INSTITUT D'AERONOMIE SPATIALE DE BELGIQUE
3, avenue Circulaire, UCCLE. BRUXELLES 18

# AERONOMICA ACTA 

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Structure of the thermosphere by M. NICOLET

FOREWORD

The paper "The structure of the thermosphere" has been written for publication in "Planetary and Space Science" and it will be published within a few months. An abstract has been presented at a session of the Upper Atmosphere Committee in Helsinki within the framework of the meetings of The International Association of Geomagnetism and Aeronomy, during the General Assembly of the International Union of Geodesy and Geophysics from July 25 till August 6, 1960. This paper has been used for the introductary lecture, $I$ gave at the International Astronautical Congress in Stockholm on August 16.

AVANT-PROPOS

Cet article "The structure of the thermosphere" a eté rédigé pour la publication dans la revue "Planetary and Space Science ${ }^{\prime \prime}$ et paraitra dans quelques mois. Un résumé a été prée senté à une séance du Comité de la Haute Atmosphère à Helsinki dans le cadre des réunions de l'Association Internationale de Géomagnétisme et d'Aéronomie lors de l'Assemblée Générale de I'Union Géodesique et Géophysique Internationale du 25 juillet au 6 aout 1960. Cet article a servi de base a 1 'expose introductif que $j^{\prime}$ ai fait au Congrès de Stockholm le 16 aoot a $l^{\prime}$ invitation de la Fédération Astronautique Internationale.

## VOORWOORD

Dit artikel : "The structure of the thermosphere" werd opgesteld ter publicatie in het tijdschrift "Planetary and Space Science" en zal binnen enkele maanden verschijnen. Een samenvatting werd ten gehore gebracht op een zitting van het "Comite de la Haute Atmosphere" te Helsinki in het raam der bijeenkomsten van de "Association Internationale de Géomagnétisme et d'Aéronomie" ter gelegenheid van de Assemblée Générale de l'Union Géodésique et Géophysique Internationale van 25 juli tot 6 augustus 1960. Dit artikel ligt eveneens aan de basis van een inleidende uiteenzetting welke ik op het Congres te Stockholm gehouden heb op 16 augustus op uitnodiging van de "Féderation Astronautique Internationale".

VORWORT

Dieser Artikel "The structure of the thermosphere" wurde geschrieben fur Herausgabe in "Planetary and Space Science" und wird in wenige Monate erscheinen werden. Eine Zusammenfassung wurde vorgestellt wahrend einer Sitzung des Komitees fur die hohere Atmosphare. in Helsinki bei der Gelegenheit der Versammlungen der Internationalen Assoziation fur Geomagnetism und Aeronomie wahrend der Generalen der Internationalen Vereinigung fur Geodesie und Geophysik vom 25. Juli zum 6. August 1960. Dieser Text diente zum Einleitungsbericht, der ich zum Internationalen Astronautischen Kongress in Stockholm am 16. August vorgestellt habe.

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## Abstract

The vertical distribution of the density in the thermosphere，deduced from satellite observations，must be explained by an increase of the scale height with altitude。 A varying gradient of the scale height cannot be interpreted by assuming an increase of the temperature gradient with altitude．An examination of the interrelationships between the absolute values of density in a dark atmosphere and diurnal conditions of heat conduction reveals that the vary－ ing gradient of the scale height above 200 km is essentially due to the decrease of the molecular weight， mg ，of the at－ mospheric constituents subject to diffusion．

In the night atmosphere the isothermy above a certain altitude $(>200 \mathrm{~km})$ is the critical factor characterizing the vertical distribution of density。 The temperature of the isothermal region，resulting from conduction，is related to the ultraviolet heating which was available during the dayo The effect of diffusion has been clearly shown by establishing a thermooisobaric relation connecting the temperature of the isothermal region with an isobaric level where atomic oxygen has a specific concentration．From observational data on the variation of the night time density at high levels，it is possible to deduce the variation of the temperature of the isothermal region。

The gradient of temperature in a sunlit atmosphere is related to the fraction of the ultraviolet sun＇s energy absorbed，which determines the magnitude of the variation
of the scale height with altitudo. Since heat transport is a function of the atomic or molecular concentrations and the square of the distance, it is shown that anomalies in the temperature gradient cannot be permanent.

1. Introduction

The effect of air resistance on the motion of an artific cial earth satellite makes it possible to derive the atmospheric density in the region of the perigee of its orbit. Formulae rea lating satellite drag to orbital elements have been derived by various authors; see for example Groves (1958-9), KingoHele (1959), Sterne (1959). Determinations of density have been made by various authors following the initial calculations of the acceleration of the first two satellites 1957 and $\beta$ (Sputniks 1 and 2), whose perigees were at about 220 km ; see for example Mullar Radio Astronomy Observatory (1957), Royal Aircraft Establishment (1957), Sterne, Folkart and Schilling (1958), Sterne and Schilling (1958), Harris and Jastrow (1958a), Jacchia (1958a, 1959a), Groves (1958a), Sterne (1958 a and b), El ${ }^{0}$ Yesberg (1958), Lidov (1958), Mikhnevich (1958), Priester, Bennewitz und Lengruesser (1958), Warwick (1959), Paetzold (1959b)。

From the spring of 1958, the satellites Vanguard $I(1958 \beta 2)$, Explorer $I(1958 \alpha)_{s}$ Explorer IV (1958e) \& and Sputnik III (1958 S), made possible an analysis of the densities at altitudes of approximately $650 \mathrm{~km}_{g} 350 \mathrm{~km}, 260 \mathrm{~km}_{8}$ and 220 km corresponding to the perigees of the above salelliteso See for example, Jacchia (1958b), Jacchia and Briggs (1958), Harris and Jastrow (1958b), Siry (1959), Sterne (19.58 a and b), Schilling, Whitney and Folkart (1958), Schilling and Whitney (1958 and 1959), Sedov (1958), KingoHele (1959a), Groves (1959a, b, c), Mikhnevich, Danilin, Repner, Sokolov (1959), Paetzold (1959d).

It is evident that the absolute values of the density
which have been deduced from these observations may vary according to the methods and satellite parameters utilized by the various authors. Thus, the drag parameter $\eta_{s}$ is dependent on the mass of the satellite $m_{S}$, on the effective cross section of the satellite $s_{S}$, and on the satellite's drag coefficient $C_{S}$, and $\eta_{S}=s_{S} C_{S} / m_{S}$ can differ according to the values used by the authors. Consequently, the absolute values of the density deduced can be different even if the formilae relating density to acceleration are the same. In addition, the values of the density do not necessarily relate to the same period in the life of a satellite and can correspond, therefore, to different physical states of the high atmosphere. In this case, the scale height associated with the density may be not appropriate. Finally, becanse the altitudes of certain satellites are not given with adequate precision the densities obtained do not correspond with the approximate altitude indicated.

From the beginning of these observations, Groves (1958a) stressed the importance of the equatorial bulge which changes the altitudes by 12.5 km between the Equator and 'latitude 50\% After Jacchia (1958b) had indicated the existence of irregularities in the acceleration of satellite 1957 ; many variations were detected and ascribed to many different causes but it was immediately obvious that the atmosphere was responsible for these irregularities. Nevertheless, the opecial effects were attributed to many causes such as discontinuities in the atmosphere at certain latitudes. For example King-Hele and Walker (1959) insisted that near
$30^{\circ} \mathrm{N}$ the effect of the irreguiaritios was caused mainly by solar disturbances. in any event, the variations of solar radiations at 20 cm and 10 cm (Priesier 1959 g Jacchia 1959b) show clearly that soiar emissions play a primary role in the variation of atmospheric density. It should be noted here that it is only possibie to enter into a detailed discussion of all the variations if the observational data are very precise and sufficient in number so as to be able to follow all the flustuations as a function of altitude。
2. Analysis of observational results

$$
\begin{aligned}
& \text { 2ot Mean yadues of the density } \\
& \text { Betore arrying out an ansiysis of the various }
\end{aligned}
$$ variations oi the density it is desirable to provide a description of the results as a whole For this reason we have summariwed the main results as shown in Figolo Before beang able to determine which variations modify the density we must decide on the average conditions. The different determination shown in figolo were made for different periods, although most of them relate to the beginning of 1953. It should be noted that the rocket results, for altitudes of the order of 200 km g give variam tions which do not appear to agree with the results dedueed from the acceleration of satellites. It appears, howeverg that the rocket measurenente of deneity carried out at about 200 km lead to values of ( 4 t 2 ) $\times 10^{-13} \mathrm{gm} \mathrm{cm}^{\alpha-3}$, although certain differences can be expected since measurements made



