Journal of Atmospheric and Terrestrial Physics, Vol. 38, pp. 1041–1046. Pergamon Press, 1976. Printed in Northern Ireland

Steady state plasmapause positions deduced from McIlwain's electric field models

J. LEMAIRE
Institut d'Aéronomie Spatiale, 3, Av. Circulaire, B-1180, Bruxelles, Belgium.

Abstract—It has recently been suggested that the plasmasphere of the Earth is peeled off near midnight (at $L = 4.5$ for $Kp = 1$–2) by 'Plasma Elements Interchange Motion' driven by centrifugal or inertial forces. MCILWAIN's (1974) convection electric field model E3H with a variable scale factor f has been used to calculate different plasmapause positions and shapes. The new theoretical models obtained with $f \simeq 1.2$ are in agreement with observations of whistlers and of the PROGNOZ satellites, for periods of very low geomagnetic activity (i.e. $Kp < 1$). Furthermore the numerical models corresponding to $f \simeq 0.7$ fit qualitatively the observations of the plasmasphere and plasmapause when the geomagnetic activity has a higher constant level (i.e. $2 < Kp < 4$). A tentative relation between f and Kp is deduced.

1. FORMATION OF THE PLASMAPAUSE

According to a recent theory the plasmapause is peeled off in the post-midnight sector by *plasma elements interchange motion** driven by centrifugal forces beyond a '*Roche-limit surface*' (LEMAIRE, 1974). The Roche-limit is defined as the surface beyond which the gravitational force and the centrifugal force† balance each other in the direction parallel to the local magnetic field **B**. The solid line in Fig. 1 running from dawn to dusk across the equatorial plane of the magnetosphere shows the section of the Roche-limit surface when the azimuthal components of the electric drift velocity ($\mathbf{E}\times\mathbf{B}/B^2$) and the local angular velocity (Ω) of the thermal plasma (<1 eV) are determined from the E3H and M2 electric and magnetic field models proposed by MCILWAIN (1974).

Field aligned hydrostatic distributions of the exospheric plasma are convectively unstable for all magnetic field lines crossing this surface; outside the Roche-limit any plasma inhomogeneity (volume element with a higher density than the background medium) tends to move away from the Earth's surface along the magnetic field line, and, eventually across electric equipotential surfaces ($\phi_E = cst$) from one magnetic field line to another. However such cross-ϕ_E interchange motions of detached plasma elements are only possible when $\sum_p$ (the integrated Pedersen conductivity) is significantly reduced below its commonly assumed infinite value.‡ As a consequence of the reduction of electric conductivity in the nightside and high latitude E-region v_{MAX} (the maximum velocity of cross-ϕ_E interchange motion) is significantly enchanced. Due to the dominant centrifugal (or inertial) forces at $L \geq 4.5$, v_{MAX} reaches values of the order of 0.5 L/hr near 0100 LT at the place of deepest penetration of the Roche-limit surface, i.e. where the plasmasphere is peeled off, and, where the plasmapause is formed (LEMAIRE, 1974, 1975).

If the electric field distribution in the magnetosphere does not change for at least 24 hr, the plasmasphere (inner shaded region in Fig. 1) will become bounded by a surface which approximately coincides with an equipotential tangent to the Roche-limit surface at its deepest penetration point.

In the plasmatrough region (unshaded in Fig. 1) the magnetic flux tubes are filled up during the day hours by evaporation from the topside ionosphere, and, by Coulomb pitch angle scattering. The plasma contents of these flux tubes are convected toward the nightside magnetosphere beyond the Roche-limit surface where enhanced interchange

* Plasma elements interchange motion is sometimes called "Magnetic flux tubes interchange motion" in the MHD or field-frozen-in approximation (SONNERUP and LAIRD, 1963).

† The centrifugal force density ($\rho u^2/R_c$) acting on a plasma rotating with a uniform angular speed $\mathbf{\Omega}$ is equal to the inertial force density $\rho\, d\mathbf{u}/dt$ or $\rho(\mathbf{u}\cdot\nabla)\mathbf{u}$ since $\mathbf{u} = \mathbf{\Omega}\times\mathbf{R}_c$. When the drift velocity $\mathbf{u} = \mathbf{E}\times\mathbf{B}/B^2$ is determined by the corotation electric field, the equatorial radius of curvature, R_c, is equal to R, the radial distance. This is also true for MCILWAIN's convection electric field distribution, at least in the post-midnight sector where the equipotential lines are nearly parallel to concentric circles in the plasmasphere and in the plasmatrough.

