Open ion-exosphere with an asymmetric anisotropic velocity distribution

by M. SCHERER (*)
Belgian Institute for Space Aeronomy
B-1180 Brussels, Belgium

Abstract. — Expressions for the number density, the escape flux, the parallel and perpendicular momentum fluxes, and the energy flux in an open ion-exosphere, are set up under the following assumptions: (a) the velocity distribution function at the exobase is given by an asymmetric bimaxwellian depending on 4 parameters; (b) along a magnetic field line the potential energy of a charged particle is a monotonic function and the magnetic field strength monotonically decreases to a constant value. A method which allows to calculate the 4 parameters of the velocity distribution for a given set of values of the state variables is outlined. Finally, the analytic formulae for the state variables are explicitely given for the special case that the velocity distribution at the exobase is a bimaxwellian.

I. Introduction

Nowadays it is generally accepted that the dynamical behaviour of the ions and electrons in the topside polar ionosphere can not be described by an hydrodynamic approach of the general transport equations. Indeed, at higher altitudes the number of collisions becomes very small and any hydrodynamic approach becomes invalid. In this collisionless region, known as the ion-exosphere, a kinetic approach is very useful [see e.g. Chiu and Schulz, 1978; Croley, Jr. *et al.*, 1978; Lemaire and Scherer, 1970, 1972a, 1973a, 1974; Whipple, Jr., 1977].

(*) Présenté par M. M. Nicolet.

The controversy between the supporters of the hydrodynamic theory and the defenders of the kinetic theory was based on a misunderstanding [Donahue, 1971; Lemaire and Scherer, 1973b]. Both methods are appropriate for the study of the dynamical behaviour of charged particles in the polar ionosphere: the hydrodynamic approach is valid in the ion-barosphere (i.e. the region where the collisions have to be taken into account), whereas the kinetic approach can be applied in the collisionfree exosphere. Between these two regions there exists a small transition region with a thickness of a few scale heights, where the number of collisions rapidly decreases. Up till now, this transition region has not yet been described successfully, and is generally neglected by introducing the assumption that the barosphere is separated from the exosphere by a sharply defined surface called the exobase or baropause.

That the hydrodynamic and kinetic approaches are complementary and not at all contradictory was illustrated by Lemaire and Scherer [1975] who calculated continuous distributions for the number density, the escape flux, the temperature, the temperature anisotropy, and the energy flux by matching at the exobase the solutions of the hydrodynamic Navier-Stokes equations for the hydrogen ions, to a simple kinetic polar wind model. Although this model calculation shows many interesting features, one of the major shortcomings is that the proton temperature anisotropy at the exobase always equals the constant value $1 - \frac{2}{\pi} \simeq 0.36$. This is due to the assumption that the velocity distribution of the hydrogen ions at the exobase is given by a pseudo-Maxwell-Boltzmann distribution:

$$f[r_0,\vec{v}(r_0)] = \mathrm{N}\left(\frac{m}{2\pi k\mathrm{T}}\right)^{3/2} \exp\left[-\frac{m}{2k\mathrm{T}}v^2(r_0)\right] \tag{1}$$

where the particles with downward directed velocities are missing.

r_0 is the radial distance of the exobase; $\vec{v}(r_0)$ is the velocity vector at the exosbase of the particle with mass m; k is the Boltzmann constant; N and T are two parameters which have to be determined so that the calculated number density and temperature at the exobase have a given value.

Once that the parameters N and T are defined, any moment of the distribution (1) has a fixed value. Therefore the escape flux and the temperature anisotropy can not be fitted on given values. This deficiency can be solved by developing a kinetic polar wind model based on a more general velocity distribution which depends on more than two parameters. The purpose of this paper is to calculate formulae for the state variables in the polar ionexosphere under the assumption that the velocity distribution at the exobase is given by an asymmetric anisotropic maxwellian.

$$f[r_0,\vec{v}(r_0)] = \pi^{-3/2} N \beta_{\parallel}^{1/2} \beta_{\perp} \exp[-\beta_{\parallel}(v_{\parallel} - u)^2 - \beta_{\perp} v_{\perp}^2] \tag{2}$$

with

$$\beta_{\parallel} = m/2kT_{\parallel}; \quad \beta_{\perp} = m/2kT_{\perp} \tag{3}$$

and where $v_{\parallel}$ and $v_{\perp}$ denote the components of the velocity vector $\vec{v}(r_0)$ respectively parallel and perpendicular to the magnetic field; i.e.

$$v_{\parallel} = v(r_0) \cos \theta(r_0); \quad v_{\perp} = v(r_0) \sin \theta(r_0) \tag{4}$$

where $\theta(r_0)$ is the pitch angle.

The asymmetric bimaxwellian (2) depends on four parameters N, u, $T_{\parallel}$ and $T_{\perp}$ which can be chosen to obtain at the exobase a given number density, flow velocity or escape flux, temperature, and temperature anisotropy. The assumptions on which the kinetic model calculations are founded are summarized in Sec. 2. For a detailed description of the kinetic approach however, we refer to Lemaire and Scherer [1971, 1975]. The number density, the particle flux, the parallel and perpendicular momentum fluxes, and the energy flux along an open magnetic field line in the exosphere are calculated in Sec. 3 for particles emerging from the barosphere. In Sec. 4 we outline how the four parameters N, u, $T_{\parallel}$ and $T_{\perp}$ can be determined. Finally, in Sec. 5 the special cases of (i) a bimaxwellian velocity distribution function ($u = 0$; $T_{\parallel} \neq T_{\perp}$); (ii) a maxwellian velocity distribution function ($u = 0$; $T_{\parallel} = T_{\perp}$); and (iii) an asymmetric maxwellian velocity distribution function ($T_{\parallel} = T_{\perp}$; $u \neq 0$) are considered.

II. The kinetic model

We assume that in the ion-exosphere the trajectories of the charged particles can be determined by the non relativistic guiding center approximation, and that the particle drift across magnetic field lines can be disregarded. Moreover we assume that along a magnetic field line, the static magnetic field is a monotonic decreasing function of the distance s (measured along the field line), and reaches a constant value at infinity. The law of conservation of energy yields the relation

$$v^2(s_0) = v^2(s) + \mathrm{R}(s) \tag{5}$$

with

$$\mathrm{R}(s) = -2\Phi[1 + \alpha - (1 + \beta)y] \tag{6}$$

where we introduced the shorthand notations

$$\alpha = Ze\varphi(s_0)/m\Phi, \quad \beta = Ze\varphi(s)/m\Phi y; \tag{7}$$

for the reduced electric potential energy respectively at the exobase s_0 and at the point s in the exosphere; Ze is the electric charge of a particle with mass m, φ is the electrostatic potential due to the small charge separation, Φ is the gravitational potential at the exobase; and finally $y = r_0/r$ is the ratio of the radial distance of the exobase level s_0, to the radial distance of the point s.

