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Some Aspects of the Solar Radiation Incident at the Top of the Atmospheres of Mercury and Venus

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A formalism has been developed for the calculation of the insolation on the planets Mercury and Venus neglecting any atmospheric absorption. For Mercury, the instantaneous insolation curves are repeated in a 2-tropical-year cycle, the distribution of the solar radiation being perfectly symmetric between both hemispheres. In addition to latitudinal variations, one observes a longitudinal effect expressed by different instantaneous insolation distributions during the course of the time; on the equator, the relative diurnal insolation variability may attain a factor of 3. The small obliquity of Venus results in a nearly symmetric solar radiation distribution with respect to the equator except at the poles, where an important seasonal effect has been found. It has to be noted that no longitudinal dependence exists. Finally, the insolation curves are repeated in a nearly half-year cycle.

1. INTRODUCTION

The knowledge of the distribution of the solar radiation incident at the top of the atmospheres of the planets of the solar system as a function of the coordinates of a surface element and time (or season) is indispensable to the study of its climatic history, energy budget and dynamical behavior.

Although the upper-boundary insolation of the Earth's atmosphere and the radiation incident on its surface have been treated extensively, it must be pointed out that theoretical investigations relative to the other planets are scarce and often incomplete. Among the papers published during the last years we cite those of Murray *et al.* (1973), Ward (1974), Vorob'yev and Monin (1975), Levine *et al.* (1977), and Brinkman and McGregor (1979).

Concerning more particularly the inner planets Mercury and Venus, the distribution of incident solar radiation has only been partially studied (Soter and Ulrichs, 1967; Han-Shou Liu, 1968; Vorob'yev and Monin, 1975). For example, the diurnal insolation has never been analyzed in detail. Moreover, it should be emphasized that the insolation variation has generally been determined as a function of the latitude of a surface element, neglecting any longitudinal effect. Taking into account the large eccentricity of the heliocentric orbit of Mercury and the slow rotation, it is obvious that the longitude of a surface element is of prime importance for various radiation problems. Although this planet has practically no atmosphere, the analysis of its insolation may be of great interest to the study of the thermal budget on its surface.

The main objective of the present paper is to analyze in detail solar radiation problems of Mercury and Venus, the nearest planets to the Sun. Another characteristic feature of those two slowly rotating planets is that they have sidereal periods of axial rotation of the same order of magnitude as the sidereal periods of revolution in their heliocentric motions. Furthermore, the inclination of the equator with respect to the planetary orbit is very small (zero for Mercury). However, two main differences have to be noted between the two planets. First, Mercury rotates on its axis in the same direction in which it revolves around the Sun, whereas Venus is the only planet in the solar system rotating in the opposite

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direction (retrograde motion). Secondly, the eccentricities of their orbits (0.00678 for Venus and 0.20563 for Mercury) represent the extreme values when listing this parameter of all planets of the solar system. Although the last two findings justify a somewhat different theoretical approach to the insolation problem of the two planets we present, in a first section, a general formalism applicable to both planets. Then, taking into account their proper conditions, we calculate the instantaneous as well as the diurnal upper-boundary insolation of the atmospheres of Mercury and Venus.

2. GENERAL FORMALISM

The instantaneous insolation is defined as the solar heat flux sensed at a given time by a horizontal unit area of the upper boundary of the atmosphere at a given point on the planet and per unit time.

In our calculations use will be made of the following parameters as illustrated in Fig. 1 (see also Ward, 1974).

(O; x, y, z) represents an equatorial coordinate system, whereas (O; x', y', z') defines an orbital coordinate system. It should be noted that in both systems the origin O is located in the center of the planet. The axes y and y' coincide and are directed toward the Sun at the epoch of the vernal equinox of the planets. The z axis contains the spin or rotation axis of the planet. **r** is the unit vector characterizing the direction of a surface element and $\mathbf{r}_\odot$ is the unit vector directed toward the Sun. λ and φ are respectively the longitude and the latitude of the surface element in the equatorial system, the y axis being taken as the origin of the longitudes (Note that the so-defined longitude is time dependent owing to the rotation of the planet). $\lambda_\odot$ and $\delta_\odot$ define the longitude and the latitude of the Sun in the same system. $\lambda'_\odot$ and λ'_p are the planetocentric longitude of the Sun and the planetocentric longitude of the planet's perihelion in the orbital system, and ϵ is the obliquity or inclination of the equator with respect to the plane of the orbit.

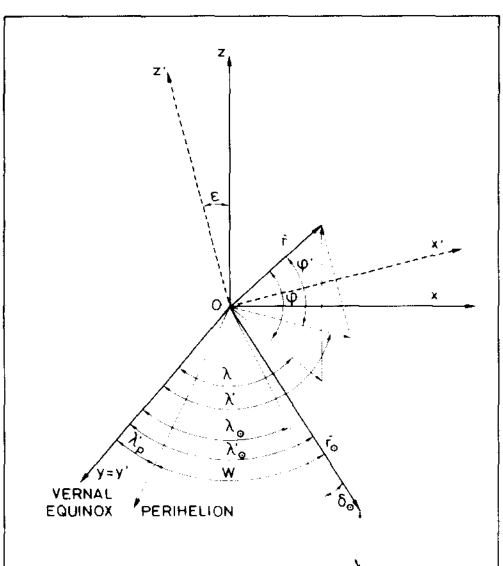

FIG. 1. Schematic representation of some parameters used in the equations for the instantaneous and diurnal insolations on Mercury and Venus.

