Field-aligned current density versus electric potential characteristics for magnetospheric flux tubes

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ABSTRACT. The field-aligned current density (J_{tot}) is a non-linear function of the applied potential difference (φ) between the ionosphere and the magnetosphere. This non-linear function has been calculated for plasma boundary conditions typical in a dayside cusp magnetic flux tube. The J-characteristic of such a flux tube changes when the temperatures of the warm magnetospheric electrons and of the cold ionospheric electrons are modified (see respectively figs. 2, 3 and figs. 5, 6); it changes also when the relative density of the warm plasma is modified (see fig. 4); the presence of trapped secondary electrons changes also the J-characteristic. The partial currents contributed by the warm and cold electrons, and by warm and cold ions are illustrated in figure 1.

The *dynamic characteristic* of an electric circuit depends on the static characteristic of *each component of the system*: i.e. the resistive ionosphere, the return current region, and the region of particle precipitation whose field-aligned current/voltage characteristics have been studied in this article.

Key words: Birkeland currents, magnetosphere-ionosphere coupling, electric fields, magnetosphere.

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INTRODUCTION

Dayside cusp magnetic flux tubes can be compared to diodes : the positively charged ionosphere is the anode, while the reservoir of warm magnetosheath particles is the cathode of the system. Like many types of electric components, such magnetospheric flux tubes have non-linear, (i.e. non-ohmic) current *versus* potential characteristics (see e.g. Knight (1973), and Lemaire and Scherer (1974), Fridman and Lemaire, 1980). These static characteristics of the magnetospheric system depend on the temperatures and densities of the collisionless electrons and ions spiralling along the magnetic field lines. Fridman and Lemaire (1980) studied the partial current carried by precipitated auroral electrons. This partial current is dominant for field-aligned potential differences larger than 50-100 V. For smaller values of this potential difference between the ionosphere and the magnetosphere, the cold electrons and ions escaping from the topside ionosphere contribute a large fraction of the total field-aligned current density. The scope of this article is more general, than our previous work since it addresses also the cases when the escaping ionospheric electrons carry almost all the return current density, when the potential differences are small.

THE KINETIC MODEL CALCULATION

Consider a high-latitude magnetic flux tube extending from the ionosphere (where the magnetic field intensity is B_I) up into the « plasma boundary layer » near the equatorial magnetopause. Along such a flux tube, magnetosheath plasma is injected from time to time (Carlson and Torbert, 1980). After a transient, corresponding to the propagation time of impulsively injected ions down to the ionosphere, the magnetosheath electrons and ions are mixed with the cold plasma of ionospheric origin consisting mainly of electrons, oxygen ions and hydrogen ions. It will be assumed that a quasi-stationary field-aligned potential drop can develop between the low altitude reference level and the high altitude equatorial region where the magnetic field intensity B_M is reduced by a factor equal to the mirror ratio $a = B_M/B_I$.

For each species in the ion-exosphere it is possible to define a distribution function (e.g. a truncated Maxwellian) which has the same first order moments as the actual velocity distribution. For the magnetosheath particles this Maxwellian (characterized by two parameters N and E related respectively to the density and the temperature at the reference level), is truncated so that there is no return flux in the upward loss cone. Indeed, it is assumed that each particle with a mirror point below the reference level is lost by collisions. On the other hand, particles which are magnetically reflected above this level, do not contribute to the net parallel flux, and *a fortiori* they do not contribute to the total field-aligned electric current density. Therefore, only magnetosheath electrons with pitch angles in the downward loss cone yield a positive upward current density. At the altitude of the reference level this current density is given by (Lemaire and Scherer, 1971)

$$J_{\text{w.e.}} = eN_{\text{w.e.}}\left(\frac{E_{\text{w.e.}}}{2\,\pi m_e}\right)^{1/2} a^{-1}\exp\left[\frac{m_e\,\phi_g - e\varphi}{E_{\text{w.e.}}}\right]\left\{1 + (a-1)\exp\left[\frac{a}{a-1}\,\frac{m_e\,\phi_g - e\varphi}{E_{\text{w.e.}}}\right]\right\} \tag{1}$$

where φ is the total electrostatic potential difference along the magnetic field line, e is the electric charge, m_e the electron mass, and ϕ_g is the gravitational potential which can generally be neglected for electrons. The subscript w.e. stands for « warm electrons ».