Moreover, from the first adiabatic invariant follows

$$\sin^2\theta(s_0) = \frac{\mathrm{B}(s_0)}{\mathrm{B}(s)} \cdot \frac{v^2(s)}{v^2(s_0)} \sin^2\theta(s) \tag{8}$$

Assuming that the potential energy, $\frac{1}{2}m\mathrm{R}(s)$, is a monotonic function, two cases have to be considered corresponding to the algebraic sign of $\mathrm{R}(s)$ [Lemaire and Scherer, 1971]. A particle for which $\mathrm{R}(s)$ is negative, will move in a potential well. Such particle will be accelerated outwardly and will escape. This occurs for the lighter positive ions such as H^+ and He^+, for which the outward directed electric force is larger than the gravitational force. On the other hand the heavy oxygen ions O^+ and the electrons emerging from the barosphere, will encounter a potential barrier and are decelerated. In this case only particles with a velocity vector inside a loss cone can escape. Particles with a velocity vector outside the loss cone are gravitationally or magnetically reflected and are called ballistic particles.

It can be shown [Lemaire and Scherer, 1971] that the region in velocity space corresponding to the class of ballistic particles is defined by:

$$
\begin{aligned}
&0 \leqslant v \leqslant v_b, \quad 0 \leqslant \theta \leqslant \pi, \quad 0 \leqslant \varphi < 2\pi; \\
&v_b \leqslant v \leqslant v_\infty, \begin{cases} 0 \leqslant \theta \leqslant \theta_m, \\ \pi - \theta_m \leqslant \theta \leqslant \pi, \end{cases} \quad 0 \leqslant \varphi < 2\pi; \\
&v_\infty \leqslant v \leqslant v_c, \begin{cases} \theta'_m \leqslant \theta \leqslant \theta_m, \\ \pi - \theta_m \leqslant \theta \leqslant \pi - \theta'_m, \end{cases} \quad 0 \leqslant \varphi < 2\pi;
\end{aligned} \tag{9}
$$

with

$$
\begin{gathered}
v_b^2 = \eta \mathrm{R}/(1-\eta), \; v_\infty^2 = -2\Phi(1+\beta)y, \\
v_c^2 = -2\Phi a(1+\alpha)/(1-a) - 2\Phi(1+\beta)y, \\
\theta_m = \arcsin\left[\eta^{1/2}(1+\mathrm{R}/v^2)^{1/2}\right], \\
\theta'_m = \arcsin\left[\mu^{1/2}(1-v_\infty^2/v^2)^{1/2}\right], \\
\eta = \mathrm{B}(s)/\mathrm{B}(s_0), \quad a = \mathrm{B}(\infty)/\mathrm{B}(s_0), \quad \mu = \eta/a = \mathrm{B}(s)/\mathrm{B}(\infty).
\end{gathered} \tag{10}
$$

The angle φ is measured in a plane normal to the magnetic field. The class of escaping particles which have to overcome the potential barrier $\frac{1}{2}m\mathrm{R}(s) > 0$ is given by:

$$
v_\infty \leqslant v \leqslant v_c, \quad 0 \leqslant \theta \leqslant \theta'_m, \quad 0 \leqslant \varphi < 2\pi \tag{11}
$$

$$
v_c \leqslant v < \infty, \quad 0 \leqslant \theta \leqslant \theta_m, \quad 0 \leqslant \varphi < 2\pi
$$

And finally, the escaping particles moving in a potential well, i.e. $\frac{1}{2}m\mathrm{R}(s) < 0$, are defined by

$$
(-\mathrm{R})^{1/2} \leqslant v < \infty, \quad 0 \leqslant \theta \leqslant \theta_m, \quad 0 \leqslant \varphi < 2\pi. \tag{12}
$$

In what follows we will assume that at the exobase, the velocity distribution function of the particles emerging from the barosphere is given by the asymmetric bimaxwellian (2). The velocity distribution in the exosphere is obtained by solving Vlasov's equation subject to the boundary condition (2). Taking into account (5) and (8) this yields:

$$
\begin{gathered}
f[r,\vec{v}(r)] = \pi^{-3/2}\mathrm{N}\beta_{\|}^{1/2}\beta_{\perp}\exp\left[-\beta_{\|}(u^2+\mathrm{R})\right] \\
\cdot \exp\left\{-\beta_{\|}\left[v^2\left(1+\frac{t-1}{\eta}\sin^2\theta\right) - 2u\left(\mathrm{R}+v^2\left(1-\frac{\sin^2\theta}{\eta}\right)\right)^{1/2}\right]\right\}
\end{gathered} \tag{13}
$$

with $t = \mathrm{T}_{\|}/\mathrm{T}_{\perp}$.

III. Moments of the velocity distribution

The state variables, defined as the physically significant moments of the velocity distribution, can be calculated by means of
(a) the number density

$$n(s) = \int f(r,\vec{v}) d^3v \tag{14}$$

(b) the escape flux

$$F(s) = \int v_{\|} f(r,\vec{v}) d^3v \tag{15}$$

(c) the parallel and perpendicular momentum fluxes

$$P_{\|}(s) = m \int v_{\|}^2 f(r,\vec{v}) d^3v \tag{16}$$

$$P_{\perp}(s) = \frac{1}{2} m \int v_{\perp}^2 f(r,\vec{v}) d^3v \tag{17}$$

(d) the energy flux parallel to the magnetic field

$$\varepsilon(s) = \frac{1}{2} m \int v^2 v_{\|} f(r,\vec{v}) d^3v \tag{18}$$

The integrations are to be taken over the appropriate three dimensional velocity space. The integration over the angle φ is straightforward since this variable does not occur in the integrand. To calculate the remaining double integrals we use the transformation formulae:

$$v = \beta_{\|}^{-1/2}[x^2 + y^2 - q(s)]^{1/2}$$

$$\theta = \mathrm{Arcsin}\, \{\eta^{1/2} x [x^2 + y^2 - q(s)]^{-1/2}\} \tag{19}$$

with

$$q(s) = \beta_{\|} R(s) \tag{20}$$

Taking into account (13), the state variables (14)-(18) become
(a) the number density:

$$n(s) = \delta C \iint \frac{A(x,y)}{B(x,y)} dx\, dy \tag{21}$$

(b) the escape flux

$$F(s) = (2 - \delta)\beta_{\parallel}^{-1/2} C \iint A(x,y)\, dx\, dy \tag{22}$$

(c) the parallel and perpendicular momentum fluxes

$$P_{\parallel}(s) = 2\delta k T_{\parallel} C \iint A(x,y)\, B(x,y)\, dx\, dy \tag{23}$$

$$P_{\perp}(s) = \delta k T_{\parallel} \eta C \iint x^2 \frac{A(x,y)}{B(x,y)} dx\, dy \tag{24}$$

(d) the energy flux parallel to the magnetic field

$$\varepsilon(s) = (2 - \delta)\beta_{\parallel}^{-1/2} k T_{\parallel} C \iint [x^2 + y^2 - q(s)]\, A(x,y)\, dx\, dy \tag{25}$$

In these formulae the following shorthand notations were introduced

$$\begin{gathered} A(x,y) = xy \exp[-tx^2 - y^2 + 2Uy] \\ B(x,y) = [px^2 + y^2 - q(s)]^{1/2} \\ C = 2\pi^{-1/2} N t\eta \exp[-U^2] \\ U = \beta_{\parallel}^{1/2} u;\; p = 1 - \eta = 1 - \frac{B(s)}{B(s_0)} > 0 \\ \delta = \begin{cases} 1 \text{ for escaping particles} \\ 2 \text{ for ballistic particles} \end{cases} \end{gathered} \tag{26}$$

The domain of integration depends on the type of particles and follows from (9), (11) and (12) and the transformation formulae (19).

