

# COLLISION FREQUENCIES AND ENERGY TRANSFER

## IONS

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**Abstract**—An investigation has been made of ion collision frequencies and energy transfer for conditions of gaseous thermal nonequilibrium. Following the methods of a development devoted to electron collisions, the problems of ion–neutral and ion–ion elastic collisions have been considered under the assumption of separate Maxwellian velocity distributions. Expressions for the nonequilibrium momentum transfer cross sections, collision frequencies, and energy transfer rates are given for neutral gases on the basis of a polarization potential and for ions on the basis of the Coulomb interaction. Next, the problem of ion–neutral resonance charge exchange is introduced in its effect upon collision cross sections and ion energy transfer rates. An analysis is made of gathered laboratory data to arrive at suitable expressions for the resonance charge exchange and momentum transfer cross sections for  $H^+$ ,  $O^+$ ,  $He^+$ ,  $N_2^+$ , and  $O_2^+$ . Using these results, resonance charge exchange and momentum transfer collision frequencies are derived. The resonance ion–neutral energy transfer rates are then presented and compared with expressions used in previous studies of ion temperatures.

## I. INTRODUCTION

In order to discuss the thermal budget between electrons and ions and neutral gas particles in the ionosphere it is necessary to know all of the energy exchange rates between the different constituents, taking into account their specific temperature dependences under conditions of thermal nonequilibrium. The problem of energy transfer from an electron gas with a Maxwellian velocity distribution as a result of elastic collisions has been discussed in a previous paper (Banks<sup>(1)</sup>) using an equation which is of general applicability to problems of elastic energy transfer. In this paper the study of energy transfer is continued with emphasis being placed upon the atmospheric ions. Thus, in Section II we enter into a discussion of the collision frequency and energy transfer equations which are valid for elastic ion–neutral and ion–ion energy transfer.

In contrast to the electron gas problem, there exists for ions a second method of energy loss by means of resonance charge exchange which, while fundamentally elastic in nature, cannot be described by the equation of collisional energy transfer. The problem of resonance charge exchange between ions and their parent neutral gas is discussed in Section III with respect to both collision frequencies and ion energy processes. Following an analysis of laboratory and theoretical cross section data numerical results for the resonance ion–neutral energy transfer rates are obtained and compared with expressions which have been used in different theoretical analyses of ion temperatures.

## II. NONRESONANT ION COLLISIONS

The equations describing collisional energy transfer have been discussed previously (Banks<sup>(1)</sup>) for the example of two gases with Maxwellian velocity distributions characterized

by the temperatures  $T_1$  and  $T_2$  and particle masses  $m_1$  and  $m_2$ . The energy transfer rate,  $dU_1/dt$ , from the first gas is given by

$$\frac{dU_1}{dt} = -3n_1 \frac{m_1 m_2}{(m_1 + m_2)^2} (kT_1 - kT_2) \bar{v}_{12} \quad (1)$$

where  $n_1$  is the first gas number density,  $k$  is Boltzmann's constant, and  $\bar{v}_{12}$  is the average collision frequency of a single particle of the first type in the second gas. Since the density, masses and temperatures are independent of the interaction forces between the different particles it is seen that it is the collision frequency which must be determined in order for the energy transfer rate between arbitrary types of gases to be known. The result is (see Banks<sup>(1)</sup>)

$$\bar{v}_{12} = \frac{4}{3} n_2 \left(\frac{8k}{\pi}\right)^{1/2} \left(\frac{T_1}{m_1} + \frac{T_2}{m_2}\right)^{1/2} \bar{Q}_D, \quad (2)$$

where  $n_2$  is the second gas number density and  $\bar{Q}_D$  is the average momentum transfer cross section, given in terms of the velocity dependent momentum transfer cross section,  $q_D$ , and the interparticle relative velocity,  $g$ , as

$$\bar{Q}_D = K^3 \int_0^\infty g^5 q_D \exp(-Kg^2) dg \quad (3)$$

$$K = \left[ \frac{2kT_1}{m_1} + \frac{2kT_2}{m_2} \right]^{-1}.$$

With equations (1), (2), and (3) it is possible to derive accurate expressions for both energy transfer rates and collision frequencies for gases of arbitrary temperature and particle mass once the velocity dependent momentum transfer cross section is known. Thus, the problem of ion elastic energy loss revolves, essentially, about the choice of proper cross sections needed to describe the interaction between the ion gas and the gas with which it is mixed. In the following analysis we will be concerned with two particular situations which are of some practical importance. The first involves the derivation of energy transfer rates and collision frequencies for ion-neutral gas mixtures under conditions where there are only elastic collisions. Next, we consider the problem of ion-ion gas mixtures when each ion species has its own separate Maxwellian velocity distribution.

### 1. Ion-neutral collisions

At low temperatures the most important ion-neutral interaction arises from an induced dipole attraction which has a potential  $\varphi = -\alpha e^2/2r^4$ . Here,  $\alpha$  is the neutral atom polarizability,  $e$  is the electric charge, and  $r$  is the radial separation. This potential corresponds to the force law acting between Maxwellian molecules and the details of the transport coefficients under conditions of thermal equilibrium are well known. In particular, it is found the collision cross section is proportional to  $T^{-1/2}$  and that the collision frequency is independent of the temperature.

For temperatures greater than 300°K, the induced dipole force of attraction is countered by a short range quantum mechanical repulsion. For sufficiently high temperatures, the polarization contribution is negligible and the collision cross section becomes nearly constant, yielding a collision frequency which varies as  $T^{1/2}$ . Unlike the polarization force, which is independent of the chemical nature of a given ion and depends only upon the

atomic polarizability of the neutral gas, the nature of the repulsive force is linked directly to the details of the ion and neutral orbital electron structure. Thus, variations in cross section are to be expected for different ions in the same neutral gas at elevated temperatures.

Because there exist virtually no data for either collision frequencies or cross sections at high temperatures, the study of ion-neutral collisions must be based upon the assumption that the polarization force is of dominating importance. A detailed evaluation of the ion-neutral interaction potential has been made by Dalgarno *et al.*<sup>(2)</sup> who terminated the polarization potential at a small atomic radius and added an elastic sphere repulsive potential to simulate the quantum repulsion effects. This theoretical cross section was then compared with reported measurements of ionic mobilities. For equal ion and neutral temperatures below 300°K the theoretical results appear to be accurate. Above 300°K, the detailed variation of the momentum transfer cross section depends upon the specific ion and neutral gas.

With there being only a few experimental data for temperatures above 300°K, it is necessary to accept the collision equations developed for the polarization force alone as representing the true interaction over the entire range of ion and neutral temperatures discussed here. Accordingly, we take the ion-neutral velocity dependent momentum transfer cross section for singly charged ions to be, following Dalgarno *et al.*<sup>(2)</sup>,

$$q_D = 2.21 \pi \left( \frac{\alpha e^2}{\mu g^2} \right)^{1/2} \text{ cm}^2, \quad (4)$$

where  $\alpha$  is the neutral gas atomic polarizability,  $\mu$  is the ion-neutral reduced mass,  $e$  is the electron charge, and  $g$  is the ion-neutral particle relative velocity.

