Ann. Géophys., t. 25, fasc. 4, 1969, p. 981 à 985.

The magnetohydrodynamical drag on artificial satellites

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RÉSUMÉ. — *On montre qu'il n'existe pas de satellites artificiels de la Terre susceptibles d'engendrer des ondes magnétohydrodynamiques par leur propre mouvement. En adoptant les conditions propres à une perturbation d'un état stationnaire on peut utiliser une approximation quasi-hydrodynamique. Par ce moyen, la théorie permet de fournir des valeurs numériques applicables à quelques satellites.*

ABSTRACT. — *It is shown that no actual artificial terrestrial satellite is able to generate magnetohydrodynamical waves by its own movement. Assuming that the observed induction drag is due to perturbations of a steady state, a quasi-hydrodynamic approximation of the problem is made. The theory gives results of the right order of magnitude for the phenomenon, given some acceptable assumptions.*

I. INTRODUCTION

Some years ago it was suggested by DRELL et al. [1] that the observed drag on large artificial balloon-type satellites, such as Echo, was mainly due to power dissipation by magnetohydrodynamic (MHD) waves, generated by the passage of the satellite through the ionosphere, and not by aerodynamical friction as is generally accepted. They based their conclusions mainly on following assumptions:

a) at Echo altitudes the influence of the neutral particles is independant of the charged ones, the influence of the latter being predominant;

b) the surface barrier for the electrons could readily be overcome by photoelectric emission.

Their formula gives good results for a particular case of Echo 1 drag measurements. However on checking the formula in other cases and for other satellites, large discrepancies were found at the Smithsonian Astrophysical Observatory [2]. By considering long rod-shaped spacecraft components, that where charged up by the charged particles of the ionosphere, CHU and GROSS [3] came to the conclusion that the values advanced by DRELL et al. were largely overestimated.

This conclusion agrees very well with that of the observers who have studied aerodynamical drag phenomena and who claim that if MDH drag exists it would be several orders of magnitude less than the aerodynamical drag. However, apart from that of DRELL et al., no serious effort has ever been made to look for observational evidence of the proposed MHD drag mechanism. Data obtained by FEA [4] for three polar satellites show that an analysis of the MHD drag could be made. By assuming the existence of an effective magnetic field of a 100 gamma it is shown that the MHD drag contributes only a very small fraction to the total drag.

II. THE DRAG MECHANISM

As a spherical metallic satellite crosses the lines of force of the geomagnetic field, an electric field will be built up in the spacecraft and will be detectable by a stationary observer. The induced electric field $\bar{E}_i$, due to the magnetic induction $\bar{B}_0$ and the velocity $\bar{v}_s$ of the satellite, will cancel the motional electric field $\bar{E}_m$:

$$\bar{E}_m = \bar{v}_s \times \bar{B}_0 \quad (1)$$

due to the Lorentz-force that produced the charge separation. As a result a co-moving observer will not experience an electric field, as explained by ALFVÉN and FÄLTHAMMAR [5], provided that the satellite is a perfect conductor. Owing to the resistivity of the spacecraft a current $\bar{J}_S$ circulates in the conductor during the build-up phase but cancels out as soon as an equilibrium is reached. However when any of the parameters governing this phenomenon is altered, the equilibrium situation is temporary destroyed and a current is restored inside the conductor. As a result a current $\bar{J}_L$ will be induced in the conducting ambient fluid; such a process will generate magnetohydrodynamic waves in appropriate circumstances, as stated by DRELL et al. The elements involved in these processes are represented schematically in Figure 1.

Owing to the fact that the medium surrounding the satellite is a weakly ionized gas, consisting of rather slow massive ions and highly mobile electrons, the spacecraft will acquire an electrostatic potential superimposed on the dipole field and, as a result, will influence strongly the flow of the charged particles in the vicinity of the probe. Being sufficiently charged up a steady state will be achieved in which there will be no net current flow towards or from the satellite, or in its neighbourhood as claimed by V. C. LIU [6]. This situation should be compared to what is happening to arc plasma generated inside a glass containment vessel. As is well known from elementary textbooks [7] there is a shield formation on the wall. Within this shield there is no current flowing towards or from the wall. In an article by LINSON [8] the conditions are discussed whenever the current from the ionosphere towards a sphere, charged up by electron emission, is equal to the emission current. As a result Alfvén-waves will no longer be generated and no MHD wave drag will be experienced. This picture remains essentially the same under conditions of active photoelectric emission; once a steady state is reached there will no longer be any MHD drag. As long as this situation has not been settled and by any perturbation of this steady state an effective electric drag field $\bar{E}_d$ will be created, that influences the motion of the satellite and by this token the motion of the charged particles in the ambient plasma. This drag field could have various causes, e.g. it could be:

1° from motional origin due to a change in velocity of the spacecraft;

2° from magnetical origin due to a variation in $\bar{B}$ (temporal or spatial);

3° from photoelectrical origin due to a variation in illuminating conditions;

4° from aerodynamical origin.

This electric drag field can always be represented as

$$\bar{E}_d = \bar{v}_s \times \Delta\bar{B} \quad (2)$$

where $\Delta\bar{B}$ is the effective magnetic-field governing the drag phenomena. Assuming that the MHD approach is applicable, we shall now apply Drell's theory but instead of his motional electric field $\bar{E}_m$, calculated a priori, we substitute the electric drag field $\bar{E}_d$, as it is defined by equation (2) and numerically deduced from observational data. With this assumptions, the dissipated power turns out to be in S I units

$$P_{\text{MHD}} = -16\, v_s^2 R^2 \frac{(\Delta B)^2}{B_0(r)} \sqrt{\pi\rho} \quad (3)$$

where r = radial distance of the satellite from the center of the Earth,
v_s = linear velocity of the satellite,
R = radius of the satellite,
$B_0(r)$ = mean value of the unperturbed magnetic field at the altitude of the satellite, and
ρ = mean atmospheric density at the altitude of the satellite.

This expression can also been written as

$$P_{\text{MHD}} = -16\, v_s^2 R^2 \frac{(\Delta\bar{B})^2}{B_0(r_0)} \left(\frac{r}{r_0}\right)^3 \sqrt{\rho\pi} \quad (4)$$

where $B_0(r_0)$ is the magnetic field at 600 km altitude, r_0 being the corresponding radial distance. The introduction of ρ simplifies the formulae and puts an upper limit on the air-drag MHD drag to ratio.

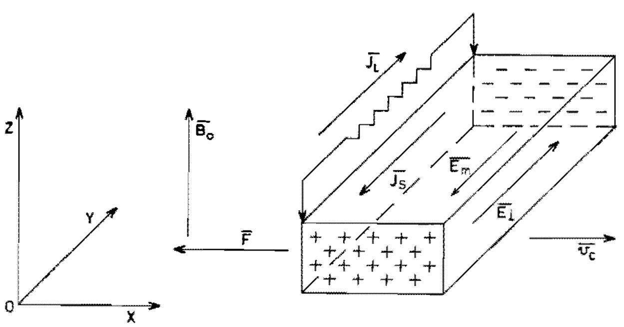

FIG. 1
Simplified geometry of the problem.

III. RELATIONSHIP BETWEEN AERODYNAMICAL AND MHD DRAG

To establish the relationship between these two kinds of drag mechanism, the following formula for the expression of the power P_A dissipated by means of aerodynamical drag can be used

$$P_A = -\frac{1}{2} C_D F S v^3 \rho \quad (5)$$

This formula applies only to a circular orbit; for other orbits additional terms ought to be considered. The interested reader is referred to KING-HELE's [9] well-known formulae. When necessary, use of these formulae will be made in this text without referring to them explicitly. In formula [5] the various symbols have the following meanings

m = mass of the satellite,
F = coefficient allowing for atmospheric rotation, according to KING-HELE [9],
S = effective cross-section of the satellite,
C_D = aerodynamical drag coefficient, and
ρ = mean atmospheric density.