Fig. 1
with rockets involve the collection of samples which can relate to sporadic conditions that are not necessarily repo resentative．As a result，values of density between 200 and 250 km as given by Mikhnevich（1958）are probably too small when compared with satollite data。

If we consider the results of LaGow et 2l。（1959）and Horowitz et al。（1957，1959），the densities，$P$ ，at 200 km g at Churchillg ioe。

1956，Nov．17，day；$\quad \rho=3.6\binom{+3}{\infty 1.5} \times 10^{-13} \mathrm{gm} \mathrm{cm}^{-3}$
1957，July 29，day；$\quad P=(6.7 \pm 2) \times 10^{\infty 13} \mathrm{gm} \mathrm{cm}^{-3}$
1958，Oct。 31 ，dey；$\quad \rho=400 . x 10^{\infty 13} \mathrm{gm} \mathrm{cm}^{-3}$（extrapolated）
1958，Feb。 24，night；$\rho=(1.3 \pm 0.6) \times 10^{\infty 13} \cdot \mathrm{gm} \mathrm{cm}^{-3}$
it is clear that the median value corresponds to the value obtained from the satellite data，$(4 \pm 2) \times 10^{\infty} 13 \mathrm{gm} \mathrm{cm}^{-3}$ 。 Furthermore，the value obtained at White Sands，of the order of $1.4 \times 10^{-13} \mathrm{gm} \mathrm{cm}{ }^{-3}$ at $200 \mathrm{~km}_{9}$ corresponds to an atmesphere， in 1957，for which the pressure was only $10^{04} \mathrm{~mm} \mathrm{Hg}$ at 100 km when it was $(3+1) \times 10^{-4} \mathrm{~mm} \mathrm{Hg}$ at Churchill。

Mikhnevitch（1958）indicating a density of $2.7 \times 10^{013} \mathrm{gm}$ 200 km is still inside of the possible variation．However， he adopts（1959）the value of $2.1 \times 10^{\infty 13} \mathrm{gm} \mathrm{cm}^{\infty 3}$ at 225 km for an atmospheric model．

It can be concluded that an average value of the order of $4 \times 10^{\infty 13} \mathrm{gm} \mathrm{om}{ }^{-3}$ represents lhe atmospheric density at 200 km during the sunspot maximum in $1958 \approx 1959$ 。 Variations leading to $(4: 2) \times 10^{-13} \mathrm{gm} \mathrm{cm}^{-3}$ can be acceptedo Extreme
values between 1 and $7 \times 10^{-13} \mathrm{gm} \mathrm{cm}^{-3}$ are not representative of latitudinal，seasonal or diurnal variations at 200 km ，but should be associated with the effect of solar activity if they are not included in the possible errors of measurement．

It must be pointed out that an analysis of the varying conditions at 200 km is to be related to the analysis of con－ ditions between 100 km and 150 km ．For example，there is a Yariation of the density of molecular oxygen by a factor of 3 according to the measurements made by Byram，Chubb and Friedman（1957）and Kupperiang Byram，Friedman and Unzicker （1958）。 Likewise considering the results obtained by Horowitz and LaGow（1957）at White Sands and by Horowitz，LaGow and Giuliani（1959）at Ft。Churchill it is clear that there is a broad range of a factor 4 in the pressure and density data at 100 km ．In other words if the value $(2.5 \pm 1.5) \times 10^{-4} \mathrm{~mm} \mathrm{Hg}$ is accepted for the pressure at 100 km ，the possibility exists that the density at 200 km is subject to a variation of $50 \%$ even if the structure of the atmosphere above 100 km is essen－ tially the same．Therefore，the data available on atmospheric density obtained by means of rockets and of satellites show that variations of the density near 200 km must be connected with the atmospheric structure in the entire region between 100 and 200 km 。 A certain variation of the density at 200 km must be explained by the boundary conditions near 100 km and by the atmospheric structure above the later altitude。

The curve in Fig．l．has been drawn following the determination of Oroves（1959a）and can be considered as an
average distribution for the first six months of 1958 corre－ sponding to a certain sunlit atmosphere．The absolute values near 200 km depend on the value which is assumed for the scale height at that altibude for example，the results of Lidov （1958）lead to values for $\rho$ between $2.4 \times 10^{-13} \mathrm{gm} \mathrm{cm}^{-3}$ to $3.2 \times 10^{-13} \mathrm{gm} \mathrm{cm}^{-3}$ if the scale heights are 50 km and 30 km ； respectively。 EliYasberg（1958）has adopted $H=25 \mathrm{~km}$ at 225 km 。 The average value of Groves（1959a）at 200 km is 46 km 。 Such differences show that variations do occur in the atmosphere above 200 km 。 But the main conclusion to be drawn from the average values of the density is that the vertical distribution must be explained by an increase of the scale height $H$ with altitude．In fact the equation for a perfect gas and the static equation indicate（Nicolet 1959a）that the density can be expressed by the relation

$$
\begin{equation*}
\frac{d \rho g}{\rho g}=\frac{1+\beta}{\beta} \frac{d H}{H} \tag{2.1}
\end{equation*}
$$

where $\beta=d H / d z$ is the gradient of the scale height and $g$ is the gravitational accelerationo The integration of equation （2．1）for a height interval sufficiently small so that $\beta$ can be considered practically constant lead＇s to
$\frac{\rho g}{\rho_{0} g_{0}}=\exp \left[-\frac{(1+\beta) z}{\frac{1}{2}\left(H+B_{0}\right)}\left\{1+\frac{1}{3}\left(\frac{\beta z}{H_{0}+H}\right)^{2}+00\right\}\right](2.2)$

In using equation（2．2）we can see that $H$ increases regularly with altitude between 150 km to 700 km ．Thus the results of Groves（1959a），corresponding to a certain sunlit atmosphere in 1958，coincide with an average model presented
by Nicolet（1959b）。 Subsequent calculations carried out by Nicolet（1960 a and b）involving an analysis of the physical conditions of an atmosphere in which the temperature is constant above 220 km lead to the important conclusion thats the vertical distribution of the density of the high atmosphere can be ex－ plained if the atmosphere in which the various constituents are subject to diffusion is isothermal above a certain level． Such an atmospheric model shows that ultraviolet radiation is involved in the heating of the atmesphere。

A recent analysis made by King－Hele and Walker（1960 b） leads to a night－time distribution in 1959 shown in Fig．2。 This Figure further shows that an isothermal atmosphere at about $1200^{\circ} \mathrm{K}$ with diffusion beginning at 150 km can represent the observations．The observational results do not differ from the model computed by Nicolet（1960b）。 In the same way， an analysis of night－time conditions by Jacchia（1960）（see Fig．2）also shows the possibility of following the atmospheric distribution for high solar activity when a temperature of the order of $1400^{\circ} \mathrm{K}$ is adopted for the isothermal layer．

The above results lead to the conclusion that there Is a thermopause and that its level is subjoct to a diurnal variation．Its altitude is maximum in a sunlit atmosphere and is minimum in a dark atmosphere．Howeverg when the scale heights corresponding to isothermal atmospheres in diffusion equilibrium are compared（Fig。3）with the empirical scale heights deduced by King－Hele and Walker（1960b）and by Jacchia （1960）considerable differences are found．KingoHele and

-10-


Fig. 3

Walker have introduced a negative gradient below 250 km and Jacchia＇s curve corresponds to an important gradient oven at 600 km 。 Such differences show that arbitrary atmospheric models can be made to follow atmospheric densities deduced from satellite observations．

2．2 Variations of the density
Jacchia（1959d）has shown that the amplitude of the fluctuations of the accelerations，proportional to $\mathrm{pH}^{1 / 2}$ ， increases with the altitude of the perigee of the satellite． In fact，the important fluctuations appear simultaneously at all altitudes and the diurnal variations are magnified between 350 and 650 km 。

The effect of the earth＇s equatorial buldge is evident． Each transit at the Equator of Explorer I corresponds to a maximum and the change in perigee height of Sputnik III leads to a variation in the acceleration。

In Fige 4 we consider data from satellite 1958 $\beta 2$
during the first half of 1959，obtained from the values deter－ mined by Jacchia（1959d）and Briggs（1959）．The solar activity is represented by daily values of solar radiation at 10.7 cm ． as observed in Canada（National Research Council）and by the daily values deduced from the three－hourly $K$ indices provided by Bartels．It is obvious，as Jacchia has shown（1959d），that there is a very close correlation between the variations of the density and those of the solar radiation as obtained using the electromagnetic radiation at 10.7 cm ．As for a magnetic


Fig. 4
activity relation to the corpuscular radiation, there is no general correspondence although, in certain cases, it is possible to see that corpuscular effects can occur during important geomagnetic disturbances. It appears, however, that a greater resolution in the orbital acceleration is needed to detect short-lived perturbations.