Using equation (3) the average momentum transfer cross section for ion-neutral collisions becomes

$$\bar{Q}_D = \frac{3\sqrt{2}}{16} \pi^{3/2} \left( \frac{4.88\alpha e^2}{\mu} \right)^{1/2} \left[ \frac{kT_i}{m_i} + \frac{kT_n}{m_n} \right]^{-1/2} \quad (5)$$

or, numerically,

$$\bar{Q}_D = 13.3 \times 10^{-14} \left( \frac{\alpha_0}{\mu_A} \right)^{1/2} \left[ \frac{T_i}{A_i} + \frac{T_n}{A_n} \right]^{-1/2} \text{ cm}^2 \quad (6)$$

where  $A_i$  and  $A_n$  are the particle masses in atomic mass units (a.m.u.),  $\mu_A$  is the reduced mass in a.m.u.,  $\alpha_0$  is the atomic polarizability in units of  $10^{-24}$  cm<sup>3</sup>, and the subscripts apply to ions and neutrals, respectively.

For conditions of equal temperatures the mass factors of equation (6) cancel and we have

$$\bar{Q}_D(T_i = T_n) = \frac{3\sqrt{2}}{16} \pi^{3/2} \left( \frac{4.88\alpha e^2}{kT_n} \right)^{1/2} \quad (7)$$

which agrees with the value used by Dalgarno *et al.*<sup>(2)</sup> in their calculations of ion mobility.

Using equations (5) and (2) the average ion-neutral collision frequency for energy transfer is obtained as

$$\bar{\nu}_{in} = 2.21 \pi n_n \left( \frac{\alpha e^2}{\mu} \right)^{1/2} \quad (8)$$

or,

$$\bar{\nu}_{in} = 2.6 \times 10^{-9} n_n \left( \frac{\alpha_0}{\mu_A} \right)^{1/2} \text{ sec}^{-1}, \quad (9)$$

independent of either gas temperature. Actually, as shown by Morse<sup>(3)</sup> although we have derived the present expression for the average momentum transfer cross section on the basis of separate Maxwellian velocity distributions, for the case of Maxwellian molecules the cross section, collision frequency, and energy transfer rate are independent of the actual velocity distribution used.

Table 1, taken from Hasted<sup>(4)</sup> and Dalgarno<sup>(6)</sup> gives values of the atomic polarizability for the atmospheric gases.

TABLE 1. POLARIZABILITIES OF NEUTRAL GASES

Neutral gas	$\alpha_0(10^{-24} \text{ cm}^3)$
N <sub>2</sub>	1.76
O <sub>2</sub>	1.60
H <sub>2</sub>	0.82
O	0.89
H	0.67
He	0.21
N	1.13

Equation (8) can be compared with an ion-neutral collision frequency which follows from the work of Chapman and Cowling<sup>(6)</sup>. In the diffusion of ions across a magnetic field, a collision interval  $\tau$  appears which can be used to define a collision frequency  $\nu$  by the relation  $\nu = 1/\tau$ . The value of  $\tau$  is related to the ion-neutral diffusion coefficient which, in turn, can be calculated directly for the polarization potential. This process yields an ion-neutral collision frequency, using the cross section of Dalgarno *et al.*<sup>(2)</sup>, given by

$$\bar{\nu}_{in} = \frac{n_i m_i + n_n m_n}{m_i m_n} 2.21 \pi (\alpha e^2 \mu)^{1/2} \quad (10)$$

where  $n_i$  is the ion density and  $n_n$  is the neutral gas number density. Comparing this result with equation (8) for the energy transfer collision frequency, it is seen that equality follows only if  $n_i = n_n$ . For other density conditions, in particular where  $n_i \ll n_n$ , the two collision frequencies are not the same. Dalgarno<sup>(7)</sup> has given ion-neutral collision frequencies based upon equation (10) under the assumption that  $n_i \ll n_n$ . For this approximation it follows that his results are smaller than those presented here by the ratio  $\mu/m_i$ . For the example of O<sup>+</sup> in N<sub>2</sub> this corresponds to a factor of 0.64.

Using the ion-neutral collision frequency given by equation (8), the rate of ion energy transfer in a neutral gas is found from equation (1) to be

$$\frac{dU_i}{dt} = -3n_i n_n \frac{2.21 \pi}{(m_i + m_n)} (\alpha \mu e^2)^{1/2} k(T_i - T_n) \quad (11)$$

or,

$$\frac{dU_i}{dt} = -6.8 \times 10^{-13} n_i n_n \frac{(\mu_A \alpha_0)^{1/2}}{(A_i + A_n)} (T_i - T_n) \text{ eV cm}^{-3} \text{ sec}^{-1}. \quad (12)$$

Appropriate values for the neutral gas polarizabilities are given in Table 1 and have been used to obtain the energy transfer rates listed in Table 2. As a consequence of the particular force law for ion-neutral collisions, the energy transfer rates depend only upon the difference in ion and neutral gas temperatures. In general there is a variation by a factor of 3.6 between the different loss rates considered. Further, except for the collisions H<sup>+</sup>-He and

He<sup>+</sup>-H, the differences in the rates are determined primarily by the varying mass factors rather than the changes in atomic polarizability. It is also noted that these ion energy loss rates are considerably larger than those found (see Banks<sup>(1)</sup>) for elastic electron-neutral collisions. As an example we may consider the O<sup>+</sup>-N<sub>2</sub> rate as, omitting common density and temperature factors,  $6.6 \times 10^{-14} \text{ eV cm}^3 \text{ }^\circ\text{K}^{-1} \text{ sec}^{-1}$ , while the e-N<sub>2</sub> rate is, at  $T_e = 1000^\circ\text{K}$ , only  $1.55 \times 10^{-16} \text{ eV cm}^3 \text{ }^\circ\text{K}^{-1} \text{ sec}^{-1}$ . This difference of a factor of 430 arises from the enhanced ion-neutral cross section and advantageous mass factors in equation (1), mitigated, of course, by the much more rapid average electron velocity.

TABLE 2. ELASTIC ION-NEUTRAL ENERGY TRANSFER RATES

Collision pair	$\frac{dU_i/dt}{n_i n_n (T_i - T_n)}$	(eV cm <sup>3</sup> sec <sup>-1</sup> °K <sup>-1</sup> )
O <sup>+</sup> -N <sub>2</sub>	$6.6 \times 10^{-14}$	
O <sup>+</sup> -O <sub>2</sub>	$5.8 \times 10^{-14}$	
O <sup>+</sup> -He	$2.8 \times 10^{-14}$	
O <sup>+</sup> -H	$3.3 \times 10^{-14}$	
H <sup>+</sup> -O	$3.5 \times 10^{-14}$	
H <sup>+</sup> -N <sub>2</sub>	$3.1 \times 10^{-14}$	
H <sup>+</sup> -He	$5.5 \times 10^{-14}$	
He <sup>+</sup> -O	$5.8 \times 10^{-14}$	
He <sup>+</sup> -H	$1.0 \times 10^{-13}$	
He <sup>+</sup> -N <sub>2</sub>	$5.3 \times 10^{-14}$	

The results given here can be compared with values which have been used in previous studies of ion temperatures in the upper atmosphere. In a recent paper Willmore<sup>(8)</sup> has adopted for the helium and oxygen ion rates the values  $3.3 \times 10^{-14}$  and  $3.6 \times 10^{-14}$  respectively. In comparing these rates with those listed in Table 2 it appears that Willmore's helium ion cooling rate, which is based upon the diffusion collision frequencies of Dalgarno<sup>(7)</sup>, is too low by a factor of 1.75 (see equation 10 and discussion). Likewise, the oxygen ion energy transfer rate is too large by a factor of 1.3.