Assuming that the total drag P_T results from aerodynamic P_A and MHD drag P_{MHD} one has the relation

$$P_T = P_A + P_{MHD} \tag{6}$$

Let us now consider the ratio

$$\xi = \frac{P_A}{P_{MHD}} \tag{7}$$

For circular orbits this ratio could be expressed by dividing equation (5) by equation (4). Remembering that

$$v_s = \sqrt{\frac{\mu}{r}}$$

one finds

$$\xi = \frac{\eta \rho^{1/2}}{(\Delta B)^2 r^{1/2}} \tag{8}$$

where

$$\eta = 3.1 \times 10^{-2} \mu^{1/2} F C_D \pi^{1/2} B_0(r_0) r_0^3 \tag{9}$$

with μ = G.M, G = universal gravity constant, and M = mass of the Earth. Obviously this ratio is independant of the volume-to-mass ratio of the satellite and hence the relative importance of the MHD drag is the same for all types of satellites. In Figure 2 the dependence of ξ on altitude and temperature for a circular orbit is shown, assuming that ΔB is of the order of 100γ and using NICOLET's atmospheric models. From Figure 2 it is clear that quite generally one can write

$$P_T = P_A + P_{MHD} = P_A\left(1 + \frac{1}{\xi}\right) \approx P_A$$

This means that generally speaking the results obtained by the aerodynamical drag theory are by no means invalidated by the existence of the MHD drag. This also means that the total drag which has been observed experimentally may be substituted for the aerodynamical drag. This will be done when satellite results are analyzed. In Figure 3 the dependence of ξ on altitude and eccentricity is shown assuming a temperature of 886 °K. From these curves it can be clearly seen that the relative importance of the MHD drag depends on temperature and increases with the altitude and with the eccentricity of the orbit.

IV. EXPERIMENTAL EVIDENCE FOR THE ACTUAL MHD DRAG FORMULA

Equation (7) offers the possibility of isolating P_{MHD} from the total drag experienced by a satellite. From equation (3) one can deduce that P_{MHD} will exhibit a power law in ρ with exponent 0.5. To verify this law one would need data from at least three satellites orbiting at widely separated altitudes. As a typical case, Explorer 19, Echo 2 and Dash 2 will be considered. Unfortunately normal hydrodynamic behaviour will not be exhibited by the magnetospheric plasma as a result of the passage of these satellites. Indeed, although the Lundquist number:

$$L = 2\sigma R B_0 \sqrt{\rho}$$

is sufficiently large, the product $\omega_H \tau$ is also large and thus it is not possible to use the MHD approach $\left(\omega_H = \frac{eB_0}{m};\right.$

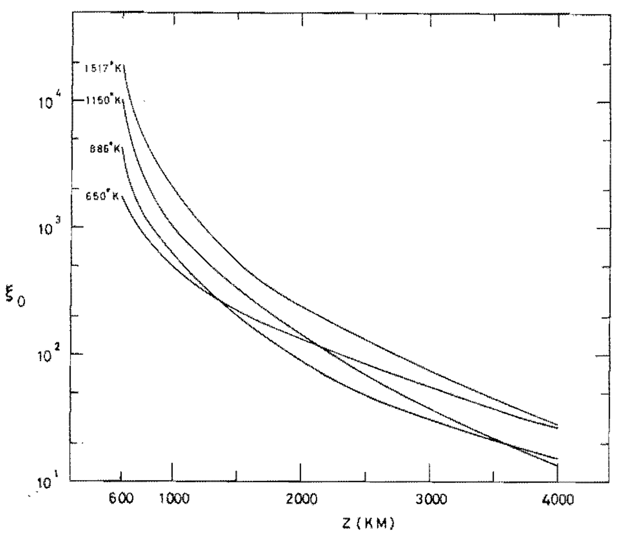

FIG. 2

Ratio ξ_0 of the aerodynamical drag to the induction drag as a function of the altitude for various exospheric temperatures at circular orbits.