The magnetosheath protons, injected downward into the magnetic flux tube yield, at the reference level, a current density given by :

$$J_p = -\,eN_p\left(\frac{E}{2\,\pi m_p}\right)^{1/2} \quad \text{for} \quad e\varphi > -\,m_p\,\phi_g\,. \tag{2}$$

The flux of these protons is small and does not contribute significantly to the total current density defined by :

$$J_{\text{tot.}} = J_{\text{w.e.}} + J_p + J_{\text{H}^+} + J_{\text{O}^+} + J_{\text{c.e.}} \tag{3}$$

The small parallel electric field resulting from the gravitational charge separation of the cold ionospheric electrons (c.e.) and the heavy oxygen ions accelerates the hydrogen ions outwards and decelerates the ionospheric electrons. As in the polar wind, the light hydrogen ions emerge from the ionosphere with a supersonic velocity and receive an additional acceleration through the large field-aligned potential drop. Their contribution to the current density is given by

$$J_{\text{H}^+} = eN_{\text{H}^+}\left(\frac{E_{\text{H}^+}}{2\,\pi m_{\text{H}^+}}\right)^{1/2} \quad \text{for} \quad e\varphi > -\,m_{\text{H}^+}\,\phi_g \tag{4}$$

and by

$$J_{\text{H}^+} = eN_{\text{H}^+}\left(\frac{E_{\text{H}^+}}{2\,\pi m_{\text{H}^+}}\right)^{1/2} a^{-1}\exp\left[\frac{m_{\text{H}}\,\phi_g + e\varphi}{E_{\text{H}^+}}\right]\left\{1 + (a-1)\exp\left[\frac{a}{1-a}\,\frac{m_{\text{H}}\,\phi_g + e\varphi}{E_{\text{H}^+}}\right]\right\} \tag{5}$$

for $e\varphi \leqq -\,m_{\text{H}^+}\,\phi_g$.

The partial current carried by the upward flowing electrons is given by :

$$J_{\text{c.e.}} = -\,eN_{\text{c.e.}}\left(\frac{E_{\text{c.e.}}}{2\,\pi m_e}\right)^{1/2} a^{-1}\exp\left[\frac{m_e\,\phi_g - e\varphi}{E_{\text{c.e.}}}\right]\left\{1 + (a-1)\exp\left[\frac{a}{1-a}\,\frac{m_e\,\phi_g - e\varphi}{E_{\text{c.e.}}}\right]\right\}. \tag{6}$$

When $e\varphi$ is much larger than the thermal energy of the cold ionospheric electrons, the deceleration is important, and the escape flux is small. However, when $e\varphi$ is smaller than or of the order of $E_{\text{c.e.}}$, this flux becomes dominant.

The oxygen ions are also accelerated outwards. However, at the altitude of the parallel electric field acceleration their density is generally so small that their net upward flux is negligible. Therefore, J_{O^+} will be ignored in the following calculations.