## 2. Ion-ion collisions

The average momentum transfer cross section for two separate ion gases, each having its own Maxwellian velocity distribution is,<sup>(1)</sup>

$$\bar{Q}_D = \frac{\pi}{2} \frac{(Z_1 Z_2 e^2)^2 \ln \Lambda}{\mu^2 \left[ \frac{kT_1}{m_1} + \frac{kT_2}{m_2} \right]^2} \quad (13)$$

where  $Z_{1,2}$  are the respective ion charges and the parameter  $\Lambda$  is given by

$$\Lambda = \frac{2\epsilon}{Z_1 Z_2 e^2} \lambda_D. \quad (14)$$

Here  $\epsilon$  is the average energy of relative motion between ions, and  $\lambda_D$ , the plasma Debye length, is given by

$$\lambda_D^{-2} = 4\pi e^2 \left[ \frac{Z_1 n_1}{kT_1} + \frac{Z_2 n_2}{kT_2} \right]. \quad (15)$$

Equation (13) can be evaluated numerically to give

$$\bar{Q}_D = \frac{4.4 \times 10^{-6} \ln \Lambda}{\mu_A^2 \left[ \frac{T_1}{A_1} + \frac{T_2}{A_2} \right]^2} \text{ cm}^2 \quad (16)$$

using the previous notation.

The average ion-ion energy transfer collision frequency is, from equation (3),

$$\bar{\nu}_{12} = \frac{4}{3} \sqrt{(2\pi)} n_2 \frac{(Z_1 Z_2 e^2)^2 \ln \Lambda}{\mu^2 \left[ \frac{kT_1}{m_1} + \frac{kT_2}{m_2} \right]^{3/2}} \quad (17)$$

or,

$$\bar{\nu}_{12} = 8.4 \times 10^{-2} n_2 \frac{(Z_1 Z_2)^2 \ln \Lambda}{\mu_A^2 \left[ \frac{T_1}{A_1} + \frac{T_2}{A_2} \right]^{3/2}} \text{ cm}^2. \quad (18)$$

To find the average collision frequency of a thermal ion in its own gas we let  $T_1 = T_2 = T_i$  and  $m_1 = m_2 = m_i$ . Thus,

$$\bar{\nu}_{12} = n_i \frac{8}{3} \sqrt{\left( \frac{\pi}{m_i} \right) \frac{Z_i^4 e^4 \ln \Lambda}{(kT_i)^{3/2}}}, \quad (19)$$

which is a factor of  $\frac{4}{3}$  larger than the ion collision frequency given by Chapman<sup>(9)</sup> for total scattering effects.

The total rate of energy transfer between two ion species having Maxwellian velocity distributions is obtained from equation (1) by using the average ion-ion energy transfer collision frequency given by equation (17). Thus,

$$\frac{dU_1}{dt} = - \frac{4 \sqrt{(2\pi)} n_1 n_2 e^4 k(T_1 - T_2) \ln \Lambda}{m_1 m_2 \left[ \frac{kT_1}{m_1} + \frac{kT_2}{m_2} \right]^{3/2}}, \quad (20)$$

or,

$$\frac{dU_1}{dt} = - \frac{2.18 \times 10^{-5} n_1 n_2 (T_1 - T_2) \ln \Lambda}{A_1 A_2 \left[ \frac{T_1}{A_1} + \frac{T_2}{A_2} \right]^{3/2}} \text{ eV cm}^{-3} \text{ sec}^{-1}, \quad (21)$$

which agrees with the result of Longmire.<sup>(10)</sup> Thus, the concept of binary collisions permits the exact description of the ion-ion energy transfer rate under the condition that each species has its own Maxwellian velocity distribution.

At this point it is now possible to make a comparison of the elastic and resonant ion-neutral energy transfer rates with the calculated ion-ion values. If we disregard the density and temperature differences it is found from Tables 2 and 11 and equation (21) that the ion-ion rates are generally larger by at least a factor of 10 for ion temperatures near 1000°K. This would imply that the ion gases are more strongly coupled to each other than to the neutral gases. However, for specific problems involving the transfer of energy in dilute gases where the different densities vary widely it is necessary to solve the heat balance equations directly in order to evaluate the assumption of a single ion temperature which is common to all the ion species present.

## III. RESONANT ION-NEUTRAL COLLISIONS

## 1. Introduction

Charge exchange exerts a significant influence upon the momentum transfer cross section and is important for both the problems of ion energy transfer and diffusion. For this process, it is found that although there is a transfer of an electric charge between two particles, each tends to retain its original kinetic energy. Thus, even though the identity of the ion has changed, the reaction is still fundamentally elastic in preserving the total kinetic energy. Charge exchange consequently becomes of particular importance to the ion energy balance since it can provide a rapid means for energetic ions to be transformed into energetic neutral particles. For many of the possible atmospheric ion-neutral collision pairs it is found that the probability for charge exchange is much larger than that for the normal gas kinetic collisions. Therefore, while an originally energetic ion could cool to the ambient gas temperature by means of multiple elastic collisions, it can lose all of its excess energy in a single charge exchange reaction.

Charge exchange also plays an important part in enhancing the magnitude of the momentum transfer cross section above those values normally associated with high temperature ion-neutral reactions. This action arises from the conversion of very slightly glancing collisions into what appear to be nearly direct impacts with a consequent backscattering. Under these circumstances there exists a relation between the charge exchange,  $q_E$ , and momentum transfer,  $q_D$ , cross sections given by Dalgarno<sup>(11)</sup> as  $q_D = 2q_E$ . This relationship is valid only for high temperatures where the ion-neutral polarization effects can be neglected.

An adequate introduction to the details of the quantum theory of resonance charge exchange can be found in Bates.<sup>(12)</sup> In general, an exact solution to the problem of determining the charge exchange cross section is not possible because of the uncertainties in the form of the energy wave functions for all possible energy states of two colliding particles. Therefore, it is necessary to resort to experimental results to determine specific resonance charge exchange cross sections.

## 2. Resonance cross sections—theory

Since we are interested in both the charge exchange and momentum transfer cross sections it must be noted that to determine the full temperature variations of the momentum transfer cross section it is not adequate to use the relation  $q_D = 2q_E$ . For temperatures below a transition region, generally in the neighborhood of 500°K, the momentum transfer cross section is determined largely by the induced dipole polarization force between the ion and the neutral atom. The appropriate cross section for this interaction is given by equation (5) as

$$\bar{Q}_D = \frac{3}{8} \pi^{3/2} \frac{(4.88 \alpha e^2)^{1/2}}{(kT_i + kT_n)^{1/2}}. \quad (22)$$

For temperatures above the transition region the polarization contribution to  $\bar{Q}_D$  diminishes rapidly and the effect of the charge exchange enhancement of the momentum transfer cross section becomes dominant. This problem has been discussed by Ghosh and Sharma.<sup>(13)</sup>

It has been shown by Dalgarno<sup>(11)</sup> that the velocity dependent charge exchange cross section,  $q_E$ , can be expressed as

$$q_E = (A - B \log_{10} \epsilon)^2, \quad (23)$$

for particle energies below 500 eV. Here  $A$  and  $B$  are constants characteristic of a given gas and  $\epsilon$  is the kinetic energy of relative motion, measured in electron volts (eV). Using