The instantaneous insolation I may now be written in the following form:

$$I = S(\mathbf{r} \cdot \mathbf{r}_\odot) \tag{1}$$

with

$$\begin{aligned} I &> 0 \quad \text{if } \mathbf{r} \cdot \mathbf{r}_\odot > 0, \\ I &= 0 \quad \text{if } \mathbf{r} \cdot \mathbf{r}_\odot \leqq 0, \\ S &= S_0/r_\odot^2 \end{aligned} \tag{2}$$

and

$$r_\odot = a_\odot(1 - e^2)/(1 + e \cos W). \tag{3}$$

S is the solar flux at an heliocentric distance $r_\odot$ and S_0 is the solar constant at the mean Sun–Earth distance of 1AU. For the calculations presented in this paper we have adopted the most reliable value of the solar constant of 1.94 cal cm^{-2} min^{-1} (or 2.79×10^3 cal cm^{-2} day^{-1}).

In expression (3), $a_\odot$, e, and W are respectively the planet's semimajor axis, the eccentricity, and the true anomaly. Table I represents, for the two planets under consideration, the numerical values of the parameters used for the computation of the

TABLE I

ELEMENTS OF THE PLANETARY ORBITS OF MERCURY AND VENUS

Planet	a (AU)	e	λ'_p (°)	ϵ (°)	T_0 (Earth days)	T (Earth days)
Mercury	0.3871	0.20563	189.22	0	87.969	58.65
Venus	0.7233	0.00678	124.43	2	224.701	243.0

instantaneous and diurnal insolations. In this table one can find also the sidereal period of revolution T_0 (tropical year) and the sidereal period of axial rotation T (sidereal day). Finally, it has to be mentioned that the argument of perihelion λ'_p is taken from Vorob'yev and Monin (1975).

The components of the unit vectors $\mathbf{r}$ and $\mathbf{r}_\odot$ are, respectively:

$$\mathbf{r} = (\cos\varphi \sin\lambda, \cos\varphi\cos\lambda, \sin\varphi)$$

and

$$\mathbf{r}_\odot = (\cos\delta_\odot \sin\lambda_\odot, \cos\delta_\odot\cos\lambda_\odot, \sin\delta_\odot).$$

With this in mind, we can write:

$$I = (S_0/r_\odot^2)[\sin\varphi\sin\delta_\odot + \cos\varphi\cos\delta_\odot\cos(\lambda - \lambda_\odot)] \quad (4)$$

or

$$I = \{S_0/[a_\odot^2(1 - e^2)^2]\}(1 + e\cos W)^2[\sin\varphi\sin\delta_\odot + \cos\varphi\cos\delta_\odot\cos(\lambda - \lambda_\odot)]. \quad (5)$$

The solar radiation I has to be expressed as a function of time, the time dependence being a function of the arguments W, $\delta_\odot$, and $(\lambda - \lambda_\odot)$. Indeed, taking $t = 0$ at the perihelion passage of the planet, the true anomaly W is given by:

$$W = n_0 t + (2e - e^3/4)\sin n_0 t + (5/4)e^2 \sin 2\, n_0 t + (13/12)e^3 \sin 3\, n_0 t, \quad (6)$$

where $n_0 = 2\pi/T_0$ designates the mean angular motion.

In relation (6), known in celestial mechanics as the equation of the center, we kept only terms up to the third degree in e. This approximation gives satisfactory results taking into account that the maximum value of the eccentricities considered here is approximately equal to 0.2 (Mercury).

Furthermore, the solar declination $\delta_\odot$ can be calculated using the following relations:

$$\sin\delta_\odot = \sin\epsilon\sin(\lambda'_p + W) \quad (7)$$

and

$$\cos\delta_\odot = (1 - \sin^2\delta_\odot)^{1/2}, \quad (8)$$

where $|\delta_\odot|$ is always smaller than 90°.

If, on the other hand, one assumes that at $t = 0$ the longitude of the surface element is denoted by λ_0, the time dependence of λ is described by the expression:

$$\lambda = \lambda_0 \pm nt$$

or

$$\lambda = \lambda_p + \Delta\lambda \pm nt, \quad (9)$$

where $n = 2\pi/T$ is the rotational angular velocity of the planet and $\Delta\lambda$ is the longitude difference between the meridian of the surface element and the meridian crossing the line of apsides at the perihelion passage of the planet which is, as already mentioned, taken as $t = 0$. Note that the minus sign is adopted for Venus (retrograde rotation).

Finally, the following set of equations can also be obtained:

$$\lambda_p = \arctan(\cos\epsilon\tan\lambda'_p), \quad (10)$$

$$\lambda_\odot = \arccos(\sec\delta_\odot\cos\lambda'_\odot), \quad (11)$$

$$\lambda'_\odot = \lambda'_p + W. \quad (12)$$

Hence, the angle $(\lambda - \lambda_\odot)$ can be determined by combining (9), (10), (11), and (12), yielding:

$$\lambda - \lambda_\odot = \arctan(\cos\epsilon \tan\lambda_p') + \Delta\lambda \pm nt - \arccos[\sec\delta_\odot \cos(\lambda_p' + W)]. \quad (13)$$

Taking into account the expressions (5)–(8) and (13), the instantaneous insolation variation at the tops of the slowly rotating planets Mercury and Venus and for a surface element characterized by the coordinates (λ_0, φ) or $(\Delta\lambda, \varphi)$ can now easily be obtained.

The diurnal insolation, I_D, can be found by integrating relation (5) numerically over a period equal to the planet's solar day $T_\odot$:

$$I_D = \int_{t=0}^{T_\odot} I\,dt. \quad (14)$$

It has to be pointed out that $I = 0$ when the condition $\mathbf{r} \cdot \mathbf{r}_\odot \leqq 0$ is fulfilled. Furthermore the length of the solar day $T_\odot$, figuring in relation (14), is defined by:

$$1/T_\odot = 1/T \pm 1/T_0 \quad (15)$$

the plus sign being used for a planet rotating in the opposite direction in which it revolves around the Sun (of which Venus is the only case). Practically, relationship (14) may also be written under the following form:

$$I_D = \int_0^{t_1} I\,dt + \int_{t_2}^{T_\odot} I\,dt, \quad (16)$$

where t_1 and t_2 correspond respectively to the time of setting and rising of the Sun; in this case I is always positive. The analytic expression of I_d, given by Ward (1974) and also by Levine *et al.* (1977) and Vorob'yev and Monin (1975) is only applicable to rapidly rotating planets where $r_\odot$ and $\delta_\odot$ are assumed to be constant in the integration of the instantaneous insolation I over a period equal to the planets solar day $T_\odot$. For slowly rotating planets this procedure is not correct when the eccentricity e or the obliquity ϵ are not negligible.