On April 30 the density has decreased, Fig. 4 , when the sun is at $90^{\circ}$ (the angle at the centre of the Earth determined by the position of the sun and of the perigee of the satellite); that is, when the Earth below the perigee is no longer sunlit. Finally, on July 31 the nocturnal effect is complete and the density is very low. These data show that the atmosphere at an altitude of 650 km is subject to a very important diurnal variation and the different analyses of Jacchia (1959d), Wyatt (1959), and of Priester and Martin (1959) show that the diurnal effect is the principal one. This strong diurnal variation is confirmed by the observed accelerations of Vanguard II 1959a, (Jacchia, 1960), for which the diurnal variation of $\mathrm{\rho H}^{1 / 2}$ is of the order of a factor of 5 between March and August 1959 (Fig。5). It is therefore necessary to consider that the altitude of the thermopause and the temperature of the isothermal atmosphere vary considerably between night and day. Under such conditions the normal heating of the upper atmosphere takes place by electromagnetic radiation. It is therefore, not necessary to look for a corpuscular effect (Bartels 1959) when investigating such a normal heating effect of the atmosphere, a hydromagnetic offect

(Dessler 1959), or any effect other than that of electromagnetic radiation, because such effects cannot be associated with a diurnal variation whose character is so pronounced at 650 km . Such effects must be linked with disturbances.

The variations of $\rho \mathrm{H}^{1 / 2}$ at an altitude of about 350 km , corresponding to the perigee of Explorer $I(1958 \sigma$ ), are considerably smaller than at an altitude of 650 km . From the beginning of January 1959, when the angular distance of the Sun reaches $180^{\circ}$ until it reaches $90^{\circ}$ at the March equinox, the density decreases slightly in agreement with the diminution in solar flux. Between March and August 1959 the increase in the acceleration occurs with the fluctuattons clearly associated with the sequence of perigees at the Equator, even if some association can be made with the solar activity.

Thus, at an altitude of 350 km , the offect of solar heating is evident and leads to a clear cut diurnal variation. The effect of the earth's equatorial buldge is of the order of $10 \%{ }^{\circ}$

When an analysis of the variations of the density is carried out at altitudes less than 300 km it is found that the diurnal effects are greatly diminished. In the analysis of the density deduced from the acceleration of $1958 \in$ (Schilling and Whitney, 1959) the diurnal effects that can be infered for the period between July and November 1958 reach a maximum of $20^{\circ} \%_{0}$ at the same time, an apparent latitude offect, causod by a change in perigee height due to the Earthis equatorial buldge, is certainly of the order of $20 \%$ The discontinuities

In the density at $50^{\circ} \mathrm{N}$ and S cannot be attributed to a lati－ tudinal variation in air density．

The analysis of the fluctuations of the Sputniks by various authors already mentioned，in particular King－Hele （1960），have shown that the diurnal effect is generally masked by other effects．In Fig． 6 the data given by Kozai（1960） for $1958 \delta 2$ are shown．It may be seen that from January to November 1959 the maximum variation is $\pm 60^{\circ} \%$ The main character of this variation corresponds to a displacement of the perigee from $24^{\circ} \mathrm{S}$ on 1 January 1959 to $65^{\circ} \mathrm{S}$ ，at the beginning of June，and to less than $10^{\circ} \mathrm{S}$ at the end of November 1959． Thus，the perigee was located in the Antarctic during the winter．Because of this，the altitude of the perigee did not diminish with time（May and Smith 1959）。 In June its al甘itude，about 225 km ，was a maximum and it is necessary to consider an increase in altitude of some 18 km with regards to the Equator．As a consequence a decrease（ $30^{\circ} \%$ to $40^{\circ} \%$ ） of $d p / d t$（see Fig．6）can be explained by the flattening of the Earth．It is，therefore，clear that the general variation observed in the acceleration of this satellite is due to ohange in the altitude of the perigee and not to a discontinuity caused by a latitude effect．

Another remarkable result is the very close association with geomagnetic storms that Jacchia（1959c）discovered for satellite $1958 \delta$ ，thanks to 2 resolution of 10 revolutions in the analysis of the acceleration。 Fig。 7 shows the variation


Fig. 6


Fig. 7
of the acceleration of $1958 \delta 1$ as given by Jacchia (1959c) with two remarkable peaks on July 9-10 and September 4-5 when a resolution of 10 revolutions is used in the analysis of observations. It is clear that such an increase of the atmospheric density is associated with the magnetic activity defined by the $I$ indices. A remarkable fact that Jacchia (1959c) has found is that the latitudes of the perigee of 1958 f I were $35^{\circ} \mathrm{N}$ and $15^{\circ} \mathrm{N}$ in July 9 and September 4 ; respectively. Furthermore, it is of interest to note that the angular distances, $\mathcal{F}$, of the perigee from the sub-solar point were $120^{\circ}$ and $75^{\circ}$; respectively, Finally, tt is clear that the electromagnetic solar radiation represented by 10.7 cm was decreasing after the occurence of its peak two or three days before. These two events show that a limited resolution corresponding to several days may smooth the transient disturbances affecting the atmospheric density。

However, if we consider the periods when the perigee was in a dark atmosphere ( $\Psi$ in Pig. 7) we can see three other increases associated with the $K$ indices ( $K>5$ ) even if the resolution is only 25 revolutions. They are June 28-30, September 24-26 and October 22-24, when the angular distance, of the porigee from the sub-solar point is greater than $90^{\circ}$. Again the corresponding latitudes of the perigee are very low; between $15^{\circ} \mathrm{N}$ and the equator from September to october.

Taking again the Smithsonian data for Sputnik II (19578) (Jacchia 1959a), it can be shown (Fig. 8) that certain variations of the density can be associated with $K \geqslant 5$ indices.


Fig. 8

Two remarkable associations are found on January 1-2 and February 9-11 when the Fangles were $80^{\circ}$ and $120^{\circ}$ and the perigee latitudes were $30^{\circ} \mathrm{N}$ and $15^{\circ} \mathrm{N}$; respectively.

Even if the observations of $1958 \delta 2$ are irregularly distributed three pronounced increases in the acceleration curve of Sputnik III (1958 ס 2) can be seeng. Fig. 6, for 27 March, 29 June and 4 September. These are associated with geomagnetic disturbances which are represented by the K indices (Bartels 1959). It should be noted that these three remarkable increases in the atmospheric density were observed during the three periods when the perigee was in the dark atmosphere. This shows again that the reactions of the atmosphere to acorpuscular effect" are more clearly distinguishable when offects on a sunIit atmosphere can be eliminatod.

If wa consider periods during which the atmosphere was sunlit (May, July, August) important magnetic disturbances were also observed, but their associated effects do not seem to be so disturbing to the behaviour of the acceleration of satellite 1958 (2。. In particular the magnetic storm of 12 May 1959 (see the $K$ indices Fig. 6) observed by Ney, Winckier and Freier (1959) in association with cosmic radiation does not show, with the low resolution of the acceleration curves, a special effect on the acceleration of the satellite. The maximum in the acceleration curve, about 8 May, is more closely associated with the maximum of the solar electromagnetic radiation。 Similarly, the maximum near 15 July at sunrise
appears to be related to the electromagnetic solar radiation， although the cosmic ray bursts described by Winckler（1959）are noteworthy。

Moreover，a comparison should be made between the magnetic storm of 16 August 1959 and the recurrent storm of 4 September（during the night for $1958 \delta 2$ ）。 Daring the first 24 hrs of the magnetic storm of 16 August，Arnoldy，Hoffman and Winckler（1960）found that about $3 / 4$ of the particles in the outer $V a n$ Allen belt had been removed and had penotrated into the atmosphere．But the effect of the＂corpuscular＂ heating of the atmosphere is probably not greater than the electromagnetic heating whilst the excoptional offect observed on 4 September is clearly a transient heating in the dark at－ mosphere．

These different examples show that is should be possible to distinguish between all of the external effects by a detailed determination，with sufficient resolution，of the acceleration of the satellites．In any event，the diurnal variation and the 28－day periodicity show that the absorption of solar electro－ magnetic radiation is the primary process for heating the atmosphere above 100 km 。 The＂corpuscular＂radiation．wilh its associated processes can easily affect the nocturnal conditions but is more difficult to detect in a sunlit atmosphere which is already heated by an increased electromagnetic radiation．Furthermore， the strong diurnal variations at the higher altitudes can mask such effectso Finally，the penetration of energy below 100 km 。 has practically no influence on the structure of the thermosphere
in that the energy involved in flares is not important compared with the total kinetic energy of the atmosphere. The most remarkable phenomenon near 200 km is the relation with the energy of the solar radiation which varies in accordance with changes in the solar activity. This is the reason that several investigators have found correlations with various indices of solar activity. Such a variation at 200 km means that the atmosphere between 100 and 200 km is affected and must correspond to a general increase of the scale height in the entire layer.