$q_D = 2q_E$ , the high temperature velocity dependent momentum transfer cross section can be given as

$$q_D = 2(A - B \ln \epsilon)^2 \quad (24)$$

It is important to note that the total cross section for momentum transfer is not equal to the sum of the charge exchange and polarization contributions. Because there exist mutual interaction effects, the polarization potential and charge exchange action combine to form a total cross section whose variation with temperature follows closely the individual cross

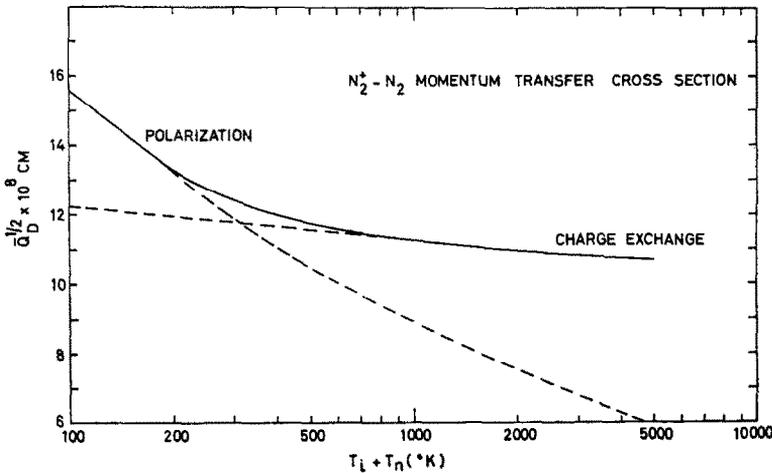


FIG. 1.  $N_2^+ - N_2$  MOMENTUM TRANSFER CROSS SECTION.

The polarization curve is based upon a molecular polarizability of  $1.76 \times 10^{-24} \text{ cm}^3$  while the charge exchange contribution is taken from the experimental data of Amme and Utterback<sup>(82)</sup> extrapolated to thermal energies.

sections in their own regions of dominance. In the transition region there is a slight enhancement of the cross section to values above those due to either polarization or charge exchange, but this extends only over a limited temperature range. Thus, for temperatures above the transition region,  $q_D$  is determined exclusively, to within 1 per cent (Knof *et al.*<sup>(14)</sup>), by charge exchange, while below this point the polarization forces are predominant. This effect illustrated in Fig. 1 for the specific example of  $N_2^+ - N_2$  collisions. Here the transition region extends, in terms of  $\Gamma = T_i + T_n$ , from 200° to 700°K and is centered about the point  $\Gamma = 340^\circ\text{K}$ . It is seen that for  $\Gamma > 750^\circ\text{K}$  there is essentially no contribution to  $\bar{Q}_D$  from the polarization.

In order to determine suitable temperature dependent average charge exchange and momentum transfer cross sections, it is necessary to average equations (23) and (24) over the ion and neutral velocity distributions. Thus,

$$\bar{Q}_D = 2\bar{Q}_E = \int \int f_i f_n q_D d^3 \bar{v}_i d^3 \bar{v}_n \quad (25)$$

where  $q_D$  is given by equation (24),  $f$  is the particle velocity distribution function,  $d^3 \bar{v}$  is a velocity space volume element, and the subscripts refer to ions and neutrals, respectively.

The solution to equation (25) for conditions where  $T_i = T_n$  has been given by Mason and Vanderslice<sup>(15)</sup> and Sheldon<sup>(16)</sup>. For situations where the ions and neutrals have separate

Maxwellian distributions with  $T_i \neq T_n$  it has been possible to generalize these results to obtain

$$\bar{Q}_E = [(A + 3.96B) - B \log_{10} \Gamma]^2 \times 10^{-16} \text{ cm}^2, \quad (26)$$

where  $A$  and  $B$ , measured in units of  $10^{-8}$  cm, are the coefficients appearing in the equation

$$q_E = (A - B \log_{10} \epsilon)^2. \quad (27)$$

Following equation (2), we now define a resonance ion-neutral collision frequency,  $\bar{v}_R$ , as

$$\bar{v}_R = \frac{4}{3} n_n \left( \frac{8k}{\pi m} \right)^{1/2} (T_i + T_n)^{1/2} \bar{Q}_D, \quad (28)$$

with  $m$  the ion or neutral particle mass. In a similar manner we define the average collision frequency for charge exchange,  $\bar{v}_E$ , as

$$\bar{v}_E = \frac{4}{3} n_n \left( \frac{8k}{\pi m} \right)^{1/2} (T_i + T_n)^{1/2} \bar{Q}_E, \quad (29)$$

the factor of  $\frac{4}{3}$  appearing since we desire to use this equation in an energy transfer equation which is analogous to equation (1), the mass factor being omitted. From equations (28) and (29), we have

$$\bar{v}_E = \frac{1}{2} \bar{v}_R, \quad (30)$$

for the temperature regime where polarization effects are negligible.

### 3. Resonance ion-neutral energy transfer

The derivation of the energy transfer rate of an ion gas under resonance conditions requires the use of the ion continuity equation with the charge exchange collision frequency. Thus, in the present section, the general equation of collisional energy loss does not apply.

The energy loss rate of an ion gas follows from consideration of the energy balance of ion production and loss. Thus,

$$\frac{dU_i}{dt} = -\{E_i n_i \bar{v}_E - E_n n_i \bar{v}_E\} \quad (31)$$

where  $E_i$  and  $E_n$  are the respective ion and neutral particle energies,  $U_i$  is total ion gas energy per unit volume, and  $\bar{v}_E$  is the charge exchange collision frequency defined by equation (29). If we apply the relations  $E_i = 3kT_i/2$  and  $E_n = 3kT_n/2$ , which apply for gases with Maxwellian velocity distributions, we obtain

$$\frac{dU_i}{dt} = -\frac{3}{2} n_i k \bar{v}_E (T_i - T_n), \quad (32)$$

or,

$$\frac{dU_i}{dt} = -1.3 \times 10^{-4} n_i (T_i - T_n) \bar{v}_E \text{ eV cm}^{-3} \text{ sec}^{-1}. \quad (33)$$

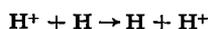
To apply the preceding results to obtain accurate values for the resonance ion-neutral collision frequencies and energy transfer rates it is necessary to know the charge exchange and momentum transfer cross sections for each ion. Thus, in the following section an analysis is made of the experimental and theoretical results for each atmospheric ion in order to arrive at satisfactory values for  $\bar{Q}_D$  and  $\bar{Q}_E$ . These quantities are then used to

obtain adequate expressions for the appropriate collision frequencies and the ion energy loss rates.