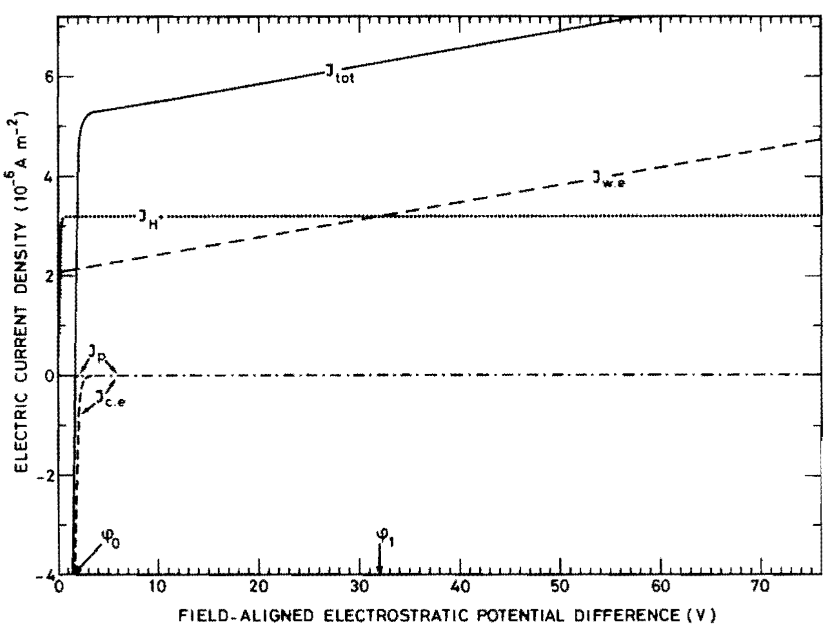

Figure 1

Partial and total field-aligned currents in a dayside cusp magnetic flux tube as a function of the applied field-aligned potential difference between the ionosphere and magnetosheath like plasma cloud. Plasma densities and temperatures used as boundary conditions are summarized in table 1.

Table 1

Parameters for the Standard Model.

Average energy (E_j), temperature (T_j) and number densities (n_j) of cold ionospheric electrons and ions at 1 000 km altitude, of warm magnetosheath electrons and protons at high altitude.

j	c.e.$^-$	O^+	H^+	s.e.$^-$	w.e.$^-$	p	
E_j	0.258	0.129	0.258	40	60.3	293	eV
T_j	3 000	1 500	3 000	4.5×10^5	7×10^5	34×10^5	K
n_j	25 000	2 000	5 000	0	10	10	cm^{-3}
$F_j(=0)$	$+ 4.3 \times 10^{11}$	—	$+ 4 \times 10^8$	0	$- 1.3 \times 10^9$	$- 6.7 \times 10^7$	$cm^{-2} s^{-1}$

F_j is the flux for each constituent when the electric potential difference (φ) is forced to be equal to zero. The parameters N_j are related to j the actual number densities (n_j) by normalisation factors described elsewhere (Lemaire and Scherer, 1971).

NUMERICAL STUDY OF THE FIELD-ALIGNED CURRENT DENSITY

The relative importance of the different partial parallel current densities as a function of the total electrostatic potential difference along the magnetic field line in an ion-exosphere populated by cold electrons, oxygen and hydrogen ions, warm electrons and protons is illustrated in figure 1. The boundary conditions at the reference level, chosen at 1 000 km, are summarized in table 1.

As in the case of non-ohmic conductors, the total electric current density $J_{tot.}$ is a non-linear function of the field-aligned electrostatic potential. When φ is much larger than $1/eE_{c.e.}$ however, $J_{tot.}$ is a linear function of φ. This property is clearly illustrated in figure 1 where the solid curve representing $J_{tot.}$, is a straight line for $\varphi > 5$ V. Fridman and Lemaire (1980) have already shown that the partial current carried by the warm electrons is a linear function of φ when $e\varphi < E_{w.e.}/a$. For magnetosheath electrons, $E_{w.e.}$ is equal to 60 eV (i.e. 10 times smaller than for plasmasheet electrons precipitating in auroral arcs), and therefore the linearity of $J_{w.e.}$ (and consequently of $J_{tot.}$) breaks down for $e\varphi > B_I/B_M . E_{w.e.} \approx 60$ keV.

Note that the ohmic like behaviour of auroral field-aligned currents was discovered experimentally by Lyons, Evans and Lundin (1979) and has been observed many times since.

At small values of φ the partial current J_{H^+} is larger than $J_{w.e.}$. The cross-over potential φ_1, where $J_{H^+} = J_{w.e.}$ can easily be obtained from equations (1) and (4). For the boundary conditions given in table 1, one obtains $\varphi_1 = 32$ V. On the other hand, there exists just one value φ_0 of the electrostatic potential, for which the total field-aligned current vanishes. In the model considered here, this value is : $\varphi_0 = 1.6$ V.

