SOME EVOLUTION AND STABILITY TRENDS DEDUCED FROM ENERGY AND WATER VAPOR BALANCE MODELS

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ABSTRACT

A hierarchy of models of climatic evolution is considered. First, an energy balance model is used to analyze the influence of systematic increase of solar energy output over the last hundred million years. Plausible scenarios of evolution of infrared cooling rates, of heat transfer coefficient and of polar temperature are constructed. Next, the dynamical coupling between humidity and temperature is considered at the level of a two-variable planetary model. The stability properties of the steady-state climatic regime of the last 250 myr are discussed both analytically and by numerical simulations.

RESUME

On examine une série de modèles d'évolution climatique. En premier lieu, on utilise un modèle de bilan énergétique pour analyser l'influence de l'accroissement systématique de la constante solaire durant les dernières centaines de millions d'années. On developpe de scénarios plausibles d'évolution des coefficients de refroidissement infra-rouge, du coefficient de transfert de chaleur ainsi que de la température polaire. Ensuite, on considère les effets du couplage dynamique humidité-température au niveau d'un modèle planétaire à deux variables et on analyse les propriétés de stabilité du régime climatique des dernières 250 millions d'années.

1. INTRODUCTION

The influence of solar output on surface temperature of the earth has been analyzed by Budyko (1969) and Sellers (1969) on the basis of the ice-albedo feedback. They found that a slight variation of the solar constant can induce climatic catastrophes associated with transitions to an ice-covered or an ice-free earth.

Recently, it was pointed out that the sun is a variable star whose energy output has systematically increased over the past billion years (Neumann and Rood, 1977). Yet, as well known, there has been no glaciation during the mesozoic and early cenozoic eras. In an attempt to resolve this apparent paradox, Sagan and Mullen (1972) invoke the possibility of an enhanced greenhouse effect (due, for instance, to an increased NH_3 concentration in the atmosphere) in the framework of a global energy balance model at the planetary scale. Moreover, by extrapolating their radiative calculations to the future they predict a catastrophic increase in temperature due to a runaway greenhouse effect fed-back positively by the increasing H_2O vapor concentration in the atmosphere.

Implicit in the above considerations is the assumption that the various coefficients appearing in the energy balance equation can be parameterized in terms of the percentage cloud cover and/or the H_2O concentration at ground level, which therefore appear to play a passive role. This is certainly reasonable for short-time predictions associated with slight variations of the thermal regime. On the other hand, in the presence of abrupt transitions that could possibly be induced by the various feedbacks present over long periods of time, this assumption is expected to break down.

The purpose of the present communication is twofold. We first attempt to analyze some global trends of climatic evolution in the past 250 myr up to the beginning of quaternary glaciations, using a model involving latitudinal energy transfer. The model presented in section 2, incorporates the effect of evolving solar output, of infrared cooling, and of energy transport. We next turn, in section 3, to the modelling of the simultaneous evolution of temperature, T and relative humidity, h on an equal footing. The resulting equations, which are considered at a planetary scale, turn out to be very difficult to analyze because of the unknown form of cloud cover and precipitation rates as a function of T and h. For this reason we limit ourselves, in section 4, to qualitative methods, particularly to a linear stability analysis of present and past climatic regimes, using some data recently compiled by Sasamori (1975). The analysis suggests the existence of certain sources of instability arising from the positive feedback of humidity on temperature and vice-versa. However, using values of parameters close to present day ones, it is found that the steady-state climatic regime remains stable. Some representative evolution trajectories of T and h are briefly discussed in section 5, whereas section 6 summerizes the results.

2. ENERGY BALANCE MODEL

The starting point is the energy balance equation of the earth-atmosphere system in the form written by North (1975a, b) :

$$C_p \frac{\partial T}{\partial t} = \bar{Q}\, S(\underset{\sim}{r}) \left[1 - \alpha(\underset{\sim}{r}, \underset{\sim}{r}_s) \right] - I(T) + \lambda \nabla^2 T \qquad (1)$$

C_p is the heat capacity, λ the heat transfer coefficient, I the infrared cooling rate, $\bar{Q}$ the solar constant, $S(\underset{\sim}{r})$ the percentage of incident flux at position $\underset{\sim}{r}$, and $\alpha(\underset{\sim}{r}, \underset{\sim}{r}_s)$ the albedo. Following Budyko (1969) α is to be approximated by a discontinuous function around $\underset{\sim}{r}_s$, the locus of the ice boundary. However, we are here interested in the climatic history of the past 250 myr or so, up to the quaternary period. It will therefore be legitimate to restrict eq. (1) to an ice-free earth and hence set :

$$\alpha(\underset{\sim}{r}, \underset{\sim}{r}_s) = \alpha_o \tag{2}$$

Moreover, we will adopt the commonly used expression for the infrared cooling rate :

$$I(T) = A + BT \tag{3}$$

where T is now expressed in degrees centrigrade, and the values of the cooling coefficients A and B include the effect of cloud cover.

