

A Hilbert-Huang transform approach to space plasma turbulence at kinetic scales

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Abstract. Heliospheric space plasmas are highly turbulent media and display multiscale fluctuations over a wide range of scales from the magnetohydrodynamic domain down to the kinetic one. The study of turbulence features is traditionally based on spectral and canonical structure function analysis. Here, we present an novel approach to the analysis of the multiscale nature of plasma turbulent fluctuations by means of Hilbert-Huang Transform (HHT). In particular we present a preliminary application of this technique to magnetic field fluctuations at kinetic scales in a fast solar wind stream as observed by Cluster mission. The HHT-energy spectrum reveals the intermittent and multiscale nature of fluctuation frequency at kinetic scales indicating that there are no-persistent and long standing frequencies.

1. Introduction

Several space plasma environments display complex dynamics characterized by multiscale processes, such as fluid and magnetohydrodynamic (MHD) turbulence. In the framework of heliospheric plasmas, turbulence is a phenomenon widely occurring in several different regions: solar wind and interplanetary medium, Earth's magnetosheath and tail central plasma sheet, etc.

Among these regions the solar wind and the interplanetary medium are surely those where the turbulence has been extensively investigated since the early era of space missions [1]. Indeed, although solar wind turbulence contains some features hardly classified within a general theoretical framework, this medium offers the best opportunity to study turbulence in collisionless plasmas, so that nowadays it is widely considered as a *natural laboratory* for the investigation and modeling of space plasma turbulence [1, 2, 3].

On the other side, the understanding and characterization of space plasma turbulence are fundamental to the modeling of plasma phenomena, such as plasma heating and acceleration, plasma transport across boundary regions, solar wind-magnetosphere coupling, etc. In the last years, it has been clearly understood that the understanding of such phenomena requires a detailed study of turbulent fluctuations of magnetic field and plasma parameters on different timescales from MHD down to kinetic domain.



Now, the study of the features of space plasma turbulence is essentially based on Fourier analysis and statistical approaches, being central issues the characterization of the spectral and scaling properties of both magnetic and velocity field increments/fluctuations. Both these approaches present some limitations and are based on some strong assumptions. For instance, Fourier analysis requires that the signals under investigation must be stationary, and that the increments/differences $\Delta x(\tau) = x(t + \tau) - x(t)$ at a timescale τ are representative of fluctuations at that scale.

However, there are several situations in which fluctuations appears to be non-stationary and, rigorously speaking, increments are only a proxy of fluctuations, since the fluctuations of a certain stochastic variable x are defined as the difference from the average value, i.e. $\delta x(t) = x(t) - \langle x(t) \rangle$, where the brackets mean some averaging procedure.

In 1998 Huang et al. [4] introduced a novel method of analysis, the *Hilbert-Huang Transform* (HHT), to characterize non-stationary signals resulting from the contribution of several oscillatory modes and embedded structures. The main feature of HHT technique stands in combining of a self-adaptive method, the *Empirical Mode Decomposition* (EMD), capable of identifying oscillatory modes and structures without using any *a priori* selected basis, with the standard Hilbert spectral analysis. The result of the application to a non stationary signal is that the signal is firstly decomposed into a set of zero-mean oscillatory modes, the *Intrinsic Mode Functions* (IMFs), which can be considered as the fluctuations at a given scale, to which successively is applied the Hilbert analysis, which allows to compute instantaneous frequency/spectrum. This allows to disentangle the traditional distinction between oscillatory modes and local fluctuations, solving the intrinsic problem of non-stationary fluctuations.

The aim of this work is to present a preliminary analysis of magnetic field fluctuations in the kinetic domain (i.e., below the ion-cyclotron frequency) using the HHT to investigate the potential of this method in characterizing the features of turbulent space plasmas down to non-MHD scales. To this end, we will present an analysis of an already studied solar wind period observed by ESA-Cluster mission.

2. The Hilbert-Huang Transform: a brief introduction

The Hilbert-Huang Transform was introduced by Huang et al. [4] in 1998 as a new tool for analysing nonlinear and non-stationary time series. HHT is indeed a very powerful method to investigate and analyze intermittent and amplitude-varying processes.

