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The Oblateness Effect on the Solar Radiation Incident at the Top of the Atmospheres of the Outer Planets

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Calculations of the daily solar radiation incident at the top of the atmospheres of Jupiter, Saturn, Uranus, and Neptune, with and without the effect of the oblateness, are presented in a series of figures illustrating the seasonal and latitudinal variation of the ratio of both insolations. It is shown that for parts of the summer, the daily insolation of an oblate planet is increased, the zone of enhanced solar radiation being strongly dependent upon the obliquity, whereas the rate of increase is fixed by both the flattening and the obliquity. In winter, the oblateness effect results in a more extensive polar region, the daily solar radiation of an oblate planet always being reduced when compared to a spherical planet. In addition, we also numerically studied the mean daily solar radiation. As previously stated by A. W. Brinkman and J. McGregor (1979, *Icarus* **38**, 479–482), it is found that in summer the horizon plane is tilted toward the Sun for latitudes less than the subsolar point, but is tilted away from the Sun beyond this latitude. It follows that the mean summer daily insolation is increased between the equator and the subsolar point, but decreased poleward of the above-mentioned limit. In winter, however, the horizon plane is always tilted away from the Sun, causing the mean winter daily insolation to be reduced. The partial gain of the mean summertime insolation being much smaller than the loss during winter season evidently yields a mean annual daily insolation which is decreased at all latitudes.

1. INTRODUCTION

It is well known that the distribution of the solar radiation incident at the top of the atmospheres of the planets of the solar system and its variability with latitude and time (or season) are of interest for various radiation problems. Although the upper-boundary insolation of the Earth's atmosphere and the radiation at its surface have been the subject of several earlier investigations, it should be emphasized that theoretical studies with respect to the other planets are rather scarce and sometimes far from complete. Among the papers published during the last fifteen years, we cite those of Soter and Ulrichs (1967), Liu (1968), Murray *et al.* (1973), Ward (1974), Vorob'yev and Monin (1975), Levine *et al.* (1977), and Van Hemelrijck and Vercheval (1981).

In the calculations made by the previous authors, the planets are assumed to be spherical. This assumption is valid for the inner planets (Mercury and Venus) and for the outer planets Mars and Pluto, the equatorial radius being equal or nearly equal to the polar radius, but is no longer acceptable for the other planets, where the flattening is not negligible. The only paper dealing with planetary solar insolation and taking into account the planet's oblateness was, to the best of our knowledge, recently published by Brinkman and McGregor (1979). In this interesting work, the direct insolation pattern of Saturn is calculated as a function of season and latitude, including both the oblateness of the planet and the effect of the shadow of the ring system. It should, however, be noted that the influence of the planet's flattening is only concisely discussed on a qualitative basis.

The main objective of the present paper is, therefore, to analyze in detail the oblateness effect on the solar radiation incident at the top of the atmospheres of the rapidly rotating planets in the solar system, Jupiter, Saturn, Uranus, and Neptune.

In a first section, we present a general

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method applicable to these four planets. Then, taking into account their orbital and planetary data, we calculate the daily insolation with and without the effect of the oblateness. Our results, presented in the form of a contour map, give the seasonal and latitudinal variation of the ratio of both insolations. Among the planets discussed in the paper, the obliquity varies from very small (Jupiter) to very large (Uranus), whereas the flattening ranges from approximately 0.021 (Neptune) to 0.1 (Saturn). As expected, the global pattern of the contour maps is strongly dependent on these two parameters.

In addition, the latitudinal variation of the mean daily solar radiations is included in a series of figures demonstrating clearly that the solar radiation of an oblate planet displays a different latitudinal distribution when compared to that of a spherical planet.

2. CALCULATION OF THE DAILY INSOLATION WITH AND WITHOUT THE OBLATENESS EFFECT

The instantaneous insolation I is defined as the solar heat flux sensed at a given time by a horizontal unit area of the upper boundary of the atmosphere at a given point on the planet and per unit time. It can be expressed as (see, e.g., Ward, 1974; Vorob'yev and Monin, 1975; Levine *et al.*, 1977; Van Hemelrijck and Vercheval, 1981)

$$I = S \cos z, \tag{1}$$

with

$$S = S_0/r_\odot^2 \tag{2}$$

and

$$r_\odot = a_\odot(1 - e^2)/(1 + e \cos W). \tag{3}$$

z is the zenith angle of the incident solar radiation, S is the solar flux at a heliocentric distance $r_\odot$, and S_0 is the solar constant at the mean Sun–Earth distance of 1 AU. For the calculations presented in this paper we took a solar constant of 1.94 cal cm^{-2} min^{-1} (or 2.79×10^3 cal cm^{-2} day^{-1}). Furthermore, in expression (3), $a_\odot$, e, and W are, respectively, the planet's semimajor axis, the eccentricity, and the true anomaly, which is given by

$$W = \lambda'_\odot - \lambda'_p, \tag{4}$$

where $\lambda'_\odot$ and λ'_p are the planetocentric longitude of the Sun (or solar longitude) and the planetocentric longitude of the planet's perihelion. Table I represents, for the four planets under consideration, the numerical values of the parameters used for the computations. In this table one can also find the obliquity ϵ, the equatorial radius a_e, the polar radius a_p, the flattening $f = (a_e - a_p)/a_e$, and the sidereal period of axial rotation T (sidereal day). Finally, it has to be mentioned that the orbital and planetary data are taken from the Handbook of the British Astronomical Association (1981) and from Vorob'yev and Monin (1975).