A. *Particles moving in a potential well* $(R(s) < 0)$

All particles are escaping. The velocity space is defined by (12) from which we deduce that the integrations (21) to (25) have to be taken over the domain

$$0 \leqslant x < \infty; \quad 0 \leqslant y < \infty$$

Hence the state variables can be calculated by means of the following formulae

(a) the number density

$$n(s) = N(t/p)^{1/2} \eta \exp(-U^2)\, h(0,\infty,S_1) \tag{27}$$

(b) the escape flux

$$F(s) = \frac{1}{4} N c_0 \eta \exp(-U^2) \cdot [1 + \sqrt{\pi}\, U \operatorname{erfex}(-U)] \tag{28}$$

with

$$c_0 = (8kT_{\parallel}/\pi m)^{1/2} \tag{29}$$

(c) the parallel and perpendicular momentum fluxes

$$P_{\parallel}(s) = NkT_{\parallel}(p/t)^{1/2}\eta \exp(-U^2)\, h(0,\infty,S_1 + S_2) \tag{30}$$

$$P_{\perp}(s) = \frac{1}{2} NkT_{\perp}(t/p)^{1/2}\eta^2 \exp(-U^2) h(0,\infty,S_1 + S_2 - S_3) \tag{31}$$

(d) the energy flux parallel to the magnetic field

$$\varepsilon(s) = \frac{1}{4} NkT_{\parallel}\, c_0\, \eta \exp(-U^2) \cdot \left\{\left[\frac{3}{2} + \frac{1}{t} - q(s) + U^2\right] \cdot [1 + \sqrt{\pi} U \operatorname{erfex}(-U)] - \frac{1}{2}\right\} \tag{32}$$

In the formulae above, $h(a,b,S)$ is a linear function of S defined by:

$$h(a,b,S) = \int_a^b y\, S(y) \exp(2Uy - y^2)\, dy \tag{33}$$

Moreover we introduced the shorthand notations

$$S_1 = \operatorname{erfex}\left(\frac{\sqrt{\pi}}{2} S_2\right)$$

$$S_2 = \frac{2}{\sqrt{\pi}}(t/p)^{1/2}[y^2 - q(s)]^{1/2} \tag{34}$$

$$S_3 = \frac{\pi}{2} S_1 S_2^2$$

$$\operatorname{erfex}(z) = \frac{2}{\sqrt{\pi}} \exp(z^2) \int_z^{\infty} \exp(-x^2)\, dx \tag{35}$$

The functions $h(0,\infty,S_i)$ with $i = 1, 2, 3$, defined by (33) and (34) can not be determined analytically; the remaining integrals have to be calculated numerically.

B. *Particles encountering a potential barrier* (R(s) > 0)

In this case the particles can be subdivided into two classes: the escaping and the ballistic particles. For each class we will calculate the state variables (21) to (25) separately.

1. *Escaping particles*

The domain of integration G_E for the escaping particles follows from (11) and (19), and is illustrated in Fig. 1 were $\nu = 1 - a$. All integrations over the variable x can be performed. This yields:

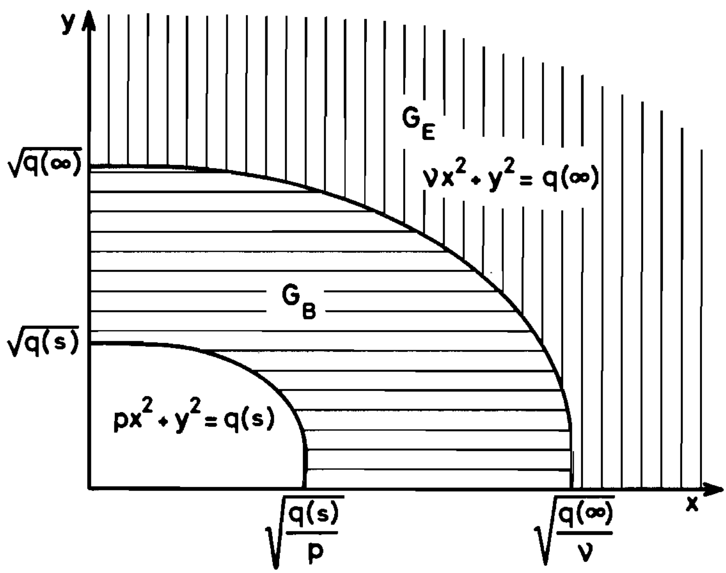

FIG. 1. — Domains of integration for the ballistic (G_B) and for the escaping (G_E) particles.

(a) the number density:

$$n^{(E)}(s) = N(t/p)^{1/2}\eta \exp(-U^2)\{h(Q,\infty,S_1) + g(0,Q,T_1)\exp[-tq(\infty)/\nu]\} \tag{36}$$

(b) the escape flux

$$F^{(E)}(s) = \frac{1}{2} N c_0 \eta \exp(-U^2)\{h(Q,\infty,1) + g(0,Q,1)\exp[-tq(\infty)/\nu]\} \tag{37}$$

(c) the parallel and perpendicular momentum fluxes

$$P_{\parallel}^{(E)}(s) = NkT_{\parallel}(p/t)^{1/2}\,\eta\,\exp(-U^2)\{h(Q,\infty,S_1 + S_2)$$
$$+\, g(0,Q,T_1 + T_2)\exp[-tq(\infty)/v] \tag{38}$$

$$P_{\perp}^{(E)}(s) = \frac{1}{2}NkT_{\perp}(t/p)^{1/2}\,\eta^2\exp(-U^2)\{h(Q,\infty,S_1 + S_2 - S_3)$$
$$+\, g(0,Q,T_1 + T_2 - T_3)\exp[-tq(\infty)/v]\} \tag{39}$$

(d) the energy flux parallel to the magnetic field

$$\varepsilon^{(E)}(s) = \frac{1}{4}NkT_{\parallel}\,c_0\,\eta\exp(-U^2)\left\{\left[\frac{1}{t} - q(s)\right]h(Q,\infty,1) + h(Q,\infty,y^2)\right.$$
$$\left. +\left[g(0,Q,y^2) + \left[\frac{1}{t} - q(s) + q(\infty)/v\right]g(0,Q,1)\right]\exp[-tq(\infty)/v]\right\} \tag{40}$$

The function $g(a,b,T)$ is linear in T and defined by

$$g(a,b,T) = \int_a^b y\,T(y)\exp\left[2Uy + \left(\frac{t}{v} - 1\right)y^2\right]dy \tag{41}$$

