MEAN MOLECULAR MASS AND SCALE HEIGHTS OF THE UPPER ATMOSPHERE (*)

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Résumé. — *La structure physique de l'atmosphère supérieure est décrite à l'aide de la masse moléculaire moyenne et des hauteurs d'échelle associées à la pression et à la densité. Toutefois, dans la thermosphère inférieure il faut préciser les conditions physiques permettant le passage d'une distribution de mélange parfait à un état d'équilibre de diffusion. Si l'hélium est en équilibre de diffusion à 120 km, le coefficient de diffusion eddy doit être inférieur à 2×10^6 cm² sec⁻¹, c'est-à-dire une valeur nettement inférieure à celle du coefficient de diffusion moléculaire.*

En utilisant des modèles atmosphériques adaptés à un cycle décroissant d'activité solaire, il est possible de mettre en évidence l'importance des variations de la masse moléculaire moyenne. On montre ainsi, comment les ceintures d'oxygène atomique, d'hélium et d'hydrogène varient au cours d'un cycle d'activité solaire. Dans la thermosphère la distribution verticale de la hauteur d'échelle atmosphérique H dépend surtout de la distribution de température. Au-dessus de la thermopause la hauteur d'échelle suit les variations de la masse moléculaire moyenne et on constate une augmentation de H lorsque la température diminue. Les variations des hauteurs d'échelles associées à la pression et à la densité se traduisent dans la distribution verticale de leurs gradients. Ainsi, le gradient β passe par un maximum aux altitudes où l'hélium et l'hydrogène deviennent respectivement les éléments principaux. D'autre part, le gradient βρ de la hauteur d'échelle associée à la densité, n'est jamais inférieur au gradient β dans la thermosphère. Au-dessus de la thermopause, βρ atteint même des valeurs nettement supérieures à β.

Abstract. — *The physical structure of the upper atmosphere is given in terms of the mean molecular mass and of the pressure and density scale heights. The physical conditions leading to a departure from a perfect mixing are discussed in the lower thermosphere. If helium is in diffusive equilibrium at 120 km, the eddy diffusion coefficient must be less than 2×10^6 cm² sec⁻¹, i.e. a value much lower than that one for the molecular diffusion coefficient.*

By using atmospheric models adapted to a decreasing solar cycle, it is possible to point out the large variations of the mean molecular mass. It is shown, how the atomic oxygen, the helium and the hydrogen belts are changing during a solar cycle. In the thermosphere, the vertical distribution of the atmospheric scale height H is mainly related to the temperature distribution. Above the thermopause, the atmospheric scale height follows the mean molecular mass variations and an increase of H is seen when the temperature decreases. The variations of the pressure and density scale heights are related to the vertical distribution of their gradients. The pressure scale height gradient β goes through a maximum at heights where helium and hydrogen respectively become the major constituent. The density scale height gradient βρ is never less than β in the thermosphere. Above the thermopause, βρ reaches values higher than β.

Резюме. — При помощи средней молекулярной маосы и высот шкалы связанных с давлением и плотно-стею описывается физичесская структура высокой атмосферы. Тем не менее, в нижней термосфере наьо уточнить физические условия позволяющие переход от распределения совершенной смеси к содтохнию диффузионного равновесия. Если гелий находится в диффузионном равновесии на 120 км, тос коэффициент турбулентной диффузии должен быть ниже 2×10^6 см² сек⁻¹, то-есть значению явно меньшему чем коэффициент молекулярной диффузии.

Употребляя атмосферные модели, применяемые к циклу убывающей солнечной активности, воз-можно выявить значение изменений средней молекулярной массы. Таким образом показывается как пояса атомного кислорода, гелия и водорода изменяются в течении цикла солнечной активности. Вертикальное распределение атмосферной высоты шкалы H зависит главным образом от распределения температуры в термосфере. Над термопаузой, высота шкалы следует изменениям средней молекуляр-ной массы, и здесь устанавливается увеличение H когда температура уменьшается. Изменения высрт шкалы, связанные с давлением и плотностью, выражаются в вертикальном распределении их гра-диент β проходит через максимум на высотах где гелий и водород соответственно становятся главн-ыми элементами. С другой стороны, градиент βρ, высоты шкалы связанной с плотностью, никаяда не является меньше градиента β в термосфере. Над термопаузой, βρ достигает даже велични овно высших чем β.

1. INTRODUCTION.

New possibilities for investigating the physical structure of the upper atmosphere have been provided by rocket soundings since 1945 and by artificial satellites since 1957. At present satellite drag analysis gives a description of the total density above 200 km during the decreasing phase of a solar cycle [1]. In the altitude range between 1000 km and 1500 km, only the satellite Echo 1 has provided drag information on the atmospheric density. Recently, FEA [2] has presented a first derivation of total densities at 3500 km from the observations of the inflated balloon satellite 1963 30 D. Elsewhere, more direct methods such as pressure measurements using gages, have been employed, notably by MIKHNEVICH et al. [3] and by SHARP et al. [4]. Recently, the aeronomical satellite Explorer 17 [5, 6, 7] was equiped with pressure gages and a mass spectrometer used for determining the composition and total pressure between 250 km and 700 km height. Rocket mass spectrometer measurements have provided very useful information [for example 8, 9] on the region between 100 km and 200 km which is essentially inaccessible to satellite investigation. On the other hand, absorption measurements [for example 10] and gas release techniques [for example 11] can also give useful information on aeronomical parameters. With these different techniques it is possible to improve and to complete theoretical results concerning the upper atmosphere structure. However, it should be mentioned that atmospheric densities deduced from satellite drag data are nearly a factor of two higher than those obtained from the mass spectrometer data now available.

Our purpose is to study the physical properties of the heterosphere by means of scale heights and their gradients. In 1936, CHAPMAN [12] introduced the atmospheric scale height H related to the total pressure p by the equation :

$$(1) \qquad \frac{1}{p}\frac{dp}{dz} = -\frac{mg}{kT} = -\frac{1}{H},$$

in which k denotes Boltzmann's constant, m the mean molecular mass, g the acceleration of gravity and T the absolute temperature at height z. With the first satellite drag data analyses [13, 14, 15, 16], another scale height H_ρ was introduced. It is related to the total mass density ρ by

$$(2) \qquad \frac{1}{\rho}\frac{d\rho}{dz} = -\frac{1}{H_\rho}.$$

Whereas the atmospheric density decreases by a factor of the order of 10^6 between ground level and 100 km, the analysis of the orbital variations of satellites shows that, above 150 km, ρ decreases much more slowly with altitude as a consequence of the increase of the atmospheric scale height. This increase of H is due to a rise in temperature and to a decrease in the mean molecular mass. Heat and mass transport, therefore, are the major factors controlling the structure of the upper atmosphere. Diffusion is particularly important because it allows the transition from a molecular mass of $28(N_2)$ to a mass of $1(H)$. It is necessary to determine the altitude above which the mixing ratio of the atmospheric constituent changes due to diffusion transport. The study of artificial sodium clouds ejected by rockets at twilight [17] and mass spectrometric measurements of the ratio $n(A)/n(N_2)$ [18, 19] have shown that the transition region from mixing to diffusion is located between 100 km and 120 km. These experimental results also seem to indicate a change which is probably associated with a time variation in the physical conditions of temperature and pressure in this transition region. It is thus impossible to fit in detail all the observations made at higher altitudes with atmospheric models based on one set of boundary conditions.