We shall regard $\bar{Q}$ as slowly varying in time according to the law suggested in Neumann and Rood (1977) :

$$\frac{1}{L}\frac{dL}{dt} \simeq \frac{12.5 \times 0.01}{1 + 1.66\, X_o - 1.66 \times 10^{-2}\, t} \tag{4}$$

where L is the luminosity of the sun, X_o is the initial hydrogen mass fraction and t is the time in billions of years.

Finally, the mean annual latitudinal distribution of radiation S(x) can be expressed in Legendre polynomials as follows (North, 1975a, b) :

$$S(x) \simeq 1 + S_2\, P_2(x) \simeq 1 - 0.482\, \frac{3\, x^2 - 1}{2} \tag{5}$$

where x is the sine of the latitude, and the factor S_2 is fitted from astronomical data.

We now insert eq. (2) to (5) into eq. (1). It is convenient to express the result in spherical coordinates. We also perform a longitudinal average and observe that the evolution of T due to planetary factors is much shorter than that arising by the evolving solar output. Hence we regard the long-term evolution of T as a sequence of quasi-steady states each one corresponding to the value of $\bar{Q}$ appropriate for a given epoch. We finally obtain :

$$\frac{d}{dx}(1 - x^2)\frac{d}{dx} I(x) - \frac{I(x)}{D} + \frac{3\bar{Q}(1 - \alpha_o)}{2D}\, S_2\, x^2 = \frac{\bar{Q}(1 - \alpha_o)}{D}\left(\frac{S_2}{2} - 1\right) \tag{6}$$

where $D = \lambda / r_o^2 B$ and r_o is the earth's radius. This equation is subject to two boundary conditions expressing the absence of heat transport at the poles and across the equator :

$$(1 - x^2)^{1/2} \left. \frac{dI}{dx} \right|_{x = 1} = (1 - x^2)^{1/2} \left. \frac{dI}{dx} \right|_{x = 0} = 0 \tag{7}$$

Eq. (6) and (7) were analyzed in some detail in a previous communication (Nicolis, 1978). The exact solution satisfying the boundary conditions is :

$$A + BT(x) \equiv I(x) = \frac{\bar{Q}(1 - \alpha_o)}{2D + \frac{1}{3}} \left(2D + \frac{1}{3} - \frac{1}{6} S_2 + \frac{1}{2} S_2 x^2\right) \tag{8}$$

From this expression one can express the equatorial temperature, corresponding to the value of $\bar{Q}$ at a given epoch as deduced from eq. (4) and the present-day value $\bar{Q} = 1.918/4 = 0.479$ cal min^{-1} cm^{-2}

$$A + BT_{eq} = \bar{Q}(1 - \alpha_o) \left(1 - \frac{S_2}{12D + 2} \right) \tag{9a}$$

or equivalently :

$$D = \frac{1}{6} \; \frac{\bar{Q}(1 - \alpha_o) \left(1 - \frac{S_2}{2}\right) - (A + B\,T_{eq})}{A + B\,T_{eq} - \bar{Q}(1 - \alpha_o)} \tag{9b}$$

Substituting (9b) into eq. (8) one can then compute the polar temperature in terms of T_{eq}, $\bar{Q}$, α_o and the infrared cooling parameters A, B in the form :

$$T_p = 3 \, \frac{\bar{Q}(1 - \alpha_o) - A}{B} - 2\,T_{eq} \tag{10}$$

From this expression we can reconstruct plausible pathways of evolution of the polar temperature as follows. We begin by requiring a more or less invariant equatorial temperature T_{eq} throughout the past 250 myr, say 25°C, in accordance with paleoclimatic data. We also argue that the cooling coefficients A, B must have been less than the present day ones through an enhanced greenhouse effect (Sagan and Mullen, 1972; Budyko, 1974, 1977). To account for such a possibility we vary A, B for each epoch, between the present-day values used in North (1975a) $A = 0.288$ cal min^{-1} cm^{-2}, $B = 0.00208$ cal min^{-1} cm^{-2}K^{-1} and values less than the present-day ones by 1% up to 10%. We also vary the albedo α_o in a similar fashion.