3. SOLAR RADIATION INCIDENT ON MERCURY

3.1. *Instantaneous Insolation*

The obliquity ϵ of Mercury being equal to zero, the formalism discussed in the previous section may be simplified. In this case we have:

$$\delta_\odot = 0$$

and

$$\lambda - \lambda_\odot = \Delta\lambda + nt - W. \quad (17)$$

Hence, the amount of solar radiation incident on Mercury can be expressed by the following equation:

$$I(\Delta\lambda, \varphi) = (S_0/r_\odot^2)\cos\varphi \cos(\lambda - \lambda_\odot). \quad (18)$$

From relation (18) it follows also that:

$$I(\Delta\lambda, \varphi)/I(\Delta\lambda, 0) = \cos\varphi. \quad (19)$$

From (19) it is obvious that the instantaneous solar radiation at a given time and at a given latitude φ can easily be determined by multiplying the insolation on the same meridian at the same moment by the correction factor $\cos\varphi$, and for a surface element located on the equator $\varphi = 0$.

Introducing the numerical values for S_0, $a_\odot$, and e (Table I) expression (18), applied to the equator, finally yields:

$$I(\Delta\lambda, 0) = 2.033 \times 10^4(1 + 0.20563 \cos W)^2 \cos(\Delta\lambda + nt - W). \quad (20)$$

Figure 2 illustrates the solar radiation incident at the top of the Mercurian atmosphere as a function of time and for three surface elements characterized by the following specific numerical values of $\Delta\lambda$: 0, −45, and −90° (as already mentioned above, $\Delta\lambda$ designates a fixed meridian on the planet).

The time variation of the instantaneous insolation extends over three sidereal periods of revolution (or tropical years) corresponding to approximately 264 Earth days. However, it should be emphasized that one insolation cycle covers two periods of revo-

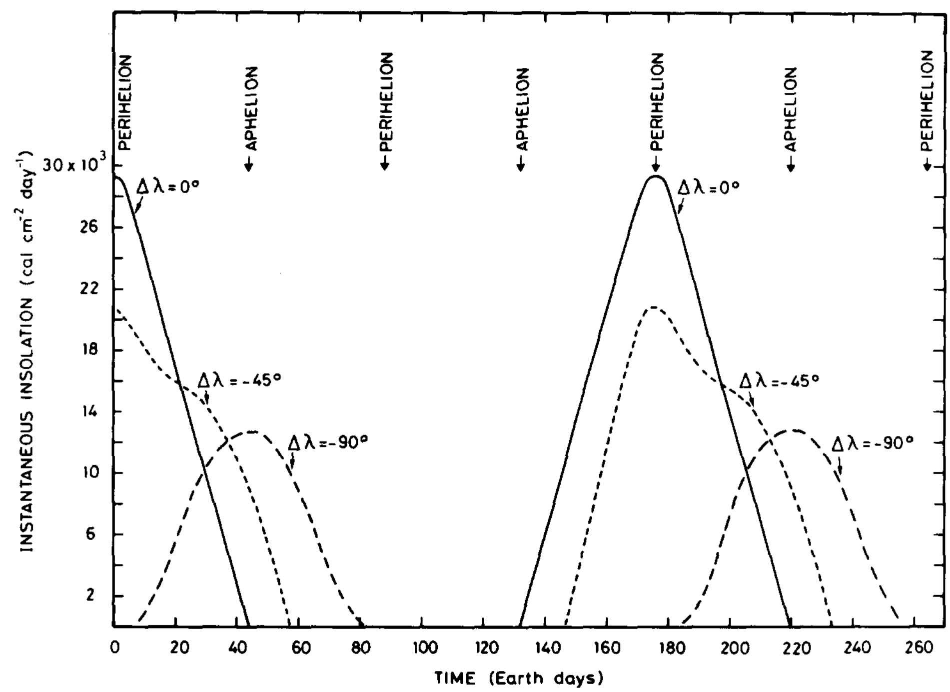

FIG. 2. Time variations of the instantaneous insolation at the top of the Mercurian atmosphere on fixed meridians characterized by the following specific numerical values of $\Delta\lambda$: 0, -45, and $-90°$. The curves are related to the equator ($\varphi = 0$). Perihelion and aphelion passages are also illustrated.

lution, application of relation (15) (with the minus sign for Mercury) showing that $T_\odot = 2T_0 \cong 176$ Earth days.

An analysis of Fig. 2 reveals that the insolation pattern as well as the maximum incident solar radiation strongly varies in passing from one meridian to another. To the best of our knowledge computations for $\Delta\lambda = 0$ have been made by Vorob'yev and Monin (1975) whereas curves representing the insolation in arbitrary units on the meridians $\Delta\lambda = 0, -75, -90$, and $-105°$ have been reported by Soter and Ulrichs (1967). It is also interesting to note that the calculations for $\Delta\lambda = -90°$ result in a bizarre distribution of solar radiation in the neighborhood of the perihelion passage of the planet showing a very short period of weak insolation preceded or followed by an even smaller period of complete darkness [see also Soter and Ulrichs (1967) and Han-Shou Liu (1968)]. An explanation for this phenomenon (not visible on Fig. 2 due to the insufficiency of the scale adopted for the ordinate) will be given later (Fig. 4).