By means of the behavior of the mean molecular mass, of the scale heights H and H_ρ as well of their gradients, we shall try to present a physical description of the heterosphere during the decreasing phase of the solar cycle.

2. DIFFUSION TRANSPORT IN THE LOWER THERMOSPHERE.

Although diffusion transport operates in the whole atmosphere, it is essential to determine its importance compared to other processes such as eddy diffusion which maintain the mixing. For example the choice of an arbitrary diffusion equilibrium level is required for the calculations of the helium abundance at high altitudes. This constituent is in fact more sensitive than the others to the choice of the equilibrium level. For instance, a decrease of 5 km in the diffusion equilibrium level corresponds to an increase of about a factor of 2 in the helium concentration in the heterosphere [20]. On the other hand, the atomic hydrogen distribution is less sensitive to the choice of the level above which the diffusion equation is applied. This difference in behaviour is essentially due to the importance of the diffusion flux. Mange [21] has shown

how this flow can affect the vertical distribution of minor constituents. By using the general diffusion equation given by CHAPMAN and COWLING [22], it is possible to show that, in the case of no net flow, the diffusion velocity w_1 for a constituent of mass m_1 is given by :

$$(3) \quad w_1 = - D \left[\frac{1}{p_1} \frac{dp_1}{dz} - \frac{1}{p} \frac{dp}{dz} \frac{m_1}{m} + \alpha_T \frac{n_2}{n} \frac{1}{T} \frac{dT}{dz} \right]$$

In this equation, D is the diffusion coefficient for the constituent of partial pressure p_1 and mass m_1 in the atmosphere characterized by the mean molecular mass m and the total pressure p. The concentration n_2 is related to the total concentration n by the relation $n_2 = n - n_1$ and α_T is the thermal diffusion factor which can be taken equal to $- 0.38$ for helium and atomic hydrogen.

For a minor atmospheric component of concentration n_1, the relation (3) can be written :

$$(4) \quad w_1 = - D \left\{ \frac{1}{n_1} \frac{dn_1}{dz} + \frac{1}{H_1} [1 + \beta_1^* (1 + \alpha_T)] \right\},$$

where β_1^* is related to the scale height gradient $\beta_1 = dH_1/dz$ by :

$$(5) \quad \beta_1^* = \beta_1 - \frac{2H_1}{r_0 + z},$$

r_0 being a reference level measured from the Earth's center and z being the altitude above that level. If there is neither loss nor production in a volume element, the vertical diffusion flow $n_1 w_1$ follows the continuity equation :

$$(6) \quad \frac{\partial n_1}{\partial t} = - \frac{\partial}{\partial z} (n_1 w_1),$$

where t represents the time.

With a mixing distribution as an initial condition for the minor constituent, is is possible, following Mange's method [23], to study the time evolution towards a diffusive equilibrium characterized by a vanishing velocity w_1. If we adopt at 100 km a pressure of 3×10^{-4} mm Hg and a temperature of 200 °K, the solution of equation (6) gives the results presented in figure 1 for a gradient $\beta = 0.2$. At a height of 120 km the mixing conditions correspond to the physical parameters used by NICOLET [24] for constructing a group of his atmospheric models. If we assume that above 120 km, the principal constituents are in diffusive equilibrium, figure 1 shows that after 10 days the helium concentration at 120 km is identical to the value obtained with a practical diffusive equilibrium beginning at 115 km. But argon reaches equilibrium more rapidly, because the practical diffusive equilibrium
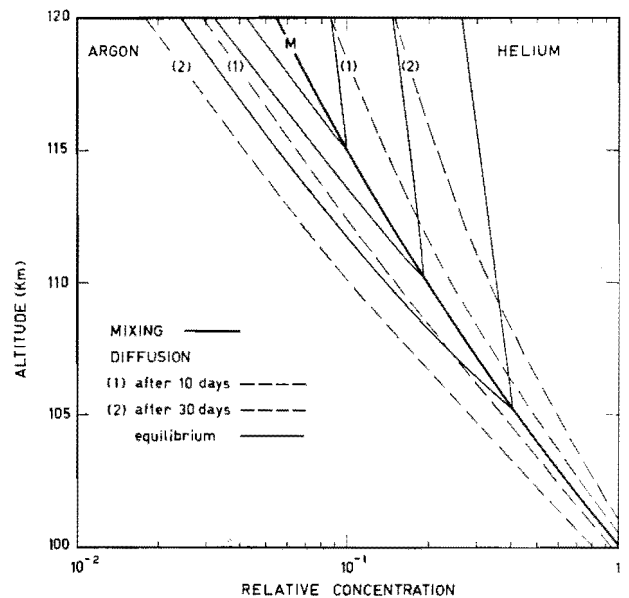

FIG. 1. — Time evolution towards diffusive equilibrium for argon and helium.

level goes down to 107 km after 10 days. This is because the difference between mixing and diffusive equilibrium distributions is smaller for argon than for helium. Since a transition zone exits between perfect mixing and complete diffusion equilibrium, figure 1 also shows that the helium concentration adopted at a practical diffusion equilibrium level will always be less than the real concentration existing at this height. The opposite situation occurs for constituents like argon for which the molecular mass is higher than the mean molecular mass associated with perfect mixing. For computations, this situation leads us to adopt an artificial level for the diffusion equilibrium always lower than the exact height above which diffusion equilibrium prevails. If an eddy diffusion flow is added to the molecular diffusion flow, one writes :

$$(7) \quad F = n_1 w_1 + n_1 w_e.$$

Following LETTAU [25], we can write the eddy diffusion velocity w_e as follows :

$$(8) \quad w_e = - \mathfrak{D} \frac{d(n_1/n)}{dz} \Big/ (n_1/n),$$

If a pressure scale height $\mathfrak{K}_1$ is introduced such that

$$(9) \quad \frac{1}{p_1} \frac{dp_1}{dz} = - \frac{1}{\mathfrak{K}_1},$$

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the vertical transport flow (7) can be written with the help of (1), (8) and (9) in the form :

$$(10) \quad F = \frac{n_1 D}{H} \left\{ \left[\frac{H}{\mathcal{H}_1} - \frac{m_1^*}{m} \right] + \Lambda \left[\frac{H}{\mathcal{H}_1} - 1 \right] \right\},$$

with

$$(11) \quad \Lambda \equiv \mathcal{D}/D$$

and

$$(12) \quad m_1^* = m_1 \left(1 + \alpha_T \beta_1^* \frac{n_2}{n} \right).$$

When $\mathcal{H}_1 = H = kT/mg$, the constituent of mass m_1 follows a perfect mixing distribution and the transport flux (10) is independent of the eddy diffusion coefficient. A value $\mathcal{H}_1 = H_1 = kT/m_1 g$ corresponds to the diffusion equilibrium, if the thermal diffusion is neglected.