Most of these variations give unacceptable values for the thermal transfer coefficient D (eq. (9b)) and/or for T_p (eq. (10)). As a matter of fact, the results are rather sensitive functions of the parameters as illustrated in Fig. 1. This already eliminates a great number of combinations

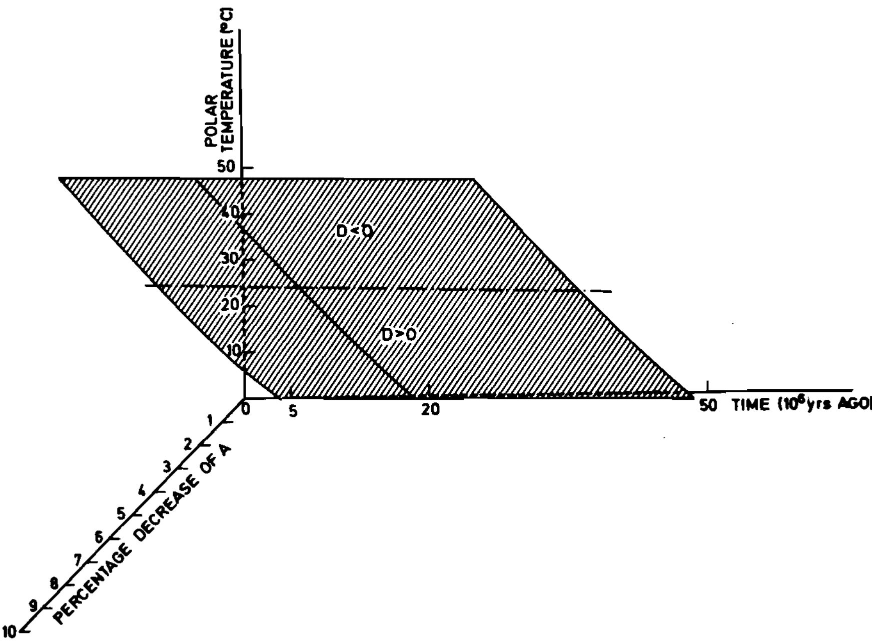

Fig. 1.- Polar temperature as a function of the percentage decrease in A and the heat influx $\bar{Q}(1 - \alpha_o)$(or equivalently, of the time in myr ago). The equatorial temperature is taken equal to 25°C.

of these parameters. Among the remaining ones we select those combinations which give an evolution of T_p toward freezing values as time evolves to the beginning of quaternary glaciations. Fig. 2 represents two pathways of evolution of T_p determined from the above described procedure. We see that past values of A, B are smaller than the present ones by a few percent whereas the heat transfer coefficient decreases systematically in time. Both trends are compatible with currently available information on paleoclimates. In particular, the decrease of D can be attributed, at least in part, to the increasingly poor equator-pole energy exchange arising from the progressive isolation of the Arctic during the last tens of millions of years (see also Budyko, 1969).

We thus obtain :

$$C_p \frac{dT}{dt} = \bar{Q}\,[1 - \lambda_o - \mu_o\, n(T, h)] - \bar{A} - \bar{B}T + (A_1 + B_1 T)\, n(T, h)$$

$$- L\,\chi\, q_s(T)\,(1 - h) + Lr(T, h) \qquad (16)$$

The main difficulty with eq. (14) and (16) lies in the occurence of the unknown functions $n(T, h)$ and $r(T, h)$. In this respect however, Sasamori (1975) has compiled data enabling the evaluation of the derivatives of these functions for present-day climatic conditions. As we show in the next section, this information can be used to make some predictions about the stability and other qualitative properties of the coupled temperature-humidity system.

4. LINEAR STABILITY ANALYSIS

Let (T_o, h_o) be a steady-state solution of eq. (14) and (16) corresponding to present-day climatic conditions or to one of the past climatic conditions depicted in the scenario of Fig. 2. We choose this as a reference state and look for the evolution in its vicinity following an initial perturbation. Such perturbations are of course inevitable in a complex system like the earth-atmosphere one. The question is whether the system will counteract them and return to the reference state (we will then say that it is asymptotically stable), or whether on the contrary the perturbations will be amplified (the reference state will then be unstable) and drive the system to a new climatic regime. Stability theory (Minorski, 1962) authorizes us to analyze this question by linearizing eq. (14) and (16) around the reference state. To this end, we set

$$\begin{aligned} T &= T_o + \delta T(t) \\ h &= h_o + \delta h(t) \end{aligned} \qquad (17)$$