The variations of the density as a function of altitude demonstrate that the diurnal variation is magnified with increasing height. It is the dominant factor at higher altitudes and must be associated with the gradient of the scale height and altitude of the thermopause。

The variation from one day to another is closely associated with solar activity and the size of the variations of the density, increasing with the altitude, depends on the energy of the solar electromagnetic radiation which is available during a 27 day period.

A corpusoular effect is evident at the time of magnetic disturbances but it is only introduced sporadically and the energies which are involved are generally less than ultraviolet energies. Seasonal and latitude offects can only be secondary in relation to the complex effects of the diurnal and solar variations.

## 3. The Constitution of the Thermosphere

The constitution of the thermosphere, that is to say of the atmosphere above 85 km , theoretically depends on the state of molecular dissociation at the highest altitudeso Indeed, the essential observation that the scale height $H$ increases with altitude necessitates an analysis of the variations of three parameters: the temperature $T_{9}$ the mean molecular mass $\bar{m}$, and the acceloration due to gravity, go

If the atmosphere above a certain altitude was in a state of complete dissociation, an increase of $H$ with altitude could only be explained by an increase of temperature with altitude. Above 300 km , we can neglect a heating effect by ultraviolet radiation in that there is practically no absorption of such radiations at very high altitudes. In this case, a very pronounced dissociation of molecular oxygen below 200 km , with a low pressure ( $10^{-4} \mathrm{~mm} \mathrm{Hg}$ ) at 100 km g leads to an atomic oxygen atmosphere. To explain the gradient of the scale height it is necessary to introduce a flux of external heat transported by conduction. Such an application has been made by Nicolet (1958a, 1959a) using Chapman's theory of an extension of the solar corona.

But an atmospheric model in which the pressure at 100 km reaches $3 \times 10^{-4} \mathrm{mmHg}$, with a small percentage of the oxygen dissociated, represents more closely the atmospheric conditions leading to a further understanding of the thermosphere。 An important temperature gradient exists below 200 km and leads to a large abundance of molecular nitrogen at high altitudes
(Nicolet 1959b, 1960). Since atomic mitrogen is a secondary constituent (Nicolet $1958 \mathrm{~b}, 1959 \mathrm{a}$ ) we may consider an atmosphere whose constitution depends essentially on the ratio $N_{2} / 0$. In a general study, Nicolet (1960) has shown how the problem of the conditions in the thermosphere can be analysed. Using the following conditions at 100 km :
$p=3 \times 10^{-4} \mathrm{~mm} \mathrm{Hg} ; T=200^{\circ} \mathrm{K} ; \rho=6.6 \times 10^{-10} \mathrm{gm} \mathrm{cm}^{-3}$ corresponding to the concentrations ( $\mathrm{cm}^{-3}$ )
$n\left(O_{2}\right)=2.2 \times 10^{12}, n\left(N_{2}\right)=1.1 \times 10^{13} ; n(0)=1.4 \times 10^{12}$ and

$$
\begin{equation*}
m=27.4 \tag{3.2}
\end{equation*}
$$

the conditions at higher altitudes can be determined if we fix the level for the beginning of diffusion and the gradient of the scale height.

In Fig. 9 are shown conditions such that diffusion. begins at 120 km and at 150 km , and the scale height has large gradients of $\beta=1.0,1.5$ or even 2.0 between 120 and 150 km and an arbitrary gradient of $\beta=0.2$ between 150 and 220 km . In order to simplify the calculations it was noted that these two gradients are almost equivalent to a variable gradient diminishing with height until about 300 km .

$$
\text { Considering Fig。 } 9 \text { we see that very different conditions }
$$ do not modify to any great extent the vertical distribution of the density. For example, the density at 200 km only varies by $\pm 25 \%$ from the mean value for the above range of variables. Consequently we can conclude that, for constant conditions



Fig. 9
at 100 km , it is easily possible to obtain the densities, deduced from the satellite observations if we consider that there is a large temperature gradient between 100 and 200 km 。 Furthermore, the vertical distribution of the density and the absolute value at 200 km are practically independent of the exact value of the temperature gradient. An increase of the order of $50 \%$ in the density at 200 km would require an increase of the order of $1000^{\circ} \mathrm{K}$ in the temperature. An increase of the density at 200 km should be caused mainly by its increase in the region of 120 km .

A similar offect explains the differences in the observations made with rockets at different periods at White Sands and Ft. Churchill. Similarly, differences obtained at the same place of observation can only result from a change of temperature in the lower thermosphere. In other words, the structure of the thermosphere depends primarily on the conditions at the limits applicable to the lower thermosphere (different densities at 100 km ). Moreover the fact of having introduced diffusion at 120 km or 150 km does not modify the above conclusions for the atmosphere at altitudes of less than 250 km 。

We can therefore conclude that: the density of the high atmosphere is essentially dependent on the solar energy absorbed below 200km. This energy determines the temperature gradient up to 400 km in a sunlit atmosphere and fixes the tomperature of the isothermal atmosphere up to the highest altitudes.

Since various temperature gradients between 120 km (E layer) and 200 km ( $F$ layer) modify only slightly the values of the density at 200 km , if constant boundary conditions are
taken at 120 km ，the small variations in the densities deduced from satellite observations near 200 km are easily explained． But it must be pointed out that variations must occur in the E layer which modify the boundary conditions for the whole thermosphere．The computations lead to the following results $120 \mathrm{~km} \rho \mathrm{P}=3.26 \times 1.0^{-11} \mathrm{gm} \mathrm{cm}^{-3} ; \quad \mathrm{T}=262^{\circ} \mathrm{K}$ $150 \mathrm{~km}, \rho=(1.5 \pm 0.03) \times 10^{-12} \mathrm{gm} \mathrm{cm}^{-3} ; 725^{\circ} \mathrm{K} \leq \mathrm{T} \leq 1650^{\circ} \mathrm{K}$ 。（3．4） Thus，it is certain that varying conditions occur inside of the E layer．

The total kinetic energy varies between $5 \times 10^{4}$ and $5 \times 10^{5}$ ergs $\mathrm{cm}^{-2}$ from about 120 km to 100 km 。 If an energy of the order of 1 erg $\mathrm{cm}^{-2} \mathrm{sec}^{-1}$ is available for the heating of the E layer，it is clear that the boundary conditions at l20km will be subject to variations resulting from solar activity． In the same way，the region of the Fi layer where the total kinetic energy is less than $5 \times 10^{4} \mathrm{erg} \mathrm{cm}^{-2}$ ，will be strongly affected by an ultraviolet heating of the order of $1 \mathrm{erg} \mathrm{cm}^{-2} \mathrm{sec}^{-1}$ ．

Below 100 km ，at 85 km for example，the kinetic energy of a vertical column is of the order of $5 \times 10^{6}$ org $\mathrm{cm}^{-2}$ and an equivalent solar energy during one day of 12 hours would require a total absorption of about $100 \mathrm{erg} \mathrm{cm}^{-2} \mathrm{sec}^{-1}$ 。 We can therefore conclude that the entire thermosphere above l00km is essentially dependent on the solar energy and its fluctuations if the energy available for heating is of the order of 1 erg $\mathrm{cm}^{-2} \mathrm{sec}^{-1}$ ．Such an energy input must be found in the X－ray and ultraviolet radiations absorbed above 100 km ．
4. Solar Radiation