In that case, the transport flux (10) can only be due to eddy diffusion.

According to the general equation (10), the condition for no downward transport is :

$$(13) \quad \Lambda \left(\frac{H}{\mathcal{H}_1} - 1 \right) \geqslant \frac{m_1^*}{m} - \frac{H}{\mathcal{H}_1}.$$

Photochemical processes can be neglected for the helium and argon distributions in the lower thermosphere. Therefore, the partial pressure scale height $\mathcal{H}_1$ must lie between the values $\mathcal{H}_1 = H$ and $\mathcal{H}_1 = H_1$. According to expression (13), figure 2 gives the values of Λ, as a function of $\mathcal{H}_1/H$, necessary for obtaining steady states characterized by no transport flow. If the following expression is adopted for the molecular diffusion coefficient :

$$(14) \quad D = \frac{3\pi}{32 Q n} \left(1 + \frac{m_1}{m} \right)^{1/2} \left(\frac{8 kT}{\pi m_1} \right)^{1/2},$$
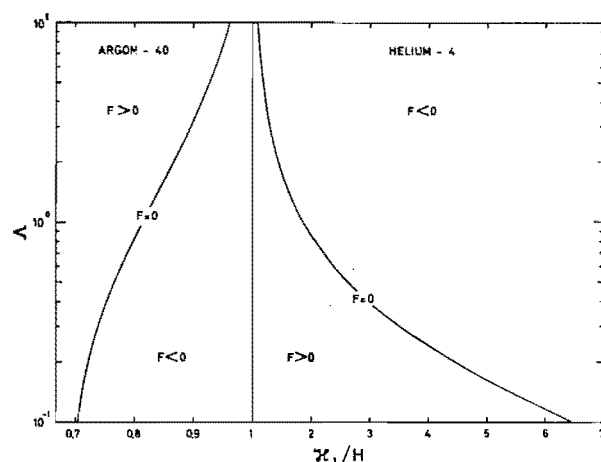
it is possible to compute values for the eddy diffu-

Fig. 2. — Values of the ratio $\Lambda = \mathcal{D}/D$ when the transport flow $F = 0$ as a function of the partial pressure scale height $\mathcal{H}_1$, expressed in atmospheric scale heights units.

TABLE I

MOLECULAR AND EDDY DIFFUSION COEFFICIENTS (cm^2 sec^{-1})
CORRESPONDING TO DIFFERENT STEADY STATES (F = 0)

z (km)	$\mathcal{H}_1/H$ D (He) [1]	1.5	2	3 EDDY DIFFUSION	4 COEFFICIENT	5	6
100	8.0×10^5	1.4×10^6	6.8×10^5	3.1×10^5	1.9×10^5	1.3×10^5	9.2×10^4
105	2.1×10^6	3.7	1.8×10^6	8.1	4.9	3.3	2.4×10^5
110	4.7	8.4	4.0	1.8×10^6	1.1×10^6	7.6	5.4
115	9.7	1.7×10^7	8.3	3.8	2.3	1.6×10^6	1.1×10^6
120	1.9×10^7	3.4	1.6×10^7	7.4	4.5	3.0	2.2
z (km)	$\mathcal{H}_1/H$ D (A) [1]	0.95	0.90	0.80 EDDY DIFFUSION	0.75 COEFFICIENT	0.70	0.69
100	3.7×10^5	2.8×10^6	1.2×10^6	3.1×10^5	1.4×10^5	2.7×10^4	8.7×10^3
105	9.5	7.4	3.0	8.0	3.6	6.9	2.2×10^4
110	2.2×10^6	1.7×10^7	6.8	1.8×10^6	8.2	1.6×10^5	5.1
115	4.5	3.5	1.4×10^7	3.8	1.7×10^6	3.3	1.1×10^5
120	8.7	6.7	2.7	7.3	3.3	6.3	2.0

[1] More precise values are not considered, since exact pressures are not known between 100 km and 120 km.

sion coefficient corresponding to a steady state. Adopting an average value for the diffusion cross section $Q = 2.8 \times 10^{-15}$ cm², the eddy diffusion coefficients given in Table I are obtained between 100 km and 120 km for the same physical conditions as in figure 1.

It can be seen from Table I that a slight departure from mixing can exist even if the eddy diffusion coefficient is greater than the molecular diffusion coefficient. But it must be pointed out that a real departure from mixing occurs only when $\mathfrak{D}$ is less than D. A value for D(A) of 8.7×10^6 cm² sec⁻¹ at 120 km and 3.7×10^5 cm² sec⁻¹ at 100 km would correspond to a ratio $\mathcal{K}_1/H = 3$ for He and $\mathcal{K}_1/H = 0.8$ for A. Those ratios $\mathcal{K}_1/H$ show again that argon reaches diffusive equilibrium more rapidly than helium. If a diffusive equilibrium is adopted for helium at 120 km, the eddy diffusion coefficient is less than 2×10^6 cm² sec⁻¹, i.e. a factor of 10 less than the molecular diffusion coefficient. This value is of the same of magnitude than the value deduced by COLEGROVE et al. [26] at 120 km.

In any case, it is clear from a complete analysis of the numerical values of Table I that an unlimited number of solutions can be adopted to define the boundary conditions at 120 km. They depend on the pressure and its variations and on the dynamical processes in the lower thermosphere.

For practical computations in the homosphere, we can write the diffusive flow F_D in the form :

$$(15) \quad F_D = 5.6 \times 10^1 \left(\frac{r_0}{r}\right)\left(1 + \frac{m}{m_1}\right)^{1/2}$$
$$\frac{Y}{H^{1/2}}\left(\frac{n_1}{n}\right) \ (\text{cm}^{-2}\ \text{sec}^{-1}),$$

where r_0 is the Earth's radius and r is the geocentric altitude. The atmospheric scale height H must be expressed in cm in formula (15). The parameter Y, which determines the flux direction, is given by :

$$(16) \quad Y = 1 - \frac{m_1}{m} - \beta^* \alpha_T.$$

In the expression (15), we used a molecular diffusion coefficient :

$$(17) \quad D = \frac{3\pi}{32 Q n}\left(1 + \frac{m_1}{m}\right)^{1/2}\left(\frac{8\,kT}{\pi\,m_1}\right)^{1/2},$$

with a mean diffusion cross section
$$Q = 2.8 \times 10^{-15} \ \text{cm}^2.$$

It is interesting to point out that formula (15) represents the maximum flux which can be suppor-

ted by molecular diffusion through the homosphere.