#### 4. Resonance experimental cross sections

A. *Atomic hydrogen.* The charge exchange reaction  $H^+ + H \rightarrow H + H^+$  has been studied extensively because of its theoretical simplicity. Dalgarno and Yadev<sup>(17)</sup> made accurate calculations using perturbed stationary state wave functions. Dalgarno<sup>(11)</sup>, in calculating the mobility of ions in parent gases, was able to reproduce essentially the same results and compare them with experiment. Gurnee and Magee<sup>(18)</sup> applied the semi-classical impact parameter method to obtain values of the charge exchange cross section

TABLE 3. ATOMIC HYDROGEN CHARGE EXCHANGE



$$q_E^{1/3} = A - B \log_{10} \epsilon$$

Source	$A \times 10^8$	$B \times 10^8$
Dalgarno <sup>(11)</sup>	6.89	0.83
Gurnee and Magee <sup>(18)</sup>	6.78	0.91
Rapp and Francis <sup>(20)</sup>	6.05	0.88
Fite <i>et al.</i> <sup>(22)</sup>	7.60	1.06

at two ion velocities. Rapp and Ortenburger<sup>(19)</sup> found their theoretical approach gave values in substantial agreement with those of Dalgarno and Yadev. Later, extensions were made by Rapp and Francis.<sup>(20)</sup>

Experimental data have been obtained by Fite *et al.*<sup>(21)</sup> by using beam techniques with ion energies as low as 400 eV. The data were later extended by Fite *et al.*<sup>(22)</sup> to a lower limit of 20 eV, giving close agreement with the previous results. The various values are listed in Table 3.

For the present purposes, the charge exchange cross section given by Dalgarno and Yadev<sup>(17)</sup> appears to represent the best compromise between experiment and theory. Thus, using equation (26) we obtain for the hydrogen ion charge exchange cross section

$$\bar{Q}_E(H^+) = [10.2 - 0.83 \log_{10} \Gamma]^2 \times 10^{-16} \text{ cm}^2, \quad (34)$$

while the average momentum transfer cross section corresponding to the high temperature limit is

$$\bar{Q}_D(H^+) = [14.4 - 1.17 \log_{10} \Gamma]^2 \times 10^{-16} \text{ cm}^2. \quad (35)$$

Here, as in the subsequent equations,  $\Gamma = T_i + T_n$ .

To determine the extent of the temperature transition region between the polarization and charge exchange dominance of the momentum transfer cross section, equations (35) and (22) have been examined. It is found that the temperature transition point where the two asymptotic effects are equal lies near  $\Gamma = 100^\circ\text{K}$ . Thus, for the case of atomic hydrogen, there is essentially no error involved in assuming that the charge exchange contribution to  $\bar{Q}_D$  is dominant at all combined temperatures above  $100^\circ\text{K}$ .

B. *Atomic oxygen.* The symmetrical charge exchange reaction  $O^+ + O \rightarrow O + O^+$  has been studied by Dalgarno<sup>(11)</sup> using a semi-classical approximation for the scattering phase shifts. Because there was considerable uncertainty in the approximation used to express the interaction energy of the ion and atom as a function of the radial separation,

accuracy was claimed only to within a factor of two. Knof *et al.*<sup>(14)</sup> have made the most detailed study of the oxygen problem by taking explicit account of twelve possible electronic states of the  $O_2^+$  pseudo-molecule. Their results are listed in Table 4, along with those of other authors. Experimentally, there is good agreement between the values given by Knof *et al.*<sup>(14)</sup> and the values obtained by Stebbings *et al.*<sup>(24)</sup>. These latter results extend down to 40 eV and, according to Knof *et al.*, the average deviation between the predicted and measured values is only 5.5 per cent. Therefore, the charge exchange cross section of Knof *et al.* will be used here.

Using equation (26) the average charge exchange cross section is

$$\bar{Q}_E(O^+) = [7.47 - 0.475 \log_{10} \Gamma] \times 10^{-16} \text{ cm}^2. \quad (36)$$

The transition region between polarization and charge exchange dominance has been investigated. Figure 2 shows the momentum transfer cross section and the extension of the

TABLE 4. ATOMIC OXYGEN CHARGE EXCHANGE

$$O^+ + O \rightarrow O + O^+$$

$$q_E^{1/2} = A - B \log_{10} \epsilon$$

Source	$A \times 10^8$	$B \times 10^8$
Knof <i>et al.</i> <sup>(14)</sup>	5.59	0.475
Dalgarno <sup>(11)</sup> (see also Dalgarno <sup>(28)</sup> )	9.43	0.588
Stebbing <i>et al.</i> <sup>(24)</sup>	5.88	0.57
Rapp and Francis <sup>(20)</sup>	6.16	0.76

asymptotic polarization and charge exchange contributions. The transition is found to take place near  $\Gamma = 470^\circ\text{K}$ . It follows from Fig. 2 that the maximum error arising from the omission of the polarization effect is 11 per cent at  $\Gamma = 470^\circ\text{K}$  and diminishes rapidly at higher temperatures. Thus, for the computation of the momentum transfer cross section, it is possible to segment the true curve of Fig. 2 into two portions: one for charge exchange and the other polarization. Hence,

$$\begin{aligned} \Gamma > 470^\circ\text{K} \quad \bar{Q}_D(O^+) &= [10.5 - 0.67 \log_{10} \Gamma]^2 \times 10^{-16} \text{ cm}^2 \\ \Gamma < 470^\circ\text{K} \quad \bar{Q}_D(O^+) &= \frac{1.79 \times 10^{-13}}{\Gamma^{1/2}} \text{ cm}^2 \end{aligned} \quad (37)$$

C. *Atomic nitrogen.* Two derivations of the nitrogen charge transfer cross section have been made. Gurnee and Magee<sup>(18)</sup> and Knof *et al.*<sup>(14)</sup> obtained the expression

$$q_E(N^+) = [5.53 - 0.46 \log_{10} \epsilon]^2 \times 10^{-16} \text{ cm}^2, \quad (38)$$

which is accepted for the present calculations.

The average charge exchange cross section is, from equation (26),

$$\bar{Q}_E(N^+) = [7.33 - 0.455 \log_{10} \Gamma]^2 \times 10^{-16} \text{ cm}^2. \quad (39)$$

Applying equations (22) and (26), it is possible to find the asymptotic charge exchange and polarization contributions to  $\bar{Q}_D$ . However, because the atomic polarizability of atomic nitrogen is only slightly larger than that for atomic oxygen and because the two charge exchange cross sections are almost identical, it is not necessary to present the data graphically. For atomic nitrogen it is found that the transition temperature corresponds to

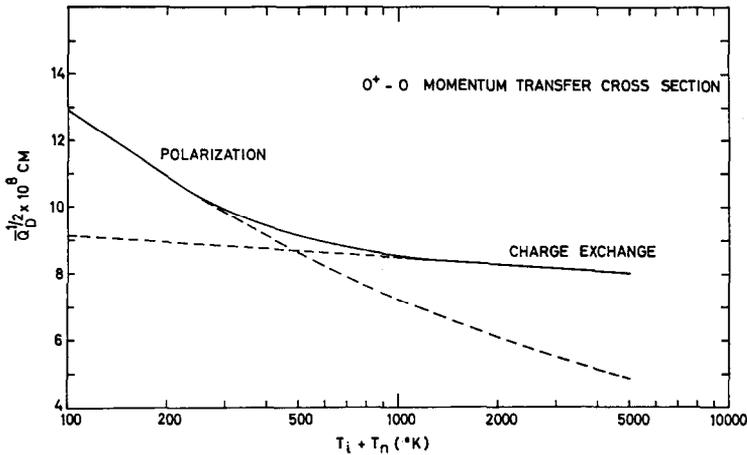


FIG. 2.  $O^+ - O$  MOMENTUM TRANSFER CROSS SECTION.

The polarization curve is based upon an atomic polarizability of  $0.89 \times 10^{-24} \text{ cm}^3$ . The charge exchange region is based upon the theoretical work of Knof *et al.*<sup>(14)</sup> expanded to include the effects of differing ion and neutral gas temperatures.