HHT consists of two successive steps: the *Empirical Model Decomposition* and the Hilbert spectral analysis [4, 5, 6]. The first step, the EMD, is indeed necessary to apply the Hilbert spectral analysis in such a way to remove possible fake results and is based on the assumption that a time series can be decomposed in a limited set of simple oscillatory modes, the *Intrinsic Mode Functions* (IMFs), which are characterized by significantly different average frequencies/periodicities. The set of IMFs is directly obtained by the time series without any *a priori* assumption via a self-adaptive method. Each IMF is characterized by two main features:

- the number of zero crossings and extrema must be the same or differ at most by one;
- the mean value of local maxima and local minima envelopes is zero.

As a result of the EMD, a time series $x(t)$ is decomposed as follows,

$$x(t) = \sum_j^N c_j(t) + res(t) \quad (1)$$

where $c_j(t)$ are the IMFs and $res(t)$ is a residue (the mean trend or a part of an oscillation characterized by a time scale longer than twice the time series under consideration). Nowadays, different methods have been developed to perform the EMD and to obtain the set of IMFs

[4, 5, 6, 7]. These methods are based on different stopping criteria of the sifting process [4] to get IMFs. Here, we make use of the Rilling et al. method [8].

Once the original time series is decomposed into the set of IMFs, it is, then, possible to apply the Hilbert spectral analysis to each IMF to compute the instantaneous frequency and amplitude. This consists in computing firstly the Hilbert transform of each IMF,

$$c_j^H(t) = \frac{1}{\pi} \mathcal{P} \int \frac{c_j(\tau)}{(t - \tau)} d\tau, \quad (2)$$

where \mathcal{P} stands for the principal Cauchy value of the integral, and successively reconstructing the correspondent analytic signal $z_j(t)$, defined as:

$$z_j(t) = c_j(t) + ic_j^H(t) = A_j(t)e^{i\theta_j(t)}, \quad (3)$$

where $A_j(t)$ represents the instantaneous amplitude and $\theta_j(t)$ is the phase. The instantaneous angular frequency $\omega_j(t)$ (or frequency $f_j(t)$) is then computed by simply differentiating the phase $\theta_j(t)$, i.e.,

$$\omega_j(t) = \frac{d}{dt}\theta_j(t) \rightarrow f_j(t) = \frac{\omega_j(t)}{2\pi}. \quad (4)$$

Using the set of the instantaneous frequencies $f_j(t)$ and amplitudes $A_j(t)$ it is possible to compute the Hilbert spectrum, defined in terms of amplitudes, $H(t, f) = A(t, f)$, or squared amplitudes, $H(t, f) = A^2(t, f)$, by contouring the the instantaneous amplitude of each IMF as a function of both time and frequency. The Hilbert spectrum represents the amplitude (energy) in a time-frequency representation.

Other relevant quantities to characterize the fluctuation field in a time series can be defined from the Hilbert spectrum. For instance, it is possible to introduce a spectral density, named the *marginal spectrum* $H(f)$, which is equivalent to the Fourier power spectral density,

$$H(f) = \frac{1}{T} \int_0^T \frac{H(t, f)}{f} dt, \quad (5)$$

the *degree of stationarity* (DS) of a frequency,

$$DS(f) = \frac{1}{T} \int_0^T \left(1 - \frac{A(t, f)}{\bar{A}(f)}\right)^2 dt, \quad (6)$$

where $\bar{A}(f)$ is the mean amplitude of the frequency f , and the *persistence* (PS) of a frequency,

$$PS(f) = \frac{\Delta T(A(f, t) > 0)}{T}. \quad (7)$$

In the framework of space studies the HHT and, in particular, the EMD has been widely used to investigate the emergence of characteristic periodicities in studies related to both solar cycle and magnetosphere dynamics (see e.g., [9, 10, 11, 12, 13, 14]). Here, we apply the full HHT method to characterize some features of the magnetic field fluctuations at non-MHD/kinetic scales.

3. Data Description

To study the features of the magnetic fluctuations at the non-MHD scales we have consider a period of fast solar wind observed by ESA-Cluster mission on January, 20, 2007 from 12:00 UT to 13:15 UT. This period is a fast ($\bar{v} \sim 600$ km/s) high- β ($\beta \sim 1.6$) solar wind stream characterized by nearly-stationary plasma conditions. Magnetic field data came from FGM and

STAFF experiments on-board of Cluster 3 satellite and are combined so to get a very high resolution (~ 450 samples per s).

Because the HHT procedure is very time consuming for very large datasets, the actual data have been down-sampled by a factor 4 so that the temporal resolution is of $\delta t \sim 8.9$ ms. Furthermore, data have been rotated from GSE reference system to the mean-field one.