For a spherical planet, the zenith angle z

TABLE I

ELEMENTS OF THE PLANETARY ORBITS OF JUPITER, SATURN, URANUS, AND NEPTUNE

Planet	$a_\odot$ (AU)	e	λ'_p (°)	ϵ (°)	a_e (km)	a_p (km)	f	T (Earth days)
Jupiter	5.2028	0.04847	58.00	3.12	71,400	67,100	0.06022	0.41
Saturn	9.539	0.05561	279.07	26.73	60,000	54,000	0.10000	0.44
Uranus	19.18	0.04727	3.02	82.14	26,000	24,500	0.05769	0.45
Neptune	30.06	0.00859	5.23	29.56	24,200	23,700	0.02066	0.66

may be written

$$\cos z = \sin \varphi' \sin \delta_\odot + \cos \varphi' \cos \delta_\odot \cos h, \quad (5)$$

where φ' is the geocentric latitude (which equals the geographic latitude φ), $\delta_\odot$ is the solar declination, and h is the local hour angle of the Sun. Note that in this case the radius vector, characterizing the direction of a surface element, is normal to the horizon plane. Furthermore, the solar declination $\delta_\odot$ can be calculated using the expression

$$\sin \delta_\odot = \sin \epsilon \sin \lambda'_\odot. \quad (6)$$

The daily insolation I_D, being defined as the amount of solar radiation incident at the top of a planetary atmosphere over the planet's day, can now be obtained by integrating Eq. (1) over time during the light time of the day and is given by

$$I_D = (ST/\pi)(h_0 \sin \varphi' \sin \delta_\odot + \sin h_0 \cos \varphi' \cos \delta_\odot), \quad (7)$$

where h_0 is the local hour angle at sunset (or sunrise) and may be determined from expression (5) by the condition that at sunset (or sunrise) $\cos z = 0$. It follows that

$$h_0 = \arccos(-\tan \delta_\odot \tan \varphi') \quad (8)$$

if

$$|\varphi'| < \pi/2 - |\delta_\odot|.$$

In regions where the Sun does not rise or, more precisely, during polar nights ($\varphi' < -\pi/2 + \delta_\odot$ or $\varphi' > \pi/2 + \delta_\odot$), we have $h_0 = 0$. Finally, in regions where the Sun remains above the horizon all day ($\varphi' > \pi/2 - \delta_\odot$ or $\varphi' < -\pi/2 - \delta_\odot$), we may put $h_0 = \pi$.

In the case of an oblate planet there is an angle $v = \varphi - \varphi'$, the so-called angle of the vertical, between the radius vector and the normal to the horizon plane; it vanishes at the equator and the poles, while elsewhere $\varphi > \varphi'$ numerically (note that the maximum value of v is flattening dependent and occurs close to latitude 45°). The angle v is related to the geocentric latitude φ' by the expression

$$v = \arctan[(1 - f)^{-2} \tan \varphi'] - \varphi'. \quad (9)$$

Defining Z as the zenith distance for an oblate planet the following relation can easily be obtained by applying the formulae of spherical trigonometry,

$$\cos Z = \cos v \cos z + \sin v \, (-\tan \varphi' \cos z + \sin \delta_\odot \sec \varphi'), \quad (10)$$

which can also be written as

$$\cos Z = (\cos z \cos \varphi + \sin v \sin \delta_\odot) \sec \varphi'. \quad (11)$$

The daily insolation of an oblate planet I_{DO} can now be obtained by integrating Eq. (1), within the appropriate time limits, where $\cos z$ has to be replaced by Eq. (10) or (11), yielding

$$I_{DO} = (ST/\pi)\{\cos v \, (h_{0_0} \sin \varphi' \sin \delta_\odot + \sin h_{0_0} \cos \varphi' \cos \delta_\odot) + \sin v \, [-\tan \varphi' \, (h_{0_0} \sin \varphi' \sin \delta_\odot + \sin h_{0_0} \cos \varphi' \cos \delta_\odot) + h_{0_0} \sin \delta_\odot \sec \varphi']\} \quad (12)$$

or

$$I_{DO} = (ST/\pi) \, [\cos \varphi \sec \varphi' \, (h_{0_0} \sin \varphi' \sin \delta_\odot + \sin h_{0_0} \cos \varphi' \cos \delta_\odot) + h_{0_0} \sin v \sin \delta_\odot \sec \varphi'], \quad (13)$$

where h_{0_0}, the local hour angle at sunset (or sunrise) for an oblate planet, is generally slightly different from h_0. As for a spherical planet, h_{0_0} may be derived from Eq. (11) by putting $\cos Z = 0$.

Hence

$$h_{0_0} = \arccos(-\tan \delta_\odot \tan \varphi). \quad (14)$$

As expected, Eq. (14) is similar to formula (8). Obviously, in areas of permanent darkness ($\varphi < -\pi/2 + \delta_\odot$ or $\varphi > \pi/2 + \delta_\odot$) and continuous sunlight ($\varphi > \pi/2 - \delta_\odot$ or $\varphi < -\pi/2 - \delta_\odot$) we have $h_{0_0} = 0$ and $h_{0_0} = \pi$, respectively.

Taking into account expressions (7) and (12) or (13), the ratio I_{DO}/I_D at the top of the rapidly rotating planets Jupiter, Saturn, Uranus, and Neptune can now easily be obtained as a function of geocentric latitude φ' and solar longitude $\lambda'_\odot$. Finally, our results are presented in the form of a ratio contour map obtained by an interpolation procedure.

3. SOME QUALITATIVE ASPECTS OF THE RATIO DISTRIBUTION OF THE SOLAR RADIATIONS

It should be pointed out that, considering the complexity of Eqs. (7) and (12) or (13), a simple analytic expression given φ' as a function of $\lambda'_\odot$ and bounding the region in which I_{DO} exceeds I_D cannot be obtained.