The longitudinal dependence of the incident solar radiation as plotted in Fig. 2 will be clarified by means of Fig. 3. The dashed lines represent the theoretical insolation curves as a function of true anomaly W (during the course of one revolution) and for various values of the angle of incidence $\zeta = \lambda - \lambda_\odot = \Delta\lambda + nt - W$ comprised between 0 and 90°. The curve for $\zeta = 90°$ coincides with the axis of abscissa. The figure shows the effect of ζ for given values of W, whereas the influence of the heliocentric distance $r_\odot$ is illustrated by the curves of constant angle of incidence. As a result, the insolation varies by a factor of approximately 2.3 between perihelion and aphelion passage for a same zenith angle ζ. Plotted are also the time scale t (at the top of the figure) and the corresponding values of the

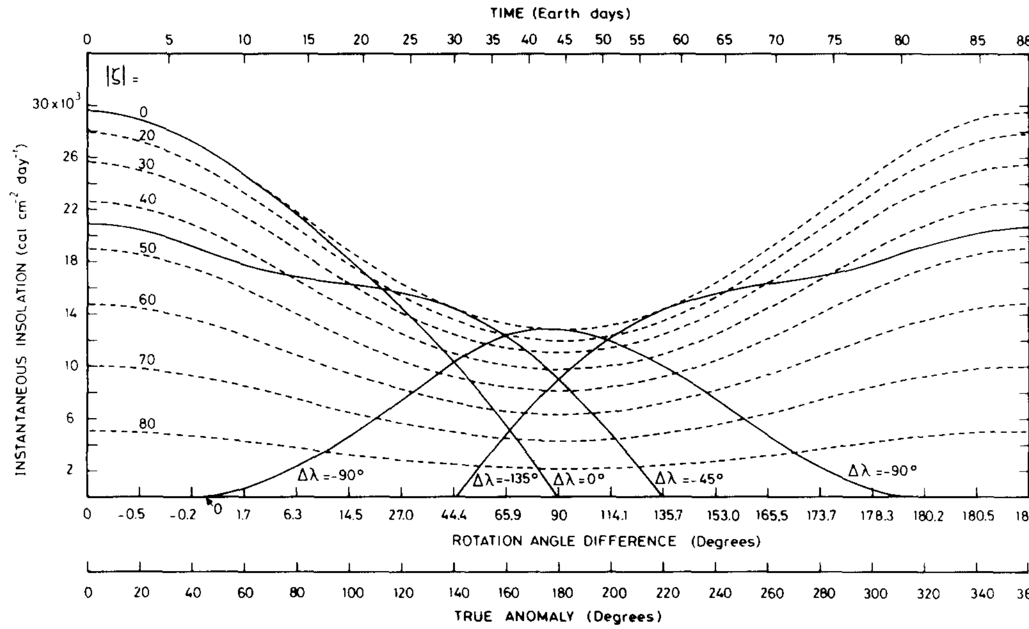

FIG. 3. Instantaneous insolation at the top of Mercury as a function of true anomaly (W), rotation angle difference ($nt - W$) or time (t). The dashed lines represent the theoretical insolation curves for various values of the angle of incidence $\zeta = \lambda - \lambda_{\odot} = \Delta\lambda + nt - W$. The solar radiation incident for surface elements characterized by longitude differences $\Delta\lambda$ equal to 0, -45, -90, and $-135°$ is given by the full lines. The curves correspond to $\varphi = 0$.

rotation angle difference ($nt - W$) obtained from the following formulae:

$$\sin E = (1 - e^2)^{1/2} \sin W / (1 + e \cos W) \quad (21)$$

and

$$t = (1/n_0)(E - e \sin E), \quad (22)$$

where E is the eccentric anomaly.

Figure 3 can be applied to any meridian $\Delta\lambda$ to obtain the solar heat flux arriving at a given time on an horizontal unit area of the upper boundary of the atmosphere at a given equatorial point on the planet Mercury. The first step for determining the corresponding insolation consists in adding algebraically $\Delta\lambda$ to the numerical value of the expression $nt - W$ leading to the angle of incidence ζ. Then, the instantaneous insolation can be found by interpolating between the curves of constant ζ. To illustrate the sensitivity of the insolation to changes in $\Delta\lambda$ we have also plotted in Fig. 3 the solar radiation incident for $\Delta\lambda = 0$, -45, -90, and $-135°$.

It is interesting to note that the knowledge of the insolation for $\Delta\lambda$ ranging from 0 to $-90°$ allows also the determination of I relative to another meridian outside the above mentioned interval; in this case, the following relations have to be taken into account:

$$I(\Delta\lambda + 180°, t + T_0) = I(\Delta\lambda, t) \quad (23)$$

and

$$I(-90° - \alpha, T_0/2 + \Delta t) = I(-90° + \alpha, T_0/2 - \Delta t). \quad (24)$$

Expression (23) indicates that the curves $I(\Delta\lambda)$ and $I(\Delta\lambda + 180°)$ are identical but phase shifted over a sidereal period of revolution T_0; indeed, during the course of one revolution T_0($\cong$88 days), the planet Mercury spins exactly 1.5 times on its axis. This means that, for example, the position of the meridian $\Delta\lambda$ at $t = 0$ will be occupied by the meridian $(\Delta\lambda + 180°)$ at a time t equal to T_0.

On the other hand, during the first half of the revolution period ($t = T_0/2 \cong 44$ days) the planet effectuates a $\frac{3}{2}$ spin–orbit coupling, implying that the meridians $(-90° + \alpha)$ and $(90° + \alpha)$ occupy at $t = 44$ days, i.e., at aphelion, symmetrical positions with respect to the Sun–Mercury direction. During the second half of the revolution period the instantaneous insolation on the meridian $(-90° - \alpha)$ at $t = T_0/2 + \Delta t$ equals the solar radiation falling on the meridian $(-90° + \alpha)$ at $t = T_0/2 - \Delta t$; hence it follows that the two insolation curves are symmetric with respect to the ordinate $t = T_0/2$ or $W = 180°$. These findings are illustrated in Fig. 3 for the curves $\Delta\lambda = -45°$ and $\Delta\lambda = -135°$ $(\alpha = 45°)$.