The diurnal variation of the atmospheric density above 250 km shows clearly that the heating of the thermosphere depends mainly on the electromagnetic radiation absorbed in the atmosphere above l' 00 km 。

Above 100 km the atmosphere absorbs radiations with wave lengths of less than 1750 A as a result of the dissociation of the oxygen molecules. The recombination of oxygen atoms mainly occurs below lookm after a downward transport and for this reason there is no important heating available from in situ resombination。 However, the energy available from the various monochromatic radiations between 1500 A and 1300 A corresponds to about 3 ergs $\mathrm{cm}^{-2} \mathrm{sec}^{-1}$, and the part transformed immediately into heat is about $0.5 \mathrm{erg} \mathrm{cm}^{-2} \mathrm{sec}^{-1}$ in the E layer. Furthermore, the total energy from X-rays absorbed in the $E$ layer doesinot exceed $0.5 \mathrm{erg} \mathrm{cm}^{-2} \mathrm{sec}^{-1}$ at the maximum of the solar cycle. In addition, the heat flow from upper levels will lead to an important effect in the $E$ layer since convection and conduction are certainly involved. We can conclude therefore that the $E$ layer is subject to varying conditions depending on the coronal flux in the $X$-ray spectrum and on the chromospheric and coronal flux in the ultraviolet spectrum.

At altitudes above the E layer, from the beginning of the Fl layer, we must take into account the absorption of radiations of wave lengths longer than 200 A. This corresponds to the entire ultraviolet spectrum involving Helium lines at $304 \mathrm{~A}^{\circ}$ and 584 A and also coronal lines of highly ionized atoms.

An energy of the order of $1 \mathrm{erg} \mathrm{cm}^{-2} \mathrm{sec}^{\infty} \mathrm{c}$ can produce temper－ ature gradients of the order of $20^{\circ} \mathrm{K}$ per km 。 Such important gradients lead to very high temperatures above 200 km 。

If we consider a steady state for an overhead sun，it is easy to show（see Section 5）that the energy E is distributed with height as follows

$$
\begin{equation*}
E=E_{\infty}+E_{u V}\left(1 \infty e^{\infty \tau}\right) \tag{4.1}
\end{equation*}
$$

in which $E_{\infty}$ denotes the energy flow at the top of the layer， $E_{u v}$ the ultraviolet solar energy available at the top of the earthis atmosphere for a certain spectral range and $\tau$ is the optical depth，which is defined by

$$
\begin{equation*}
\tau=K_{\lambda} \int_{z}^{\infty} \mathrm{n} \mathrm{dz} \tag{4,2}
\end{equation*}
$$

Since the heating must be proportional to the energy absorbed it is evident that the temperature gradient will decrease with height up to a certain altitude where the optical depth is negligible。

If there is no external heat flow，the sunlit atmosphere must be isothermal where there is no absorption of solar radiation。 Since $X$－rays and ultraviolet radiations are absorbed between the $E$ layer to the $F$ layer the temperature must increase con－ tinuously and no decrease can be expected in the whole thermo－ sphere。

In order to study the variation of the short ultram violet radiations and of $X$－rays one refers to measurements of the centimeter or decimeter electromagnetic radiations emitted by the solar corona．Nicolet（1960b）has compared the results
obtained at various observatories from 3 to 30 cm iand has shown that the general behavior of the solar activity is the sane for the entire spectral region. The mean daily solar fluxes from 8 to 15 cm are subject to identical variations. The extremes of 3 cm and 30 cm differ from 10 cm by an extent which is much less than the fluctuations. In other words, the largest flueuations of the solar radiation from 3 cm to 30 cm are observed at about locm although the complete spectrum varies, under the same conditions, with the solar activity.

Since Priester and Martin (1959) and Jacchia (1960), after having used locm (1959d), have based their analysis of satellite accelerations on the solar energy of 20 cm wavelength observed at Berlin, it is important to compare the observational data at 10 cm and 20 cm . Fig. 10 shows that, in 1958, the solar radiation at 10.7 cm measured at $0 t$ tawa does not indicate a general variation which is as pronounced as that for the solar radiation at 20 cm measured at Berlin。 In fact, during 1958, the relation between the extremes of radiation at 20 cm is of the order of 5 while that for 10.7 cm (which must be a maximum) is only about 2. Since Priester and Martin (1959) and Jacchia (1960) are able to fnllow the general variation of the almuspheric densities deduced from satellite measurements by using the 1958 measurements at 20 cm , it is quite strange that such a good correlation has been obtained. There is no way to account for a special behavior at 20 cm , with a variation of a factor of 5 in 1958, when the entire spectrum botween 3 cm and 30 cm does not vary by more than.a factor of 2 。


Fig. 10

Fig. 11 shows that, in 1959, the solar radiations measured at 10.7 cm in Ottawa and at 20 cm in Berlin are subject to the same general variations and that the ratio Berlin/Ottawa of the solar fluxes, taking as a unit the mean $\nabla a l u e$ for 1959 , does not vary by more than $420 \% / 0^{\circ}$ The data on radio radiations at 20 cm , therefore, introduce a systematic error when used in 1958 to compare the variations of the atmospheric density with that of the solar flux.

In conclusion, when errors incalibration of radio measurements have been eliminatea one can say that in general the variations of the activity are represented by all wave lengths between 3 and 30 cm . The most pronounced range of the variations is given by the spectral region centered on locm. Thus the observations at Ottawa at 10.70 m , made during 1958 and 1959, and indicating a maximum variation of a factor 2.2 , fix the maximum of the variation of the mean daily value for all the spectral region from 30 m to 30 cm ioe of the slowly varying component of the sun's radiomemissiono

Since the temperature and its vertical distribution in the chromosphere and corona remain normal in the active regione emitting the derimeter radiations, it is convenient to consider as the basic radation of the sun heating the upper atmosphere the mirimum value of the ridio fiux. Hence we may assume that from july to November 1958 the basic radiation $S$ at 10.7 cm was never less ihan 200 units (ratte $m^{-2}$ (cyele-sec) ${ }^{-1}$ $x 10^{-22}$ ), while in 1959, from the beginning of september to the end of November, it was less bian a:0 reaching 150 at the


Fig. 11
beginning of September.
Such differences must correspond to variations of the solar emission in the ultraviolet region and particularly in the X-ray spectrum。 It is evident that the variation in amplitude of the radio enission cannot be directly proportional to that of the entire iltraviolet spectrum or of the spectrum at the shortest wavelengths. The difficulty of obtaining an exact relation between the optical and radio ranges is oesy to understand sinca such radiations originate between the photosphere and corona, i.e. from the lower and cooler part of the solar atmosphers up to the normal high temperature corona. Nevertheless, it is true that the terrestrial atmosphere between l00km and 200 km is subject to the variable heating resulting from the absorption of ultraviolet radiation and X-rays. The implication is that the thermosphere is heated by all radiations for which the absorption cross-section is greater than $10^{-19} \mathrm{~cm}^{2}$. There js no particular terrestrial layer above 100 km which is especially heated, since the energy is distributed with height accordirg to (4.1).

Other sources of heating than electromagnetic radiation depend on the energy which is available. The use of the energy of the radiation belt in one form or another is Iimited by the total energy avajlable. This, according to Dessier and Vestine (1960), is $6 \times 10^{22}$ erg and is about a factor of 10 greater than the energy involved in a magnetic storm; $10^{22}$ ergs according to Chapman and Bartels (1940). On the other hand, Arnoldy, Hoffman and Wirckler (1960) have observed, during the
first phase of a geomagnetic stormg a loss of about twoothirds of the energy of the outex belt. Such energies place limits on the energy which would be immediately available from the radiation belt and we agree with Bates (1959) that the time period would be too short if "corpuscular" energy must pass through the adatatop belt.

With a total eriergy of $6 \times 1.0^{22}$ ergs, the maximum energy atailabie for the earth ${ }^{8}$ s atmosphere cannot be more than $10^{\text {fif }}$ erg cm ${ }^{2}$. Ar energy supplied to the atmosphere of tho order of 1 erg $\mathrm{cm}^{-2} \mathrm{sec}^{\mathrm{m}}$, which would be given by electrons of the oder of jokev according to Krassovsky (1960), will lead to the total energy of the radiation belt in about 3 hours. The hypothesis pat forward by Krassovsky is very difficult to aceept since the energy of the radiation should be remende several times a dayo Consequently, various suggestions oi the normal heatang of the upper atmosphere or of the crentron of anated atmosphere in a definite region of the auronal wore ty the chamaling of tharged particles through the radialisn bejt iannot be accepted. Hence the deduction of Jastrow (i959) that two atmospheres can exist simultaneously ioe a lom latitude atmosphere with a temperature of the order of aboul $1000^{\circ} \mathrm{K}$ and one at a high latitude mith a temperature of about $2000^{\circ} \mathrm{K}$, has littile plausibility.