Therefore, in this section, a qualitative study will be given on the behavior of the ratio distribution I_{DO}/I_D for some particular conditions in the northern hemisphere, both for summer ($0 < \lambda'_\odot < 180°$) and winter ($180° < \lambda'_\odot < 360°$) periods. It should, however, be emphasized that the results presented are also valid for the southern hemisphere.

3.1. Summer

It is interesting to note that in regions of permanent daylight ($h_0 = h_{0_0} = \pi$) the isocontours I_{DO}/I_D parallel the lines of constant geocentric latitude and that the solar radiation I_{DO} is always greater than I_D.

Indeed, by putting $h_0 = h_{0_0} = \pi$ in Eqs. (12) and (7) we obtain

$$\begin{aligned} I_{DO} &= ST \sin \delta_\odot (\cos v \sin \varphi' + \sin v \cos \varphi') \\ &= ST \sin \delta_\odot \sin (v + \varphi') \\ &= ST \sin \delta_\odot \sin \varphi \quad (15) \end{aligned}$$

and

$$I_D = ST \sin \delta_\odot \sin \varphi'. \quad (16)$$

Dividing (15) by (16) yields

$$I_{DO}/I_D = \sin \varphi / \sin \varphi'. \quad (17)$$

Equation (17) clearly indicates that in regions where the Sun remains above the horizon all day, the ratio of both insolations is independent on the solar longitude $\lambda'_\odot$. Second, from the condition that $\varphi > \varphi'$ (except at the equator and the poles) it follows that the solar radiation of an oblate planet, in the area considered, is increased with respect to the solar radiation of a spherical planet.

Furthermore, it can be mathematically demonstrated that $\sin \varphi / \sin \varphi'$ is a monotonically decreasing function in the geocentric latitude interval $(0, \pi/2)$, with a maximum value equal to $(1 - f)^{-2}$ for $\varphi' = 0$ and a minimum value equal to unity for $\varphi' = \pi/2$.

Practically, the minimum value of the geocentric latitude φ' for which there still exists a day with permanent sunlight can be found from the equality $\varphi' = \pi/2 - \epsilon$, ϵ being the largest solar declination $\delta_\odot$ occurring at $\lambda'_\odot = \pi/2$. As a consequence, the maximum value $(I_{DO}/I_D)_{max}$ can be determined from (17) by putting $\varphi' = \pi/2 - \epsilon$.

In this case we obtain

$$(I_{DO}/I_D)_{max} = \sin \{\arctan [(1 - f)^{-2} \cot \epsilon]\} / \cos \epsilon, \quad (18)$$

stating that $(I_{DO}/I_D)_{max}$ is dependent on the flattening as well as on the obliquity.

Another point of interest regards the length of the day, hereafter defined as the time interval between rising and setting of the Sun. It is clear that the so-defined quantity is proportional to the hour angle at sunset or sunrise.

From (8) and (14), with the condition that $\varphi > \varphi'$ and, in summer, $\delta_\odot > 0$, it follows that

$$(h_{0_0})_{SUMMER} > (h_0)_{SUMMER}. \quad (19)$$

From Eq. (19) it can be seen that, in summer, the length of the day of an oblate planet is always greater than the length of the day of a spherical planet except, of course, in the region of permanent sunlight, where $h_{0_0} = h_0 = \pi$.

We can derive an equation governing the time evolution in summer of the function $\cos Z / \cos z$ at local noon ($h = 0$) when the Sun is on the meridian plane. From Eqs. (5), by putting $h = 0$, and (11), where $v = \varphi - \varphi'$, and after some rearrangements, we have

$$\cos Z/\cos z = \cos(\varphi - \delta_\odot)/\cos(\varphi' - \delta_\odot). \quad (20)$$

This expression also follows immediately from the fact that at local noon the zenith distance of the Sun(Z or z) is algebraically equal to the difference between the latitude (φ or φ') and the solar declination $\delta_\odot$.

It can be mathematically proven that, in summer, the isocontour $\cos Z/\cos z = 1$ coincides closely with the curve expressing the solar declination of the Sun ($\delta_\odot$) as a function of its planetocentric longitude ($\lambda'_\odot$). Moreover, it can be demonstrated that $\cos Z/\cos z > 1$ for latitudes between the equator and the subsolar point (or $\varphi' < \delta_\odot$) and that $\cos Z/\cos z < 1$ elsewhere ($\varphi' > \delta_\odot$). From (19) and owing to the fact that at local noon $\cos Z > \cos z$ for $\varphi' < \delta_\odot$, it may be concluded that at any time of the day $Z < z$. In other words, in the region bounded by the equator and the solar declination curve, the horizon plane is tilted toward the Sun (see also Brinkman and McGregor, 1979). Hence in this area, both the length of the day and $\cos Z$ being enhanced, it is obvious that, in this particular region, the solar radiation of an oblate planet is increased with respect to the insolation of a spherical planet. Outside this region, particularly in its neighborhood, it is *a priori* difficult to predict whether the upper-boundary insolation is increased or decreased, the oblateness effect on the solar radiation depending on the relative importance of the ratio of both the lengths of the day (or h_{0o}/h_0) and the cosines of the zenith distances ($\cos Z/\cos z$).

In conclusion, we found, on a qualitative basis, two obviously distinguished regions where $I_{DO} > I_D$: the first, near the poles, coincides with the area of permanent sunlight, and the second, in the equatorial region, is limited by the seasonal march of the Sun. Note that these two zones are symmetric with respect to the solar longitude $\lambda'_\odot = \pi/2$. For latitudes between the subsolar point and the region where the Sun remains above the horizon all day, the ratio $\cos Z/\cos z$ is decreasing (with $\cos Z < \cos z$), whereas h_{0o}/h_0 is increasing with the condition that $h_{0o} > h_0$. Whether or not the two above-mentioned regions are linked depends, as already stated, on the relative effect of those two ratios and can only be determined by computation of the expression I_{DO}/I_D. In Section 4 we will see that the isocontours $I_{DO}/I_D = 1$ coincide remarkably well with the two branches of a hyperbola.