Recently, several authors [27, 28] adopted a helium concentration at 120 km higher than the value proposed by KOCKARTS and NICOLET [20, 29]. As the mixing conditions are variable in the lower thermosphere, this arbitrary increase could be justified during certain periods. In fact, NICOLET [30] has shown that in order to increase the total helium content by a factor of 2 by means of the flux (15), the following times are necessary as a function of height :

85 km	90 km	95 km	100 km
—	—	—	—
3 months	1 month	2 weeks	1 week

From Table I it can also be seen that a decrease of the eddy diffusion coefficient from 6.8×10^5 cm² sec⁻¹ to 1.9×10^5 cm² sec⁻¹ at 100 km can lead to an increase of a factor 2 for the total helium content. Therefore rocket observations must be analyzed by keeping in mind the variable transition region which exists between mixing and diffusion equilibrium. While at present, all the theoretical informations mainly concern minor components such as helium, argon and hydrogen, it is clear that for the principal constituents, the transport processes are also very important in the lower thermosphere. For all these reasons, it is necessary to fix the boundary conditions at 120 km in order to deduce the most important features of the physical structure of the upper atmosphere, but keeping in mind that the physical structure (p, T, ρ) changes in the region of 100 km.

3. MEAN MOLECULAR MASS FOR A DECREASING SOLAR CYCLE.

NICOLET [24] presented the first atmospheric model taking into account of diffusion processes and of heat conduction transport [31]. HARRIS and PRIESTER [32] adopted the same boundary conditions at 120 km in constructing a set of models as a function of local time for different solar activity levels. In order to fit the mass spectrometric measurements, JACCHIA [27] as well as HARRIS and PRIESTER [28] recently modified the boundary conditions at 120 km. Table II gives the adopted set of conditions.

Nicolet's boundary conditions are adopted here and it is assumed that the principal components are in diffusive equilibrium above the height of 120 km.

TABLE II

BOUNDARY CONDITIONS AT 120 km

T (°K)	$n(N_2)$ (cm^{-3})	$n(O_2)$ (cm^{-3})	$n(O)$ (cm^{-3})	$n(He)$ (cm^{-3})	REFERENCE
324	5.8×10^{11}	1.2×10^{11}	7.6×10^{10}	1.9×10^7	NICOLET [24]
355	4.0×10^{11}	7.5×10^{10}	7.6×10^{10}	3.4×10^7	JACCHIA [27]
355	4.0×10^{11}	7.5×10^{10}	7.6×10^{10}	2.4×10^7	HARRIS and PRIESTER [28]

HEDIN and NIER [33] showed that this hypothesis is in agreement with the mass spectrometric measurements [9], at least during periods of low solar activity. The vertical distribution of an atmospheric constituent with concentration n_i is given by :

$$(18) \qquad \frac{dn_i}{dz} + \frac{n_i}{H_i}[1 + \beta_i^*(1 + \alpha_T)] = 0.$$

The thermal diffusion factor α_T is negligible for O, O_2 and N_2. However, if we omit the term in α_T for helium, we obtain at high altitudes concentration too low by a factor of 1.4 to 2.4 according to the thermopause temperature [34]. Atomic hydrogen distribution will be computed by considering a diffusion flow [20, 29] of the order of 2.5×10^7 cm^{-2} sec^{-1} which corresponds to a concentration of the order of 10^7 cm^{-3} at an altitude of 100 km.

The mean molecular mass m can be written :

$$(19) \qquad m = \sum n_i m_i / \sum n_i,$$

where m_i and n_i respectively are the mass and the concentration. The sum must be taken over all the atmospheric components. In order to study the mean molecular mass evolution during a decreasing solar cycle, the annual mean isothermal temperatures of Table III, will be adopted, obtained by using NICOLET's relationships [35] between the temperature and the solar flux at 8 cm.

TABLE III

ANNUAL MEAN VALUES FOR DAY-AND NIGHTTIME TEMPERATURES BETWEEN 1958 AND 1964

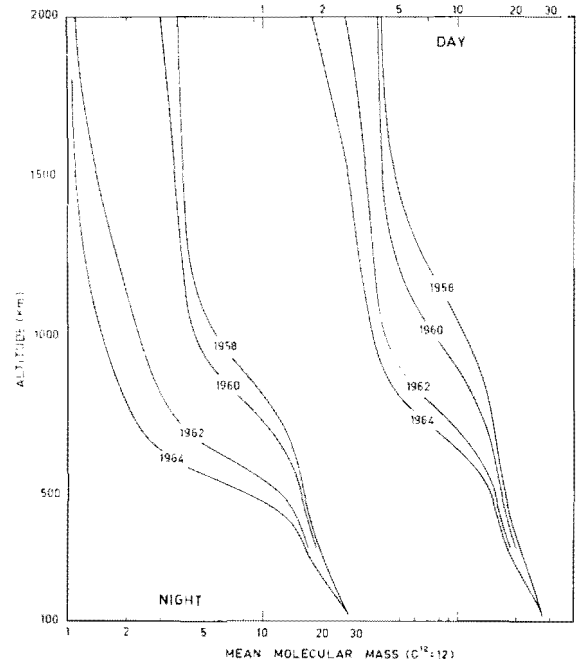| YEAR	1958	1960	1962	1964
T_{day} (°K)	1600	1350	1000	900
T_{night} (°K)	1200	1050	750	650

FIG. 3. — Mean molecular mass for day- and nightime conditions during decreasing solar cycle 1958-1964.

These temperatures should be correct to ± 50 °K. Figure 3 shows the mean molecular mass distribution for daytime and nighttime conditions during the period 1958-1964. It can be seen that the mean molecular mass changes more rapidly with height at low temperature. This means that the transition between the atomic oxygen, the helium and the hydrogen belts is more abrupt during a minimum of solar activity. In 1958, the nighttime mean molecular mass reached a value of 4 in the neighbourhood of 1250 km. As the temperature decreases, the mean molecular mass becomes smaller and approaches 1 rapidly. Therefore, the thickness of the helium belt introduced by NICOLET [36] is smallest during minimum solar activity and atomic hydrogen becomes the major constituent. At all levels of solar activity, the largest variations in m occur in the isothermal region.

TABLE IV

INFLUENCE OF A PRESSURE CHANGE AT 120 km ON THE MEAN MOLECULAR MASS

z (km)	T (°K)	$n(O)$ (cm^{-3})	$n(O_2)$ (cm^{-3})	$n(N_2)$ (cm^{-3})	ρ (gm cm^{-3})	$m(C^{12} = 12$
120	291 325 355	7.60×10^{10}	1.18×10^{11}	5.83×10^{11}	3.54×10^{-11}	27.4
150	629 641 653	1.11×10^{10} 1.28 1.45	5.45×10^{9} 6.67 7.90	3.59×10^{10} 4.33 5.07	2.25×10^{-12} 2.71 3.16	25.9 26.0 26.1
200	766 768 772	2.61×10^{9} 3.10 3.59	3.66×10^{8} 4.66 5.73	3.30×10^{9} 4.13 5.00	2.42×10^{-13} 2.99 3.58	23.2 23.4 23.5
300	798 798 798	2.73×10^{8} 3.26 3.81	4.18×10^{6} 5.34 6.66	6.57×10^{7} 8.23 1.01×10^{8}	1.05×10^{-14} 1.28 1.52	18.3 18.5 18.6

This means that diffusion is the dominant process involved in modifying the atmosphere above the thermopause level. The curves of figure 3 were computed without changing the boundary conditions at 120 km. According to NICOLET [37] a total density range of $(3.5 \pm 1.0) \times 10^{-13}$ gm cm^{-3} can be obtained at 200 km by assuming a pressure variation of only $\pm$ 10 per cent at 120 km and by changing the gradient β by $\pm$ 0.1. Table IV shows the effect of such a pressure variation alone on the mean molecular mass without changing the gradient β at 120 km. Table IV has been constructed for an isothermal temperature of 800 °K, i.e. a value similar to that deduced from the mass spectrometer measurements made by NIER et al. [9] on 6 June 1963. Although the concentration are affected by a change of $\pm$ 10 per cent in the temperature at 120 km, the mean molecular mass is practically unchanged. The same result is obtained by changing the gradient β at 120 km between 0.3 and 0.6.