‡ In the MHD or field-frozen-in approximation $\sum_p \simeq \infty$. The maximum velocity of interchange motion is zero since, $v_{MAX} \alpha \Sigma_p^{-1}$; the radial velocity of a plasma element in an external force is limited by the rate of dissipation of the potential energy by Joule heating in the E-region.

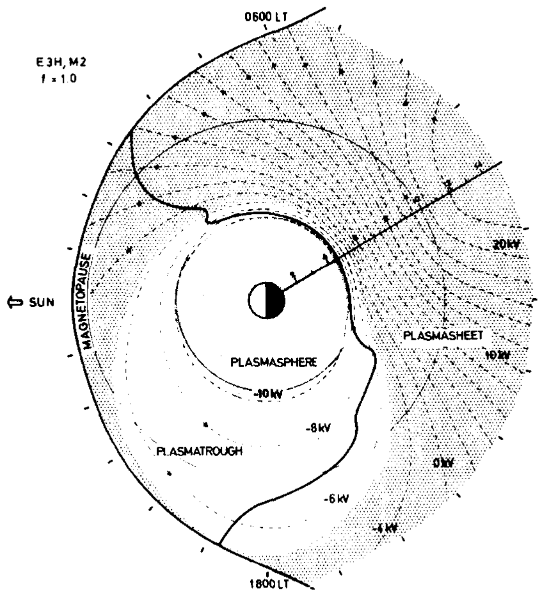

Fig. 1. The Plasmapause for McILWAIN's (1974) electric field model E3H ($f = 1$); the dashed lines are the equatorial sections of the electrostatic potentials; the solid line across the magnetosphere is the equatorial section of the Roche-limit surface where the gravitational and centrifugal forces balance each other along the magnetic field direction. The boundary of the inner shaded region is tangent ot the Roche-limit surface near midnight; the inner boundary of the outer shaded region is the last closed electric equipotential surface, i.e. the Alfvèn layer for zero-energy electrons and ions.

motion will transport the (partially) filled elements beyond the last closed equipotential into the outermost region of the magnetosphere (outer shaded region in Fig. 1). Once it has penetrated into this region (which approximately corresponds to the plasmasheet) the thermal plasma invariably will be convected on an open magnetic flux tube of the polar cusp (or cleft) where finally it is released into the magnetosheath along the flanks of the magnetopause. The drawings in Fig. 2 illustrates the evolution of a plasma element convecting from the dayside plasmatrough toward the polar cusp merging magnetic field lines. The outer edge of the plasmatrough is perhaps related to the substorm injection boundary introducted by McILWAIN (1974) and MAUK and McILWAIN (1974); it corresponds to the zero-energy-Alfvèn layer, i.e. the limit of the forbidden region for cold electrons and ions convecting earthward in the plasmasheet.

A more detailed description of the three regions illustrated in Fig. 1 (plasmasphere, plasmatrough and plasmasheet) is given elsewhere (see, LEMAIRE, 1975).

2. PLASMAPAUSE FOR DIFFERENT ELECTRIC FIELD MODELS

From the previous description it appears that the position of the plasmapause is determined by the minimum radial distance of a Roche-limit surface, i.e. by the convection velocity (in the midnight