3.2. *Winter*

During the winter season, running from the autumnal equinox over winter solstice to vernal equinox and spanning 180°, two main characteristic features are found. On one hand, the length of the day of an oblate planet (h_{0o}) is always smaller than the length of the day of a spherical planet (h_0). This finding can be evaluated by Eqs. (8) and (14), where the solar declination $\delta_\odot$ is reckoned negative. On the other hand, it can be proven that $\cos Z < \cos z$ over the entire winter season. Indeed, expression (11) can be rewritten as

$$\cos Z = (\cos\varphi \cos z/\cos\varphi') + (\sin v \sin\delta_\odot/\cos\varphi'). \quad (21)$$

From the conditions $\sin v > 0$, $\cos\varphi' > 0$, and $\sin\delta_\odot < 0$ it follows that the second term on the right-hand side of Eq. (21) is always negative. Hence

$$\cos Z < (\cos\varphi \cos z/\cos\varphi'). \quad (22)$$

The multiplication factor ($\cos\varphi/\cos\varphi'$), always being smaller than unity, finally yields

$$\cos Z < \cos z. \quad (23)$$

Thus in winter, as previously stated by Brinkman and McGregor (1979), the horizon plane is tilted away from the Sun, causing both $\cos Z$ and the length of the day to be reduced. Consequently, the daily solar radiation of an oblate planet is always reduced when compared to that of a spherical planet.

4. DISCUSSION OF THE RATIO DISTRIBUTION OF THE SOLAR RADIATIONS

As already mentioned in the introduction, and following the method adopted by Levine *et al.* (1977) in presenting the incident solar radiation in contours of calories per square centimeter per planetary day as a function of geocentric latitude and season, we calculated both I_D and I_{DO}. When comparing our results of I_D with those of the previous authors, a fairly good agreement was noticed. Concerning the computations relative to I_{DO}, it is found that, owing to the small flattening of the planets, the global insolation pattern is only slightly different with respect to the solar radiation distribution for a spherical planet. Note that the calculations of I_D and I_{DO} are not included in this paper.

Finally, the ratio distribution I_{DO}/I_D is presented in a series of figures, showing very clearly the oblateness effect on the upper-boundary insolation of the atmospheres of Jupiter, Saturn, Uranus, and Neptune.

4.1. Jupiter

Application of Eqs. (7) and (12) or (13) leads to the isopleths illustrated in Fig. 1, where values of constant ratio distribution I_{DO}/I_D are given on each curve.

As for all planets, the effect of the oblateness results in a more extensive polar region (dotted-dashed lines) and, as already demonstrated in Section 3.1, in two regions of increased solar radiation (dashed lines). It should be emphasized that the curves $I_D = 0$ and $I_{DO} = 0$ practically coincide (the maximum difference attaining approximately 0.4° at $\lambda'_{\odot} = 270°$) and that the two zones where $I_{DO} > I_D$ are not linked (of which Jupiter is the only case). These two effects are ascribed to the very small obliquity (3.12°) of the planet. Furthermore, it follows from Eq. (18) that in the region of permanent sunlight $(I_{DO}/I_D)_{max}$ is negligible.

An analysis of Fig. 1 reveals that, over nearly the entire Jovian year, the ratio distribution of both insolations closely parallels the lines of constant geocentric latitude. In summer, it is seen that if φ' increases from about 20 to approximately 60°, the ratio distribution decreases from about 0.98 to 0.92 with a bulge at $\lambda'_{\odot} = 90°$. In winter, I_{DO}/I_D falls from 0.98 at 20° to 0.85 in the latitude interval 70–80°. At higher latitudes, especially near the region where the Sun does not rise, we have $I_{DO} = 0$ and $I_D \neq 0$.

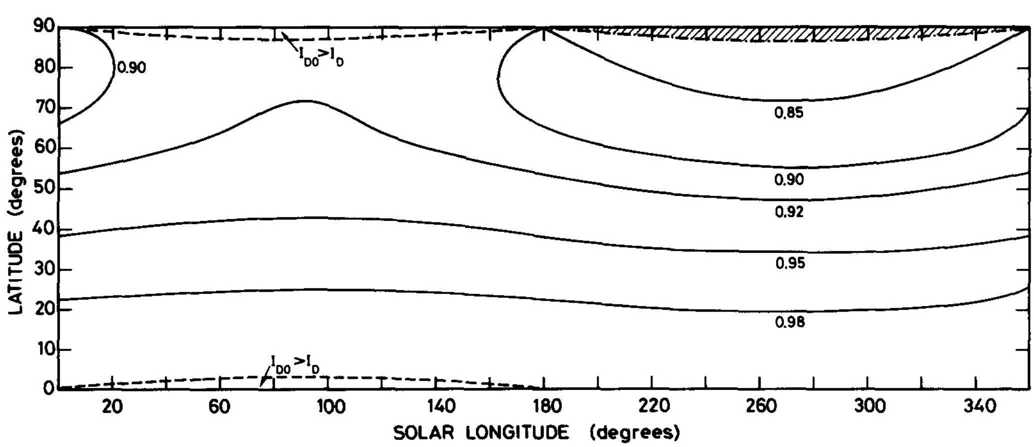

FIG. 1. Seasonal and latitudinal variation of the ratio of the daily insolation with (I_{DO}) and without (I_D) the oblateness effect at the top of the atmosphere of Jupiter. Solar declination (lower part) and the region where the Sun does not set in the case of a spherical planet (upper part) are represented by the dashed lines. The areas of permanent darkness are shaded and bounded by the dotted-dashed lines. Values of I_{DO}/I_D are given on each curve.