For better understanding of the variations shown in figure 3, the mean molecular mass given by equation (19) may be considered as the sum of the masses of the constituents each weighted by its relative abundance.

Figures 4 and 5 represent the relative abundances for the atmospheric constituents during 1958 and 1964. For the daytime conditions in 1958, just after the maximum of solar activity, atomic oxygen was still the major constituent between 400 km
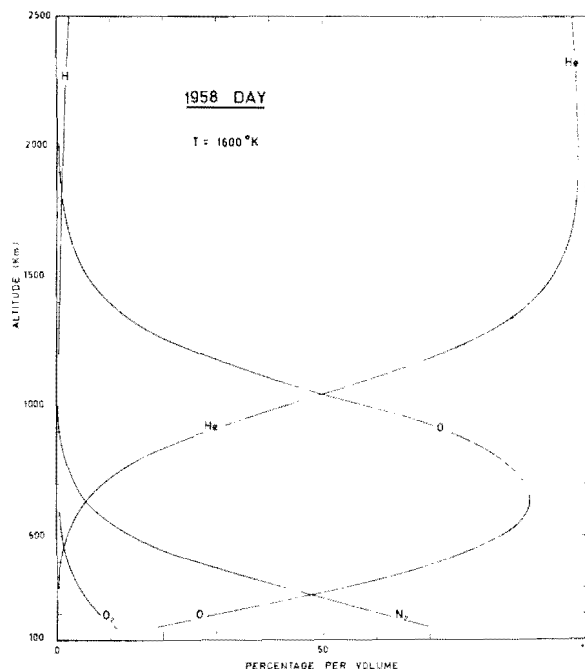

FIG. 4. — Vertical distribution of the relative abundances of the atmospheric constituents for averages daytime conditions in 1958.

and 800 km. The nighttime conditions for 1964 show a thinner atomic oxygen belt with a maximum around 350 km. On figure 4 it can be seen that helium becomes the major component above 1000 km whereas the atomic hydrogen relative abundance only reaches a few per cent at 2500 km.
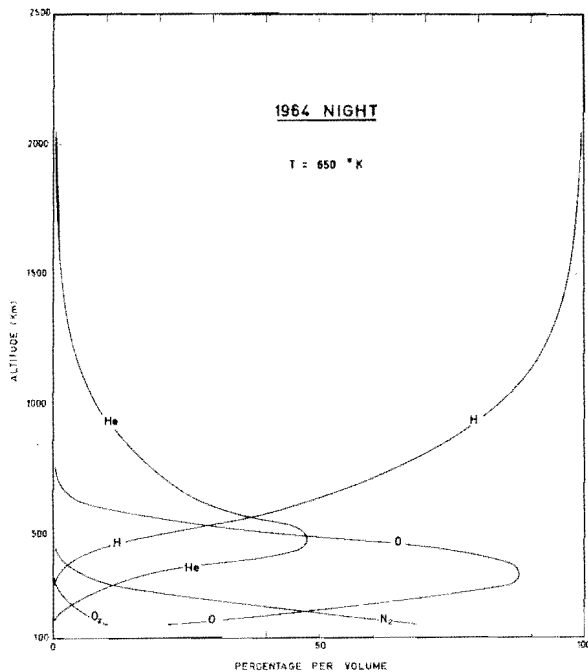
FIG. 5. — Vertical distribution of the relative abundances of the atmospheric constituents for average nighttime conditions in 1964.

For very low solar activity, the helium belt becomes rather thin and atomic hydrogen is already the major constituent at an altitude of the order of 1000 km. So, we may conclude that the mean molecular mass variations are largest during minimum solar activity.
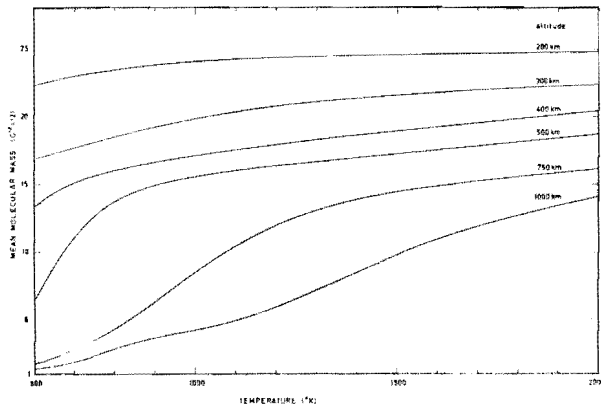

FIG. 6. — Distribution of the mean molecular mass as a function of the thermopause temperature at various levels between 200 km and 1000 km.

In figure 6, the mean molecular mass has been plotted as a function of the thermopause temperature for a few altitudes between 200 km and 1000 km. It is again apparent that the largest variations occur in the isothermal region. For

instance, at 200 km the mean molecular mass is (23.0 ± 1.5) for the whole solar cycle ; but, at 1000 km, values between 1 and 14 are possible depending on the thermopause temperature.

4. ATMOSPHERIC SCALE HEIGHT AND ITS GRADIENT β.

An experimental determination of the atmospheric scale height H is rather complex because it requires an exact knowledge of the slope of the vertical pressure distribution, or a simultaneous measurement of the temperature and of the mean molecular mass. Figures 7 and 8 show the vertical distribution of H for daytime and nighttime conditions during the period 1958-1964. In the thermosphere, H is influenced by both temperature and mean molecular mass variations. The temperature effect, however, dominates at heights lower than 400 km, and the scale height H decreases with decreasing temperature in this region. Above the thermopause level, the situation becomes more complex because of the great variations in the mean molecular mass. Even when the scale height H is less than 100 km at the thermopause, it can reach values as high as 1000 km at altitudes above 400 km and above 500 km ; H increases when the solar activity decreases (see Table V). The increase of H above the thermopause level is due to the diffusion effect, corresponding to a rapid decrease of the mean molecular mass with altitude (see fig. 3).

Above 500 km, the atmospheric scale height becomes comparable with the particles mean free path and thus the hydrostatic equation is no longer valid.