Figure 1 shows the three magnetic field components for the selected time interval in the mean-field reference system. Looking to the traces plotted in Figure 1 it is evident how the magnetic field fluctuations are more pronounced in the perpendicular plane than in the parallel direction. This feature is confirmed by variance analysis ($\sigma_{\perp}^2/\sigma_{\parallel}^2 \sim 26$) and suggests that the turbulent fluctuation field is essentially 2D.

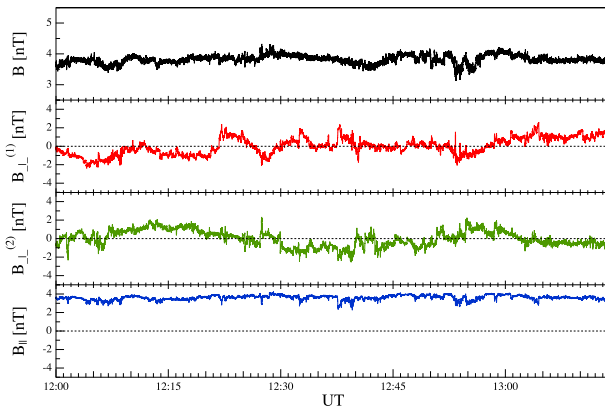


Figure 1. The magnetic field measurements relative to the period of fast solar wind observed by Cluster 3 satellite in the mean-field reference system.

4. Analysis and Results

As a first step of the HHT analysis, we apply the EMD to each time series of the magnetic field component and the magnetic field intensity. We get 18 and 19 IMFs for the magnetic field components and intensity, respectively. Each of this IMF is characterized by a mean characteristic time scale (periodicity) T_j that can be computed by measuring the average time interval between two successive zero-crossings of each IMFs. Figure 2 shows the characteristic periodicities T_j of the IMFs for the three components and the intensity. An interesting feature of EMD of the magnetic field components is that the components time series are decomposed into a set of IMFs characterized by the same periodicities.

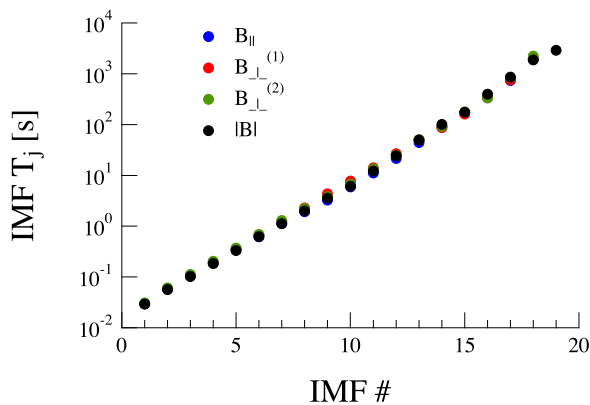


Figure 2. The characteristic periodicities T_j of the IMFs of the magnetic field components and intensity vs. the IMF mode number (IMF#).

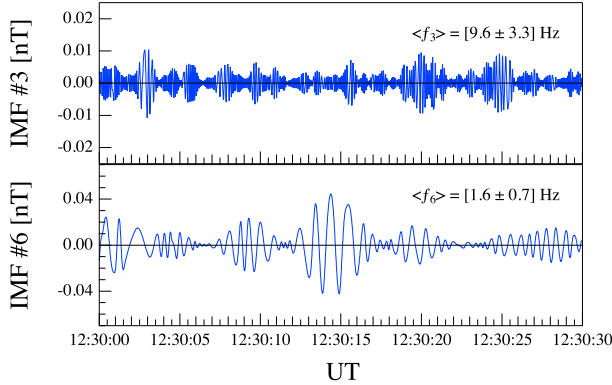


Figure 3. Samples of IMFs #3 and #6 of the magnetic field parallel component at timescales shorter than that corresponding to the ion-cyclotron frequency $f_\Omega \sim 0.06$ Hz. Here, $\langle f_j \rangle$ denotes the mean value of the local frequency $f_j(t)$ computed by Hilbert transform.