Figure 3 also reveals that in the vicinity of the perihelion ($t = 0$ and $t = 88$, respectively) the numerical value of the expression $(nt - W)$ is slightly negative for $0° < W < 45°$ and scarcely above 180° for $315° < W < 360°$. The reason for this phenomenon is that in the neighborhood of perihelion the Mercurian angular velocity of revolution $(\dot{W})$ exceeds the rotational angular velocity (n) of the planet on its own axis (see also Soter and Ulrichs, 1967; Han-Shou Liu, 1968; Turner, 1978). This effect is illustrated in Fig. 4, the curve representing the variability of $\dot{W}$ being obtained by Kepler's second law.

$$\dot{W} = n_0(1 - e^2)^{1/2}(a/r_\odot)^2 \tag{25}$$

with

$$(a/r_\odot)^2 = 1 + e^2/2 + (2e + \tfrac{3}{4}e^3)\cos M + \tfrac{5}{2}e^2\cos 2M + (13/4)e^3\cos 3M, \tag{26}$$

where M is the mean anomaly.

Note that in expression (26) we kept only terms up to the third degree in e.

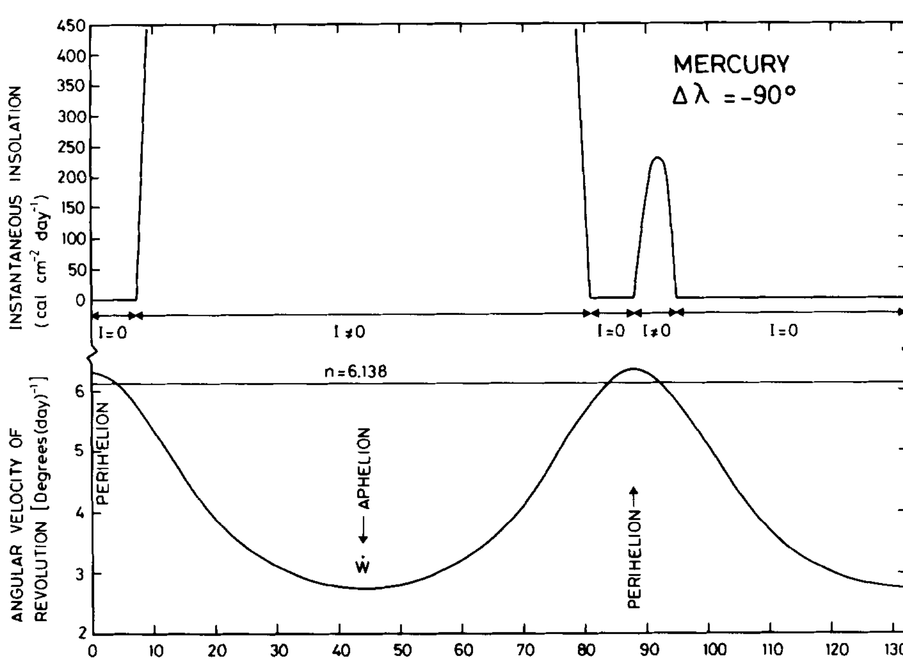

FIG. 4. Mercurian angular velocity of revolution $(\dot{W})$ and rotational angular velocity (n) of the planet on its own axis as a function of time (lower part of the figure). The upper part shows the time variability of the instantaneous insolation at the top of the planet for $\varphi = 0$ (equator).

As a consequence of this effect an interesting feature, already pointed out previously, regards the meridian $\Delta\lambda = -90°$: at the perihelion passage of the planet it is found that a very short period of weak insolation is preceded or followed by an even smaller period of complete darkness. This suprising phenomenon is clearly demonstrated in the upper part of Fig. 4. Indeed, by analyzing the instantaneous insolation curve it can be seen that a surface element is only illuminated from the eighth day ($t = 8$) with an insolation period extending to day 81 followed by a time interval of complete darkness of approximately 7 days. At the next perihelion passage of the planet, which occurs at day 88, the surface element considered here is insolated again for nearly 7 days. Hence, the solar radiation is incident until day 95. At this moment the surface element considered returns into the nonilluminated hemisphere.

3.2. *Latitudinal Variation*

It should be emphasized that in the theory of insolation presented above we only have considered a surface element located at the equator ($\varphi = 0$). However, taking into account expression (19), the instantaneous insolation received at any latitude can also easily be calculated as a function of time on a fixed meridian or as a function of any meridian at a given time.

For example, Fig. 5 illustrates the distribution of the instantaneous insolation on the meridian ($\Delta\lambda = 0$) crossing the line of apsides at the perihelion passage of the planet taken as $t = 0$. The incident solar radiation is given in contours of calories per square centimeter per day as a function of