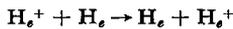
$\Gamma = 550^\circ\text{K}$  and that the error at this point in neglecting the polarization contribution is 10 per cent. Thus, for the present purposes,  $\bar{Q}_D$  can be taken as

$$\Gamma > 550^\circ\text{K} \quad \bar{Q}_D(N^+) = [10.3 - 0.645 \log_{10} \Gamma]^2 \times 10^{-16} \text{ cm}^2$$

$$\Gamma < 550^\circ\text{K} \quad \bar{Q}_D(N^+) = \frac{2.0 \times 10^{-13}}{\Gamma^{1/2}} \text{ cm}^2. \quad (40)$$

**D. Helium.** Like atomic hydrogen, helium has been the object of intensive theoretical and experimental study. A paper by Rapp and Francis<sup>(20)</sup> summarizes the results of twenty-one experiments and four theoretical studies.

TABLE 5. HELIUM CHARGE EXCHANGE



$$q_B^{1/2} = A - B \log_{10} \epsilon$$

Source	$A \times 10^8$	$B \times 10^8$
Cramer and Simons <sup>(25)</sup>	5.25	0.74
Dalgarno <sup>(11)</sup>	5.39	0.62
Rapp and Francis <sup>(20)</sup>	4.51	0.68
Gurnee and Magee <sup>(18)</sup>	5.02	0.64
Hasted <sup>(26)</sup>	5.14	0.55
Chanin and Biondi <sup>(27)</sup>	4.93	0.82

Dalgarno<sup>(11)</sup> calculated both ion mobility and charge transfer cross sections and compared them with the experimental mobility data of Chanin and Biondi.<sup>(27)</sup> Agreement was found to be within 20 per cent at the higher temperatures. Rapp and Francis<sup>(20)</sup> applied an empirical two state model which gives good accord with the experimental data for energies above 200 eV. Cramer and Simmons<sup>(25)</sup> were able to measure the charge exchange cross section at energies down to 16 eV. These data agree well with the earlier results of Hasted<sup>(26)</sup> and Dillon *et al.*<sup>(28)</sup> The correspondence is very good between the various authors

with a spread of 15 per cent at 1000°K, if the results of Rapp and Francis are disregarded. The data of Cramer and Simons listed in Table 5 are used here as representing the charge exchange cross section.

The average charge exchange cross section for helium may be taken as

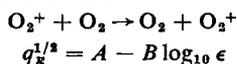
$$\bar{Q}_E(\text{He}^+) = [8.17 - 7.4 \log_{10} \Gamma]^2 \times 10^{-16} \text{ cm}^2. \quad (41)$$

For the momentum transfer cross section it is found that the transition temperature lies below  $\Gamma = 200^\circ\text{K}$  and virtually no error is involved in neglecting the polarization effect. Consequently, we take

$$\bar{Q}_D(\text{He}^+) = [11.6 - 1.04 \log_{10} \Gamma] \times 10^{-16} \text{ cm}^2. \quad (42)$$

E. *Molecular oxygen*. For the reaction  $\text{O}_2^+ + \text{O}_2 \rightarrow \text{O}_2 + \text{O}_2^+$  there appear to be no theoretically derived values for the charge exchange cross section. However, several molecular

TABLE 6. MOLECULAR OXYGEN CHARGE EXCHANGE



Source	$A \times 10^8$	$B \times 10^8$
Stebbing <i>et al.</i> <sup>(30)</sup>	5.32	0.65
Amme and Utterback <sup>(32)</sup>	5.37	0.54
Potter <sup>(31)</sup>	3.82	0.36

beam experiments have been conducted with energies below 100 eV giving results which can be extrapolated to thermal energies.

Ghosh and Sheridan<sup>(29)</sup> obtained cross section data for ion energies below 100 eV which agree well with the later measurements of Stebbings, *et al.*<sup>(30)</sup> The earlier work of Potter<sup>(31)</sup> does not appear to be accurate in comparison, since the cross sections given are a factor of two less than the above authors. Amme and Utterback<sup>(32)</sup> repeated the experiment of Stebbings *et al.* while making a close determination of the dependence of the cross sections upon the degree of ionic excitation. At high electron impact energies there was a significant decrease in the charge transfer cross section, indicating that the resonance behaviour was easily destroyed by small energy defects between the excited ions and the neutral gas. Table 6 summarizes the various results.

For the present calculations the data of Amme and Utterback<sup>(32)</sup> are accepted. The charge exchange cross section is

$$\bar{Q}_E(\text{O}_2^+) = [7.51 - 5.4 \log_{10} \Gamma] \times 10^{-16} \text{ cm}^2. \quad (43)$$

The relative importance of the polarization and charge exchange effects in determining  $\bar{Q}_D$  can be seen in Fig. 3 where the transition temperature is 1600°K. This relatively high temperature appears to be a consequence of the small charge exchange cross section for molecular oxygen. The maximum error associated with the neglect of the polarization effect at the transition temperature is 10 per cent. Thus, we may approximate the momentum transfer cross section by

$$\Gamma > 1600^\circ\text{K} \quad \bar{Q}_D(\text{O}_2^+) = [10.6 - 0.765 \log_{10} \Gamma]^2 \times 10^{-16} \text{ cm}^2$$

$$\Gamma < 1600^\circ\text{K} \quad \bar{Q}_D(\text{O}_2^+) = \frac{2.4 \times 10^{-13}}{\Gamma^{1/2}} \text{ cm}^2. \quad (44)$$

F. *Molecular nitrogen*. Theoretical data for charge exchange in  $N_2$  have been given by Gurnee and Magee<sup>(18)</sup> and are listed in Table 7.

A number of experimental studies have been conducted, but the problem of ionic excitation has been responsible for inconsistent values. Stebbings *et al.*<sup>(30)</sup> have made extensive measurements of the charge transfer cross section and have analyzed many of the previous

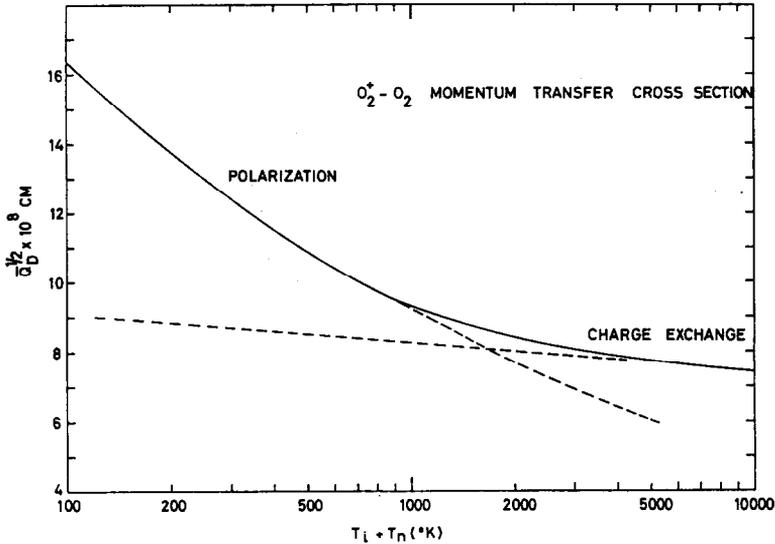
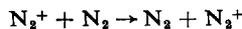


FIG. 3.  $O_2^+ - O_2$  MOMENTUM TRANSFER CROSS SECTION.