An influm of $2 \times 10^{11}$ electrons $\mathrm{cm}^{-2} \mathrm{sec}^{\mathrm{m}}$ leading to a heat source $Q=4 \times 10^{\infty 16} \mathrm{cal} \mathrm{cm}^{-3} \mathrm{sec}{ }^{-1}$ at $300 \mathrm{~km}_{8}$ as computad by Jastrows aorresponds to an injection into the thermosphere which is not observer even during auroraso Meredith (1960)
has given 2 specific example oi a $f$ fux between 2 ari $6 \times 10^{7}$ electrons $\mathrm{cm}^{-2} \mathrm{sec}^{-1}$ of about 10 keV inside of a diffuse aurora. Such an example shows that normal heating of the atmosphere by corpuscular radiation must be excluded. However, sporadic conditions currespunding to geomegnetic storme and loading to energite of $10^{22}$ orgs may lead to appreciable transiont heating since it corresponds to 0.5 erg $\mathrm{cm}^{-2} \mathrm{sec}^{-1}$ for the entire earth during one hour or about oneatwentieth of the earth's surfaco during 24 hours.

Dossler (1959) has put forward the view that the nomal heating of the thermosphere is due to hydromagnetio heating. Against this view it can be stated that the diurnal varietion of the temperature of the thermosphere cannot be explained by this processo But the increased orbital acceleration of satellites observed during geomagnetic storms at any latitude does not exclude a direct heating of the atmosphere outside of the airoral zone。 The hydromagnetic heating, therePore, must be kept among the hypotheses noeded to explain a heating at very low latitudes during geomagnetia storme. A final decision cannot be reached until the variations of the atmospheric densjty are properly explored by using physieal parameters such as the temperature and pressure in addition to the density and the seale height.
5. Transport of heat by conduction
5.1 The equations of conduction

The solar radiation hoats the themosphere at varifous albitudes, according to the values of the coefficients of
absorption which range from $10^{-19} \mathrm{~cm}^{2}$ to $10^{-16} \mathrm{~cm}^{2}$ 。 It is，there－ fore，appropriate to investigate the behaviour of the atmosphere under the effect of the conduction of heat depending on the gradient of temperature。

The flux density of heat，$E$ ，can be written

$$
\begin{equation*}
E=\infty \lambda_{c} \operatorname{grad} T \tag{5.1}
\end{equation*}
$$

where $T$ is the temperature and $\lambda_{C}$ is the thermal conductivity。 If $E$ is expressed in erg $\mathrm{cm}^{-2} \mathrm{sec}^{\infty 1}, \lambda_{c}$ is given in $\operatorname{erg} \mathrm{cm}^{-1} \mathrm{sec}^{\infty} \mathrm{deg}^{-1}$ and is related to the coefficient of viscosity，$\mu_{g}$ by the equation （Chapman and Cowling，1939）

$$
\begin{equation*}
\lambda_{c}=\mathbf{f} \mu c_{V} \tag{5.2}
\end{equation*}
$$

Where $c_{v}$ is the specific heat at constant volume and fepresents a numerical value equal to about 2.5 for spherical molecules （monatomic gas）and can be equal to log for diatomic molecules．

The viscosity $\mu$ can be expressed as

$$
\begin{equation*}
\mu=\frac{5}{16}\left(\frac{\pi k m T}{\pi \sigma^{2}}\right)^{I / 2} \tag{5,3}
\end{equation*}
$$

where $\sigma$ is the atomic radius，$k$ Boltzmann ${ }^{\circ} s$ constant and $m$ the atomic masso This formula can be used in the high atmoo sphere（Nicolet，1960），and the values used are，for atomic hydrogen，$\sigma=2.0 \times 10^{-8} \mathrm{~cm}$ ，

$$
\begin{equation*}
\mu(H)=6.8 \times 10^{-6} T^{1 / 2} \tag{5.4}
\end{equation*}
$$

atomic oxygen，$\sigma=2.4 \times 10^{-8} \mathrm{~cm}$ ，

$$
\begin{equation*}
\mu(0)=1.9 \times 10^{-5} T^{1 / 2} \tag{50.5}
\end{equation*}
$$

molecular nitrogen and oxygen，$\sigma=3.3 \times 10^{-8} \mathrm{~cm}$

$$
\begin{equation*}
\mu\left(\mathrm{N}_{2} ; \mathrm{O}_{2}\right)=1.3 \times 1.0^{-5} \mathrm{~T}^{1 / 2} \tag{5.6}
\end{equation*}
$$

In using equations (5.2) and (5.3) the thermal conduetivity can be denoted by

$$
\begin{equation*}
\lambda_{C}=A T^{1 / 2} \tag{5.7}
\end{equation*}
$$

where $A$ is a constant, depending on the atmospheric conatituents. The numerical value of this constant is, in arg $\mathrm{cm}^{-1} \sec ^{-1} \mathrm{dog}^{-3 / 2}$,

$$
\begin{align*}
& \mathrm{A}(\mathrm{H})=2.1 \times 10^{3}  \tag{5.8}\\
& \mathrm{~A}(0)=3.6 \times 10^{2}  \tag{5.9}\\
& \mathrm{~A}\left(\mathrm{O}_{2}, \mathrm{H}_{2}\right)=1.8 \times 10^{2} \tag{5.1.0}
\end{align*}
$$

Since $\lambda_{c}$ is a function of the temperature, we introduce a new variable defined by

$$
\begin{equation*}
\theta=\int_{T_{2}}^{T} \frac{\lambda}{\lambda_{2}} d T \tag{5.11}
\end{equation*}
$$

leading to the following relation, by using (5.7),

$$
\begin{equation*}
\theta=\frac{2}{3} \frac{T^{3 / 2}-T_{2}^{3 / 2}}{T_{2}^{I / 2}} \tag{5.12}
\end{equation*}
$$

Using (5.7) and (5.12) equation (5.1) can now be writton as

$$
\begin{equation*}
\mathbb{E}=-A \mathrm{P}_{2}^{1 / 2} \operatorname{grad} \theta \tag{5.1.3}
\end{equation*}
$$

The continuity equation is written as follows

$$
\begin{equation*}
\rho C_{V} \frac{\partial T}{\partial K}+d I \nabla E=P=I \tag{5.14}
\end{equation*}
$$

in which $P$ denotes the production of heat per unit of time and

$$
-40 \infty
$$

by unit of volume and $L$ the loss of heat per unit of time and volume. Considering the heat capacity per unit of volume, $\rho c_{v}$ where $\rho$ is the density, equation (5.14) can be written as

$$
\begin{equation*}
\frac{\partial \theta}{\partial t_{i}}=\frac{A T^{1 / 2}}{\rho_{V}^{Q}} \nabla^{2} \partial+(P-L) \frac{T^{1 / 2}}{\rho e_{V} T_{2}^{1 / 2}} \tag{5.15}
\end{equation*}
$$

where the coefficient $A I^{1 / 2} / P c_{V}$ is the thermal diffusivity which, if $n$ is the atomic concentration, can be written

$$
\begin{equation*}
a(T, n)=A_{1} T^{1 / 2} / n \tag{5.16}
\end{equation*}
$$

where $A_{1}$ is a constant which has the following values $\left(\mathrm{cm}^{-1} \mathrm{sec}^{-1} \mathrm{deg}^{-1 / 2}\right.$ )

$$
\begin{array}{ll}
A_{1}(H) & =1.0 \times 10^{19} \\
A_{1}(0) & =1.75 \times 10^{18}  \tag{5.18}\\
A_{1}\left(N_{2}, O_{2}\right) & =503 \times 10^{17}
\end{array}
$$

As a result, the differential equation (5.15) for
the conduction becomes:

$$
\begin{equation*}
\frac{\partial \theta}{\partial t}=\frac{A_{1} I^{I / 2}}{n} \nabla^{2} \theta+\frac{A_{1} T^{I / 2}}{n_{A} T_{2}^{1 / 2}} \quad(P-I) \tag{5.20}
\end{equation*}
$$