Figure 3 shows two samples of 30 s of the magnetic field parallel component at two timescales corresponding to frequencies above the ion-cyclotron frequency $f_\Omega \sim 0.06$ Hz. This plot shows the quasi-periodicity of the IMFs and the inherent intermittent character of fluctuations at these scales. Indeed, the IMFs show periods of relative small amplitude punctuated by intervals with a very large amplitude. However, although the traces seem nearly periodic, the local frequency $f_j(t)$, computed by Hilbert transform, is not constant from one point to another, but spread over a wide interval. For example, for the two IMFs shown in Figure 3 the interval $(\langle f \rangle - 2\sigma_f, \langle f \rangle + 2\sigma_f)$ is $f_3 \in (3.0, 16.2)$ Hz and $f_6 \in (0.2, 3.0)$ Hz, respectively.

Figure 4 shows the Hilbert energy spectrum of the parallel (left panel) and perpendicular (right panel) components for a short time interval of 30 min. The Hilbert spectrum of the perpendicular components is the total one computed as the sum of the two Hilbert spectra of the perpendicular components. The spectra show a singular character with non-persistent frequencies. This suggests that the inherent character of the fluctuation field seems to be more compatible with fluctuations instead of standing waves, as it should be for a regime of strong turbulence.

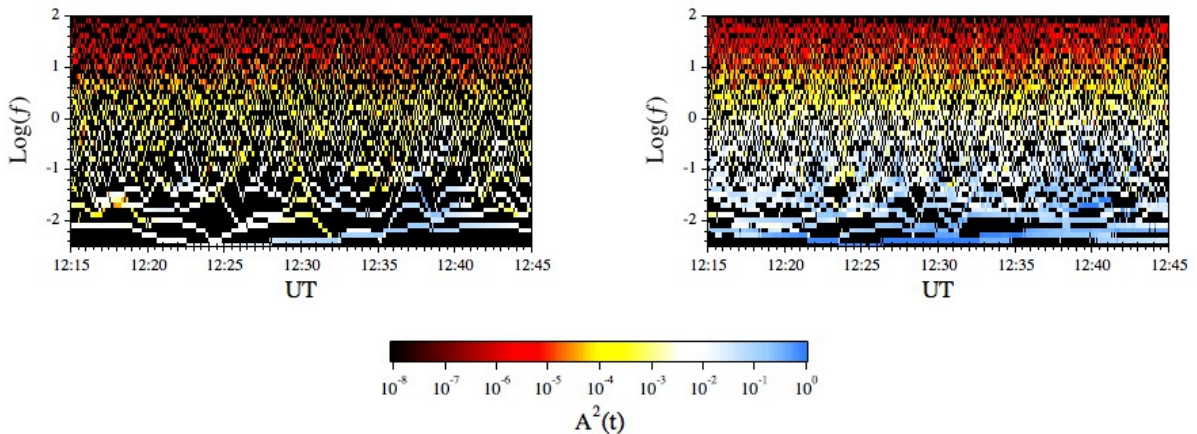


Figure 4. Hilbert energy spectra of the parallel (left panel) and perpendicular (right panel) components, respectively.

This point is confirmed by the degree of stationarity $DS(f)$ and the persistence degree $PS(f)$ as shown in Figure 5. Indeed, the persistence of the observed frequencies both in the MHD and kinetic domain do not exceed the 40%, clearly indicating that the fluctuation field is made by

pulsations localized in time (or in the space assuming the Taylor’s hypothesis).

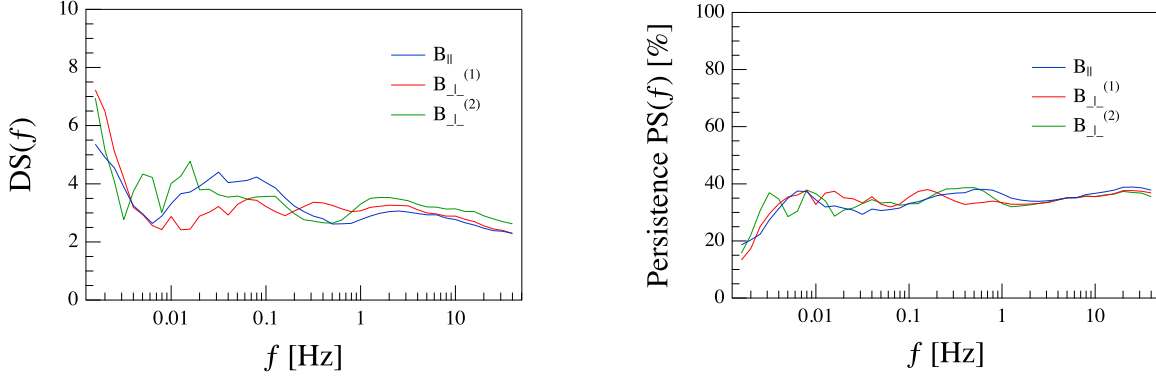


Figure 5. The stationary degree $DS(f)$ (left panel) and the persistence degree $PS(f)$ (right panel) for the parallel and perpendicular directions.