The polarization curve is based upon a molecular polarizability of  $1.98 \times 10^{-24}$  cm<sup>3</sup>. The charge exchange curve is based upon the experimental data of Amme and Utterback<sup>(32)</sup> extrapolated to thermal energies.

experimental studies in an effort to determine the sources of error. Amme and Utterback<sup>(32)</sup> have recently been able to show that the work of Stebbings *et al.* leads to cross sections which are too low as a consequence of excited molecular ions which are able to destroy the resonance behaviour of the charge exchange process. They repeated the experiment using low energy electrons to produce the nitrogen ions found a considerable increase in the charge exchange cross section.

TABLE 7. MOLECULAR NITROGEN CHARGE EXCHANGE



$$q_E^{1/2} = A - B \log_{10} \epsilon$$

Source	$A \times 10^8$	$B \times 10^8$
Ammé and Utterback <sup>(32)</sup>	7.36	0.68
Stebbing <i>et al.</i> <sup>(30)</sup>	6.18	0.43
Gurnee and Magee <sup>(18)</sup>	8.28	0.58
Potter <sup>(31)</sup>	6.05	1.26

Using the results of Amme and Utterback<sup>(32)</sup>, the charge exchange cross section at thermal energies is taken as

$$\bar{Q}_E(N_2^+) = [10.1 - 0.68 \log_{10} \Gamma]^2 \times 10^{-16} \text{ cm}^2. \quad (45)$$

The behaviour of the momentum transfer cross section is shown in Fig. 1 where a transition temperature of  $\Gamma = 340^\circ\text{K}$  is indicated. The maximum error involved in neglecting polarization is found to be 9 per cent at this point. Accepting this uncertainty, the momentum transfer cross section is

$$\Gamma > 340^\circ\text{K} \quad \bar{Q}_D(N_2^+) = [14.2 - 0.92 \log_{10} \Gamma]^2 \times 10^{-16} \text{ cm}^2. \quad (46)$$

### 5. Resonance ion-neutral collision frequencies

With the derivation of the appropriate cross sections equations (28) and (29) can now be used to find the charge exchange and momentum transfer collision frequencies. The former

TABLE 8. AVERAGE ION-NEUTRAL CHARGE EXCHANGE COLLISION FREQUENCIES

Species	$\nu_B$ (sec <sup>-1</sup> )
H <sup>+</sup> , H	$9.5 \times 10^{-13} n(\text{H}) \Gamma^{1/2} [14.4 - 1.17 \log_{10} \Gamma]^2$
O <sup>+</sup> , O	$2.3 \times 10^{-13} n(\text{O}) \Gamma^{1/2} [10.5 - 0.61 \log_{10} \Gamma]^2$
N <sup>+</sup> , N	$2.6 \times 10^{-13} n(\text{N}) \Gamma^{1/2} [10.3 - 0.64 \log_{10} \Gamma]^2$
He <sup>+</sup> , He	$4.4 \times 10^{-13} n(\text{He}) \Gamma^{1/2} [11.6 - 1.04 \log_{10} \Gamma]^2$
O <sub>2</sub> <sup>+</sup> , O <sub>2</sub>	$1.7 \times 10^{-13} n(\text{O}_2) \Gamma^{1/2} [10.6 - 0.76 \log_{10} \Gamma]^2$
N <sub>2</sub> <sup>+</sup> , N <sub>2</sub>	$1.8 \times 10^{-13} n(\text{N}_2) \Gamma^{1/2} [14.2 - 0.96 \log_{10} \Gamma]^2$

TABLE 9. REDUCED CHARGE EXCHANGE COLLISION FREQUENCIES

$$\log_{10} \Gamma = \log_{10} (3000^\circ\text{K})$$

Species	$\bar{\nu}_B$ (sec <sup>-1</sup> )
H <sup>+</sup> -H	$1.6 \times 10^{-10} n(\text{H}) \Gamma^{1/2}$
O <sup>+</sup> -O	$2.1 \times 10^{-11} n(\text{O}) \Gamma^{1/2}$
N <sup>+</sup> -N	$2.3 \times 10^{-11} n(\text{N}) \Gamma^{1/2}$
He <sup>+</sup> -He	$4.4 \times 10^{-11} n(\text{He}) \Gamma^{1/2}$
O <sub>2</sub> <sup>+</sup> -O <sub>2</sub>	$1.5 \times 10^{-11} n(\text{O}_2) \Gamma^{1/2}$
N <sub>2</sub> <sup>+</sup> -N <sub>2</sub>	$2.9 \times 10^{-11} n(\text{N}_2) \Gamma^{1/2}$

TABLE 10. AVERAGE ION-NEUTRAL RESONANCE MOMENTUM TRANSFER COLLISION FREQUENCIES

Species	$\bar{\nu}_B$ (sec <sup>-1</sup> )
H <sup>+</sup> , H	$\Gamma > 100^\circ\text{K} \quad 1.9 \times 10^{-13} n(\text{H}) \Gamma^{1/2} (14.4 - 1.15 \log_{10} \Gamma)^2$
O <sup>+</sup> , O	$\Gamma > 470^\circ\text{K} \quad 4.7 \times 10^{-13} n(\text{O}) \Gamma^{1/2} (10.5 - 0.67 \log_{10} \Gamma)^2$
	$\Gamma < 470^\circ\text{K} \quad 8.6 \times 10^{-10} n(\text{O})$
N <sup>+</sup> , N	$\Gamma > 550^\circ\text{K} \quad 5.2 \times 10^{-13} n(\text{N}) \Gamma^{1/2} (10.3 - 0.64 \log_{10} \Gamma)^2$
	$\Gamma < 550^\circ\text{K} \quad 1.0 \times 10^{-9} n(\text{N})$
He <sup>+</sup> , He	$\Gamma > 100^\circ\text{K} \quad 8.7 \times 10^{-13} n(\text{He}) \Gamma^{1/2} (11.6 - 1.04 \log_{10} \Gamma)^2$
O <sub>2</sub> <sup>+</sup> , O <sub>2</sub>	$\Gamma > 1600^\circ\text{K} \quad 3.4 \times 10^{-13} n(\text{O}_2) \Gamma^{1/2} (10.6 - 0.76 \log_{10} \Gamma)^2$
	$\Gamma < 1600^\circ\text{K} \quad 8.2 \times 10^{-10} n(\text{O}_2)$
N <sub>2</sub> <sup>+</sup> , N <sub>2</sub>	$\Gamma > 340^\circ\text{K} \quad 3.6 \times 10^{-13} n(\text{N}_2) \Gamma^{1/2} (14.2 - 0.96 \log_{10} \Gamma)^2$

are of direct importance to the problem of resonance ion-neutral energy transfer while the latter can be applied to the study of ion diffusion. The results for the charge exchange collision frequencies of the atmospheric ions are listed in Table 8. In order to compare the various values it is convenient to adopt the approximation that  $\Gamma = 3000^\circ\text{K}$  in the logarithm. This process leads to errors of less than 10 per cent for  $1200 < \Gamma < 4000^\circ\text{K}$ . Table 9 lists these reduced charge exchange collision frequencies and shows that there is an order of magnitude difference between the extremum values.

The resonance ion-neutral collision frequencies for momentum transfer have been obtained and are listed in Table 10. For values of  $\Gamma$  below the transition temperature it has

been assumed that only the polarization effect is present. Hence, a given gas can be characterized by two different functional forms for  $\bar{v}_R$ . These results indicate significant differences exist between the different ions but that for atoms with similar electronic structures, such as O and N, nearly the same values are found.