Consequently, in this relatively small zone I_{DO}/I_D drops very rapidly to zero. Furthermore, it is interesting to note that in summer, respectively in winter, the curves are perfectly symmetric with respect to the summer and winter solstices.

In conclusion, it is particularly evident from Fig. 1 that the insolation is reduced, except in two extremely small zones.

4.2. *Saturn*

The ratio of the solar radiations at the top of Saturn's atmosphere is illustrated in Fig. 2. When comparing Fig. 1 with Fig. 2 it is obvious that the general pattern of the two contour maps is explicitly different.

In summer, the two regions of enhanced solar radiation are joined by curves roughly similar to the two branches of a hyperbola. The effect of the oblateness can clearly be seen to increase the insolation over extensive parts of the summer hemisphere, especially near summer solstice, where a rise of the incident solar radiation on the order of 3% has been found. Concerning more particularly the area of continuous sunlight, application of Eq. (18) leads to a maximum I_{DO}/I_D value of 1.037. Figure 2 also reveals that in the neighborhood of the equinoxes the loss of insolation is most important for the midlatitude regions.

In winter, as stated previously, the effect of the flattening results in a more extensive polar region; the larger flattening gives rise to a maximum difference, occurring at winter solstice, of approximately 5° as shown in the figure. The solar radiation significantly decreases in passing from equator latitudes to mid- and polar latitudes. For example, if, at $\lambda'_{\odot} = 270°$, φ' increases from 10 to 40°, the ratio I_{DO}/I_D decreases from 0.95 to about 0.60. Moreover, at the same solar longitude and for $\varphi' = 50°$ the solar radiation is reduced by more than a factor of 2.

4.3. *Uranus*

At $\epsilon > 45°$ (of which Uranus is the only case), the Arctic circle, bounding the polar region in which there are days without sunset, lies inside the tropical zone (in which there are days on which the Sun reaches the zenith). As a consequence, the very large obliquity of Uranus (82.14°) results in an increased solar radiation over, roughly speaking, the entire summer season (Fig. 3).

Introducing the numerical values for a_e, a_p, and ϵ (Table I) Eq. (18) yields $(I_{DO}/I_D)_{max} = 1.124$. This value approaches very closely the maximum theoretical value equal to $(1 - f)^{-2}$ (=1.128). It follows that the obliquity and the oblateness of Uranus

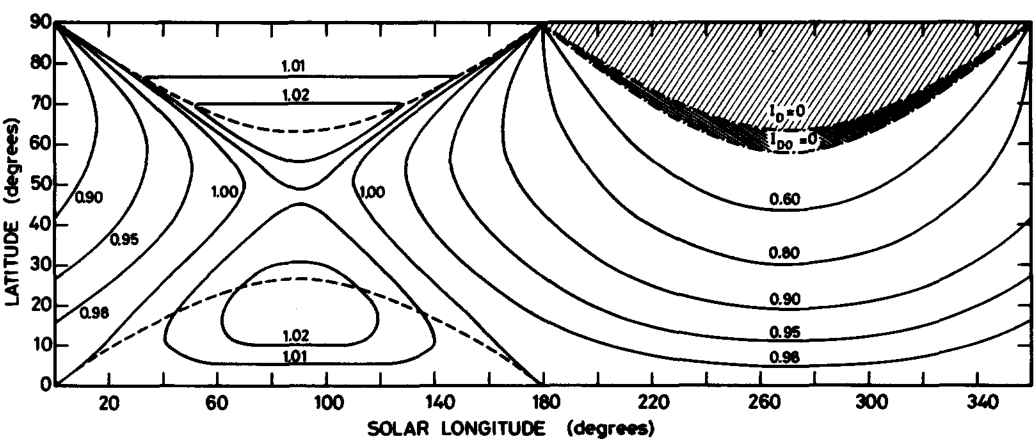

FIG. 2. Seasonal and latitudinal variation of the ratio of the daily insolation with (I_{DO}) and without (I_D) the oblateness effect at the top of the atmosphere of Saturn. See Fig. 1 for full explanation.

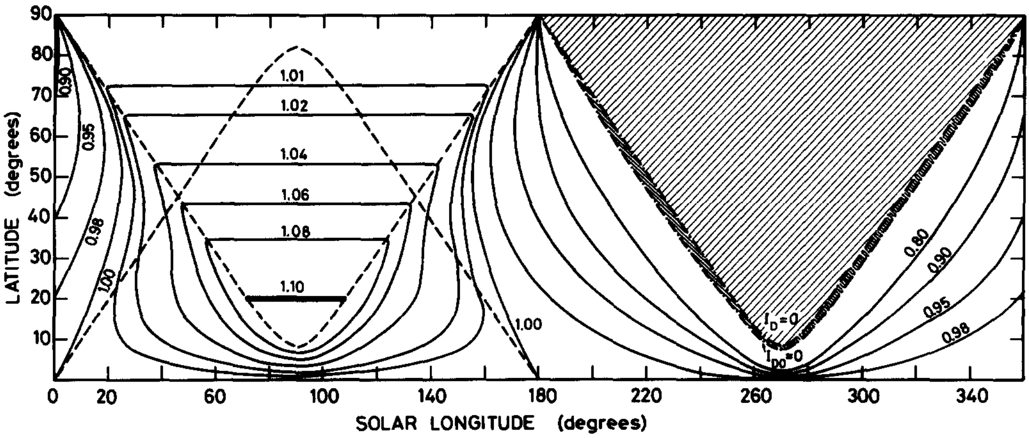

FIG. 3. Seasonal and latitudinal variation of the ratio of the daily insolation with (I_{DO}) and without (I_D) the oblateness effect at the top of the atmosphere of Uranus. See Fig. 1 for full explanation.

cause near the equator a gain of insolation of approximately 12%, decreasing systematically to about 1% at a geocentric latitude of 70°.