TABLE V

ATMOSPHERIC SCALE HEIGHT (km) AS A FUNCTION OF THE THERMOPAUSE TEMPERATURE

	TEMPERATURE (°K)					
z (km)	2000	1750	1500	1250	1000	650
400	92	84	76	66	56	43
500	105	96	86	75	64	76
750	132	121	111	107	128	323
1000	164	162	179	223	270	496

For practical computations, however, it can be shown that with a critical level depending on temperature, the mean molecular mass computed with
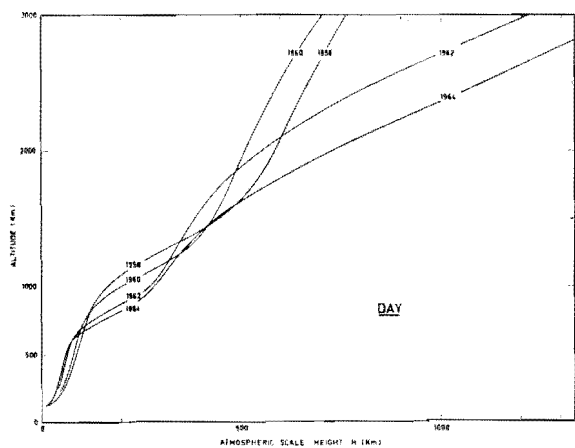
FIG. 7. — Vertical distribution of the atmospheric scale height H for daytime conditions from 1958 to 1964.
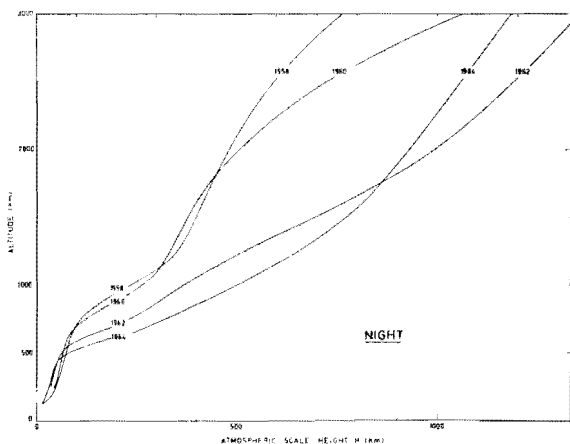

FIG. 8. — Vertical distribution of the atmospheric scale height H for nighttime conditions from 1958 to 1964.

an exospheric distribution is nearly identical to that resulting from applying the hydrostatic equation at heights below 3000 km. The scale heights presented in figures 7 and 8 thus still have physical meaning even at altitudes above 500 km.

According to figures 7 and 8, it is clear that the atmospheric scale height is highly variable, and, it is useful to study the behaviour of the gradient $\beta = d\mathrm{H}/dz$. As we have no analytical expression for the temperature distribution with height, one sees that in the thermosphere the gradient β must be computed by means of finite differences between atmospheric scale heights. However, for an isothermal atmosphere in diffusive equilibrium, it is possible to establish an expression which allows us to compute β at a given height. In this case, the gradient can be written :

$$(20) \qquad \beta = \mathrm{H}\left[-\frac{1}{m}\frac{dm}{dz} + \frac{2}{r_0 + z}\right],$$

where r_0 is the Earth's radius and z is the altitude. For an isothermal atmosphere in diffusive equilibrium we have further :

$$(21) \qquad \frac{dn}{dz} = -\sum \frac{n_i}{\mathrm{H}_i}$$

and

$$(22) \qquad \frac{d\rho}{dz} = -\sum \frac{\rho_i}{\mathrm{H}_i}.$$

The sum is taken over all the atmospheric constituents distributed with their own scale height $\mathrm{H}_i = k\mathrm{T}/m_i\, g$. With the help of relations (19), (21) and (22), the gradient β is given by :

$$(23) \quad \beta = \mathrm{H}\left[\frac{1}{\rho}\sum\frac{\rho_i}{\mathrm{H}_i} - \frac{1}{n}\sum\frac{n_i}{\mathrm{H}_i} + \frac{2}{r_0 + z}\right].$$

Above the thermopause, this formula has been used for computing the gradients β presented in figures 10 to 13 in the next section. Within the isothermal region β can reach and even exceed 0.6 for all solar cycle conditions. These high values only result from the diffusion whereas at 150 km a gradient $\beta = 0.6$ would correspond to a temperature increase of the order of 20°K/km. Elsewhere, the greatest values of β occur for low solar activity, i.e. when the mean molecular mass
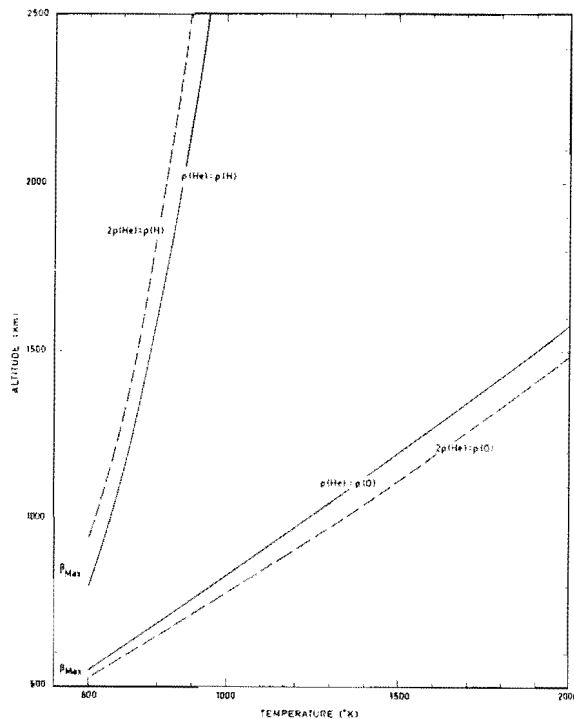

FIG. 9. — Altitudes of β peaks as a function of the thermopause temperature. The dotted curves correspond to an increase by a facto r of 2 of helium concentra tion.

variation with height is largest. Finally, for a given temperature, the vertical distribution of β goes at least through one maximum between 500 km and 3000 km. If one considers two constituents and if one neglects the terms proportionnal to $2/(r_0 + z)$, one can show [38] the maxima happen to be at height where $\rho(\text{He}) = \rho(\text{O})$ and $\rho(\text{H}) = \rho(\text{He})$. In figure 9, the altitudes of the maxima are given as a function of the thermopause temperature. Physically, the maxima of the atmospheric scale height gradient characterize the heights at which helium and hydrogen successively become the major atmospheric component. Figure 9 also shows the effect of an increase by a factor of two in the helium concentration. In this case, the thickness of the atomic oxygen belt is reduced from 25 km to 100 km depending on the thermopause temperature.

Atomic hydrogen only becomes the major constituent at much higher altitudes. It is apparent that the helium belt persists for low solar activity even when the helium is not increased. Only the extent of the helium layer diminishes with decreasing solar activity and its thickness decreases to about 200 km during solar minimum conditions.

