SOLAR VARIABILITY AND STOCHASTIC EFFECTS ON CLIMATE*

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Abstract. The effect of fluctuations on a simple energy balance model is examined. A new, long characteristic time scale referring to the passage between different stable climatic states is identified. It is shown that a weak external forcing whose period is comparable to this scale enables the system to switch between different states with a high probability. The connection with glaciation cycles is pointed out.

1. Introduction – The Importance of Fluctuations

The most important single element affecting the energy budget of the Earth-atmosphere system is, without any doubt, the solar output. It has been recognized for a long time that the response to this output is far from being a passive one, obeying to a simple proportionality law (see e.g., North *et al.*, 1981). Rather, it reflects the existence of numerous feedbacks related to the inherently nonlinear dynamics of the Earth-atmosphere system, such as the ice-albedo feedback, the effect of clouds, and so forth.

As well known, nonlinear dynamical systems may give rise to a variety of instability and transition phenomena. (Nicolis and Prigogine, 1977). In the last decade numerous authors pointed out that this possibility may be at the origin of the climatic transitions that have occurred in the past. A detailed analysis of several climatic models has substantiated this conjecture to the extent that it has established the existence of bifurcations on such models. It has however left open the answer to two most important questions: (i) how can the system transit between the states available through bifurcation? In the usual analysis the various stable states are fairly far apart, and the passage between them requires giant perturbations which are difficult to conceive. (ii) What is the time scale of climatic change? The general trend of models available so far is to predict scales which are far too short as compared to the scales of major climatic episodes such as, say, the Quaternary glaciations. Especially crucial in this latter context, is the inability to reproduce the 100 000 yr dominant periodicity of glaciation cycles.

The purpose of this paper is to show that the answer to these questions rests in the *variability* of both the solar output and the *internal dynamics of the Earth-atmosphere system*. It is hardly necessary to insist, in a meeting like this, on the justification of the variability of the first kind. As regards the internal variability, the main idea is that in a complex system like the Earth-atmosphere one there are continuous imbalances between the rates of the various processes going on. Such imbalances are perceived by the system as a noise around the deterministic evolution

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and are called *fluctuations*. An individual fluctuation is, typically, a localized small amplitude event. Yet in a potentially unstable system even small random disturbances associated with fluctuations will sooner or later drive the system to a new regime. In principle therefore, we expect that the analysis of fluctuations should provide the answer to questions (i) and (ii) raised above.

In Section 2 we outline the formalism of Markovian processes, appropriate for the study of fluctuations. Section 3 is devoted to a brief representation of the major results.

2. Stochastic Formulation

Let $\bar{x}$ denote a set of climatic variables obeying to a closed equation of evolution. A typical example is the surface temperature T averaged over space coordinates, within the framework of a 'zero-dimensional' (0-d) climate model. In the absence of fluctuations $\bar{x}$ is supposed to obey to the following equation of evolution:

$$\frac{\mathrm{d}\bar{x}}{\mathrm{d}t} = f(\bar{x}, \lambda, t) = f_0(\bar{x}, \lambda) + \varepsilon f_1(\bar{x}, \lambda, t) . \tag{2.1}$$

Here f is an appropriate nonlinear rate function, and λ stands for a set of characteristic parameters such as mean annual solar influx, albedo and so forth. This function is decomposed into a part, f_0, corresponding to an autonomous evolution and a part, f_1, describing the effect of some external forcing proportional to ε. An interesting example of the latter is the variation of insolation associated with the variation of eccentricity of the Earth's orbit.

Of special interest for our work are cases where the steady-state solutions of Equation (2.1) in the absence of forcing, $\varepsilon = 0$,

$$f_0(\bar{x}_s, \lambda) = 0 \tag{2.2}$$

are multiple and see their stability properties change as the parameters λ take different values.