Substituting into eq. (14) and (16), expanding the right hand side in Taylor series around (T_o, h_o) and neglecting quadratic or higher terms, we obtain :

$$C_p \frac{d\delta T}{dt} = \Bigg[- \bar{Q}\,\mu_o \left(\frac{\partial n}{\partial T}\right)_o - \bar{B} + (A_1 + B_1 T_o)\left(\frac{\partial n}{\partial T}\right)_o + B_1 n_o$$

$$+ L\left(\frac{\partial r}{\partial T}\right)_o - \chi\, q_s(T_o)\, L \frac{5{,}385}{(273 + T_o)^2}\,(1 - h_o) \Bigg]\,\delta T$$

$$+ \Bigg[- \bar{Q}\,\mu_o \left(\frac{\partial n}{\partial h}\right)_o + (A_1 + B_1 T_o)\left(\frac{\partial n}{\partial h}\right)_o$$

$$+ L\left(\frac{\partial r}{\partial h}\right)_o + \chi\, q_s(T_o)\, L\Bigg]\,\delta h \qquad (18a)$$

$$N\left(\frac{d\delta h}{dt} + h_o \frac{5{,}385}{(273 + T_o)^2} \frac{d\delta T}{dt}\right) = \frac{1}{q_s(T_o)} \left[\frac{5{,}385}{(273 + T_o)^2} r_o - \left(\frac{\partial r}{\partial T}\right)_o \right] \delta T$$

$$- \left[\chi + \frac{1}{q_s(T_o)} \left(\frac{\partial r}{\partial h} \right) \right] \delta h \qquad (18b)$$

We have set $n_o = n(T_o, h_o)$, $r_o = r(T_o, h_o)$. Note that T_o is to be calculated by integrating expression (8) over latitude, whereas h_o, r_o are determined from the steady-state conditions $E(T_o, h_o) \simeq r(T_o, h_o)$. Adopting again the quasi-steady state picture discussed in section 2, we may regard the coefficients of δT and δh in eqs. (18) as time-independent. Hence we seek for solutions of the form

$$\begin{aligned} \delta T &= \widehat{\delta T}\, e^{\omega t} \\ \delta h &= \widehat{\delta h}\, e^{\omega t} \end{aligned} \qquad (19)$$

and compute ω from the characteristic equation. If it turns out that $\mathrm{Re}\omega > 0$ for at least one of the roots of this equation. (T_o, h_o) will be unstable. If $\mathrm{Re}\omega < 0$ for both roots, then (T_o, h_o) will be asymptotically stable.

To simplify notation we write (18a), (18b) in the form

$$\begin{aligned} \omega\widehat{\delta T} &= \alpha\widehat{\delta T} + \beta\widehat{\delta h} \\ \omega\widehat{\delta h} + \omega h_o \frac{5{,}385}{(273 + T_o)^2} \widehat{\delta T} &= \gamma\widehat{\delta T} + \varepsilon\widehat{\delta h} \end{aligned} \qquad (20)$$

where α, β, γ, ε are defined by comparing eq. (20) to eqs. (18). The characteristic equation then reads :

$$\omega^2 - \left(\alpha + \varepsilon - \frac{5{,}385\, h_o}{(273 + T_o)^2} \beta\right) \omega + (\alpha\varepsilon - \beta\gamma) = 0$$

or $$\omega^2 - T\omega + \Delta = 0 \qquad (21)$$

Depending on the signs and relative magnitudes of T and Δ we will have monotonic or oscillatory damping, oscillatory instabilities or saddle point behavior. Moreover, the sign of α, β, γ, ε will give us the way humidity and temperature feed back into their own rate of change or on the rate of change of the other variable. On inspecting the complete expressions for these coefficients, eqs. (18), one could then see, for example, how the cloud cover acts on a global scale to affect the system's dynamics. This analysis is carried out in the next section.