Moreover the following shorthand notations were introduced

$$T_1 = \mathrm{erfex}\left(\frac{\sqrt{\pi}}{2}T_2\right)$$
$$T_2 = \frac{2}{\sqrt{\pi}}(t/p)^{1/2}[\tau y^2 + X^2]^{1/2} \tag{42}$$
$$T_3 = \frac{\pi}{2}x_1 \cdot S_2^2$$

$$Q = [q(\infty)]^{1/2};\; X^2 = \frac{p}{v}q(\infty) - q(s);$$

$$v = 1 - a = 1 - \frac{B(\infty)}{B(s_0)} > 0;\; \tau = 1 - \frac{p}{v} = \frac{B(s) - B(\infty)}{B(s_0) - B(\infty)} > 0$$

The integrations over the y variable, which still remain in the formulae (36)-(40) can only be calculated analytically for the escape flux (37) and the energy flux (40). Indeed, after some tedious calculation we obtain:

$$h(Q,\infty,1) = \frac{1}{2}[1 + \sqrt{\pi}\,U\,\mathrm{erfex}(Q - U)]\exp[U^2 - (Q - U)^2] \tag{43}$$

$$h(Q,\infty,y^2) = \frac{1}{2}\left\{1 + Q^2 + QU + U^2 + \sqrt{\pi}\,U\left(\frac{3}{2} + U^2\right)\mathrm{erfex}\,(Q - U)\right\} \cdot \exp\left[U^2 - (Q-U)^2\right] \tag{44}$$

$$g(0,Q,1) = \begin{cases} \dfrac{1}{2}\dfrac{v}{t-v}\left\{\left[1 - \sqrt{\pi}\dfrac{U}{\kappa}\mathscr{D}\left(\kappa Q + \dfrac{U}{\kappa}\right)\right]\exp\left[2UQ + \left(\dfrac{t}{v} - 1\right)Q^2\right] - \left[1 - \sqrt{\pi}\dfrac{U}{\kappa}\mathscr{D}\left(\dfrac{U}{\kappa}\right)\right]\right\} & \text{if } \kappa^2 \equiv \dfrac{t}{v} - 1 > 0 \\ \dfrac{1}{2U}\left[\dfrac{1}{2U} + \left(Q - \dfrac{1}{2U}\right)e^{2UQ}\right] & \text{if } t = v \\ \dfrac{1}{2}\dfrac{v}{t-v}\left\{\left[1 + \sqrt{\pi}\dfrac{U}{\rho}\mathrm{erfex}\left(\rho Q - \dfrac{U}{\rho}\right)\right]\exp\left[2UQ + \left(\dfrac{t}{v} - 1\right)Q^2\right] - \left[1 + \sqrt{\pi}\dfrac{U}{\rho}\mathrm{erfex}\left(-\dfrac{U}{\rho}\right)\right]\right\} & \text{if } \rho^2 \equiv 1 - \dfrac{t}{v} > 0 \end{cases} \tag{45}$$

with

$$\mathscr{D}(z) = \frac{2}{\sqrt{\pi}}\exp(-z^2)\int_0^z \exp(t^2)\,dt \tag{46}$$

and finally

$$g(0,Q,y^2) = \begin{cases} \dfrac{1}{2\kappa^2}\left\{\left[Q^2 + \dfrac{1}{\kappa^2}\left(\dfrac{U^2}{\kappa^2} - 1 - UQ\right) + \sqrt{\pi}\dfrac{U}{\kappa^3}\left(\dfrac{3}{2} - \dfrac{U^2}{\kappa^2}\right)\mathscr{D}\left(\kappa Q + \dfrac{U}{\kappa}\right)\right] \cdot \exp\left[2UQ + \kappa^2 Q^2\right] - \dfrac{1}{\kappa^2}\left[\dfrac{U^2}{\kappa^2} - 1 + \sqrt{\pi}\dfrac{U}{\kappa^3}\left(\dfrac{3}{2} - \dfrac{U^2}{\kappa^2}\right)\mathscr{D}\left(\dfrac{U}{\kappa}\right)\right]\right\} & \text{if } \kappa^2 \equiv \dfrac{t}{v} - 1 > 0 \\ \dfrac{1}{2U}\left[\dfrac{3}{4U^3} + \left(Q^3 - \dfrac{3Q^2}{2U} + \dfrac{3Q}{2U^2} - \dfrac{3}{4U^3}\right)e^{2UQ}\right] & \text{if } t = v \\ \dfrac{1}{2\rho^2}\left\{\dfrac{1}{\rho^2}\left[\dfrac{U^2}{\rho^2} + 1 + \sqrt{\pi}\dfrac{U}{\rho}\left(\dfrac{3}{2} + \dfrac{U^2}{p^2}\right)\mathrm{erfex}\left(-\dfrac{U}{\rho}\right)\right] - \left[Q^2 + \dfrac{1}{\rho^2}\left(\dfrac{U^2}{\rho^2} + 1 + UQ\right) + \sqrt{\pi}\dfrac{U}{\rho^3}\left(\dfrac{3}{2} + \dfrac{U^2}{\rho^2}\right)\mathrm{erfex}\left(\rho Q - \dfrac{U}{\rho}\right)\right] \exp\left[-\rho^2 Q^2 + 2UQ\right]\right\} & \text{if } \rho^2 \equiv 1 - \dfrac{t}{v} > 0 \end{cases} \tag{47}$$

All the other functions h and g in the formulae (36), (38) and (39) have to be calculated numerically

2. *Ballistic particles*

The domain of integration G_B in the xy-plane for the class of ballistic particles can be determined by (9) and (19), and is illustrated in fig. 1. Since for the ballistic particles $\delta = 2$, the escape flux (22) and the energy flux parallel to the magnetic field (25) will be zero. For the remaining state variables the integrations over the x variable can be performed. This yields:
(a) the number density:

$$n^{(B)}(s) = 2N(t/p)^{1/2}\eta \exp[-U^2 - t\,q(s)/p]\{l(0,Q,W_1) - l(\sqrt{q(s)}, Q, W_2)\} \tag{48}$$

(b) the parallel and perpendicular momentum fluxes

$$\begin{aligned} P_{\parallel}^{(B)}(s) = 2NkT_{\parallel}(p/t)^{1/2}\eta \exp(-U^2)\{[l(0,Q,W_1) \\ - l(\sqrt{q(s)}, Q, W_2)]\exp[-t\,q(s)/p] \\ + h(\sqrt{q(s)}, Q, S_2) - g(0,Q,T_2)\exp[-tq(\infty)/v]\} \end{aligned} \tag{49}$$

$$\begin{aligned} P_{\perp}^{(B)}(s) = NkT_{\perp}(t/p)^{1/2}\eta^2 \exp(-U^2)\{[l(0,Q,W_1 - W_3) \\ - l(\sqrt{q(s)},Q,W_2 - W_4)] \\ \cdot \exp[-tq(s)/p] + h(\sqrt{q(s)},Q,S_2) - g(0,Q,T_2)\exp[-tq(\infty)/v]\} \end{aligned} \tag{50}$$

where $l(a,b,W)$ is a linear function in W defined by

$$l(a,b,W) = \int_a^b y\,W(k)\exp\left[2Uy + \left(\frac{t}{p} - 1\right)y^2\right]dy \tag{51}$$