As discussed in the introduction, the deterministic description must often be supplemented with information concerning the fluctuations. We denote their effect by a random force $F(t)$ and assume the latter to be x-independent and define a *white noise* (Wax, 1954):

$$\begin{aligned} \langle F(t) \rangle &= 0 , \\ \langle F(t)F(t') \rangle &= q^2 \delta(t - t') , \end{aligned} \tag{2.3}$$

where the average is taken over the appropriate statistical ensemble. Conditions (2.3) are expected to be reasonable because of the local character of fluctuations, as a result of which the system should rapidly loose the memory of the past states. Equation (2.1) is now to be replaced by the stochastic differential equation

$$\mathrm{d}x_t = f(x_t, \lambda, t) + F(t)\,\mathrm{d}t . \tag{2.4}$$

As well known Equations (2.3)–(2.4) are equivalent to a *Fokker – Planck equation* with nonlinear friction (drift) coefficient and constant diffusion coefficient:

$$\frac{\partial P(x,t)}{\partial t} = -\frac{\partial}{\partial x} f(x,\lambda,t)P(x,t) + \frac{q^2}{2}\frac{\partial^2 P(x,t)}{\partial x^2}, \tag{2.5}$$

where $P(x, t)$ is the probability density for having the value(s) x of the state variable(s) at time t.

In the sequel we illustrate the application of Equation (2.5) to the global (0-d) energy balance model. In this case x is the surface temperature, f the energy budget divided by the heat capacity (yr^{-1} K). The variance q^2 is then to be measured in ($\mathrm{yr}^{-1}\,\mathrm{K}^2$).

3. Results

A. AUTONOMOUS EVOLUTION

We first discuss the behavior of Equation (2.5) in the absence of external forcing, $\varepsilon = 0$. As the coefficients become then time-independent the equation admits a steady-state solution of the form (Nicolis and Nicolis, 1981):

$$P_s(x) = \left\{ \int_0^\infty \mathrm{d}x \exp\left[-\frac{2}{q^2} U_0(x)\right]\right\}^{-1} \exp\left[-\frac{2}{q^2} U_0(x)\right]. \tag{3.1}$$

This expression features the *climatic potential* $U_0(x)$ defined through the relation

$$U_0(x,\lambda) = -\int^x \mathrm{d}\xi\, f_0(\xi,\lambda). \tag{3.2}$$

The lower limit in the integral is immaterial, as it corresponds to the addition of an arbitrary constant to $U_0(x)$ which cancels in Equation (3.1) anyway.

For typical values of the parameters λ, the underlying 0-d deterministic model admits three steady-state solutions (Crafoord and Källén, 1978): two stable states T_+, T_- corresponding respectively to the present-day and to a deep-freeze climate, and an unstable state, T_0, separating the first two ones. This property is reflected by the existence of *two minima* and *one maximum* of the climatic potential $U_0(x)$. A sensitivity analysis of U_0 in parameter space further reveals that the depth of the minima can be changed. According to Equation (3.1), the deepest minimum will correspond to the most probable state. It is therefore legitimate to consider such a state as the dominant climatic regime. We have defined the conditions under which the present-day or the deep-freeze climate will dominate. These two situations are separated by a *coexistence* region corresponding to specific relations between parameter values in which these two climates are equally dominant. Figure 1 represents qualitatively the probability function in this region.