5. DENSITY SCALE HEIGHT AND ITS GRADIENTS β_ρ.

After the analysis of the atmospheric scale height H and its gradient β, it is useful to study the behaviour of the scale height H_ρ which is associated with the total density determination based on the rate of decay of artificial satellites orbits. KING-HELE and REES [39] developed two methods for finding H_ρ either from the decrease in a satellite's perigee height or from the decrease in the orbital period of a satellite in a small eccentricity orbit. MAY [40] deduced density scale heights H_ρ from a numerical analysis of the ratio between the variations of the perigee geocentric altitude and the variations of the eccentricity.

Using the relations $p = nk\text{T}$ and $\rho = nm$, the pressure and density variations may be written in the following form :

$$(24) \qquad \frac{dp}{p} = \frac{dn}{n} + \frac{d\text{T}}{\text{T}} = -\frac{dz}{\text{H}}$$

and

$$(25) \qquad \frac{d\rho}{\rho} = \frac{dn}{n} + \frac{dm}{m} = -\frac{dz}{\text{H}_\rho}.$$

TABLE VI

Ratio H/H_ρ between 200 km and 3000 km
for day and nighttime conditions from 1958 to 1964

z (km)	1958 DAY	1958 NIGHT	1960 DAY	1960 NIGHT	1962 DAY	1962 NIGHT	1964 DAY	1964 NIGHT
200	1.24	1.17	1.21	1.14	1.13	1.10	1.12	1.09
250	1.15	1.12	1.14	1.11	1.10	1.09	1.10	1.09
300	1.12	1.10	1.11	1.10	1.09	1.08	1.09	1.07
350	1.11	1.09	1.10	1.08	1.08	1.06	1.07	1.06
400	1.10	1.08	1.08	1.07	1.07	1.07	1.06	1.12
450	1.09	1.06	1.07	1.06	1.06	1.11	1.06	1.28
500	1.07	1.06	1.06	1.06	1.07	1.22	1.09	1.61
750	1.09	1.25	1.16	1.42	1.50	1.66	1.64	1.64
1000	1.30	1.57	1.53	1.39	1.31	1.43	1.22	1.55
1250	1.56	1.19	1.38	1.10	1.11	1.55	1.20	1.38
1500	1.37	1.06	1.10	1.09	1.14	1.52	1.31	1.20
1750	1.13	1.05	1.04	1.14	1.21	1.38	1.44	1.10
2000	1.04	1.07	1.03	1.21	1.31	1.24	1.54	1.04
2250	1.02	1.10	1.04	1.30	1.42	1.14	1.55	1.02
2500	1.02	1.14	1.05	1.39	1.50	1.08	1.49	1.01
2750	1.02	1.19	1.07	1.48	1.55	1.04	1.40	1.00
3000	1.02	1.25	1.09	1.54	1.55	1.03	1.30	1.00

By combining both equations, the relationship between H and H_ρ is obtained :

$$(26) \qquad H_\rho = \frac{H}{1 + \beta - \dfrac{2H}{r_0 + z}} \cdot$$

This expression is valid throughout the whole atmosphere and shows that H_ρ is never greater than H because the gradient β is always greater or equal to $2H/(r_0 + z)$. When atomic hydrogen becomes the major constituent, H and H_ρ are identical. But, in the heterosphere, the ratio H/H_ρ varies rapidly and may reach values greater than 1.5 as can be seen from Table VI.

When the density scale heights H_ρ have been determined from satellite observations [39, 40], it is possible to compute the atmospheric scale heights by numerical integration of (26) or by adopting a hypothesis for the value of β. Such a procedure must, however, be carefully considered because of the great variability of β.

The variations of H_ρ are similar to those of the atmospheric scale heights presented in figures 7 and 8, and it is useful to study the behaviour of the gradient, $\beta_\rho = dH_\rho/dz$. By using relation (26), one obtains the following expression :

$$(27) \quad \beta_\rho = \frac{\beta}{1 + \beta - \dfrac{2H}{r_0 + z}} \left[1 - H\rho \left(\frac{\beta'}{\beta} + \frac{2H}{\beta(r_0 + z)^2} - \frac{2}{r_0 + z} \right) \right],$$
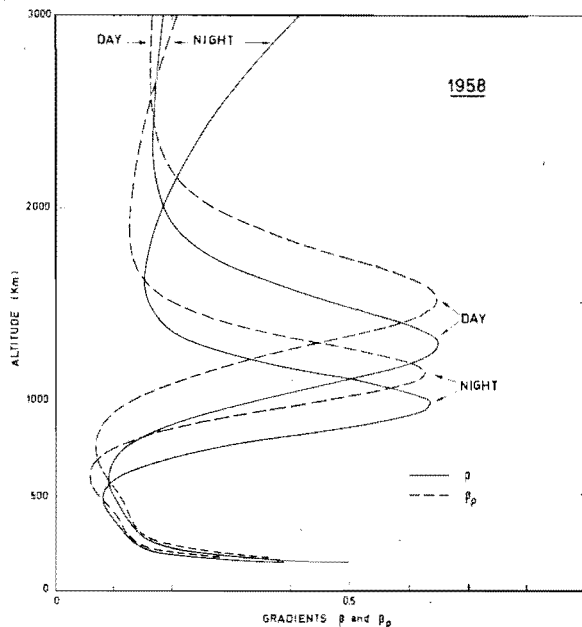

FIG. 10. — Vertical distribution of the gradients β and β_ρ for average day- and nighttime conditions in 1958.
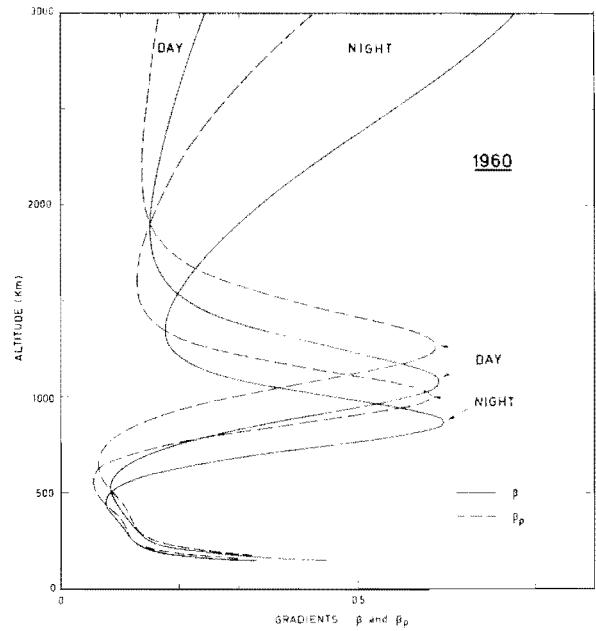