By using the variable $T$ instead of introducing the temperature parameter $\theta$ g equation (5.20) would bo non-linear. In fact the problem can always be studied by considering a condition at the Imits $\theta_{2}=0$ for all fixod values of $T=T_{2}$ 。

$$
\text { If } \partial \theta / \partial t=0 \text {, equation }(5.20) \text { becomes Poisson's }
$$

equation which can be applied to the steady state

$$
\begin{equation*}
\nabla^{2} \theta+\frac{\mathrm{P}-\mathrm{L}}{\mathrm{~A}_{2} \mathrm{~T}_{2}}=0 \tag{5.21}
\end{equation*}
$$

If, on the other hand, there is no loss or production of heat at the centre of the volume being studied then (5.20) corresponds to the Laplace equation

$$
\begin{equation*}
\nabla^{2} \theta=0 \tag{5.22}
\end{equation*}
$$

When there is cooling by conduction with no production or loss of heat inside of the volume, equation (5.20) should be written

$$
\begin{equation*}
\frac{\partial \theta}{\partial t}=\frac{A_{1} T^{I / 2}}{n} \nabla^{2} \theta \tag{5.23}
\end{equation*}
$$

In which, for ease of calculation, we can take a mean value of $A_{1} T^{1 / 2}$ and, thus, the thermal conduction depends, essentially, upon the value of the concentration $n$.

## $\therefore$ 5.2 Steady state with no sourcelcor: 10ss

## of heat inside of the volume.

In a sphere where the temperature is a function only
of the radius $r$, the solution of $(5.20)$ is

$$
\begin{equation*}
\theta=\theta_{1} \frac{r-r_{2}}{r_{I}-r_{2}} \times \frac{r_{2}}{r} \tag{5.24}
\end{equation*}
$$

if $\theta_{2}=0$ at a distance $r=r_{2}$ from the center of the sphere and $\theta_{1}$ at $r=r_{1}$ According to (5:12), the distribution of the
temperature becomes, (5.24),

$$
\begin{equation*}
\frac{T^{3 / 2}-T_{2}^{3 / 2}}{T_{1}^{3 / 2}-T_{2}^{3 / 2}}=\frac{r-r_{2}}{r_{1}-r_{2}} \frac{r_{1}}{r} \tag{5.25}
\end{equation*}
$$

The heat flow $E_{r}$ is therefores $(5.8)$ and (5.25),

$$
\begin{equation*}
E_{r}=\frac{r_{2}}{r} \frac{2}{3} A \frac{T^{3 / 2}-T_{2}^{3 / 2}}{r-r_{2}} \tag{5.26}
\end{equation*}
$$

An immediate application is a nighttime atmosphere where there is no heating by ultraviolet radiation at a sufficiently high altitude. In an atomic oxygen atmosphere if the heat flow has the following values at 500 km

$$
0.1 \quad 0.2 \quad 0.5 \mathrm{erg} \mathrm{~cm}^{-2} \mathrm{sec}^{-1}
$$

the corresponding values of the temperatures obtained at 700 km are
$\begin{array}{lll}1250^{\circ} \mathrm{K} ; \Delta T=350 & 1550^{\circ} \mathrm{K} ; \Delta T=650 & 2300^{\circ} \mathrm{K} ; \Delta T=1400 \\ 1850^{\circ} \mathrm{K} ; \Delta T=250 & 2100^{\circ} \mathrm{K} ; \Delta T=500 & 2900^{\circ} \mathrm{K} ; \Delta T=1300\end{array}$ if the temperatures $T_{2}$ at 300 km are $900^{\circ} \mathrm{K}$ and $1600^{\circ} \mathrm{K}$; respectively。

These numerical data show how the temperature would vary if an external heating were involvedn However, since no direct determination of the temperature can be obtained from satellite data, it is necessary to consider (5.1) when the scale height $H$ is used instead of To With (5.9) and (5.10), the equations are

$$
\begin{equation*}
E(0)=-0.817 \times 10^{-3}(\mathrm{~g} / 900)^{3 / 2} \mathrm{H}^{1 / 2} \frac{\mathrm{dH}}{\mathrm{~d} z} \tag{5.27}
\end{equation*}
$$

for atomic oxygen, and

$$
\begin{equation*}
E\left(\mathrm{~N}_{2}, \mathrm{O}_{2}\right)=-0.945 \times 10^{-3}(\mathrm{~g} / 900)^{3 / 2} \mathrm{H}^{1 / 2} \frac{\mathrm{dH}}{\mathrm{dz}} \tag{5.28}
\end{equation*}
$$

for air, when $g=900 \mathrm{~cm} \mathrm{sec}^{-1}$ is adopted for an altitude of 280 km . If we consider the nighttime data of Jacchia (1960) between 600 and 700 km , it is clear that the onergy which is required to maintain such a gradient according to (5.27) and (5.28), is

$$
\begin{array}{r}
\mathrm{E}(600-700 \mathrm{~km})=(0.28 \pm 0.02) \mathrm{erg} \mathrm{cos}^{-2} \mathrm{sec}^{-1} \mathrm{night} \tag{5.29}
\end{array}
$$

However, it has been shown (see Fig. 2) that at the highsst levels it is possible to interpret the satellite observations concerned with atmospheric drag in terms of an isothermal atmosphere subject to diffusion.

As far as the daytime data are concerned, Jacchia (1960) has deduced a rore pronounced gradient which leads, in the same range of aititudes, to

$$
\begin{equation*}
E\left(603-700 \mathrm{~km}_{\mathrm{day}} \geqslant 1 \mathrm{erg} \mathrm{~cm}^{-2} \mathrm{soc}^{-1}\right. \tag{5.30}
\end{equation*}
$$

As a matter cif fact, such a deduction corresponds to an increase of $E$ with altitude, since near $600 \mathrm{~km} E \simeq 1$ erg $\mathrm{cm}^{-2} \mathrm{sec}^{-1}$ and near $700 \mathrm{kza} \mathrm{E}>2.3$ org cm ${ }^{-2} \mathrm{sec}^{-1}$.

If at 700lom, a difference of about 1 erg $\mathrm{cm}^{-2} \mathrm{sec}^{-1}$
distinguishes the difference between a sunlit and daris atmosphere, $(5.29)$ and (5.30), it would be necessary to essume that electromagnetic zadiation can be absorbed at such high altitudes. The maximum possible total number of atoms in a vertical column, in using Jacchia's (1960) data, cannot be more than $8 \times 10^{14} \mathrm{~cm}^{-2}$. Since any absorption cross-section cannot be more than $10^{-1,6} \mathrm{~cm}^{2}$, the optical depth cannot be more than 0.08 and the ultraviolet energy needed would be more than 12.5 erg cm ${ }^{-2}$ sec $^{-1}$. Furthermore,
since the absorption crossosection must be less than $10^{\infty 16} \mathrm{~cm}^{2}$, the ultraviolet radiation required should be more than $50 \mathrm{erg} \mathrm{cm}^{-2} \mathrm{sec}^{-1}$ and such an energy will lead to temperature gradients below the unit optical depth of more than $500^{\circ} \mathrm{K}$ per km 。

In fact, the gradients of temperature for $400^{\circ} \mathrm{K} \leqslant \mathrm{T} \leqslant$ $1600^{\circ} \mathrm{K}$ are given, E in erg $\mathrm{cm}^{-2} \mathrm{sec}^{-1}$, by

$$
\begin{equation*}
\left(\frac{d T}{d z}\right)_{k m}=(10 \pm 3) E \tag{5.31}
\end{equation*}
$$

for an atomic oxygen atmosphere, and

$$
\begin{equation*}
\left(\frac{\mathrm{dT}}{\mathrm{~d} \mathrm{z}}\right)_{\mathrm{km}}=(20 \pm 6) \mathrm{E} \tag{5.32}
\end{equation*}
$$

for an undissociated atmosphere. The scale height gradients, $\beta$, are, under the same conditions; respectively

$$
\begin{equation*}
\beta(0)=(0.51 \pm 0.10) \quad E \tag{5.33}
\end{equation*}
$$

and $\beta\left(N_{2}, O_{2}\right)=(0.46 \pm 0.10) \quad E$
It is clear that $E$ cannot be very different between day and night at highest altitudes since it would require a higher ultraviolet energy than is available from the sun. A corpuscular effect will not lead to a difference between day and night, and any external heating cannot explain such a difference. It must be concluded, therefore, that there is no physical process to be found to explain an increase of the temperature at such altitudes. An increase of the scale height gradient can Ve found only where the laws of diffusion can be employed; in fact at very high altitudes the gradient decreases and finally the scale height becomes constant.