Moreover we introduced the shorthand notations

$$W_1 = \operatorname{erf}\left(\frac{\sqrt{\pi}}{2}T_2\right);\ W_2 = \operatorname{erf}\left(\frac{\sqrt{\pi}}{2}S_2\right) \tag{52}$$

$$W_3 = \frac{\pi}{2}W_1 S_2^2;\ W_4 = \frac{\pi}{2}W_2 S_2^2$$

with

$$\mathrm{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z \exp(-t^2)\,dt \tag{53}$$

IV. Determination of the parameters N, u, $T_{\parallel}$ and $T_{\perp}$

The in Sec. 3 calculated state variables depend on the parameters N, u, $T_{\parallel}$ and $T_{\perp}$ of the velocity distribution function (2). These can be determinated by expressing that at the exobase the calculated number density $n(s_0)$, flow speed $w(s_0)$, temperature $T(s_0)$ and temperature anisotropy $a(s_0)$ are equal to given values. The flow speed is defined by

$$w(s) = F(s)/n(s) \tag{54}$$

and the temperature and temperature anisotropy can be calculated by means of

$$T(s) = \frac{1}{3}[T_{\parallel}(s) + T_{\perp}(s)] \tag{55}$$

$$a(s) = T_{\parallel}(s)/T_{\perp}(s) \tag{56}$$

with

$$T_{\perp}(s) = P_{\perp}(s)/k\,n(s) \tag{57}$$

$$T_{\parallel}(s) = [P_{\parallel}(s) - mw(s)\,F(s)]/kn(s) \tag{58}$$

The number density and the parallel and perpendicular momentum fluxes at the exobase can be expressed by means of analytical formulae. Indeed from (21), (23), (24) and (26) follows:

(a) the number density

$$n(s_0) = \delta\,C \iint x \exp[-tx^2 - y^2 + 2Uy]\,dx\,dy \tag{59}$$

(b) the parallel and perpendicular momentum fluxes

$$P_{\parallel}(s_0) = 2\delta k T_{\parallel} C \iint xy^2 \exp[-tx^2 - y^2 + 2Uy]\,dx\,dy \tag{60}$$

$$P_{\perp}(s_0) = \delta k T_{\parallel} C \iint x^3 \exp[-tx^2 - y^2 + 2Uy]\,dx\,dy \tag{61}$$

For the escaping particles the domains of integration are the same as given in Sec. 3, whereas for the ballistic particles it can be shown that the domain of integration is determined as the region inside the ellipse $\nu x^2 + y^2 = q(\infty)$ for which $x \geqslant 0$ and $y \geqslant 0$. Some tedious calculations give the following results.

A. *Particles moving in a potential well* $(\mathrm{R}(s) < 0)$

(a) the number density

$$n(s_0) = \frac{1}{2}\mathrm{N}\,\mathrm{erfc}(-\mathrm{U}) \tag{62}$$

with $erfc(z) = \exp(-z^2)\, erfex(z) = 1 - erf(z)$ (63)
and (b) the parallel and perpendicular momentum fluxes

$$\mathrm{P}_{\|}(s_0) = \mathrm{N}k\mathrm{T}_{\|}\left[\frac{\mathrm{U}}{\sqrt{\pi}} + \left(\frac{1}{2} + \mathrm{U}^2\right)\mathrm{erfex}(-\mathrm{U})\right]\exp(-\mathrm{U}^2) \tag{64}$$

$$\mathrm{P}_{\perp}(s_0) = \frac{1}{2}\mathrm{N}k\mathrm{T}_{\perp}\,\mathrm{erfc}(-\mathrm{U}) \tag{65}$$

After substitution of these results in (55)-(58) the temperature $\mathrm{T}(s_0)$ and the temperature anisotropy $a(s_0)$ at the exobase can be calculated by means of

$$\mathrm{T}(s_0) = \frac{1}{3}\mathrm{T}_{\perp}\left[(2+t)\,\mathrm{erfex}(-\mathrm{U}) - \frac{2t}{\pi\,\mathrm{erfex}(-\mathrm{U})} - \frac{2}{\sqrt{\pi}}t\mathrm{U}\right]/\mathrm{erfex}(-\mathrm{U}) \tag{66}$$

$$a(s_0) = t\left\{1 - \frac{2\mathrm{U}}{\sqrt{\pi}\,\mathrm{erfex}(-\mathrm{U})} - \frac{2}{\pi[\mathrm{erfex}(-\mathrm{U})]^2}\right\} \tag{67}$$

Substitution of (28) and (62) in (54) yields the flow speed at the exobase:

$$w(s_0) = \frac{1}{2}c_0[1 + \sqrt{\pi}\,\mathrm{erfex}(-\mathrm{U})]/\mathrm{erfex}(-\mathrm{U}) \tag{68}$$

For a given set of values $n(s_0)$, $w(s_0)$, $\mathrm{T}(s_0)$ and $a(s_0)$ the parameters N, u, $\mathrm{T}_{\|}$ and $\mathrm{T}_{\perp}$ can be determined by means of the system of four transcendental equations (62), (66), (67) and (68). This will be discussed in more detail in a forthcoming paper.

B. *Particles encountering a potential barrier* $(R(s) \geqslant 0)$

1. *Escaping particles*

(a) the number density

$$n^{(E)}(s_0) = \frac{1}{2} N[\operatorname{erfc}(Q - U) + K_1] \tag{69}$$

with

$$K_1 = \begin{cases} \dfrac{1}{\kappa}\mathscr{D}\left(\kappa Q + \dfrac{U}{\kappa}\right)\exp[-(Q-U)^2] - \dfrac{1}{\kappa}\mathscr{D}\left(\dfrac{U}{\kappa}\right)\exp[-U^2 - tq(\infty)/v] \\ \qquad \text{for } \kappa^2 \equiv \dfrac{t}{v} - 1 > 0 \\ \dfrac{1 - e^{-2UQ}}{\sqrt{\pi}U}\exp[-(Q-U)^2] \quad \text{for } t = v \\ -\dfrac{1}{\rho}\operatorname{erfex}\left(\rho Q - \dfrac{U}{\rho}\right)\exp[-(Q-U)^2] + \dfrac{1}{\rho}\operatorname{erfex}\left(-\dfrac{U}{\rho}\right) \\ \qquad \exp[-U^2 - tq(\infty)/v] \quad \text{for } \rho^2 \equiv 1 - \dfrac{t}{v} > 0 \end{cases} \tag{70}$$