FIG. 11. — Vertical distribution of the gradients β and β_ρ for average day- and nighttime conditions in 1960.
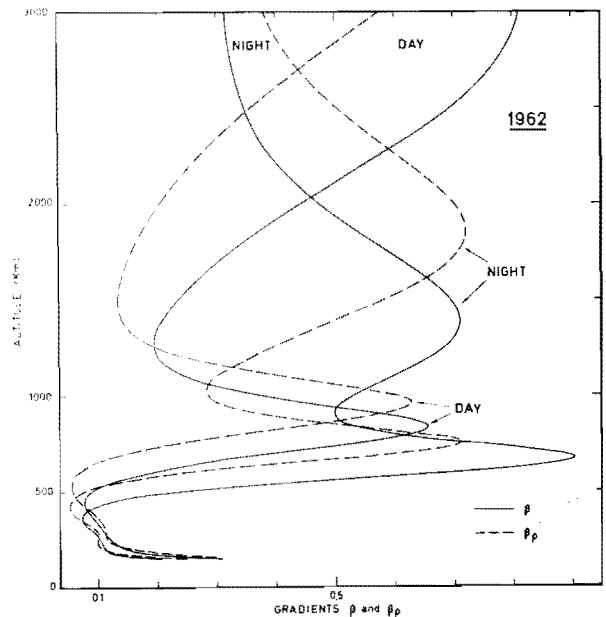

FIG. 12. — Vertical distribution of the gradients β and β_ρ for average day- and nighttime conditions in 1962.

where β' is the derivative of the gradient β with respect to height. Although this expression is valid for the whole atmosphere, we shall only use it above the thermopause level where an analytical expression is available for β' [38]. In the thermosphere β_ρ must be computed from finite differences. Figures 10 to 13 show the gradients β and β_ρ for the solar cycle conditions between 1958 and 1964. It can be seen that the gradient β_ρ reaches
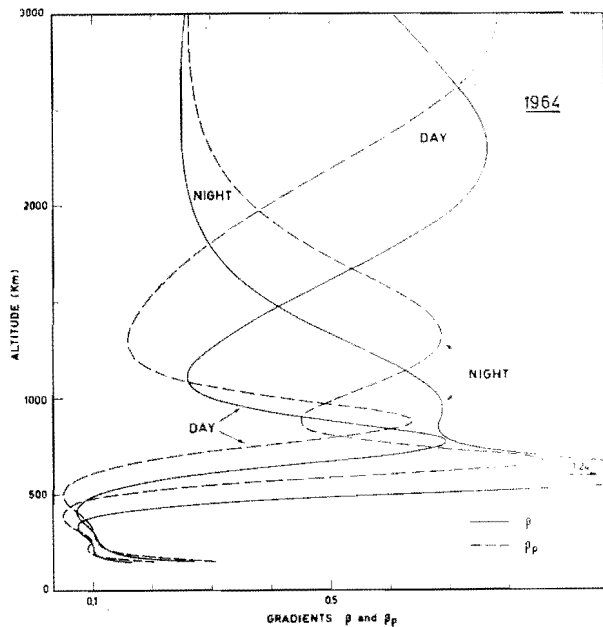
FIG. 13. — Vertical distribution of the gradients β and β_ρ
for average day- and nighttime conditions in 1964.

values of the same order as the atmospheric scale
height gradient. Elsewhere, β_ρ also goes through
one or two maxima in the height range between
500 km and 3000 km. Finally, it is important to
note that the gradient β_ρ is not less than β in the
thermosphere. Above the thermopause β_ρ is even
greater than β in a certain height range. This is a
direct consequence of formula (27) according to
which β_ρ may be greater than β when the deriva-
tive β' is negative i.e. when the gradient β decreases
with height. Figures 10 to 13 also show that the
variations of β_ρ are larger during solar minimum
conditions. Again, the diffusion process is the
leading factor controlling the structure of the iso-
thermal heterosphere and important differences
appear in the absolute values of β and β_ρ as a func-
tion of the thermopause temperature.

CONCLUSION

By using atmospheric models adapted to a decrea-
sing solar cycle, it is shown, in terms of the mean
molecular mass and of the scale heights H and H_ρ,
how diffusion transport leads to important varia-
tions in the structure of the upper atmosphere.
An analysis of the general mass transport equa-
tion shows that any level adopted for the begin-
ning of the molecular diffusion directly depends
on the ratio between the coefficients for eddy diffu-
sion and molecular diffusion, as well as on the initial

distribution for the considered constituent. For
example, if helium is assumed to be in diffusive
equilibrium at 120 km, the eddy diffusion coeffi-
cient must be less than 2×10^6 cm^2 sec^{-1} at that
height. Experimental and theoretical studies are
still necessary for analyzing with enough precision
the physical conditions of the transition region
between perfect mixing and molecular diffusion.
Particulary, it is necessary to know the pressure
at 100 km and its variations in order to study the
time variations of the boundary conditions to be
adopted at 120 km for constructing atmospheric
models. Before knowing all those physical para-
meters, however, it is necessary to describe physi-
cal characteristics of the terrestrial upper atmos-
phere.
Firstly, the mean molecular mass variations
are more pronounced in the isothermal region
and for low temperature. A pressure variation
of $\pm$ 10 per cent at 120 km has almost no effect
on the mean molecular mass, whereas it changes
the total density at 200 km between 2.4×10^{-13}
and 3.6×10^{-13} gm cm^{-3}. The large variation
in m above the thermopause level results directly
from the diffusion effect. The thickness of the
atomic oxygen and helium belts decreases with the
solar activity. For minimum solar activity, the
helium belts becomes rather thin and atomic hydro-
gen is already the major component at a height of
the order of 1000 km.
In the thermosphere, the atmospheric scale height
depends both on temperature and mean molecular
mass variations. Temperature is however the
dominant factor and so, at a given height, H
decreases with the solar cycle. Above the ther-
mopause level, the great variability of m causes
more complex changes of H and the atmospheric
scale height increases when the solar activity dimi-
nishes. The variations of the atmospheric scale
height with altitude are also due to diffusion effects
leading to a sharp decrease in the mean molecular
mass above the thermopause. The variations of H
are presented in the behaviour of the gradient β
which goes through at least one maximum between
the thermopause level and 3000 km. The maxima
of β have a physical meaning because they occur
at the levels where $\rho(He) = \rho(O)$ and $\rho(H) = \rho(He)$,
i.e. at the heights where helium and hydrogen suc-
cessively become the major component. In addi-
tion, it is interesting to note that above the ther-
mopause, the gradient β reaches values higher than
those usually adopted at 150 km where β can be
considered as proportionnal to the temperature
gradient. So, diffusion produces scale height gra-

dients larger than those resulting from the ultra-violet heating in the thermosphere.

The density scale height H_ρ shows a behaviour similar to that of the atmospheric scale height, and the gradient β_ρ is strongly height dependent. Contrary to what is usually assumed, β_ρ is never less than the gradient β in the thermosphere. H_ρ, however, is never higher than the atmospheric scale height.

In conclusion, it can be said that the complex structure of the isothermal atmosphere is essentially due to diffusion which permits important variations in the mean molecular mass and the related physical parameters.

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