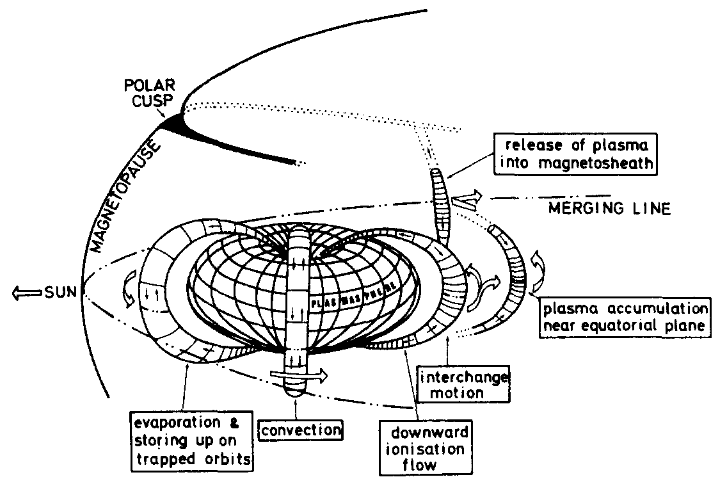

Fig. 2. Illustration of a field-aligned plasma element drifting in the plasmatrough. Evaporation from the topside ionosphere and Coulomb pitch angle scattering in the dayside ion-exosphere concur to store cold charged particles on trapped orbits with high altitudes miror points. The electric drift leads these particles around the plasmasphere toward the nightside region and beyond the Roche-limit surface. As a consequence of the reduction of the integrated Pedersen conductivity after sunset, interchange motion can transport plasma elements across equipotential surfaces at a significant rate. Therefore the cold trapped plasma accumulated near the equatorial plane moves outwardly into the region of 'open' equipotentials, and, finally it reaches the magnetopause where the plasma is released from the magnetosphere along the merging polar cusp magnetic field lines.

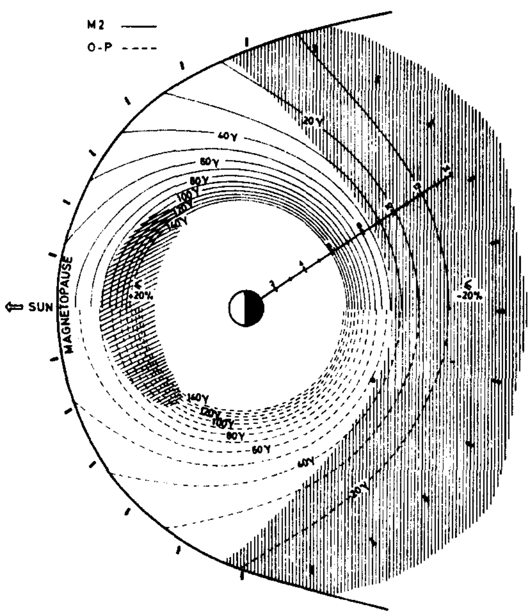

Fig. 3. Comparison of magnetic field models. The solid curves (upper part) are isointensity lines of McIlwain's (1972) magnetic field model M2. The dashed curves (lower part) correspond to Olson and Pfitzer's (1974) model (abreviated version). The shaded zones indicate the areas where the relative difference of the magnetic field intensities $[(B_{M2}-B_{O-P})/B_{M2}]$ is larger than +20% and −20%.

sector) which is a function of the magnetospheric electric field and magnetic field intensities.

In most of our model calculations we adopted McIlwain's (1972) magnetic model M2:

$$B(\gamma)=6-24\cos\varphi+18\cos^2\varphi/(1+1728/R^3)+31{,}000/R^3 \quad (1)$$

where φ is the local time angle (LT), and R is the radial distance in Earth radii. The lines of equal magnetic field intensity for the model M2 are shown by the solid curves in Fig. 3. This model differs from Olson and Pfitzer (1974) model (abreviated version) shown by dashed lines in the lower part of Fig. 3. The shaded areas correspond to the equatorial regions where the relative disagreement measured by $(B_{M2}-B_{O-P})/B_{M2}$ exceeds −20% and +20%. It can be seen that in the plasmasphere and plasmapause regions (i.e. $R<5$–6) the M2 model used in the present paper is rather conservative and differs not too much from a dipole configuration.

The electric equipotential (dashed lines in Fig. 1) are determined by

$$\phi(\mathrm{kV})=10-92\left(\frac{B}{31{,}000}\right)^{1/3}+\sum_{i=1}^{6}\sum_{j=1}^{20}A_{ij}\exp\{-a_i(B-B_i)^2-b_j[1-\cos(\varphi-\varphi_j)]\} \quad (2)$$

where the constants B_i (in γ), $a_i=\ln 2/d_i$ (in γ^{-2}), φ_j (in LT hours), $b_j=\ln 2/(1-\cos C_j)$ and A_{ij} (in kV) can be deduced from Table 1 for the model