In summer, an important ratio distribution difference exists in that Uranus displays only one maximum whereas for the other planets two maxima are found (contrast Fig. 3 with Figs. 2 and 4). Figure 3 also clearly illustrates that in the region where the Sun remains above the horizon all day, the ratio I_{DO}/I_D is independent on the solar longitude $\lambda'_\odot$.

In winter, as for all planets, the insolation is reduced, the rate of change being extremely rapid near the winter solstice, but less sensitive near the equinoxes. Finally, it is worth noting that the maximum difference of the Arctic circles $I_D = 0$ and $I_{DO} = 0$ occurs at $\lambda'_\odot = 225°$ and $\lambda'_\odot = 315°$ with a value of about 3.4°; at the winter solstice this difference is rather small (~0.9°).

4.4. *Neptune*

The ratio pattern of Neptune is plotted in Fig. 4. Comparison of Fig. 2 with Fig. 4 illustrates some obvious geometric similarities, especially in summer, the two branches of the hyperbola of both planets roughly coinciding. This is due to the fact

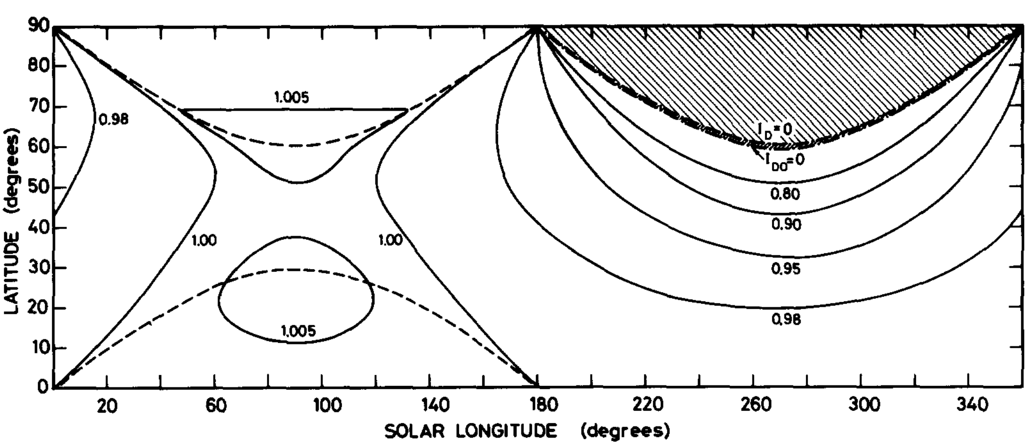

FIG. 4. Seasonal and latitudinal variation of the ratio of the daily insolation with (I_{DO}) and without (I_D) the oblateness effect at the top of the atmosphere of Neptune. See Fig. 1 for full explanation.

that the obliquities are approximately equal. On the other hand, a striking difference exists concerning the maximum value of the ratio of both insolations (1.010 for Neptune and 1.037 for Saturn). This is easily explained by observing that the flattening of Neptune (0.02066) is much smaller than the value for Saturn (0.1).

In winter, the solar radiation very slowly decreases with increasing geocentric latitude φ'. This effect is also ascribed to the small value of f. Another point about the curves is that the isocontours $I_{DO} = 0$ and $I_D = 0$ practically coincide, the maximum difference being only on the order of 1°.

In conclusion, the sensitivity of the curves to changes in the obliquity and the flattening is clearly illustrated in Figs. 1–4.

5. DISCUSSION OF CALCULATION OF THE MEAN DAILY INSOLATIONS

In the previous sections we discussed the effect of the oblateness on the daily insolation. Here we examine the influence of the flattening on the mean (annual, summer, and winter) daily insolations. It should, however, be emphasized that a detailed physical analysis of the insolation distribution curves is beyond the scope of the present work (see, e.g., Levine *et al.*, 1977).

5.1. Jupiter

The oblateness dependence on the mean daily insolations as a function of geocentric latitude is given in Fig. 5. Curves corresponding to a spherical planet are indicated by the full lines, whereas calculations related to an oblate planet are shown by the dashed lines. It has to be pointed out that the mean annual daily insolation distribution on Jupiter, assumed as a spherical planet, is in reasonable agreement with the results of Levine *et al.* (1977), this conclusion also being valid for the other planets discussed in this work.

Concerning more particularly the mean summertime insolations, hereafter denoted as $(\bar{I}_D)_S$ and $(\bar{I}_{DO})_S$, the importance of the

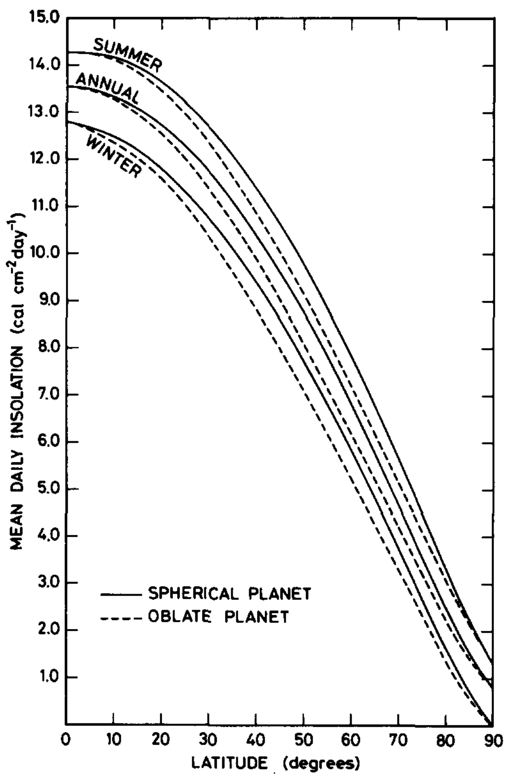

FIG. 5. Latitudinal variation of the mean daily insolations at the top of the atmosphere of Jupiter.