When φ tends to φ_0, the downward current carried by the escaping ionospheric electrons balances the upward current which is mainly due to outward flowing H^+ ions. The early hydrodynamic and kinetic polar wind model were based on the assumption that the net field-aligned current was zero (Banks and Holzer, 1969; Lemaire and Scherer, 1970). This condition was used explicitely by Lemaire and Scherer (1970, 1973) to determine the field-aligned potential difference along open magnetic field lines.

In the present model the sharp decrease of $J_{tot.}$ in the vicinity of φ_0 is due to the high sensitivity of $J_{c.e.}$ to the small change of the electrostatic potential barrier that the ionospheric electrons with energies between 0.1 and 1.0 eV have to overcome in order to escape. The current which is transported by the cold electrons of ionospheric origin is illustrated by the dashed curve in figure 1.

Figure 2 shows the total current density $J_{tot.}$ *versus* the applied potential difference for three different velocity distributions of the warm electrons corresponding to $T_{w.e.} = 14 \times 10^5$ K; 7×10^5 K and 3.5×10^5 K (i.e. $E_{w.e.} = 120$ eV; 60 eV and 30 eV, respectively). The static characteristics plotted in figure 2, for magnetospheric cusp field lines are similar to those deduced earlier by Lemaire and Scherer (1974) for the nightside auroral field lines. The extension of these characteristics into the negative current domain is shown in figure 3.

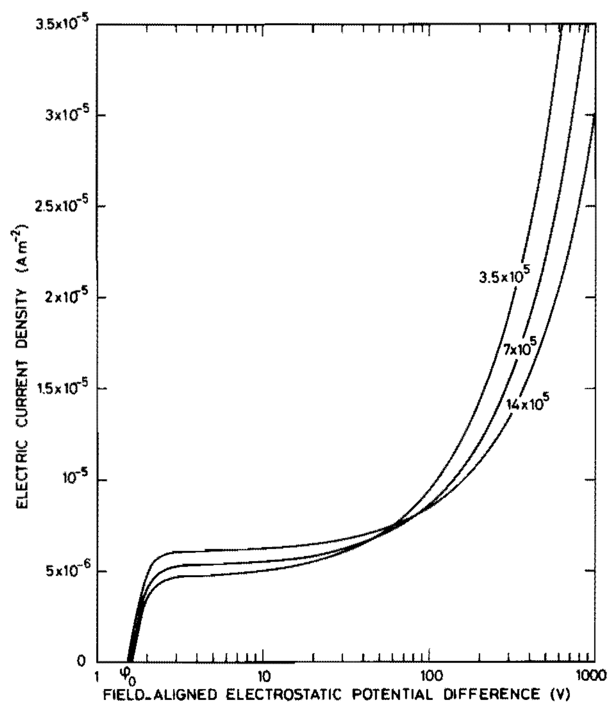

Figure 2

Influence of a change in the temperature ($T_{w.e.} = 3.5 \times 10^5$; 7×10^5 ; 14×10^5 K) of warm magnetospheric electrons on the $J_{tot.}$-characteristic of a dayside cusp magnetic flux tube. The other number densities and temperatures are summarized in table 1. Note the non linearity of the characteristics except in a limited range of the potential difference.

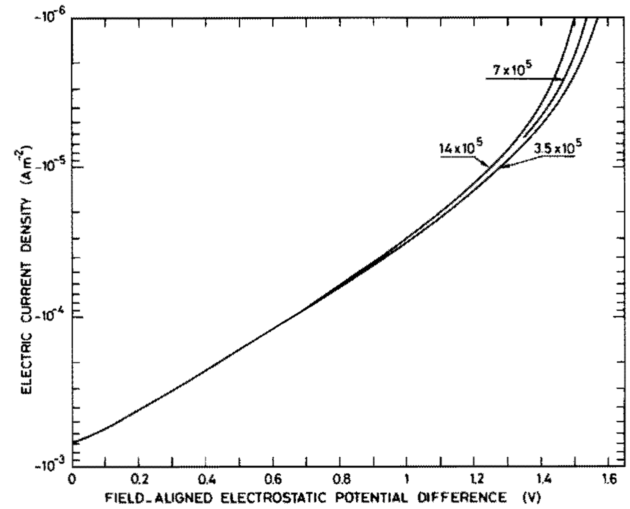

Figure 3
Extension of the characteristics curves of figure 2, for field-aligned potential differences smaller than 1.6 Volt. Note the large negative (return) current densities obtained at small values of the potential difference.