5. RESULTS

Eqs. (18) to (21), have been evaluated numerically as follows. Values of $\bar{Q}$ and of the infrared cooling coefficients are chosen for various epochs according to a particular scenario, for instance that represented by curve b) of Fig. 2. Next, the derivatives

$$\left(\frac{\partial n}{\partial T}\right)_o , \quad \left(\frac{\partial n}{\partial h}\right)_o , \quad \left(\frac{\partial r}{\partial T}\right)_o , \quad \left(\frac{\partial r}{\partial h}\right)_o$$

are computed from the "sensitivity factors" recently evaluated in Sasamori (1975). The remaining factors μ_o, L, χ are taken from thermodynamic data and from Budyko (1974). A first result is that, throughout the past 250 myr, the coefficient α remains negative. According to eq. (18a) this means that for a fixed relative humidity, the thermal regime itself tends to be stable*. A similar property holds for the humidity equation, namely $\varepsilon < 0$. Note that, from eq. (18a), the coefficient α itself contains a purely thermal contribution and a contribution due to humidity. The latter turns out to be even larger in absolute value than the purely thermal one. Thus, the direct effect of humidity on temperature amounts to a strong negative feedback.

The situation is very different with the coupling coefficients β and γ, which turn out to be both positive and large. In other words the dynamical coupling between humidity and temperature amounts to a strong positive feedback.

Potentially, the competition between these two opposing tendencies-stabilizing trend through α and ε, and a destabilizing one through β and γ-can give rise to a breakdown of stability of the reference state. Yet, on numerically evaluating the coefficients one finds that the factors T and Δ in the characteristic equation (21) are, respectively, negative and positive with $T^2 - 4\,\Delta > 0$. This means that both roots of this equation, say ω_1 and ω_2 (with $|\omega_1| < |\omega_2|$) are real and negative. According to stability theory (Minorski, 1962) the steady state (T_o, h_o) is therefore stable and behaves like a node.

The next point of interest concerns time scales. It appears that $|\omega_1|$, which is of the order of 10^{-8} min^{-1}, is smaller than $|\omega_2|$ by a factor of at least 10^3. Thus, one of the stable modes relaxes to zero at a relatively fast scale, whereas the other one evolves more slowly at a geological time scale. More precisely, the way these two modes are superposed

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* The results persist even when $\frac{\partial n}{\partial T}$ is varied in the range - 0.01 to - 0.03. This corroborates an idea developed by Budyko (1974, 1977) and Cess (1976) that cloudiness feedback is not particularly effective in affecting the thermal regime.

in the time development of $\delta T(t)$ and $\delta h(t)$ is given by the equations :

$$\delta T(t) = \frac{-\beta\,\delta h_o + (\omega_2 - \alpha)\,\delta T_o}{\omega_2 - \omega_1}\, e^{\omega_1 t} + \frac{\beta\,\delta h_o + (\alpha - \omega_1)\,\delta T_o}{\omega_2 - \omega_1}\, e^{\omega_2 t} \tag{22}$$

$$\delta h(t) = \frac{\omega_1 - \alpha}{\beta}\,\frac{-\beta\,\delta h_o + (\omega_2 - \alpha)\,\delta T_o}{\omega_2 - \omega_1}\, e^{\omega_1 t} + \frac{\omega_2 - \alpha}{\beta}\,\frac{\beta\,\delta h_o + (\alpha - \omega_1)\,\delta T_o}{\omega_2 - \omega_1}\, e^{\omega_2 t} \tag{23}$$

where δT_o, δh_o are the initial values of the perturbations.