oblateness effect is evident from Fig. 5. Indeed, for latitudes between 60 and 80° as much as 8% of the mean summer daily insolation is lost through the flattening effect. For latitudes less than 60° this effect is of decreasing significance. Another interesting phenomenon (not visible in Fig. 5 due to the insufficiency of the scale adopted for the ordinate) is that for latitudes between the equator and the subsolar point (~3°), $(\bar{I}_{DO})_S > (\bar{I}_D)_S$ (see also Brinkman and McGregor, 1979). However, owing to the small obliquity of Jupiter, the increase of insolation is practically nonexistent.

In winter, as stated previously, the daily insolation of an oblate planet I_{DO} is always reduced when compared to that of a spherical planet I_D. Consequently, $(\bar{I}_{DO})_W < (\bar{I}_D)_W$ at any latitude, the difference ranging from about 6% at 40° to approximately 20% at polar region latitudes.

Finally, it is obvious that the mean annual daily insolations $(\bar{I}_D)_A$ and $(\bar{I}_{DO})_A$ can be found by the expressions

$$(\bar{I}_D)_A = [(\bar{I}_D)_S T_S + (\bar{I}_D)_W T_W]/T_O \quad (24)$$

and

$$(\bar{I}_{DO})_A = [(\bar{I}_{DO})_S T_S + (\bar{I}_{DO})_W T_W]/T_O, \quad (25)$$

where T_S, T_W, and T_O are, respectively, the length of the northern summer, the length of the northern winter, and the sidereal period of revolution (tropical year). Application of Eqs. (24) and (25) leads to a maximum difference of as much as 10% near the pole.

5.2. Saturn

For the case of Saturn, we neglected the shadow effect of the ring system (Brinkman and McGregor, 1979).

In summer, the increase of insolation for latitudes less than the subsolar point (~27°) is clearly demonstrated in the upper part of Fig. 6, the maximum difference occurring at a latitude of about 10° and reaching a value on the order of 1%. Beyond the subsolar point $(\bar{I}_{DO})_S < (\bar{I}_D)_S$, and the maximum loss of insolation is found at midlatitudes and is approximately 3%.

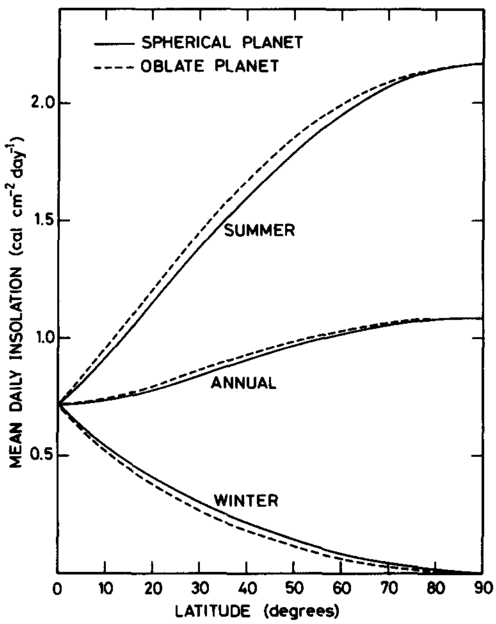

FIG. 7. Latitudinal variation of the mean daily insolations at the top of the atmosphere of Uranus.

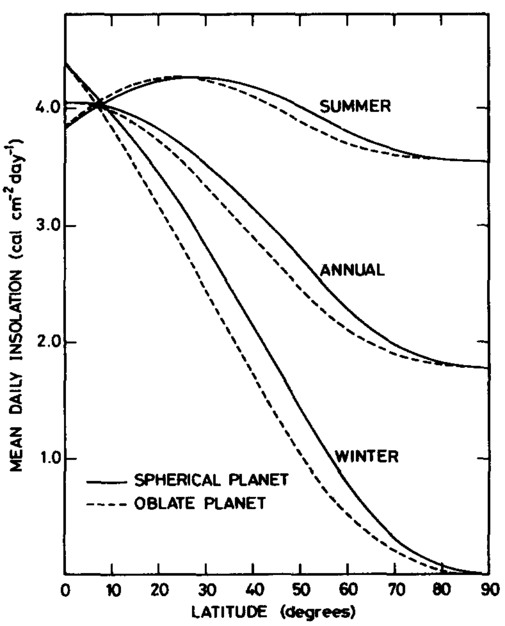

FIG. 6. Latitudinal variation of the mean daily insolations at the top of the atmosphere of Saturn.

The difference between the mean winter insolations increases with increasing latitude and may attain values of about 30% and more at polar region latitudes.

Concerning the mean annual daily insolation, the maximum difference is equal to about 10% at φ' ~50°.

5.3. Uranus

The very large obliquity of Uranus results in an increased mean summertime insolation over practically the entire northern hemisphere except at latitudes between the subsolar point (~82°) and the north pole, where it is found that $(\bar{I}_D)_S$ is scarcely above $(\bar{I}_{DO})_S$. The gain of insolation is of most importance for latitudes between 20 and 30° with values of about 5% (Fig. 7).

In winter, especially at high latitudes, as much as 20% of the mean winter daily insolation is lost through the oblateness effect. Furthermore, it is obvious that the mean annual daily insolation will be reduced by an amount determined by the two opposite summer and winter effects. It follows that the decrease of insolation is much smaller than that in winter season, taking a maximum value approaching 2.5% at latitudes near 30–40°.