Moving from the Hilbert energy spectra, we evaluate the marginal energy spectra which is equivalent to the Fourier power spectrum. Figure 6 shows the marginal spectra in the parallel and perpendicular directions to the mean field.

Two different spectral regimes, characterized by different spectral exponents, can be identified above and below the spectral break $f_b \sim 0.3$ Hz. This frequency break f_b is compatible with the ion cyclotron frequency f_Ω considering the effect of Doppler shift. Furthermore, below the frequency break f_b all the spectra tend to the typical MHD Alfvénic turbulence spectrum ($S(f) \sim f^{-3/2}$), while above f_b , i.e., in the kinetic domain, the spectra are approximately $\sim f^{-8/3}$. This value seems to be compatible with several different kinetic turbulent regimes (e.g., compressible Hall-MHD turbulence, EMHD-turbulence, kinetic Alfvén wave turbulence), which predict a spectral slope near $\sim -7/3$.

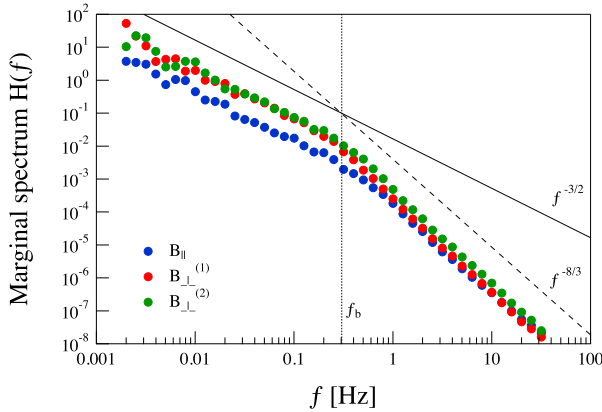


Figure 6. The marginal Hilbert energy spectra in the parallel and perpendicular directions to the mean field. The frequency break is $f_b \sim 0.3$ Hz.

An interesting feature emerging from the marginal spectra is that going to the kinetic scale the asymmetry between perpendicular and parallel spectra reduces. This can be evaluated introducing a sort of pseudo-dimension $D(f)$ defined as,

$$D(f) = \frac{1}{\max(H_j(f))} \sum_j H_j(f), \quad (8)$$

where j is the component index (i.e., parallel (B_{\parallel}) and the two perpendicular components $B_{\perp}^{(1,2)}$) Figure 7 shows the behavior of this quantity computed using the marginal spectra. The most interesting feature emerging from this plot is how the dimensionality (or, if we prefer, the anisotropy) of the fluctuation field tends to increase (decrease) going to the kinetic domain. We underline how magnetic fluctuations tends to be 3D going toward electron inertial scale, i.e. towards a regime of isotropic fluctuation field.

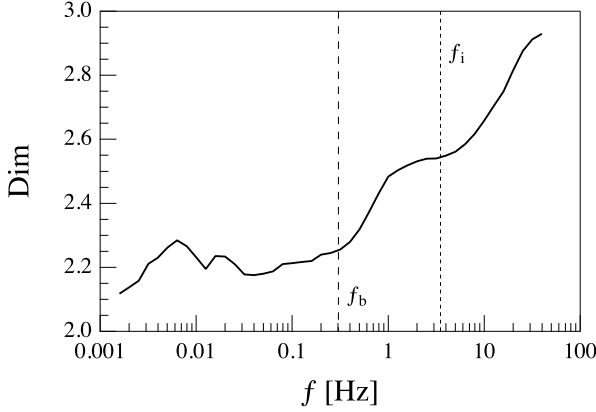


Figure 7. The pseudo-dimension $D(f)$ as a function of the frequency f . Here, f_b and f_i are the frequency break and the one corresponding to the ion-inertial scale η_i computed using the Taylor's hypothesis.