(b) The parallel and perpendicular momentum fluxes

$$P_{\parallel}^{(E)}(s_0) = NkT_{\parallel}\left\{\left[\frac{U+Q}{\sqrt{\pi}} + \left(\frac{1}{2} + U^2\right)\operatorname{erfex}(Q-U) + K_2(Q)\right] \right.$$
$$\left. \exp[-(Q-U)^2] - K_2(0)\exp[-U^2 - tq(\infty)/v]\right\} \tag{71}$$

$$P_{\perp}^{(E)}(s_0) = \frac{1}{2}NkT_{\perp}\left\{[\operatorname{erfex}(Q-U) + K_3(Q)]\exp[-(Q-U)^2]\right.$$
$$\left. - K_3(0)\exp[-U^2 - tq(\infty)/v]\right\} \tag{72}$$

with

$$K_2(z) = \begin{cases} \dfrac{1}{\kappa^3}\left[\dfrac{\kappa^2 z - U}{\kappa\sqrt{\pi}} + \left(\dfrac{U^2}{\kappa^2} - \dfrac{1}{2}\right)\mathscr{D}\left(\kappa z + \dfrac{U}{\kappa}\right)\right] \\ \qquad \text{for } \kappa^2 \equiv \dfrac{t}{v} - 1 > 0 \\ [1 - 2Uz + 2U^2z^2 - e^{-2Uz}]/(2\sqrt{\pi}U^3) \quad \text{for } t = v \\ -\dfrac{1}{\rho^3}\left[\dfrac{\rho^2 z + U}{\rho\sqrt{\pi}} + \left(\dfrac{U^2}{\rho^2} + \dfrac{1}{2}\right)\operatorname{erfex}\left(\rho z - \dfrac{U}{\rho}\right)\right] \\ \qquad \text{for } \rho^2 \equiv 1 - \dfrac{t}{v} > 0 \end{cases} \tag{73}$$

$$
K_3(z) = \begin{cases} \dfrac{L}{\kappa}\mathscr{D}\left(\kappa z + \dfrac{U}{\kappa}\right) - \dfrac{t(\kappa^2 z - U)}{\sqrt{\pi}\nu\kappa^4} & \text{for } \kappa^2 \equiv \dfrac{t}{\nu} - 1 > 0 \\ \dfrac{1+z^2}{\sqrt{\pi}} \cdot \dfrac{1 - e^{-2Uz}}{U} - K_2(z) & \text{for } t = \nu \\ -\dfrac{L}{\rho}\operatorname{erfex}\left(\rho z - \dfrac{U}{\rho}\right) + \dfrac{t(\rho^2 z + U)}{\sqrt{\pi}\nu\rho^4} & \text{for } \rho^2 \equiv 1 - \dfrac{t}{\nu} > 0 \end{cases} \tag{74}
$$

$$
L = 1 + \frac{t}{2\,(1-\nu)} + \frac{t}{\nu} q(\infty) - \frac{t}{\nu\kappa^4} U^2 \tag{75}
$$

2. *Ballistic particles*

It can be shown that the number density, and the parallel and perpendicular momentum fluxes for the class of ballistic particles are given by

$$
n^{(B)}(s_0) = N \operatorname{erfc}(-U) - 2\,n^{(E)}(s_0) \tag{76}
$$

$$
P_{\parallel}^{(B)}(s_0) = 2NkT_{\parallel}\left[\frac{U}{\sqrt{\pi}} + \left(\frac{1}{2} + U^2\right)\operatorname{erfex}(-U)\right]\exp(-U^2) - 2P_{\parallel}^{(E)}(s_0) \tag{77}
$$

$$
P_{\perp}^{(B)}(s_0) = NkT_{\perp} \operatorname{erfc}(-U) - 2P_{\perp}^{(E)}(s_0) \tag{78}
$$

where $n^{(E)}(s_0)$, $P_{\parallel}^{(E)}(s_0)$ and $P_{\perp}^{(E)}(s_0)$ are the corresponding state variables for the escaping particles, respectively given by (69), (71) and (72).

The total number density, escape flux and parallel and perpendicular momentum fluxes are given by

$$
n(s_0) = n^{(E)}(s_0) + n^{(B)}(s_0) \tag{79}
$$

$$
F(s_0) = F^{(E)}(s_0) \tag{80}
$$

$$
P_{\parallel}(s_0) = P_{\parallel}^{(E)}(s_0) + P_{\parallel}^{(B)}(s_0) \tag{81}
$$

$$
P_{\perp}(s_0) = P_{\perp}^{(E)}(s_0) + P_{\perp}^{(B)}(s_0) \tag{82}
$$

which allows to calculate the flow speed $w(s_0)$, the temperature $T(s_0)$ and the temperature anisotropy $a(s_0)$ by means of the relations (54)-(58). Since all the state variables at the exobase still depend on Q or $q(\infty)$ (which is proportional to the height of the potential barrier) the parameters N, u, $T_{\parallel}$ and $T_{\perp}$ can not be determined as easily as for the particles moving in a potential well. The height of the potential barrier depends on the different kinds of constituents of the ion-exosphere, and can be calculated in a self consistent way by expressing that the total ion escape flux equals the electron escape flux [Lemaire and Scherer, 1971]. Therefore the parameters N, u, $T_{\parallel}$ and $T_{\perp}$ corresponding to the velocity distribution (2) of one kind of particles will not only be determined by the particle density $n(s_0)$, the flow speed $w(s_0)$, the temperature $T(s_0)$, and the anisotropy $a(s_0)$ of the considered particles, but also by the values at the exobase of the state variables of all other sorts of particles building up the ion-exosphere.

V. Some special velocity distribution functions

In Sec. 3 we calculated the state variables under the assumption that the velocity distribution was given by a general asymmetric anisotropic Maxwellian, depending on four parameters N, u, $T_{\parallel}$ and $T_{\perp}$. In what follows we will consider some special cases of this velocity distribution, corresponding to some particular values of the parameters.

(i) The bimaxwellian velocity distribution function ($U = 0$, $T_{\parallel} \neq T_{\perp}$)

The bimaxwellian or anisotropic velocity distribution depends on three parameters N, $T_{\parallel}$ and $T_{\perp}$. The formulae for the state variables can be deduced from the formulae of Sec. 3, in which $U = 0$. As a consequence of this the y-integrations can be calculated analytically.