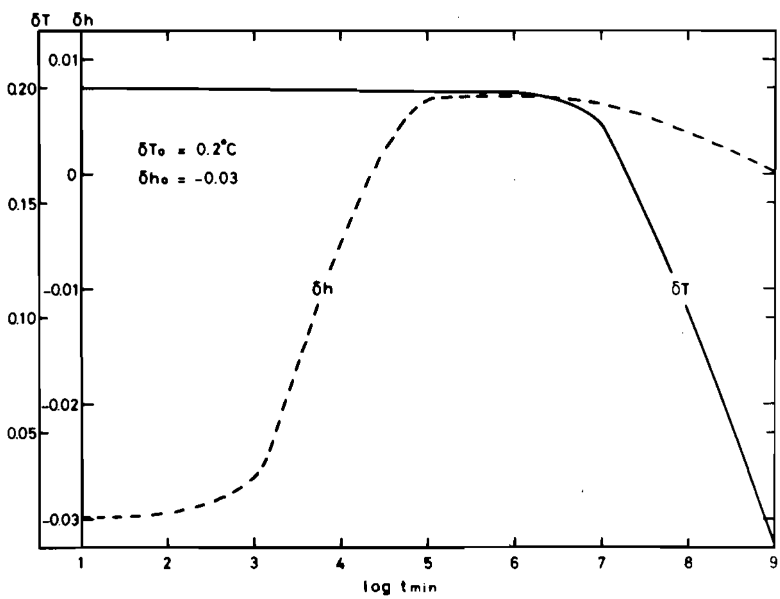

Fig. 3.- Time evolution of temperature and relative humidity perturbations for δT_o = 0.2°C, δh_o = - 0.03. The following values are chosen : Q = 0.479 cal cm^{-2} min^{-1}, $\bar{A}$ = 3.24 x 10^{-1} cal cm^{-2} min^{-1}, A_1 = 6.94 x 10^{-2} cal cm^{-2} min^{-1}, $\bar{B}$ = 3.24 x 10^{-3} cal cm^{-2} min^{-1} K^{-1}, B_1 = 2.31 x 10^{-3} cal cm^{-2} min^{-1} K^{-1}, T_o = 15°C, n_o = 0.5, r_o = 1.8 x 10^{-4} g cm^{-2} min^{-1}, χ = 7.5 x 10^{-2} g cm^{-2} min^{-1}, h_o = 0.77, μ_o = 0.26, C_p = 3.5 x 10^5 cal cm^{-2}

Fig. 3 and Fig. 4 describe the time course of temperature and humidity as given by eqs. (22) and (23) for some representative values of initial conditions. A significant feature is the occurence of overshoots (or undershoots) before the stage of systematic decrease of the perturbations and the ultimate stabilization toward the reference state is reached.

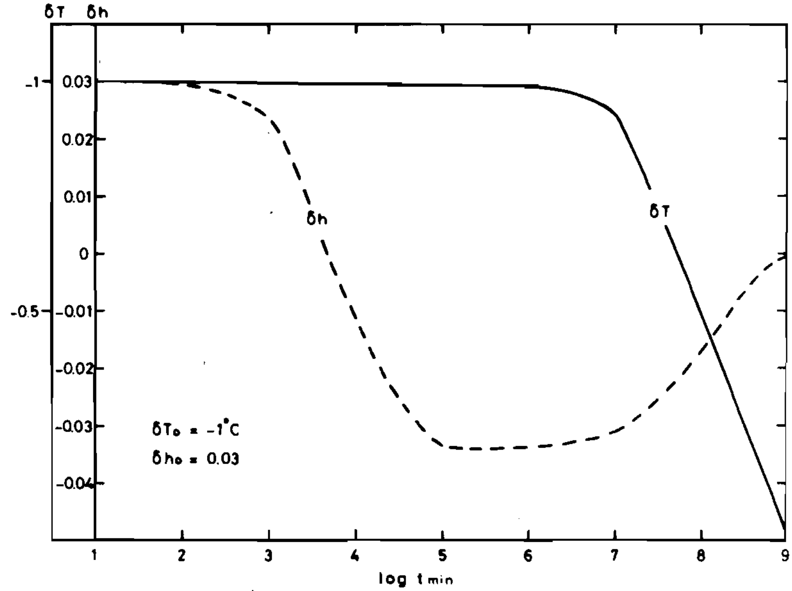

Fig. 4.- Same phot as in Fig. 3, but for initial conditions $\delta T_o = -1°C$, $\delta h_o = 0.03$.

6. CONCLUDING REMARKS

We have seen that simple energy balance models are capable of reproducing some general trends of past climatic evolution. At each epoch the latter is characterized by a pronounced thermal stability, although in a long time scale it is slowly modulated by the sun's evolving energy output.

The situation remains stable when a two-variable description in terms of temperature and humidity is adopted. It is found that, despite the temperature-humidity positive feedback, the climate system is characterized by an inherent stability. As a result, there is a tendency to evolve back to the steady-state regime (T_o, h_o), although the transient behaviour may present some interesting features like the occurence of overshoots.

In carrying out the numerical simulations reported in Section 5, specific values of the various parameters had to be adopted. Moreover, the values of the "sensitivity factors" leading to the evaluation of the derivatives of n and r compiled by Sasamori, have been utilized for past climatic conditions as well. It is not impossible that the stability will be compromized when some of the parameters will vary in rather wide ranges of values remote from present-day conditions. Unfortunately,one cannot be more specific at this time because of the scarsity of paleoclimatic data regarding cloudiness n and condensation rate r.

It would be interesting to project the analysis into the future to see how the temperature-humidity feedback is modified by the systematic increase of the solar constant. Similarly, a more realistic model of two variables including latitudinal transport and/or the possibility of ice boundary is likely to add novel features. Finally, the dependence of the cooling coefficients A, B on the distribution of water vapor should eventually be taken into account following, for instance, the model developed by Cess (1974). In future work we hope to report on these points.

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