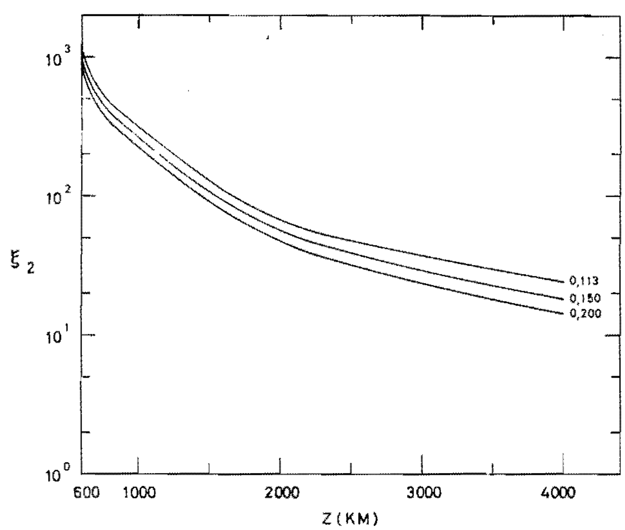

FIG. 3

Ratio ξ_2 of the aerodynamical drag to the induction drag as a function of the altitude at 886 °K for slightly elliptical orbits of various eccentricities.

$\tau = \frac{1}{\nu}$ where ν is the total collision frequency). This means that artificial terrestrial satellites, even of the size of Echo, are unable to generate MHD waves.

However, in many cases of transport problems in a plasma which does not satisfy the MHD conditions, the theory can still be used as an approximation. In such circumstances it is called the quasi-hydrodynamic approximation by GINZBURG [10]. Considering the large uncertainties in the data entering into the formulae, the use of the quasi-hydrodynamic approximation seems fully justified in attemting to explain the observed phenomena.

The data relative to the satellites to be considered are displayed in table I. Some of them are borrowed from FEA [4]. As one can see, almost all the spacecraft have nearly polar orbits and they revolve at widely separated altitudes. As for C_D, various choices are possible according to COOK [11]. After trying various combinations, the result finally obtained was a very good fit assuming $F = 1$, $B = 100\gamma$, and $C_D = 2.4$, considering everywhere suprathermal flow of the air around the satellite. Less good agreement is found when thermal flow is assumed. Poor agreement is obtained when one assumes suprathermal flow below 800 km and thermal flow above this altitude.

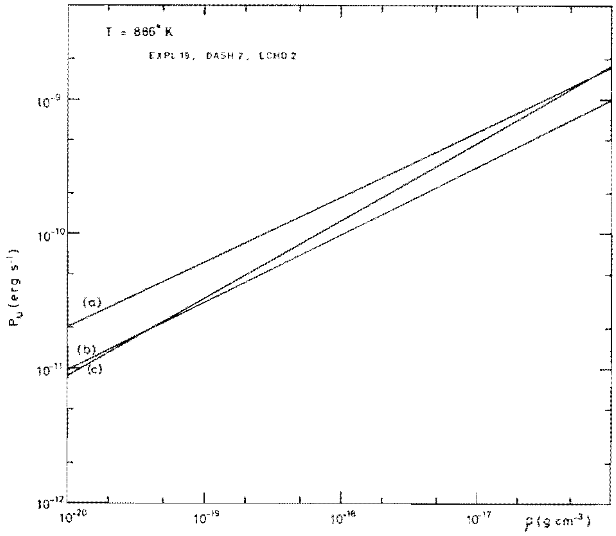

FIG. 4

Induction drag P_u per unit effective cross-section, unit satellite velocity, unit magnetic field and a magnetic driving field of 100 γ deduced from the observations of the satellites Explorer 19, Dash 2 and Echo 2 assuming (a) suprathermal flow, (b) thermal flow and (c) suprathermal flow below 800 km and thermal flow above 800 km.

TABLE I

Data concerning the various satellites.

N Satellite	1 Explorer 19	2 Echo 2	3 Calsphere 1	4 Dash
F	1	0.98	0.98	1
S/m cm²/g	13.9	57.6	1.04	40
m g	7×10^3	2.56×10^5	980	1.2×10^3
Z_p 10^8 cm....	0.596	1.124	1.054	3.119
Z_a 10^8 cm....	2.365	1.206	1.085	4.28
a 10^8 cm	7.88	7.565	7.47	10.1
e...........	0.113	0.005	0.002	0.058
H_ρ 10^6 cm ...	8.0	39.3	35.0	120.7
ρ g/cm³.	9×10^{-18}	5.55×10^{-19}	6.1×10^{-19}	1.6×10^{-20}
v_s 10^5 cm/s ...	7.9713	7.2624	7.3228	6.6608
$2R$ cm	365	4.1×10^3	36	240
B_0 gauss	0.473111	0.38063	0.38934	0.187822
τ 10^3 s.......	6.98	6.55	6.43	10.12
C_D	2.4	2.8	2.8	4
I...........	78.6°	81.5°	89.9°	88.4°