A. *Particles moving in a potential well*

The analytical expressions for the functions $h(0,\infty,S_i)$, with $i = 1, 2, 3$, entering in (27), (30) and (31) can be calculated quiet easily. This yields

(a) the number density

$$n(s) = \frac{1}{2}N\alpha\left\{\operatorname{erfex}(V_x) - (p/t)^{1/2}\operatorname{erfex}\left[V_x(t/p)^{1/2}\right]\right\} \tag{83}$$

(b) the parallel and perpendicular momentum fluxes

$$P_{\parallel}(s) = \frac{1}{2}NkT_{\parallel}\alpha\left\{\operatorname{erfex}(V_x) - (p/t)^{3/2}\operatorname{erfex}\left[V_x(t/p)^{1/2}\right] + \frac{2}{\sqrt{\pi}}\left(1 - \frac{p}{t}\right)V_x\right\} \tag{84}$$

$$P_\perp(s) = \frac{1}{2} N k T_\perp \alpha \{\alpha \operatorname{erfex}(V_x) - \eta V_x/\sqrt{\pi}$$

$$- (p/t)^{1/2} M_1 \operatorname{erfex}[V_x(t/p)^{1/2}]\} \tag{85}$$

with

$$\alpha = \eta/\left(1 - \frac{p}{t}\right); \ V_x = [-q(s)]^{1/2}$$

$$M_1 = \alpha + \frac{1}{2}\eta + t\eta\, q(s)/p \tag{86}$$

The escape flux and the energy flux follow respectively from (28) and (32)

$$F(s) = \frac{1}{4} N c_0 \eta = \frac{1}{2} N(2kT_\parallel/\pi m)^{1/2}\eta \tag{87}$$

$$\varepsilon(s) = \frac{1}{4} N k T_\parallel c_0 \eta [1 + t^{-1} - q(s)] \tag{88}$$

B. *Particles encountering a potential barrier*

1. *Escaping particles*

The functions $h(Q,\infty,S_i)$ and $g(0,Q,T_i)$, with $i = 1, 2, 3$, entering in the formulae (36), (38) and (39) can be defined analytically for $U = 0$. This yields after some tedious calculations.
(a) the number density:
for $t \neq v$

$$n^{(E)}(s) = \frac{1}{2} N\alpha \{[\operatorname{erfex}(V_c) + Z(V_c)] \exp[-q(\infty)]$$

$$- [(p/t)^{1/2} \operatorname{erfex}[X(t/p)^{1/2}] + Z(X)] \exp[-tq(\infty)/v]\} \tag{89}$$

and for $t = v$

$$n^{(E)}(s) = \frac{1}{2} N\alpha \exp[-q(\infty)] \{\operatorname{erfex}(V_c)$$

$$- (p/t)^{1/2} \operatorname{erfex}[X(t/p)^{1/2}] + \frac{2}{\sqrt{\pi}}(V_c - X)\} \tag{90}$$

where we introduced the following shorthand notations

$$V_c = [q(\infty) - q(s)]^{1/2} \tag{91}$$

$$Z(x) = \begin{cases} \gamma \mathscr{D}(x/\gamma) & \text{with } \gamma = (\tau/\kappa^2)^{1/2} \text{ if } \kappa^2 \equiv \frac{t}{v} - 1 > 0 \\ \\ -\beta \operatorname{erfex}(x/\beta) & \text{with } \beta = (\tau/\rho^2)^{1/2} \text{ if } \rho^2 \equiv 1 - \frac{t}{v} > 0 \end{cases} \tag{92}$$

(b) the parallel and perpendicular momentum fluxes
for $t \neq v$

$$P_{\parallel}^{(E)}(s) = \frac{1}{2} N k T_{\parallel} \alpha \left\{ \left[\operatorname{erfex}(V_c) + \frac{2}{\sqrt{\pi}} \frac{(t-p)}{(t-v)} V_c - \frac{\tau v}{t-v} Z(V_c) \right] \right.$$
$$\exp[-q(\infty)] - \left[(p/t)^{3/2} \operatorname{erfex}[X(t/p)^{1/2}] + \frac{2}{\sqrt{\pi}} \frac{v(t-p)}{t(t-v)} X \right.$$
$$\left. \left. - \frac{\tau v}{t-v} Z(X) \right] \exp[-tq(\infty)/v] \right\} \tag{93}$$

$$P_{\perp}^{(E)}(s) = \frac{1}{2} N k T_{\perp} \alpha \left\{ \left[\alpha \operatorname{erfex}(V_c) - \frac{t\eta V_c}{(t-v)\sqrt{\pi}} + M_2 Z(V_c) \right] \right.$$
$$\exp[-q(\infty)] - \left[M_1 (p/t)^{1/2} \operatorname{erfex}[X(t/p)^{1/2}] - \frac{v\eta X}{(t-v)\sqrt{\pi}} + M_2 Z(X) \right]$$
$$\left. \exp[-tq(\infty)/v] \right\} \tag{94}$$

with

$$M_2 = \alpha + \frac{t\eta}{2(t-v)} + \frac{t\eta}{v-p} V_c^2 \tag{95}$$

and for $t = v$

$$P_{\parallel}^{(E)}(s) = \frac{1}{2} N k T_{\parallel} \alpha \exp[-q(\infty)] \left\{ \operatorname{erfex}(V_c) - (p/t)^{3/2} \operatorname{erfex}[X(t/p)^{1/2}] \right.$$
$$\left. + \frac{2}{\sqrt{\pi}} \left[V_c - pX/t + \frac{2}{3}(V_c^3 - X^3) \right] \right\} \tag{96}$$

$$P_\perp^{(E)}(s) = \frac{1}{2} N k T_\perp \alpha \exp[-q(\infty)] \Big\{ \alpha \,\mathrm{erfex}(V_c)$$

$$- M_1 (p/t)^{1/2} \,\mathrm{erfex}[X(t/p)^{1/2}]$$

$$+ \frac{2}{\sqrt{\pi}} \left[\alpha V_c + \frac{2}{3} \alpha V_c^3 - M_1 X - \frac{2}{3} \left(\frac{1}{2} + \frac{1}{\tau} \right) \frac{\eta t}{p} X^3 \right] \Big\} \tag{97}$$

The escape flux and the energy flux parallel to the magnetic field can be determined respectively from (37) and (40), in which the results (43)-(45), (47) for $U = 0$ are substituted. This yields for $t \neq v$

$$F(s) = \frac{1}{4} N c_0 \eta \{ t \exp[-q(\infty)] - v \exp[-t q(\infty)/v] \} / (t - v) \tag{98}$$

$$\varepsilon(s) = \frac{1}{4} N k T_\parallel c_0 \eta \left\{ \left[1 + \frac{1 + t V_c^2}{t - v} + \frac{a v}{(t - v)^2} \right] \exp[-q(\infty)] \right.$$

$$\left. - \frac{v}{t - v} \left[\frac{1}{t} + \frac{a}{t - v} + \frac{q(\infty)}{v} - q(s) \right] \exp[-t q(\infty)/v] \right\} \tag{99}$$

and for $t = v$:

$$F(s) = \frac{1}{4} N c_0 \eta [1 + q(\infty)] \exp[-q(\infty)] \tag{100}$$

$$\varepsilon(s) = \frac{1}{4} N k T_\parallel c_0 \eta \left\{ [1 + q(\infty)] \left(1 + \frac{1}{t} + V_c^2 \right) \right.$$

$$\left. - q(\infty) \left[1 + \frac{t - 1}{2t} q(\infty) \right] \right\} \exp[-q(\infty)] \tag{101}$$