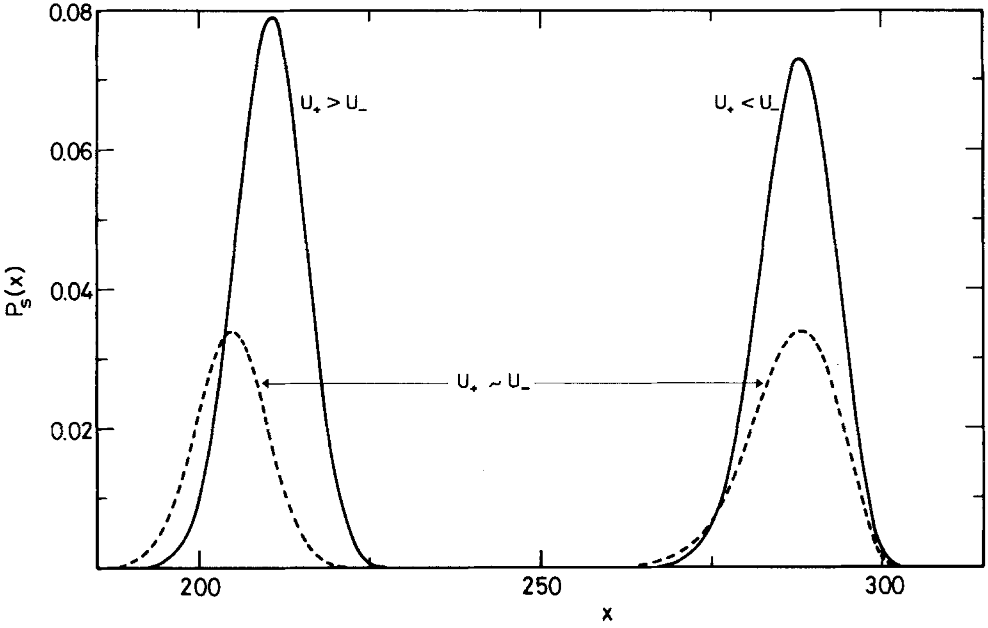

Fig. 1. Steady-state probability distribution $P_s(x)$ for three representative cases. Note that, in the second case where $P_s(x)$ is a two-hump distribution, the two climates T_+ and T_- are equally dominant.

The next question to be asked concerns the transition between different stable climatic states. In the presence of fluctuations such transitions can occur spontaneously at any instant, but typically they will require a long time period because the system will have to overcome the *barrier* corresponding to the unstable state T_0. In terms of the potential $U_0(x)$ this barrier is measured by the difference

$$\Delta U_{0\pm} = U_0(T_0, \lambda) - U_0(T_\pm, \lambda)\,. \tag{3.3}$$

The mean transition time τ has been evaluated using a method similar to Kramers' evaluation of chemical reaction rates (Nicolis and Nicolis, 1981). For a transition over the barrier starting from T_+, the result is

$$\tau \sim \pi(-U_0''(T_0, \lambda)U_0''(T_+, \lambda))^{-1/2} \exp\left[\frac{2}{q^2}\Delta U_{0+}(\lambda)\right]. \tag{3.4}$$

For small values of the variance q^2 compared to the height of the potential barrier ΔU_{0+}, this time becomes exceeding long. We have explored systematically the parameter space and found values of q^2 and λ for which τ can become of the order of 10^4 and 10^5 yr, which is precisely in the range of characteristic times of glaciations.

B. EFFECT OF SOLAR VARIABILITY

We now consider the case where the autonomous evolution is perturbed by a small external forcing. One example is the effect of sunspot cycle which, despite an inherent noise, shows an approximate 11-yr periodicity. Another example is the slight change in the mean annual energy influx (~0.1%) arising from the variation of the eccentricity of the Earth's orbit with a periodicity of about 10^5 yr (e.g. see Imbrie and Imbrie, 1980). In either case, to simplify the analysis as much as possible we describe these nearly periodic variations in the form

$$Q \sim Q_0(1+\varepsilon \sin \omega t)\,. \tag{3.5}$$

The unperturbed solar constant is taken to be $Q_0 = 340\ \mathrm{Wm}^{-2}$.