When the electrostatic potential becomes smaller than φ_0, the potential barrier for the cold ionospheric electrons is reduced and their escape flux is drastically enhanced. For small field-aligned electrostatic potential differences (0.5 eV) the current carried by the cold electrons, is at least two orders of magnitude greater than the partial currents carried by the other constituents.

For a vanishing electrostatic potential, the escape flux of the ionospheric electrons is given by the Jeans evaporation flux formula for neutral particles of mass m_e, flowing out of the Earth's gravitational field. Hence the maximum for the electron escape flux is approximately given by

$$(F_{c.e.})_{max} = N_{c.e.}\left(\frac{E_{c.e.}}{2\pi m_e}\right)^{1/2}. \quad (7)$$

Applying the boundary conditions in table 1 yields a downward current of -0.7 mA/m^2. Field-aligned currents of such a large density have never been observed in the magnetosphere. Moreover, since $\mu_0 J = (\text{curl } B)$, such a large parallel current would cause an angular change in the magnetic field direction, greater than 1.5° per km in the direction of $\vec{B}$. This is almost two orders of magnitude greater than the normally observed spatial changes of the magnetic declination angles at 1 000 km altitude. As a consequence it can be concluded that actual field-aligned electron fluxes are always much smaller than the Jeans escape flux (7) and that is never equal to zero or negative.

Outside the areas where warm magnetospheric electrons are accelerated and precipitated into the atmosphere, the return currents transport cold thermal ionospheric electrons out of the ionosphere. It is clear from figures 1 and 3, that a slight reduction of the electrostatic potential below φ_0 is enough to drag out of the ionosphere the large return current required to close the magnetosphere-ionosphere electric circuit.

In regions where negative return currents flow downwards i.e. for $\varphi < \varphi_0$, the Ohmic form of the $(J - \varphi)$ characteristic is no longer valid. Indeed, from figure 3, it can be seen that the absolute value of the current increases when the potential decreases. This is clearly in contradiction with Ohm's law according to which the total current is directly proportional to the electrostatic potential.

Furthermore, it is not necessary to assume, as in the models of Chiu *et al.* (1981), that the field-aligned potential difference changes sign when the electric current reverses direction at the frontier of the precipitation region and the return current area. The extension of the characteristic curves into the negative potential domain, would give much too large return current densities as can be deduced from figure 3. Indeed, if the potential became negative (i.e. if the ionosphere were at a negative potential with respect to the magnetosphere) all the thermal electrons of the ionosphere would be accelerated out of the ionosphere, and all the ions would be precipitated. This process (mechanism) would build up an inversely directed charge-separation electric field opposing the applied negative field-aligned potential.

In figure 4 the $(J - \varphi)$ characteristics are plotted for 4 different densities of the magnetosheath particles i.e. : 5 cm^{-3}, 10 cm^{-3}, 20 cm^{-3} and 50 cm^{-3}. Since, $J_{w.e.}$ is directly proportional to the magnetosheath electron density, the total parallel electric current is enhanced when the warm electron density is increased.

Figure 5 illustrates the influence of an increase of the temperature of the ionospheric electrons corresponding to an increase of the parameter $T_{c.e.}$ from 2 000 K to