2. *Ballistic particles*

The functions $l(0,Q,W_i)$ and $l(\sqrt{q(s)},Q,W_1)$ with $i = 1, 3$, $h(\sqrt{q(s)}, Q,S_2)$, and $g(0,Q,T_2)$ which enter in the right hand side of (48), (49) and (50) can be defined analytically if $U = 0$. Hence the number density and the parallel and perpendicular momentum fluxes of the ballistic particles can be calculated by means of analytical expressions. Calculating the integrals which define l, h and g is, however, very cumbersome. These tedious calculations can be avoided by defining the functions $\mathcal{N}(s)$, $\mathcal{P}_{\parallel}(s)$ and $\mathcal{P}_{\perp}(s)$ as follows

$$\mathcal{N}(s) = 2\mathrm{C} \iint\limits_{\mathrm{G_E+G_B}} \frac{xy}{\mathrm{B}(x,y)} \exp\left[-tx^2 - y^2\right] dx\,dy \tag{102}$$

$$\mathcal{P}_{\parallel}(s) = 4k\mathrm{T}_{\parallel}\mathrm{C} \iint\limits_{\mathrm{G_E+G_B}} xy\,\mathrm{B}(x,y) \exp\left[-tx^2 - y^2\right] dx\,dy \tag{103}$$

$$\mathcal{P}_{\perp}(s) = 2k\mathrm{T}_{\parallel}\eta\mathrm{C} \iint\limits_{\mathrm{G_E+G_B}} \frac{x^3y}{\mathrm{B}(x,y)} \exp(-tx^2 - y^2)\, dx\,dy \tag{104}$$

where $\mathrm{B}(x,y)$ and C are given in (26), and $\mathrm{G_E}$ and $\mathrm{G_B}$ are the domains of integration respectively for the escaping and ballistic particles (see Fig. 1). The double integrals in the righthand side of (102), (103) and (104) can be calculated easily and lead to the formulae.

$$\mathcal{N}(s) = \mathrm{N}\alpha\{\exp[-q(s)] - (p/t)^{1/2}\exp[-tq(s)/p]\} \tag{105}$$

$$\mathcal{P}_{\parallel}(s) = \mathrm{N}k\mathrm{T}_{\parallel}\alpha\{\exp[-q(s)] - (p/t)^{3/2}\exp[-tq(s)/p]\} \tag{106}$$

$$\mathcal{P}_{\perp}(s) = \mathrm{N}k\mathrm{T}_{\perp}\alpha\{\alpha\exp[-q(s)] - \mathrm{M}_1(p/t)^{1/2}\exp[-tq(s)/p]\} \tag{107}$$

where α and M_1 are given by (86).

From the definitions (102)-(104) and the relations (21), (23) and (24) follows that

$$n^{(\mathrm{B})}(s) = \mathcal{N}(s) - 2n^{(\mathrm{E})}(s) \tag{108}$$

$$\mathrm{P}_{\parallel}^{(\mathrm{B})}(s) = \mathcal{P}_{\parallel}(s) - 2\mathrm{P}_{\parallel}^{(\mathrm{E})}(s) \tag{109}$$

$$\mathrm{P}_{\perp}^{(\mathrm{B})}(s) = \mathcal{P}_{\perp}(s) - 2\mathrm{P}_{\perp}^{(\mathrm{E})}(s) \tag{110}$$

where $n^{(\mathrm{E})}(s)$, $\mathrm{P}_{\parallel}^{(\mathrm{E})}(s)$ and $\mathrm{P}_{\perp}^{(\mathrm{E})}(s)$ are respectively given by (89) and (90), (93) and (96), and (94) and (97).

(ii) Maxwellian velocity distribution function ($\mathrm{U} = 0$, $\mathrm{T}_{\parallel} = \mathrm{T}_{\perp}$)

In this case we can deduce the formulae for the state variables directly from the expressions obtained for a bimaxwellian velocity distribution by putting $t = 1$. The resulting formulae are much simplified since

$$\alpha = 1; \quad t - v \equiv 1 - v = a > 0$$

and therefore: $\mathrm{Z}(x) = \gamma\mathcal{D}(x/\gamma)$ with $\gamma = (\tau v/a)^{1/2}$.

The formulae obtained in this way are in compliance with the results obtained in earlier model calculations [Lemaire and Scherer, 1970].

(iii) The asymmetric maxwellian velocity distribution ($U \neq 0$, $T_{\parallel} = T_{\perp}$)

Substituting $t = 1$ in the general formulae deduced in Sec. 3 we recover the earlier calculated results for an ion-exosphere model with an asymmetric distribution [Lemaire and Scherer, 1972b].

VI. CONCLUSIONS

Assuming that at the exobase the velocity distribution function of the particles emerging from the barosphere is given by an asymmetric anisotropic velocity distribution function the number density, the escape flux, the parallel and perpendicular momentum fluxes and the energy flux along an open magnetic field line have been calculated. Moreover we have shown that the earlier determined formulae for the polar wind state variables [Lemaire and Scherer, 1970, 1972b] can be obtained as a special case of the present model ion-exosphere in which the temperature anisotropy of the protons at the exobase is no longer restricted to the constant value $1 - \frac{2}{\pi}$.

REFERENCES

CHIU, Y. T. and SCHULZ, M., 1978. Self-consistent Particle and Parallel Electrostatic Field Distributions in the Magnetospheric-Ionospheric Auroral Region, *J. Geophys. Res.*, (to be published).

CROLEY, D. R. Jr., MIZERA, P. F. and FENNELL, J. F., 1978. Signature of a Parallel Electric Field in Ion and Electron Distributions in Velocity Space, *J. Geophys. Res.*, (to be published).

DONAHUE, T. M., 1971. Polar Ion Flow: Wind or Breeze?, *Rev. Geophys. Space Phys.*, **9**, 1-9.

LEMAIRE, J. and SCHERER, M., 1970. Model of the polar ion-exosphere, *Planet. Space Sci.*, **18**, 103-120.

LEMAIRE, J. and SCHERER, M., 1971. Simple model for an ion-exosphere in an open magnetic field, *Phys. Fluids*, **14**, 1683-1694.

LEMAIRE, J. and SCHERER, M., 1972a. The effect of photoelectrons on kinetic polar wind models, *Bull. Cl. Sci. Acad. Roy. Belg.*, **58**, 502-512.

LEMAIRE, J. and SCHERER, M., 1972b. Ion-exosphere with asymmetric velocity distribution, *Phys. Fluids*, **15**, 760-766.
LEMAIRE, J. and SCHERER, M., 1973a. Plasma sheet particle precipitation: A kinetic model, *Planet. Space Sci.*, **21**, 281-289.
LEMAIRE, J. and SCHERER, M., 1973b. Kinetic Models of the Solar and Polar Winds, *Rev. Geophys. Space Phys.*, **11**, 427-468.
LEMAIRE, J. and SCHERER, M., 1974. Exospheric models of the topside ionosphere, *Space Sci. Rev.*, **15**, 591-640.
LEMAIRE, J. and SCHERER, M., 1975. Contribution à l'étude des ions dans l'ionosphère polaire, *Aeronomica Acta* A-147.
WHIPPLE, E. C. Jr., 1977. The Signature of Parallel Electric Fields in a Collisionless Plasma, *J. Geophys. Res.*, **82**, 1525-1531.