Table 1. Model E3H

j	A_{1j}	A_{2j}	A_{3j}	A_{4j}	A_{5j}	A_{6j}	φ_j	C_j
1	6.49	3.26	2.30	1.12	−0.24	0.17	4	2
2	1.37	0.83	0.38	0.48	−0.17	0.13	6	2
3	1.73	0.84	0.36	0.50	−0.26	0.08	8	2
4	0.69	−0.13	−0.31	0.16	−0.28	−0.03	10	2
5	0.90	−0.49	−0.53	−0.09	−0.41	−0.17	12	2
6	0.06	−1.04	−0.88	−0.37	−0.46	−0.26	14	2
7	−0.96	−1.78	−1.11	−0.64	−0.62	−0.38	16	2
8	−2.30	−1.78	−1.28	−0.53	−0.60	−0.32	18	2
9	−3.20	−2.13	−1.75	−0.89	−0.83	−0.43	20	2
10	−0.78	−1.09	−0.06	−0.63	0	−0.17	21	1
11	1.00	−1.10	−0.29	−0.49	−0.27	−0.19	22	1
12	0.59	0.07	−0.04	0.03	−0.16	−0.05	22.5	0.5
13	1.20	0.28	0.17	0.13	−0.20	−0.05	23	0.5
14	2.24	0.77	0.57	0.29	−0.29	−0.08	23.5	0.5
15	2.64	1.13	0.94	0.40	−0.23	−0.06	0	0.5
16	3.47	1.67	1.38	0.53	−0.22	−0.06	0.5	0.5
17	2.48	1.27	1.05	0.38	−0.12	−0.03	1	0.5
18	2.00	1.04	0.84	0.30	−0.08	−0.02	1.5	0.5
19	5.67	2.92	2.28	0.76	−0.13	−0.02	2	1
20	2.23	1.00	0.72	0.30	−0.03	0.06	3	1
i	1	2	3	4	5	6		
B_i	0	40	100	180	280	400		
d_i	30	50	70	90	110	130		

E3H (taken from McILWAIN, 1974). A similar expansion describes McIlwain's earlier model E3 (see, McILWAIN, 1972).

Both electric field distributions E3 and E3H have been proposed for steady geomagnetic activity corresponding to $Kp = 1$—2. Both models predict quite the same plasmapause positions for nearly all local time angles except for $2000\,\mathrm{LT} < \varphi < 2300\,\mathrm{LT}$. The E3 model predicts a slightly more prominent bulge in this range of local times (compare Fig. 1 and LEMAIRE's, 1974 Fig. 3).

To obtain better correspondence to Kp values outside the range of $1 < Kp < 2$, McILWAIN (1974) introduced a scale factor f such that a family of different electric field models can be derived by the conversion $B_i' = B_i/f^3$, $a_i' = a_i f^6$ and $A_{ij}' = A_{ij}/f$. The E3H model illustrated in Fig. 1, actually corresponds to $f = 1$. For f smaller than 1 the convection electric field is enhanced, and, the dawn-dusk asymmetry of the closed equipotential surfaces are reinforced as can be seen by the dashed lines in Fig. 4 for $f = 0.70$.

When f is larger than 1 the convection electric field decreases everywhere and the corotation electric field (second term in r.h.s. of equation 2) prevails at larger radial distances, as illustrated by the more circular equipotential lines of Fig. 5 obtained for $f = 1.3$. The equatorial section of the

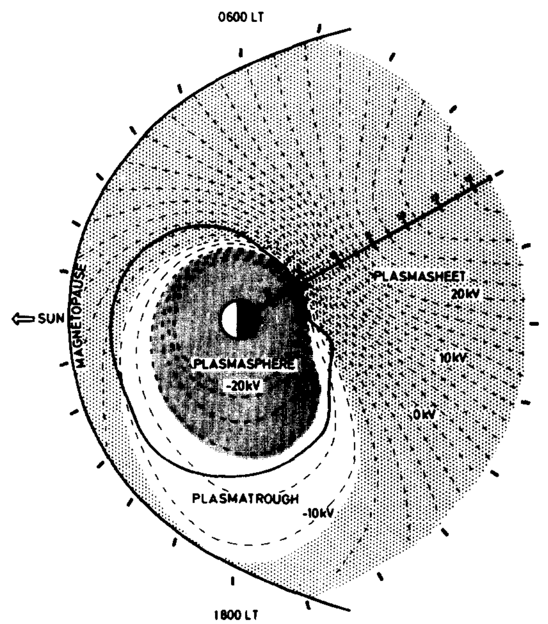

Fig. 4. The plasmapause for McILWAIN'S (1974) electric field model E3H ($f = 0.7$). (See and compare with Fig. 1.) The dusk side bulge of the plasmapause is enhanced as during extended periods of moderate or high geomagnetic activity ($Kp > 2$).