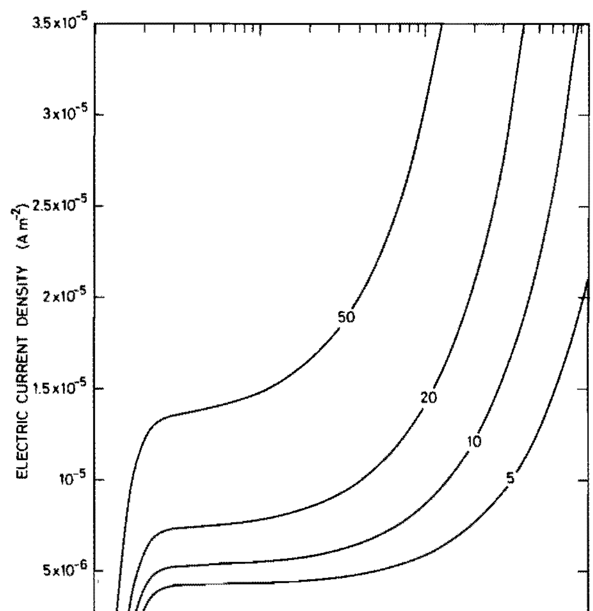

Figure 4
Influence of a change in the number density ($n_{w.e.}$ and n_p = 5 ; 10 ; 20 ; 50 cm^{-3}) of the warm magnetosheath plasma on the J-characteristic of a dayside cusp magnetic flux tube. The other number densities and temperatures are summarized in table 1. Note the enhancement of J_{tot}, the total current (J_{tot}) resulting from the increased precipitation flux of warm electrons.

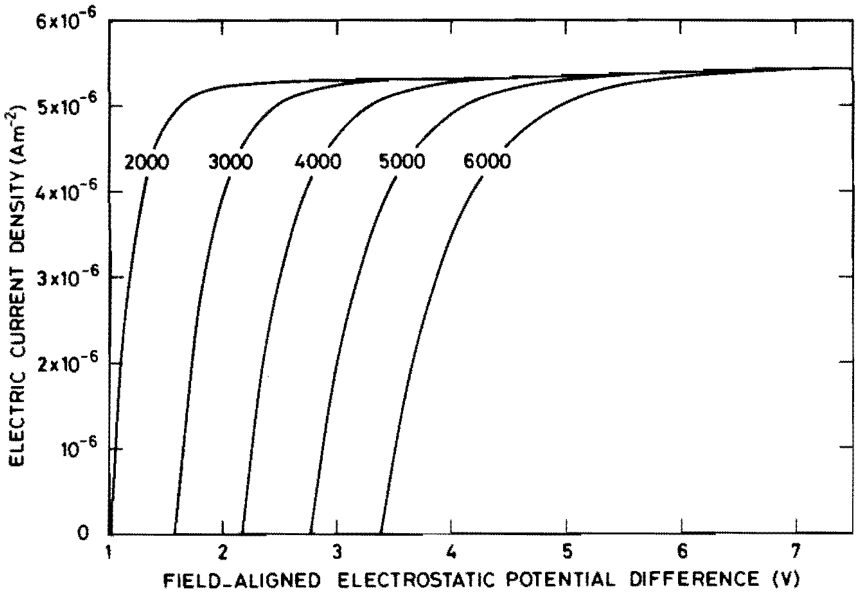

Figure 5

Influence of a change in the cold electron temperature ($T_{c.e.}$ = 2 000; 3 000; 4 000; 5 000; 6 000 K) on the J-characteristic of a dayside cusp magnetic flux tube. The other number densities and temperatures are the same as for the standard model (see table 1). Note that the value of the field-aligned potential for which the total current vanishes, increases when the temperature of the ionospheric electrons is increased.

6 000 K. A larger average thermal velocity of the cold electrons enhances their escape flux. A larger potential φ_0, corresponding to the zero-current condition, is then needed to reduce $J_{c.e.}$ so that it becomes comparable to the current carried by the upward accelerated cold ionospheric ions. On the contrary, when the temperature of the hydrogen ions increases, the value of φ_0 is decreased slightly. This is shown in figure 6 for the parameter values T_{H_+} = 1 500 K, 3 000 K, and 6 000 K.

The $(J - \varphi)$ characteristic of open magnetospheric flux tubes can also be modified by the fluxes of the heavier ions, such as He^{++}, present in the magnetosheath plasma, or O^+ which is the major constituent in the ion-exosphere below 2 000-3 000 km altitude.