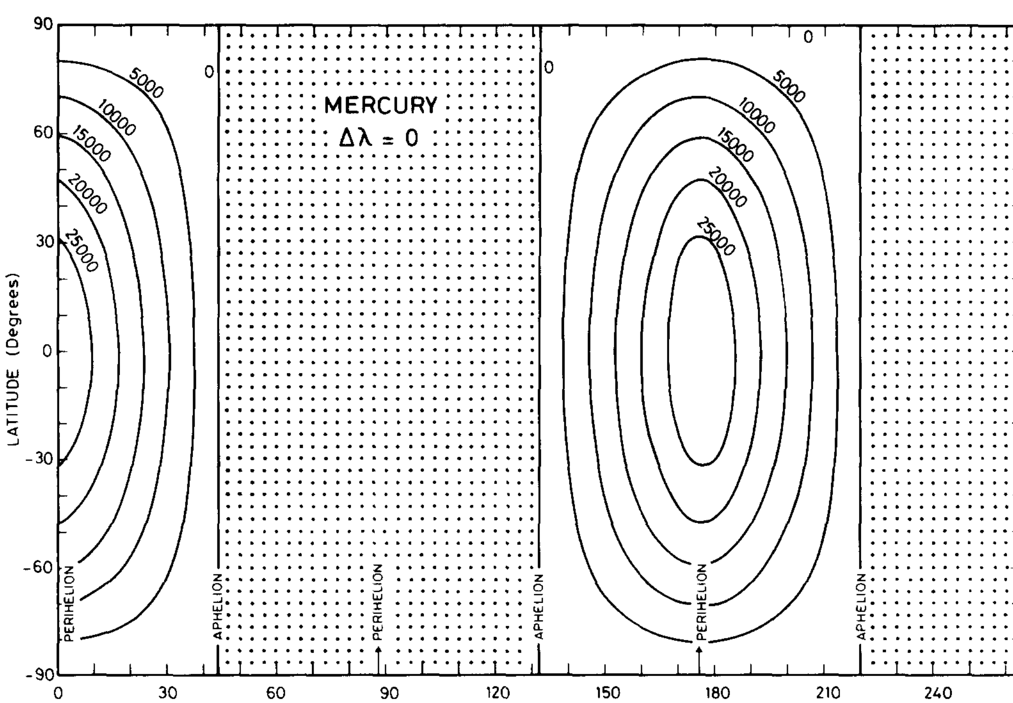

FIG. 5. Distribution of the instantaneous insolation at the top of Mercury on the meridian ($\Delta\lambda = 0$) crossing the line of apsides at the perihelion passage of the planet taken as $t = 0$. The incident solar radiation is given in contours of calories per square centimeter per day as a function of latitude (φ) and time (t) taken over 1.5 Mercurian solar days or approximately 264 Earth days. Areas of permanent darkness are dotted.

planetary latitude and time (taken over 1.5 Mercurian solar days or 264 Earth days). The ratio $T_0/T_\odot$ being about $\frac{1}{2}$, it is easily to see that the insolation curve should be repeated in a 2-Mercurian year cycle.

From the figure, it can be seen that the maximum solar radiation is incident on the equator at the first and third perihelion passage of the planet with a value of about 3×10^4 cal cm^{-2} (day)$^{-1}$. For exactly half a solar day (88 Earth days) all parts of the meridian $\Delta\lambda = 0$ are in perpetual darkness, whereas a surface element located at the poles never receives solar radiation. Furthermore, because of the zero obliquity, the distribution of the radiation incident on the Mercurian atmosphere is perfectly symmetric with respect to the equator.

When comparing our results with those of Vorob'yev and Monin (1975) some agreements as well as one striking difference are noticed. On one hand, the insolation pattern is similar and the calculated maximum radiation incident is of the same order of magnitude. On the other hand, there exists a discrepancy as to the position of the maximum. Obviously, this maximum has to occur at the perihelion passage of Mercury when the Sun is on the meridian plane but this is not the case according to the paper published by the previous authors. It is difficult to judge whether the computing algorithm or a mistake in drawing the figure is responsible for the observed disagreement.

3.3. Diurnal Insolation

For completeness, we also have investigated the diurnal insolation I_D obtained by integrating numerically expression (16) over time during the light time of the day. The integration limits may be determined from the following relations:

$$nt_2 - W(t_2) = 270^\circ - \Delta\lambda \qquad (27)$$

and

$$nt_1 - W(t_1) = 90^\circ - \Delta\lambda. \qquad (28)$$

In Fig. 6 we have plotted the variability

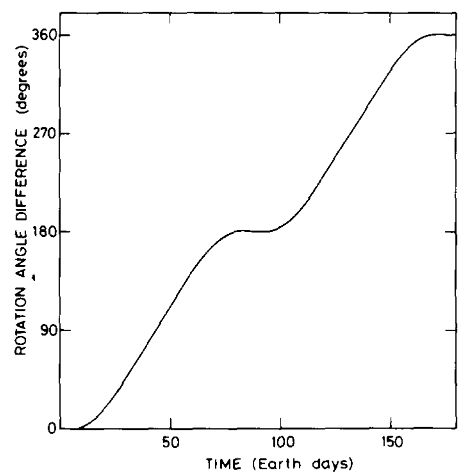

FIG. 6. Rotation angle difference ($nt - W$) for Mercury as a function of time (t).

of the function $f(t) = nt - W(t)$ over a time period equal to one Mercurian solar day and where $W(t)$ is given by formula (6).

The procedure to obtain t_1 and t_2 is as follows. For a fixed meridian $\Delta\lambda$ one calculates the corresponding functions $f(t_2)$ and $f(t_1)$ given by expressions (27) and (28). Then, t_1 and t_2 can easily be found from Fig. 6. The accuracy obtained with this graphical method to evaluate the diurnal insolation is sufficiently high considering the small value of the solar radiation incident in the neighborhood of the lower and upper time limits.

The variability of the diurnal insolation on the equator as a function of the longitude difference between the meridian of a surface element and the meridian crossing the line of apsides at the perihelion passage of the planet is given in Fig. 7. It can be seen that the diurnal insolation decreases by nearly a factor of 3 as the longitudinal difference increases from 0 to 90°. This effect has, to the best of our knowledge, never been reported previously.

