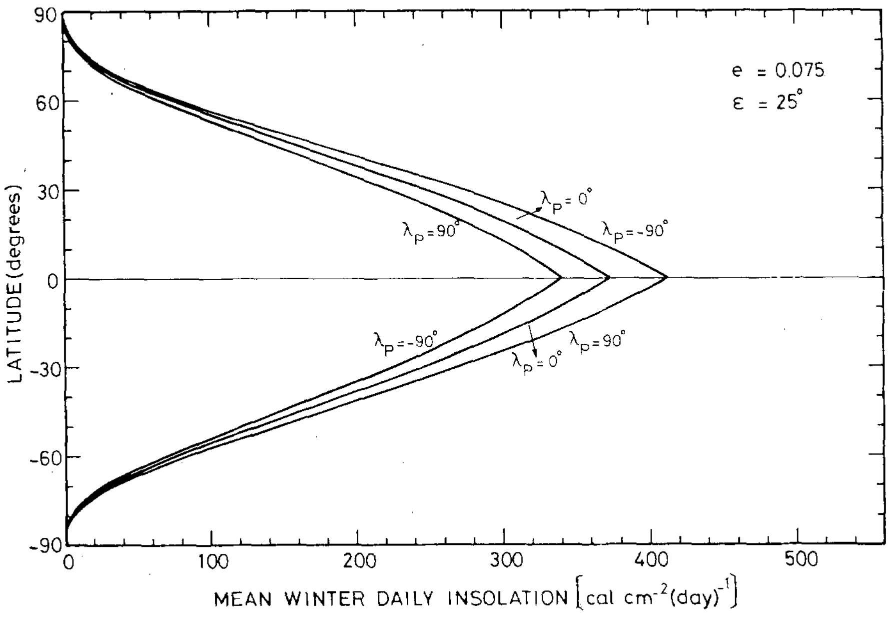

Fig. 8. Latitudinal variation of the mean winter daily insolation for various values of the longitude of perihelion and with $e = 0.075$ and $\epsilon = 25°$.

Finally, the long-term mean winter daily insolation changes are shown in Figure 8. The upper curves depict a substantial loss of $(\bar{I}_D)_W$ in going from $\lambda_p = -90$ to $\lambda_p = 90°$; the lower plots manifest a gain of solar energy over the same longitude of perihelion interval. Although the distribution curves are more closely spaced at polar latitudes than at mid- and equatorial ones, the percentage differences between the curves corresponding to the two extreme values of λ_p are identical to those obtained in Figure 7, i.e. they are constant over the entire latitudinal interval, the rise or loss of insolation being as much as 21 and 17%.

3. Summary and Conclusions

In the preceding sections emphasis is placed on the influence of orbital element variations on the mean summer and winter daily insolations on the planet Mars.

The principal results of this study may be summarized by the following statements:

(1) Although (first configuration) $(\bar{I}_D)_A$ varies by only about 1% between the two extreme values of the eccentricity, the mean seasonal insolutions are much more sensitive (of the order of 20%) to changes in e, the percentage differences being independent on the latitude.

(2) For a circular orbit (second case), and in contrast to the first situation, the fluctuations are found to be latitude dependent. In going from the minimum to the maximum value of ϵ, the behavior of the percentage difference distribution of $(\bar{I}_D)_S$ and $(\bar{I}_D)_A$ is roughly similar (−7% at the equator and over 100% at the poles). The critical latitudes past which $(\bar{I}_D)_S$ and $(\bar{I}_D)_A$ at $\epsilon = 35^\circ$ are greater than those at $\epsilon = 15^\circ$ are respectively situated near 15 and 40°. The mean winter daily insolations at high obliquities are always smaller than at lower obliquities, the maximum percentage difference attaining about 60% in the 60–80° latitude interval.

(3) The insolation variations arising from fluctuations in λ_p are characterized by a percentage difference remaining constant as a function of latitude. Although the precession of the perihelion position has no influence on the daily insolation averaged over a year, $(\bar{I}_D)_S$ and $(\bar{I}_D)_W$ are markedly affected by the time of perihelion passage, the seasonal average insolation variations being nearly similar to those obtained when the eccentricity oscillates between 0 and 0.15 (first configuration).

In conclusion, we believe that long-term changes in the mean summer and winter daily insolations on Mars caused by variations of the Martian orbit, obliquity and equinoctial precession has to be taken into account for a better understanding of some aspects of climate changes on Mars. Although our qualitative and even our quantitative knowledge of the quasi-periodic climate changes on Mars is quite well developed (see e.g. Toon *et al.*, 1980; Carr, 1982; Cutts, 1982 and Pollack and Toon, 1982) it is obvious that much remains to be examined in order to improve our understanding about the past and the present weather and climate on Mars.

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References

Brouwer, D. and Van Woerkom, A. J. J.: 1950, *Astron. Pap. Amer. Ephemeris Naut. Alm.* **13**, 81.
Carr, M. H.: 1982, *Icarus* **50**, 129.
Cutts, J. A. and Lewis, B. H.: 1982, *Icarus* **50**, 216.
Levine, J. S., Kraemer, D. R., and Kuhn, W. R.: 1977, *Icarus* **31**, 136.

Murray, B. C., Ward, W. R., and Yeung, S. C.: 1973, *Science* **181**, 260.

Pollack, J. B. and Toon, O. B.: 1982, *Icarus* **50**, 259.

Toon, O. B., Pollack, J. B., Ward, W. R., Burns, J. A., and Bilski, K.: 1980, *Icarus* **44**, 552.

Van Hemelrijck, E.: 1982a, *Icarus* **51**, 39.

Van Hemelrijck, E.: 1982b, 'The Oblateness Effect on the Solar Radiation Incident at the Top of the Atmosphere of Mars', in *Proceedings of the Workshop on the Planet Mars, Leeds, 1982.* ESA special publication, SP-185, 59–63.

Van Hemelrijck, E.: 1982c, *Icarus* **52**, 560.

Van Hemelrijck, E.: 1982d, *Bull. Acad. R. Belg., Cl. Sci.* **68**, 675.

Van Hemelrijck, E.: 1983a, 'The Oblateness Effect on the Extraterrestrial Solar Radiation', *Solar Energy* (in press).

Van Hemelrijck, E. and Vercheval, J.: 1981, *Icarus* **48**, 167.

Vorob'yev, V. I. and Monin, A. S.: 1975, *Atmos. Ocean. Phys.* **11**, 557.

Ward, W. R.: 1973, *Science* **181**, 260.

Ward, W. R.: 1974, *J. Geophys. Res.* **79**, 3375.

Ward, W. R.: 1979, *J. Geophys. Res.* **84**, 237.

Ward, W. R., Burns, J. A., and Toon, O. B.: 1979, *J. Geophys. Res.* **84**, 243.

Wilson, R. C.: 1982, *J. Geophys. Res.* **87**, 4319.