### 6. Resonance ion-neutral energy transfer

The charge exchange collision frequencies given in Table 8 have been used to derive the ion energy loss rates listed in Table 11. A simplification has again been made, however,

TABLE 11. RESONANCE ION-NEUTRAL ENERGY TRANSFER RATES  
(eV cm<sup>-3</sup> sec<sup>-1</sup>)

H	$\frac{dU_i}{dt} = -1.4 \times 10^{-14} n(\text{H}^+)n(\text{H})(T_i + T)^{1/2}(T_i - T)$
O	$\frac{dU_i}{dt} = -2.1 \times 10^{-15} n(\text{O}^+)n(\text{O})(T_i + T)^{1/2}(T_i - T)$
N	$\frac{dU_i}{dt} = -2.1 \times 10^{-15} n(\text{N}^+)n(\text{N})(T_i + T)^{1/2}(T_i - T)$
He	$\frac{dU_i}{dt} = -4.0 \times 10^{-15} n(\text{He}^+)n(\text{He})(T_i + T)^{1/2}(T_i - T)$
O <sub>2</sub>	$\frac{dU_i}{dt} = -1.4 \times 10^{-15} n(\text{O}_2^+)n(\text{O}_2)(T_i + T)^{1/2}(T_i - T)$
N <sub>2</sub>	$\frac{dU_i}{dt} = -2.7 \times 10^{-15} n(\text{N}_2^+)n(\text{N}_2)(T_i + T)^{1/2}(T_i - T)$

by noting that the collision cross sections for resonance charge exchange vary only slightly for the normal range of ionospheric ion and neutral temperatures. Thus, the average value  $\Gamma = 3000^\circ\text{K}$  has been chosen for the logarithmic term of the cross section.

These results can be compared with values adopted in several studies of charged particle temperatures. Hanson<sup>(33)</sup> has used an oxygen ion energy loss rate of

$$\frac{dU_i(\text{O}^+ - \text{O})}{dt} = -1.82 \times 10^{-13} n(\text{O}) n(\text{O}^+) (T_i - T_n) \text{ eV cm}^{-3} \text{ sec}^{-1}, \quad (47)$$

based upon the collision frequencies of Dalgarno<sup>(7)</sup>. Analysis of this equation indicates that it was derived for a temperature of  $300^\circ\text{K}$  and, further, that it was based upon the momentum transfer cross section rather than the charge exchange cross section. However, because Dalgarno's momentum transfer cross section is too large by a factor of 2.7-3.0 (Knof *et al.*<sup>(14)</sup>), the final result is larger than the present value by the factors 2.2-1.4 over the temperature range  $1600 < \Gamma < 3600^\circ\text{K}$ .

Another calculation of the oxygen ion cooling rate has been made by Dalgarno.<sup>(34)</sup> He finds

$$\frac{dU_i(\text{O}^+ - \text{O})}{dt} = -4.47 \times 10^{-14} n(\text{O}) n(\text{O}^+) (T_i - T) \text{ eV cm}^{-3} \text{ sec}^{-1}. \quad (48)$$

Comparing this equation with that obtained from Table 11, it is seen that the present energy transfer rate is larger by the factor  $4.7 \times 10^{-2} \Gamma^{1/2}$  which, for temperatures in the range  $1600^\circ < \Gamma < 3600^\circ\text{K}$ , varies between 1.9-2.8.

Willmore<sup>(8)</sup> has calculated the effect of helium resonance charge exchange upon the cooling of helium ions. His result is

$$\frac{dU_i(\text{He}^+ - \text{He})}{dt} = -1.19 \times 10^{-13} n(\text{He}^+) n(\text{He}) (T_i - T) \text{ eV cm}^{-3} \text{ sec}^{-1}. \quad (49)$$

From Table 11, the corresponding equation from the present data is larger than equation (49) by the factor  $3.4 \times 10^{-2} \Gamma$ . For temperatures between  $1600^\circ\text{K} < \Gamma < 3600^\circ\text{K}$ , this factor varies between 1.4 and 2.0. At higher temperatures an increasing divergence will be noted.

Further calculations of resonance ion-neutral energy transfer rates are not known to have been made.

#### IV. SUMMARY

The primary purpose of this paper has been to present a consistent development of the problems of ion energy transfer and collision frequencies. Knowledge of these factors is needed for any detailed study of the thermal behaviour of ions in a dilute, partially ionized plasma. In the first section emphasis was placed upon ion-neutral and ion-ion collisions where resonance charge exchange is not an important factor. Using a general equation of energy transfer, adequate expressions for collision frequencies and energy transfer rates were obtained. For ion-neutral collisions it was necessary to use a momentum transfer cross section appropriate to an ion-neutral polarization interaction. Thus, while the present results are probably accurate for temperatures below  $300^\circ\text{K}$ , the true behaviour at high temperatures must remain in doubt until adequate experimental studies have been made.

The ion-ion collision frequency and energy transfer rate have been computed directly from the Coulomb potential. Under conditions of equivalent particle densities the ion-ion energy transfer rate is found to be an order of magnitude larger than the listed ion-neutral rates.

The problem of resonance collisions between an ion and its parent neutral particle has been discussed. Appropriate equations have been developed for the thermal nonequilibrium collision frequencies for both charge exchange and momentum transfer. It was shown that ion energy transfer rates for charge exchange must dominate at temperatures above the polarization-charge exchange transition region. For all of the ions, with the exception of  $\text{O}_2$ , there is little error involved in ignoring the polarization contribution to  $\bar{Q}_D$  in the atmosphere.

In general, both the nonresonant and resonant ion-neutral energy transfer rates show only a small dependence upon the gas temperatures. The resonance behaviour leads to rates which are somewhat larger than those found for elastic collisions.

Since the ion-neutral energy transfer rates are generally much larger than the electron-neutral rates and, in a similar fashion, electron-ion energy transfer is much slower than ion-ion energy exchange, for an external source giving heat to electrons in a weakly ionized plasma there will be a complex set of relations needed to arrive at equilibrium values of the electron, ion and neutral gas temperatures.

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**Резюме**—Было сделано исследование частоты соударений и переноса энергии для условий теплового неравновесия газов. Следуя методам по разработке соударений электронов, проблемы упругих соударений между нейтральными ионами-ионами и ионами-ионами были рассмотрены на предположении существования отдельных максвеллиевых распределений скорости. Выражения для неравновесия поперечных сечений переноса толчка, частоты соударений и скорости переноса энергии даются для нейтральных газов на основе потенциала поляризации, а для ионов—на основе кулоновского взаимодействия. Затем, вводится проблема обмена резонансным зарядом между ионами и нейтральными ионами, т.е. влияние такого обмена на сочтения соударений и скорость переноса энергии ионов. Производится анализ комплектованных лабораторных данных, чтобы выработать подходящие выражения для обмена резонансным зарядом и поперечных сечений переноса импульса, для  $H^+$ ,  $O^+$ ,  $He^+$ ,  $N_2^+$ , и  $O_2^+$ . При помощи этих результатов, вычисляются обмен резонансным зарядом и частота соударений в переносе толчка. Скорости переноса энергии резонанса ионы-нейтральные ионы тогда представляются и сопоставляются с выражениями, применявшимися в предыдущих исследованиях ионов.