The value of $J_{He^{++}}$ is always a small fraction of J_p, and J_{O^+} is generally much smaller than J_{H^+}. However, when the height of the electrostatic shock is below 2 000-3 000 km, the number of O^+ ions which are accelerated can exceed the flux of the H^+ ions, but both ionospheric ion-fluxes remain negligible compared to the partial current carried by magnetosheath electrons impulsively injected in the dayside cleft ionosphere.

The $(J - \varphi)$ characteristics of open magnetospheric flux tubes can also be modified by the presence of secondary electrons in the zone of particle precipitation. To evaluate the effect of these backscattered electrons, we assumed that they escape with a Maxwellian velocity distribution — the downward loss cone being empty — and that they have lost half of the kinetic energy they had before penetrating into the atmosphere. The net downward current ($J_{s.e.}$) transported by these backscattered secondary electrons can then be calculated by means of equation (6) where $E_{c.e.}$ is replaced by $E_{s.e.}$ and $N_{c.e.}$ by $N_{s.e.}$. In figure 7 the dashed characteristic has been obtained by assuming that $N_{s.e.} = 0$, and therefore $J_{s.e.} = 0$; the solid curve shows the total field-aligned current (including $J_{s.e.}$) when $N_{s.e.} = 5\ cm^{-3}$, $E_{s.e.}$ = 40 eV, the other boundary conditions being the same as those of table 1. The addition of a population of secondary electrons reduces the total upward current density.

DISCUSSION AND CONCLUSIONS

Stationary solutions fail to be quasi-neutral when the field-aligned potential exceeds a critical value (see e.g. : Stern, 1981). When φ exceeds this critical value, there is a multiplicity of different solutions corresponding to the same boundary conditions. The system is then probably unstable, and can jump or oscillate from one solution to another. Not all these solutions are quasi-neutral. The partial and total electric current densities are the same, however, since the boundary conditions in the ionosphere remain unchanged. Therefore, the characteristic curves described above are appropriate

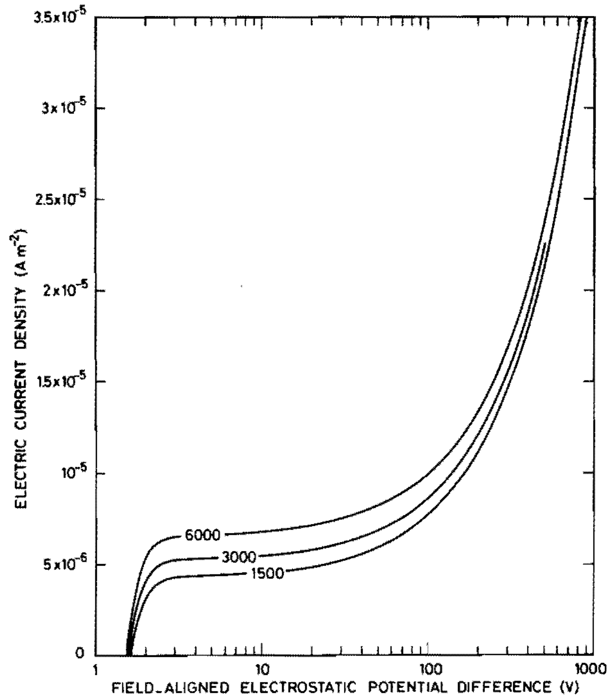

Figure 6

Extension of the ($J_{tot.}$) characteristics of figure 5, for higher values of the field-aligned potential. Note the increase of the total upward current when the partial (downward) current carried by the ionospheric electrons is enhanced with $T_{ce.}$.

even when the parallel electric field becomes non-stationary in the electrostatic shock region. A non-stationary electric potential distribution is likely to develop electrostatic turbulence in the region of oscillating shocks or double layers. Magnetospheric electrons can be scattered while they traverse the acceleration region as a result of interaction with the electrostatic waves generated at the shock. It is generally assumed that the magnetic moment of the electrons and ions is conserved during the traversal of the narrow region where they are accelerated by a large parallel electric field. Nevertheless this can fail to be true when the electrostatic waves have a large amplitude in the direction normal to the magnetic field direction. In this case the perpendicular energy of the traversing particles may be modified, and their magnetic moment will then be changed non-adiabatically. As a result of this non-adiabatic pitch angle scattering, the net flux of precipitated magnetospheric electrons will be reduced. Indeed, strong wave particle interactions reduce $J_{w.e.}$ (i) by scattering particles into the upward directed loss cone which has been assumed empty in the present calculations and (ii) by increasing the small pitch angles of precipitating electrons so that they become trapped.