We have analyzed the deterministic and the stochastic response of the Earth-atmosphere system to the forcing described by Equation (3.5), at the level of a 0-*d* energy balance model. For a small value of ε, typically 0.001, the deterministic response is negligible. In contrast, the *stochastic response can be dramatically amplified,* provided that the transition time scale (Equation (3.4)) matches the time scale of the external periodicity, $2\pi/\omega$. Under these conditions the probability density at the most probable value varies as (Nicolis, 1981)

$$P(T_+, t) = \hat{N}_+ \sin(\omega t + \phi) + N_{0+}\,, \tag{3.6}$$

where

$$\hat{N}_+ \sim \frac{1}{(1+\omega^2\tau^2)^{1/2}}\, G(\lambda) \tag{3.7}$$

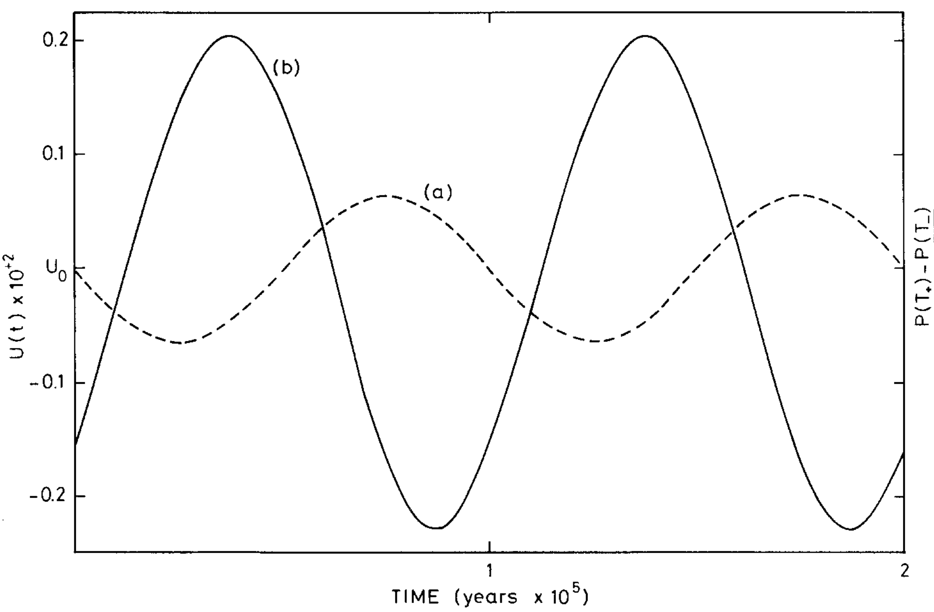

Fig. 2. Curve (a): Time dependence of the potential difference $U(t) = U(T_+, t) - U(T_0, t)$, subject to a periodic forcing with $\omega = 2\pi/10^5$ and $\varepsilon = 0.001$, simulating the variation of the eccentricity of the Earth's orbit. Curve (b): Time evolution of the difference of the probabilities of the two stable states divided by their common steady-state value $P_s = P(T_+) = P(T_-)$, in the presence of the forcing represented in curve (a).

and

$$\phi \sim -\text{arctg}\,(\omega\tau)\,. \tag{3.8}$$

The function $G(\lambda)$ turns out to be exponentially small in all cases, except when the two states T_+ and T_- are equally dominant (referred to as 'coexistence' in Section 3A). On the other hand, the factor multiplying this function is usually very small since as we mentioned in the previous subsection, τ tends to be very large. Typically therefore the amplitude of the stochastic response N_+ is negligible, compared to the steady-state level N_{0+}. There is however an exception to this rule, namely when the two characteristic times τ and $2\pi/\omega$ are comparable. In that case the factor multiplying $G(\lambda)$ is of order 1 and the stochastic response to the periodic forcing becomes comparable to the steady state value N_{0+}.

Figure 2 provides an illustration of these results in the case of a forcing having a very long periodicity ($\sim 10^5$ yr). We have found therefore a mechanism enabling the system to capture such long period signals and use them to lower the value of the potential barrier and perform a transition to another state with an appreciable probability. The connection between these results and the glaciation cycles is tempting.

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