For the meaning of F, S, m, H_ρ, ρ, R, and C_D the reader is referred to the text

Z_P height of the perigee

Z_a height of the apogee

e eccentricity of the orbit

v_s linear velocity of the satellite

τ period

B_0 geomagnetic field strength at the perigee altitude

H_ρ density scale height

I inclination of the orbit

On computing the power P_u,

$$P_u = \frac{P_A B_0(r)}{16\pi^{1/2}\xi v_s^2 R^2} \tag{10}$$

dissipated as a result of MHD drag for unit mass, cross-section, velocity and magnetic field, one obtains the curves in Figure 4. The P_A values are deduced from observational data by Fea. The curve (*a*) corresponds to the case of the suprathermal flow and assuming $C_D = 2{,}4$, the curve (*b*) does so assuming thermal flow and the curve (*c*) represents the case of suprathermal flow below 800 km and thermal flow above 800 km. In Figure 4 the curves are calculated for a mean temperature of 886 °K which has been chosen because of the very low solar activity at the moment when the drag measurements were made. The equations of the various curves obtained by a least squares best fit, are:

a) $\log P_u = 0.483 \log \rho - 0.102$;
b) $\log P_u = 0.502 \log \rho - 0.972$;
c) $\log P_u = 0.579 \log \rho + 0.515$.

Drag data from Calsphere 1 were also available for a somewhat later period. By extrapolating these data, so as to obtain data corresponding to the conditions prevailing at the moment that the drag data for the other satellites were obtained, it was found that they fitted rather well with the earlier data. The results are shown in Figure 5.

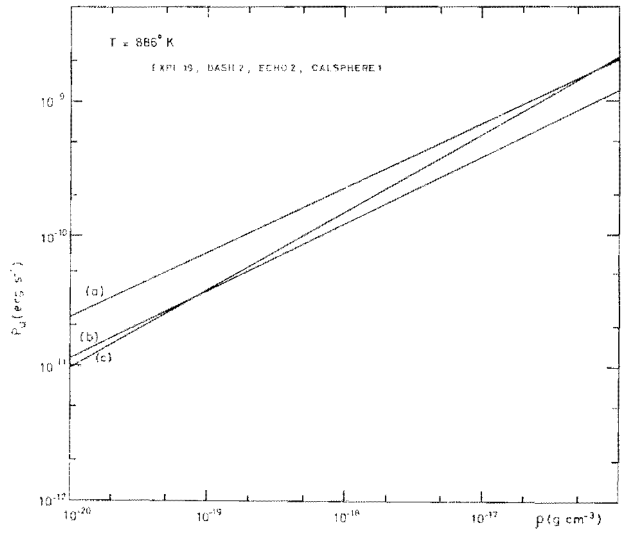

FIG. 5

Induction drag P_u per unit effective cross-section, unit satellite velocity, unit magnetic field and a magnetic driving field of 100 γ, deduced from the observations of the satellites Explorer 19, Dash 2, Echo 2 and including extrapolated data for Calsphere 1, assuming:

(a) suprathermal flow, (b) thermal flow, and (c) suprathermal flow below 800 km and thermal flow above 800 km.

V. CONCLUSIONS

From the results discussed above one can conclude that for 886 °K the MHD-drag shows a dependence on ρ such as one would expect from the theory. It has also been shown that the importance of the MHD drag relative to the aerodynamical drag is independent of the mass-to-volume ratio of the satellite. Finally it is emphasized that the relative importance of the MHD drag depends on both altitude and temperature. It is at least an order of magnitude smaller than the aerodynamical drag, a result which confirms the opinion of the observers of aerodynamical drag phenomena on artificial satellites.

ACKNOWLEDGMENT

The information given by my colleague J. Vercheval was greatly appreciated as is also the encouragement of Dr. L. Jacchia of the Smithsonian Astrophysical Observatory.

Manuscrit reçu le 1er août 1969.

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