As last step of our analysis, we investigate the scaling features of the fluctuation field at the kinetic scale using the Hilbert spectrum. This can be done computing the q^{th} -order pseudo-structure function, $S_q^*(f)$, using the frequency instead of the scale separation, i.e.,

$$S_q^*(f) = \langle A(t, f)^q \rangle_{\Omega_T} \sim f^{-\zeta(q)}, \quad (9)$$

where the averaging procedure is done over the time sub-interval, Ω_T , in which the frequency is present, i.e., $\Omega_T = \{\Delta t \in [0, T] \mid A(t, f) \neq 0\}$. The definition of the pseudo-structure function $S_q^*(f)$ is analogous to the standard structure function, $S_q(\tau) = \langle |x(t + \tau) - x(t)|^q \rangle$, once it is assumed that $A(t, f) \simeq |x(t + \tau) - x(t)|$ and $\tau \sim 1/f$. A similar procedure has been also adopted to analyze intermittency in homogeneous turbulence [15].

Figure 8 shows the behavior of the scaling exponent $\zeta(q)$ as a function of the moment order q . A linear trend is found for all the three directions, parallel and perpendicular to the mean field. Anyway, different scaling exponent are recovered indicating an anisotropy of the scaling feature parallel and perpendicular. The observed linear scaling of $\zeta(q)$ as a function of q suggests that at the kinetic scales the PDFs of Hilbert amplitudes, A_f , are characterized by mono-fractal scaling features. This is, indeed, the case as clearly shown in Figure 9 where we report the PDFs of the Hilbert amplitudes, A_f (where A_f stands for $= A(t, f)$), scaled to the standard deviation σ_{A_f} in the frequency range from 1 Hz to 20 Hz.

PDFs of scaled Hilbert amplitudes A_f collapse into a single master curve, which is compatible for the occurrence of mono-fractal scaling feature, i.e., there is no evidence of an intermittent cascading process which should be characterized by multi-fractal scaling features.

5. Summary and Conclusions

In this very preliminary work we have investigated the potentiality of the full Hilbert-Huang Transform approach in characterizing the properties of turbulent space plasmas. In particular, we have applied the full method and not limited our work only to the first step, the EMD analysis. This have allowed to characterize the intermittent nature of the fluctuation field clearly showing that there are no-standing frequencies at all the investigated MHD and kinetic scales.

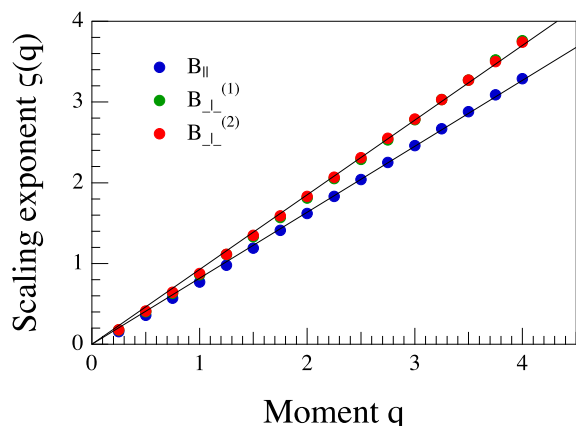


Figure 8. The scaling exponent $\zeta(q)$ as a function of the moment order q .

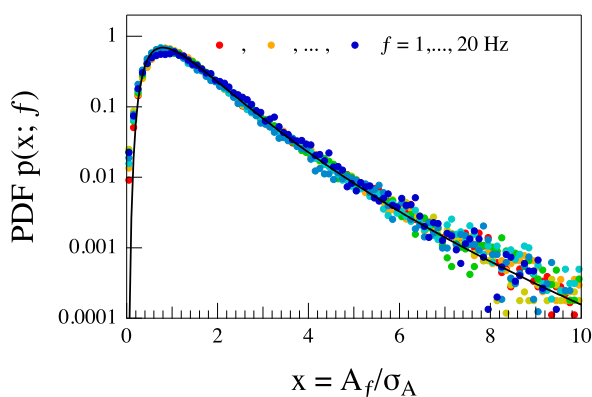


Figure 9. Data collapsing of the PDFs of the Hilbert amplitudes, A_f , scaled to the standard deviation σ_{A_f} . The black curve is a Log-Normal PDF.

The results suggest that the fluctuation field at kinetic scale is essentially characterized by a mono-fractal set of scale invariant fluctuations. Furthermore, we have found an increases of dimensionality from 2 to 3 going toward the electron inertial scale. In other words, we are in presence of an increase of isotropy of the fluctuation field toward the electron kinetic scales.

Clearly, this is only a first application of the HHT method, which has many other unexplored potentialities to investigate turbulence such as a multiscale variance analysis of the real fluctuation field.

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