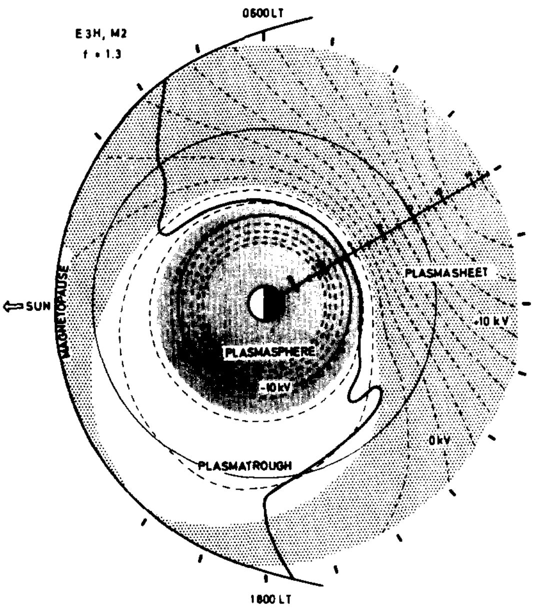

Fig. 5. The plasmapause for McILWAIN's (1974) electric field model E3H ($f = 1.3$). (See and compare with Fig. 1.) The plasmasphere and plasmapause extend at larger radial distances in the noon direction than in the midnight sector. A similar noon–midnight asymmetry is observed by whistlers and satellites during extended periods of very low geomagnetic activity ($Kp < 1$).

Roche-limit surface as well as the steady state plasmapause and the Alfvèn layer have been determined as described in the previous section. The new position of these boundaries are shown in Figs. 4 and 5 for $f = 0.7$ and $f = 1.3$. The magnetopause boundaries in Figs. 1, 4 and 5 are determined by

$$B(\gamma) = f_{MP}(16 - 35\cos\varphi + 12\cos^2\varphi) \qquad (3)$$

where f_{MP} is a scale factor accounting qualitatively for the change in the magnetopause position with geomagnetic activity (BRIDGE *et al.*, 1965; EGIDI *et al.*, 1970; FAIRFIELD, 1971; GRINGAUZ *et al.*, 1975). For $f_{MP} = 1$, 1.3 and 0.9 the magnetopause distances at subsolar point are equal to 10.7 R_E, 8.7 R_E and 12.2 R_E respectively as in Figs. 1, 4 and 5.

A whole sequence of steady state plasmapause positions for successive values of f is shown in Fig. 6 where the central shaded area corresponds to the plasmasphere for $f = 1$.

Finally, Fig. 7 shows the theoretical position of the plasmapause as a function of the scale factor f, at 4 different LT angles.

3. DISCUSSION

Let us consider with McILWAIN (1974) that the sequence of E3H models obtained by changing the

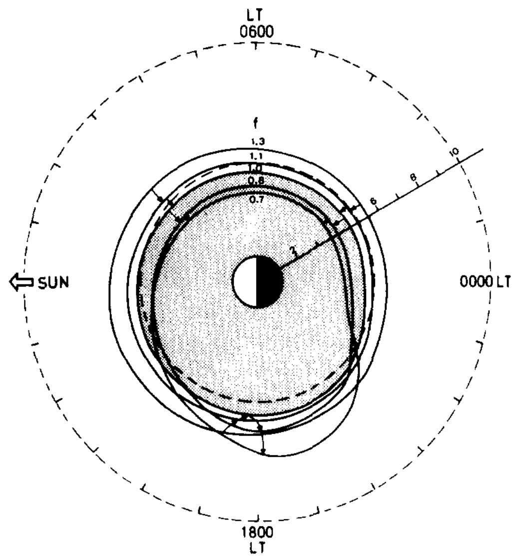

Fig. 6. Plasmapause position as a function of local time and as a function of the scale factor f of the E3H electric field model. When f decreases from 1.3 to 0.7 the shape of the plasmapause changes from a large eccentric circle to a highly asymmetric 'droplet' with a bulge in the 2000 LT direction. Similar deformations of the plasmasphere and plasmapause have been observed when Kp takes constant values from 0 to 4. The shaded area corresponds to the plasmasphere for $f = 1.0$.