5.4. *Neptune*

The mean daily solar radiation incident on Neptune is given in Fig. 8.

When comparing Fig. 6 with Fig. 8, one agreement, as well as one striking difference, is noticed. On one hand, the shape of the curves is qualitatively similar, the numerical value of the obliquity of Neptune being close to the value for Saturn. On the other hand, a discrepancy exists concerning

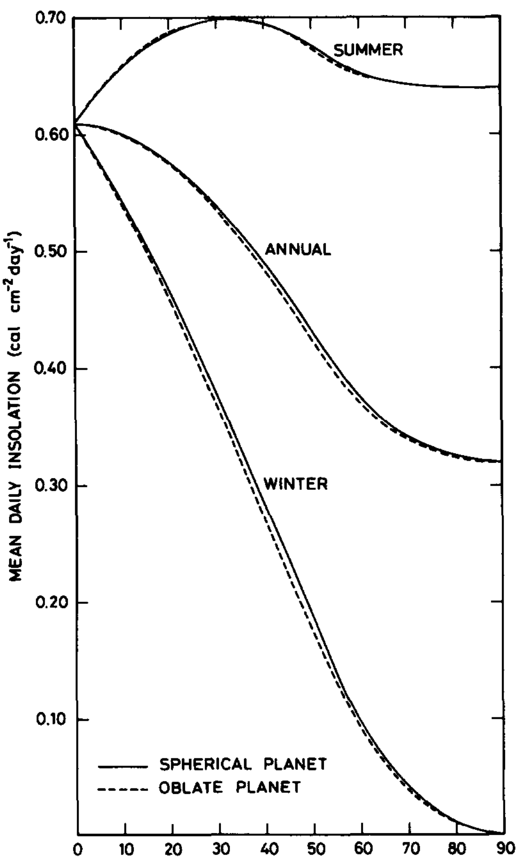

FIG. 8. Latitudinal variation of the mean daily insolations at the top of the atmosphere of Neptune.

the maximum differences between the curves with and without the oblateness effect, the flattening of Neptune (~0.02) being much smaller, by a factor of about 5, than that of Saturn (0.1). So, in summer, we obtained only +0.3% and −0.5% at about 20 and 50°, respectively. In winter, the solar energy is reduced by 8% near 70°. Finally, the mean annual daily insolation is increased by approximately 1.5% in the latitude region 40–60°. It is interesting to note that the above-mentioned maxima occur at practically the same geocentric latitudes as those for Saturn.

6. CONCLUSIONS

In the preceding sections we investigated the influence of the oblateness on the solar radiation incident on Jupiter, Saturn, Uranus, and Neptune. As a result of this study, it follows that the flattening causes significant variations in both the planetary-wide distribution and the intensity of the daily insolation.

In summer, the daily insolation is increased over periods strongly dependent upon the obliquity. This phenomenon is particularly evident from Figs. 1–4. Indeed, it can be seen that the very small obliquity of Jupiter (ϵ = 3.12°) results in two extremely small zones of enhanced solar radiation, whereas for Uranus (ϵ = 82.14°) the region where $I_{DO} > I_D$ extends over almost all of the summer hemisphere. Furthermore, from the obliquities of Saturn and Neptune being only slightly different, it follows that the two areas of increased solar energy nearly coincide.

According to Eq. (18), the maximum value of I_{DO}/I_D in the region of permanent sunlight is a function of both the obliquity and the flattening. This ratio is equal to 1.00033, 1.037, 1.124, and 1.010 for Jupiter, Saturn, Uranus, and Neptune, respectively. Although for Jupiter the gain of insolation is negligible, it should be emphasized that for the other planets and for relatively brief periods the oblateness effect plays an obvious role.

In winter, the effect of the flattening results in a more extensive polar region, the maximum difference of the Arctic circles varying from about 0.4 (Jupiter) to approximately 5° (Saturn). The insolation is always reduced and the curves of constant ratio I_{DO}/I_D roughly parallel the boundary of the polar night except in the neighborhood of the equinoxes. At the winter solstice, the rate of change of I_{DO}/I_D with geocentric latitude is very sensitive both to the obliquity and the flattening. When comparing Jupiter with Uranus, both having nearly the same flattening but with obliquities representing the extreme values, it can be seen that for Jupiter a loss of insolation of 15% occurs over the latitude interval 70–80°, whereas for Uranus the above-mentioned decrease is already reached in the vicinity of the equator. On the other hand, comparison of Saturn and Neptune with approximately the same obliquity reveals that the gradient of I_{DO}/I_D is strongly dependent upon the flattening. More precisely, for latitudes between the equator and 40°, I_{DO}/I_D falls from unity to 0.6 (Saturn), whereas it drops only to 0.9 in the case of Neptune. It should be noted that near the equinoxes the loss of insolation is of decreasing importance.

Finally, we have also studied the latitudinal variation of the mean daily insolations. It is found that for latitudes equatorward of the subsolar point, the mean summer daily insolation of an oblate planet is increased when compared to that of a spherical planet. Although for Jupiter the gain of insolation is extremely small, the maximum increase is equal to 1, 5, and 0.3% for Saturn, Uranus, and Neptune, respectively.

The mean wintertime insolation, however, is always decreased, the maximum loss of insolation being much higher than the mean summertime increase. For example, at polar region latitudes as much as 20 and 30% of the mean winter daily insolation is lost through the oblateness effect for Jupiter and Saturn, respectively. The partial gain of the mean summertime insolation being much smaller than the loss of solar energy during the winter season evidently results in a mean annual daily insolation which is reduced over the entire latitude region.

In conclusion, we believe that the effect of the oblateness must be taken into account in studies related to the radiation and energy budget and the dynamical behavior of the planets discussed in this paper.

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