4. SOLAR RADIATION INCIDENT ON VENUS

4.1. Instantaneous Insolation

The instantaneous insolation variation at the top of the planet Venus and for a sur-

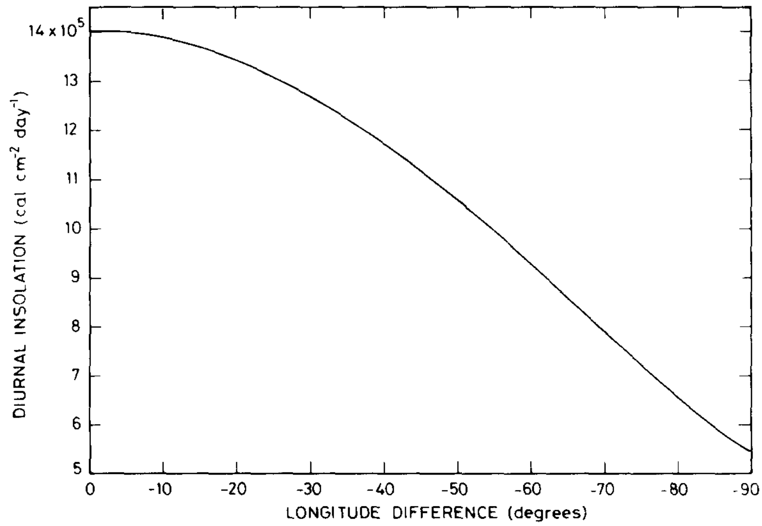

FIG. 7. Diurnal insolation at the top of Mercury as a function of the longitude difference $\Delta\lambda$. The curve corresponds to the equator.

face element characterized by the coordinates (λ_0, φ) or $(\Delta\lambda, \varphi)$ can also be determined from relations (5)–(8) and (13).

Introducing the numerical values for S_0, $a_\odot$, and e (Table I), expression (5) may be written under the following form:

$$I = 5.340 \times 10^3(1 + 0.00678 \cos W)^2[\sin \varphi \sin \delta_\odot + \cos \varphi \cos \delta_\odot \cos(\lambda - \lambda_\odot)]. \quad (29)$$

It should be pointed out that, taking into account the small value of the eccentricity of the orbit ($e = 0.00678$), formula (6) may be simplified. Keeping only terms up to the first degree in e, the true anomaly W is now, in a very good approximation, given by:

$$W = n_0 t + 2e \sin n_0 t. \quad (30)$$

Following Vorob'yev and Monin (1975) we first calculated the time variation, over 3 Venusian solar days ($\cong$350 Earth days), of the instantaneous insolation on the meridian ($\Delta\lambda = 0$) crossing the line of apsides at the planet's perihelion passage. Application of expression (29) leads to the isocontours shown in Fig. 8.

From an analysis of the solar radiation it may be concluded that the maximum is incident near the equator with a value of approximately 5.4×10^3 cal cm^{-2} (day)$^{-1}$. This maximum occurs at the first perihelion passage ($t = 0$), whereas the position of the other maxima is slightly shifted with respect to the next aphelion or perihelion passages. It can be seen that for about half a solar day ($\cong$58 Earth days) all parts of the Venusian meridian $\Delta\lambda = 0$ are in permanent darkness except for high latitudes where the periods of nonillumination (or illumination) increase in both hemispheres from half a solar day to approximately one solar day at the poles. Owing to the very small obliquity, the insolation distribution is quasi-symmetric with respect to the planet's equator. At the poles, however, this symmetry vanishes completely.

Furthermore, it is obvious that the ratio T_0/T ($\cong$0.92) cannot be expressed, as for Mercury, by any simple rational fraction although the numerical value is only slightly different from unity. It follows, as already stated by Vorob'yev and Monin (1975), that the instantaneous insolation pattern is only quasi-periodic in nature and that it should roughly be repeated in a half-

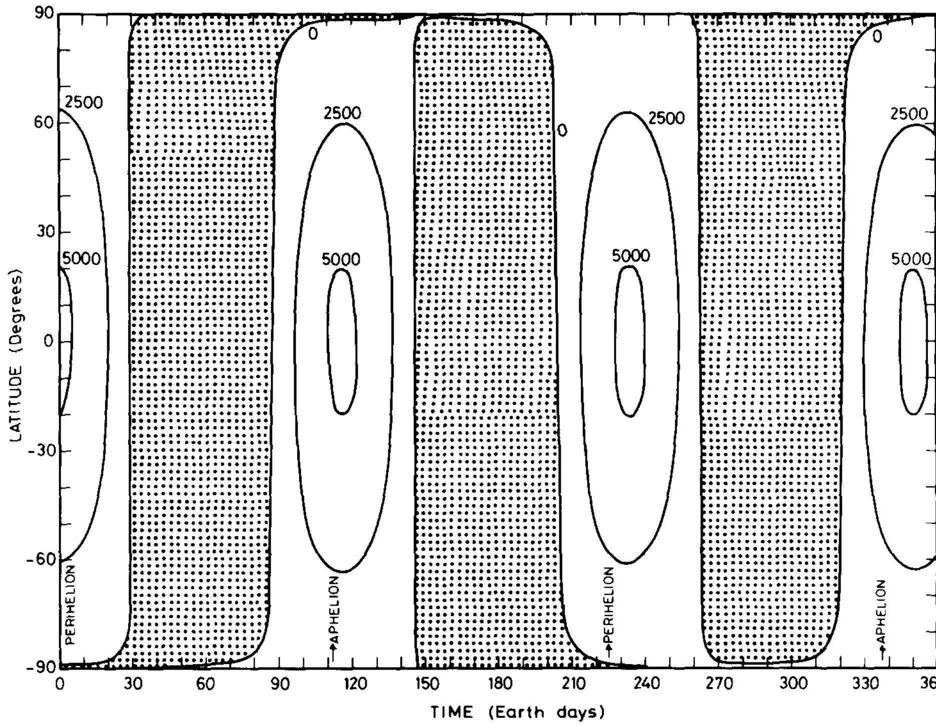

FIG. 8. Distribution of the instantaneous insolation at the top of Venus on the meridian ($\Delta\lambda = 0$) crossing the line of apsides at the perihelion passage of the planet taken as $t = 0$. The incident solar radiation is given in contours of calories per square centimeter per day as a function of latitude (φ) and time (t) taken over 3 Venusian solar days or approximately 350 Earth days. Areas of permanent darkness are dotted.

year cycle. The deviation from rigorous periodicity is, however, small since both the eccentricity and the obliquity are small.