scale factor f corresponds to the large scale electric field distribution in the magnetosphere for different constant geomagnetic conditions, i.e. for different values of the Kp-index. From the sequence of plasmapause positions and shapes illustrated in Figs. 6 and 7 it can be concluded that larger

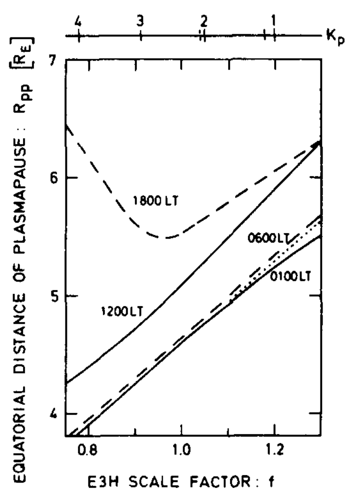

Fig. 7. The equatorial distances of the plasmapause at midnight, dawn, noon and dusk as a function of the scale factor f. The noon–midnight asymmetry of the plasmapause increases when f increases. The dawn–dusk asymmetry increases when f decreases.

(smaller) values of f are associated to smaller (larger) values of Kp. Indeed:

(i) the observed plasmapause distance in the post-midnight sector decreases when Kp increases,

$$L_{pp} = 5.7 - 0.47(Kp)_{-12} \quad \text{for} \quad Kp < 5 \tag{4}$$

where $(Kp)_{-12}$ is the maximum value of Kp for the 12 preceding hours (Carpenter and Park, 1973). A similar decrease of the post-midnight plasmapause is predicted by the theoretical models when f decreases (see Figs. 6 and 7);

(ii) early whistlers observations by Carpenter (1966) have shown that during quiet times (i.e. low Kp) the plasmapause tends to a more circular shape as is shown in Figs. 5 and 6 for $f = 1.3$.

(iii) a noon–midnight asymmetry of the plasmasphere has recently been found by Carpenter and Selly (1975) from very quiet time whistler drift paths observations. Gringauz and Bezrukikh (1975) deduced also from PROGNOZ observations that after a long period of low Kp the noon plasmapause is located at significantly larger L values than in the midnight sector. Using McIlwain's electric field model E3H for $f = 1.3$ a similar noon-midnight asymmetry is predicted.

(iv) when magnetic activity increases the noon–midnight asymmetry of the plasmasphere and plasmapause decreases (Gringauz and Bezrukikh, 1975; Carpenter and Selly, 1975). This tendency is also seen in Figs. 6 and 7 where $L(1200\,\text{LT}) - L(0100\,\text{LT})$ decreases when f decreases.

(v) it is known since a long time that the plasmapause has a bulge in the dusk sector during constant moderate and high geomagnetic activity. (Carpenter, 1966). Recently Park (1975) presented very clear evidence that during moderate substorm activity the whistler paths in the outer plasmasphere move to higher L shells in the afternoon and evening sector from 1600 LT to 2000 LT. This indicates that the equipotential lines in the afternoon-dusk region bulge out toward the 2000 LT direction as predicted by McIlwain's E3H model when the scale factor is less than 1, (see Fig. 6). Consequently, the dawn–dusk asymmetry of the plasmapause increases for increasing values of the Kp-index and for decreasing values of f.

The qualitative agreements between these (whistler and satellite) observations and our theoretical results can be considered as new arguments for the present theory of plasmapause formation as well as tor McIlwain's electric field model E3H with variable scale factor f. From the present discussion it can also be concluded that the E3H models with large values of f correspond best to the magnetospheric electric field distribution for low Kp-index, and conversely the smallest values of f(i.e. $f = 0.7$–0.8) correspond to the larger values of Kp.