The effects of wave-particle interactions, or of the contribution of J_{O_+} to the field-aligned fluxes, are corrections which still have to be worked out quantitatively. Nevertheless, the values of the fluxes deduced within the framework of an ideal collisionless kinetic theory are adequate zero order approximations, good enough to facilitate a grasp of the basic physical processes involved, and appropriate to illustrate the effect of a change in the boundary conditions in the ionosphere or in the magnetosphere.

A magnetic flux tube in which currents flow out of the ionosphere into a hot magnetospheric plasma cloud constitutes a non-ohmic component of a global circuit. The DC potential generator is at high altitude, the load is the ionosphere where the transverse Pedersen currents flow horizontally through a resistive medium. The return current flowing outside the precipitation region forms another non-ohmic part of the magnetosphere-ionosphere electric system. This circuit has dynamic characteristics which differ from the static characteristics discussed in the present paper.

The perpendicular electric fields produced by the magnetospheric generator (e.g. the $\vec{V} \times \vec{B}$ field of a moving plasma cloud, or the charge separation electric field between two plasma regions of different temperature) are due to potential differencies (φ_M) between adjacent magnetic field lines; φ_M is equal to the sum of (i) the field-aligned potential in the precipitation zone (φ), (ii) the potential drop along field lines in the return current area ($\varphi' \simeq$ 1-1.5 V), and (iii) the ionospheric potential difference between these field lines (φ_I). The net field-aligned current $J_{tot.}$ increases when φ_M increases. The curve $J_{tot.}(\varphi_M)$ is the dynamic characteristic of the system. But since φ_M is not equal to φ, this dynamic characteristic of the circuit differs from $J_{tot.}(\varphi)$, the static characteristic corresponding to one isolated flux tube which is only one single component of the global magnetosphere-ionosphere circuit.

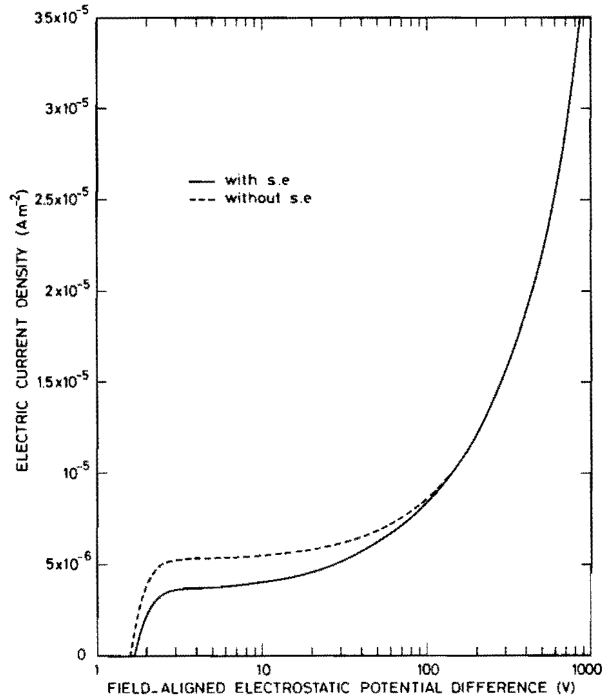

Figure 7

Effect of secondary electrons on the J-characteristics of a dayside magnetic flux tube. The dashed line corresponds to the standard model in which non secondary electrons were taken into account. The solid line illustrates the results when secondary electrons of 40 eV average energy are assumed to have a density of $n_{s.e.} = 5\ cm^{-3}$ at the 1 000 km. Note that the presence of escaping secondary electrons reduces the total upward current for values of the potential difference which are not much larger than the average energy of these additional outflowing electrons.

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