Our results are compared with the data of Vorob'yev and Monin (1975). The insolation curves are in reasonable agreement at all latitudes but obviously differ at the polar regions. Although the maximum radiation is identical, there does exist a difference relative to its position. Again, it is difficult to elucidate the problem of phase shift.

For completeness, analogous calculations of the insolation as a function of time and latitude and for various values of $\Delta\lambda$ were made. Comparison of a series of figures, not presented in this paper, reveals that the solar radiation difference is extremely small on passage from one meridian to another. This effect is ascribed to the small eccentricity ($e = 0.00678$) of Venus.

4.2. *Diurnal Insolation*

Finally, the diurnal insolation I_D as a function of latitude and for a fixed meridian ($\Delta\lambda = 0$) was studied by numerical integration of relation (16). It should be noted that the upper and lower time limits were taken from Fig. 8.

The diurnal insolation I_D for the first solar day, arbitrary taken from $t = 0$ to $t = T_{\odot}$, is represented in Fig. 9. From this figure it can be seen that the maximum diurnal insolation occurs over the equator with a value of approximately 2×10^5 cal cm^{-2} (day)$^{-1}$. A significant hemispheric asymmetry exists in that there is considerably more insolation over the southern polar regions than over the northern polar regions during this solar day. For comparison, the diurnal insolation received at the South and the North Pole

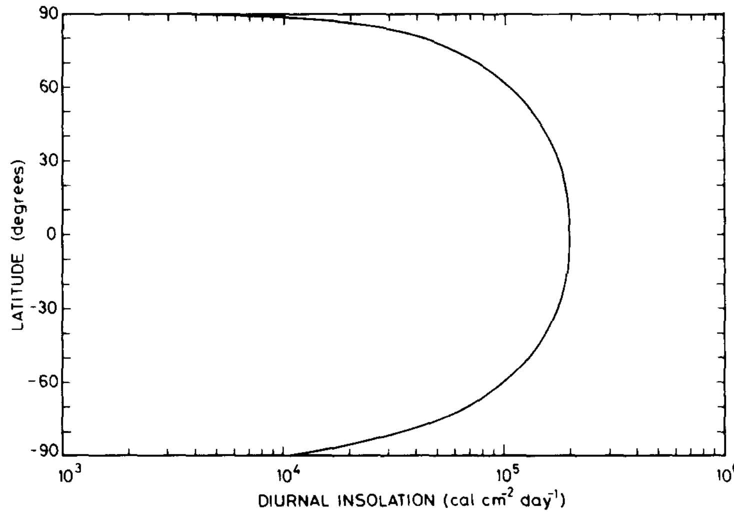

FIG. 9. Diurnal insolation at the top of Venus ($\Delta\lambda = 0$) for the first solar day, arbitrary taken from $t = 0$ to $t = T_\odot$ as a function of latitude (φ).

amounts to about 1.1×10^4 and 2.9×10^3 cal cm^{-2} (day)$^{-1}$ respectively for the specific day mentioned above.

This phenomenon is attributed to the fact that the total amount of the diurnal insolation, especially at polar region latitudes, is strongly dependent upon the arbitrary chosen lower time limit of the so-called first solar day. Indeed, integration of expression (16) over a period equal to the planet's solar day $T_\odot$ but for lower time limits $t \neq 0$ (where $t = 0$ was taken at the planet's perihelion passage) yields numerical values different from those obtained for $t = 0$. For example, if $t = 0$ is taken at the time of sunrise at the South Pole, the Sun remains above its horizon over a whole solar day, whereas the North Pole is in permanent darkness. On the other hand, if $t = 0$ corresponds to the time of maximum instantaneous insolation on one of the poles, the diurnal insolation on both poles is approximately equal.

6. CONCLUSIONS

The solar radiation incident at the top of the atmospheres of the planets Mercury and Venus depends on a certain number of parameters related to the heliocentric orbits of those two planets and also to the position of the surface element sensed by this radiation. The intensity of the radiation at a given time is fixed by the heliocentric distance of the planet and by the zenith angle of the Sun.

For Mercury, the effect of the heliocentric distance causes a relative variation by a factor of about 2.3 between perihelion and aphelion position; this result associated with the slow rotation period of Mercury on its own axis implies a great diversity in the instantaneous insolation curves obtained at the equator on points of different longitudes; pertaining to the diurnal insolation, a similar conclusion can be drawn, the maximum relative variation attaining a factor of about 3. Moreover, in the neighborhood of the perihelion passage, the large eccentricity of Mercury leads to an angular velocity of revolution exceeding the rotational angular velocity of the planet on its own axis. As a consequence of this effect, a very short period of weak insolation is preceded or followed by an even smaller period of complete darkness. The above-mentioned phenomena were not found for the planet Venus, the orbital eccentricity being nearly equal to zero. On the other hand, the small obliquity of its orbit give rise to an important seasonal effect in the vicinity of the poles. On Mercury, this effect is nonexistent, the distribution of the incident solar radiation being perfectly symmetric with respect to the equator. This symmetrical behavior, which one practically observes also outside the polar regions of Venus, yields to an identical rate of decrease of the insolation with increasing latitude in both hemispheres.

The periodicity of the instantaneous insolation curves is fixed by the length of the solar day of the planets: exactly 2 tropical years in the case of Mercury and approximately half a tropical year for Venus. It should be emphasized that, in reality, the diurnal insolation variations on Venus are only quasi-periodic in nature when taking into account that both the eccentricity of the orbit and the obliquity are nonzero, although very small.

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