Acknowledgements—The author wishes to thank Dr. C. E. McIlwain who provided him with the computer subroutines of his model E3H. It is a pleasure to acknowledge Prof. M. Nicolet for his valuable support during the preparation of this paper. The author appreciated the discussion and comments of Drs. A. Nishida, M. Petit and M. Blanc.

REFERENCES

Bridge H., Egidi A., Lazarus A., Lyon E. and Jacobson L. 1965 *Space Research* V, p. 969. North-Holland, Amsterdam.
Carpenter D. L. 1966 *J. geophys. Res.* **71**, 693.
Carpenter D. L. and Park C. G. 1973 *Rev. Geophys. Space Phys.* **11**, 133.
Carpenter D. L. and Seely N. T. 1975 (Submitted to) *J. geophys. Res.*
Egidi A., Formisano V., Palmiotto F., Saraceno P. and Moreno G. 1970 *J. geophys. Res.* **75**, 6999.
Fairfield D. H. 1971 *J. geophys. Res.* **76**, 6700.
Gringauz K. I. and Bezrukikh V. V. 1975 Paper presented at Symposium on *Physics of Plasmapause*, Grenoble, September.
Gringauz K. I., Zastenker G. N. and Khokhlov M. Z. 1975 *Cosmic Res.* **12**, 815.
Lemaire J. 1974 *Planet. Space Sci.* **22**, 757.
Lemaire J. 1975 *Annls Géophys.* t 31, fasc. **1**, 175.
Mauk B. H. and McIlwain C. E. 1974 *J. geophys. Res.* **22**, 3193.
McIlwain C. E. 1972 *Earth Magnetospheric Processes* (Edited by B. M. McCormac), p. 268. Reidel, Dordrecht, Holland.
McIlwain C. E. 1974 *Magnetospheric Physics* (Edited by B. M. McCormac), p. 143. Reidel, Dordrecht, Holland.
Olson W. P. and Pfitzer K. A. 1974 *J. geophys. Res.* **79**, 3739.
Park C. G. 1975 (submitted to) *J. geophys. Res.*
Sonnerup B. U. O. and Laird M. J. 1963 *J. geophys. Res.* **68**, 131.

APPENDIX

A tentative relationship between f and Kp

It has been shown above that McIlwain's (1974) E3H model with a variable scale factor f describes, at least qualitatively, the plasmapause positions as a function of the local time angle and as a function of geomagnetic activity. Therefore it is tempting to relate the scale factor f to the value of the geomagnetic index Kp. Such a relation can in principle be deduced from correlated observations of the plasmapause position at a fixed local time, φ. Equation (4) illustrates such a relationship between the maximum Kp index for the 12 preceeding hours and the observed position of the plasmapause near midnight.

On the other hand, according to the theory, the plasmapause position at 0100 LT is approximately given by the linear relation

$$R_{pp}(0100\,\mathrm{LT}) = 1.1 + 3.5f. \qquad (A1)$$

This relation is illustrated in Fig. 7 by the dotted curve. It fits well the theoretical curve (solid line, 0100 LT) for $0.75 < f < 1.2$.

Combining equations (4) and (A1), and neglecting the slight difference between the equatorial distances (R_{pp}) and McIlwain's parameter (L_{pp}) one obtains

$$f = 1.31 - 0.134(Kp)_{-12} \qquad (A2)$$

which is approximately applicable in the range $0 < Kp < 4$ due to the limitations of equations (4) and (A1).

It would be interesting to deduce similar relationships from observations of the plasmapause at other different local time angles, and compare them to (A2). If all these relations $f = f(Kp)$ would be for instance nearly identical we would be able to conclude that the large scale magnetospheric electric field distribution can be represented very precisely by McIlwain's E3H model and that its change with Kp can actually be described by the variation of *one* single parameter, f. It is however very unlikely that the variation of only one parameter is sufficient to describe the complex changes of the large scale magnetospheric electric field distribution as a function of geomagnetic activity. But such a test would certainly be useful to estimate in which way and how much the actual electric fields departs from the model E3H. It would also be interesting to conduct such a test to verify the proposed theory for the plasmapause formation under constant geomagnetic conditions. Unfortunately the magnetic activity is rarely constant for 24 consecutive hours; therefore time dependent models of the magnetospheric electric field and plasma convection must be the next goal of our efforts to describe the detailed plasmapause position as a function of local time and changing magnetic activity conditions.