Another aspect of a special behavior deduced from observational data is the gradient of the scale height (see Fig. 3) adopted by King-Hele and Walker (1960) near 200km. The gradient is negative between $210-220 \mathrm{~km}, \beta=-0.3$; it is positive between 200-210km, $\beta=+0.4$. Using formulas (5.27) and (5.28), a thin layer near 200 km should lead to an upward heat flow of ( $0.80 \pm 0.06$ ) erg $\mathrm{cm}^{-2} \mathrm{sec}^{-1}$ and a downward heat flow of ( $1.15 \pm 0.08$ ) erg $\mathrm{cm}^{-2} \mathrm{sec}^{-1}$, i.e. a heat loss of about $2 \mathrm{erg} \mathrm{cm}{ }^{-2} \mathrm{sec}^{-1}$ for a layer of thickness less than lokm. The total energy lost in one hour by such a layer would be more than its total kinetic energy. This shows that such a discontinuity cannot be a peranant feature in the thermosphere. We shall see later that such an artificial gradient would have a very short life time. In any case, it can be shown (see Fig. 3) that the vertical distribution of the density can be explained in an isothermal atmosphere subject to diffusion without such an anomaly in the scale height gradient.

### 5.3 Steady state with heating by ultra-

Violet radiation and loss by infrared
radiation
In a steady state with one dimension the equation (5.21)
is

$$
\begin{equation*}
\frac{\partial^{2} \theta}{\partial z^{2}}+\frac{P-L}{A T_{2}^{I / 2}}=0 \tag{5.35}
\end{equation*}
$$

or $\quad \frac{\partial E}{\partial z}+P=L$

$$
-46 \infty
$$

Considering an atmosphere subject to a heating by several radiations, the production of heat $P\left(\mathrm{~cm}^{-3} \mathrm{sec}^{-1}\right)$ is

$$
\begin{equation*}
P=\operatorname{InK} E_{u v} e \int_{z}^{\infty} n K d z d z=\Sigma E_{u v} e^{-\tau} d \tau \tag{5.37}
\end{equation*}
$$

in which K is the appropriate absorption cross-section of the radiation $E_{u v}$ of wavelength $\lambda$ by atoms of concentration $n_{0}$ Tis the optical depth defined by

$$
\begin{equation*}
d \tau=-n K d z \tag{5.38}
\end{equation*}
$$

leading to

$$
\begin{equation*}
\tau=\int_{z}^{\infty} n x d z \tag{5.39}
\end{equation*}
$$

In the thermosphere, the principal infrared radiator is atomic oxygen, according to Bates ${ }^{\circ}$ process (1954),

$$
\begin{equation*}
o\left({ }^{3} P_{1}\right) \rightarrow o\left({ }^{3} P_{2}\right)+h v(\lambda=63 \mu) \tag{5.40}
\end{equation*}
$$

since molecular nitrogen and oxygen have no dipole。 The loss of heat ( $\mathrm{cm}^{-3} \mathrm{sec}^{-1}$ ) is therefore (in org $\mathrm{cm}^{-3} \mathrm{sec}^{-1}$ )
$L=\operatorname{Rn}(0)=n(0) \frac{1.68 \times 10^{-18} e^{-228 / T}}{1+0.6 \times e^{-228 / T}+0.2 \times e^{-325.3 T}}$
leading to

$$
\begin{align*}
& L(E \text { layer })=(5 \pm 1) \times 10^{-19} \mathrm{n}(0) \mathrm{erg} \mathrm{~cm}  \tag{5.42a}\\
& L(\mathrm{~F} \text { lager })=(8 \pm 0.5) \times 10^{-19} \mathrm{sec} \mathrm{~m}^{-1}(0) \mathrm{erg} \mathrm{~cm}^{-3} \mathrm{sec}^{-1} \quad(5042 \mathrm{~L})
\end{align*}
$$

Using (5.37) and (5.40), integration of (5.36) leads to

$$
\begin{equation*}
E=E_{\infty}+E_{u \nabla}\left(I_{\infty} e^{-\tau}\right)+\mathbf{R} n(0) H(0)(g / \bar{g}) \tag{5.43}
\end{equation*}
$$

in which $F_{\infty}$ denotes the external heat available for the layer,
and $\bar{R}$ and $\bar{g}$ are mean values.

Since the absorption cross section in the ultraviolet range is $K>10^{-17} \mathrm{~cm}^{2}$ and $\mathrm{B}<10^{-18}$ erg sec ${ }^{-1}$,

$$
\begin{equation*}
E_{u v}\left(1-e^{-\tau}\right)>\bar{R} n(0) H(0) \tag{5.44}
\end{equation*}
$$

and above the unit optical depth, it is clear that the radiation loss is also negligible. A loss of heat of the order of 1 erg $\mathrm{cm}^{-2} \mathrm{sec}^{-1}$ requires $2 \times 10^{18} \mathrm{~cm}^{-2}$ atoms of oxygen, ide. a layer of lokm with concentration of $2 \times 10^{12}$ atoms $\mathrm{cm}^{-3}$ which corresponds to an altitude of the order of looks. It can be concluded that the loss of heat at $63 \mu$ occurs mainly inside of the $E$ layer, where the radiation is certainly of the order of one erg $\mathrm{cm}^{-2} \mathrm{sec}^{-1}$. In the F layer the total loss of a vertical column would be of the order of 0.1 erg $\mathrm{cm}^{-2} \mathrm{sec}^{-1} \mathrm{~g}$ thus the loss of heat by radiation may be neglected in a sunlit amon sphere above the $E$ layer when the ultraviolet energy available is at least 1 erg $\mathrm{cm}^{-2} \mathrm{sec}^{-1}$. On the other hand, any ultraviolet energy of the order of 0.1 erg $\mathrm{cm}^{-2} \mathrm{sec}^{-1}$ absorbed above the level of the $F_{1}$ peak is of the order of the heat loss by atomic oxygen since $K \simeq 10 \mathrm{R}$ 。

The integration of (5.43) with the use of (5.27) and (5.28) may lead to a determination of the possible increases of the scale height ( $H_{\infty}$ ) up to the isothermal layer. The result can be written as follows, neglecting the variation of $g$,

$$
H_{\infty}^{1 / 2}=H_{0}^{1 / 2}+\frac{E_{\infty}}{2 A} \log \tau_{0} / \tau
$$

$$
+\frac{\mathrm{Euv}}{2 \mathrm{~A}}\left[\log \tau_{0}=\mathrm{Ei}\left(-\tau_{0}\right)+0.57722\right]
$$

$$
-\frac{\mathbb{K}}{2 A}\left[n_{0}(0) \quad H_{0}(0)-n(0) H(0)\right]
$$

where $A$ is the constant of（5．27）or（5．28）。 Adopting $\tau_{0} x l$ ，near the absorption peak，the effect of the ultraviolet radiation on the scale height can be shown by the following examples：


In＿other words，the scale height varies by a factor of 2 between the absorption peak and the beginning of the isothermal layer when the ultraviolet energy available for heating is of the order of 2 erg $\mathrm{cm}^{\mathrm{m}} \mathrm{sec}^{-1}$ 。 Since diffusion is also involved，it is evident that scale heights greater than 100 km can be reachedif the ultraviolet energy is of the order of 1 erg $\mathrm{cm}^{-2} \mathrm{sec}^{-1}$ ．

Evidence favouring an important gradient of temperature in a sunlit atmosphere is shown by the occurence of pronounced diurnal variations of the density。 An exact vertical distribution of the temperature cannot he obtained oince the solar spectrum in the ultraviolet must be known in all the details and also the exact，varying，ratio $N_{2} / 0$ ．However，the diminution in the temperature gradient should be indicated by a law of the form

$$
\begin{equation*}
\frac{d T}{d z} \propto \Sigma E_{u v}\left(I-e^{-\tau}\right) \tag{5.46}
\end{equation*}
$$

for an overhead sun．

It is possible to obtain some idea of the conditions which are required to reach the observed densities at 200 km and scale heights deduced from (5.45). If we assume, for example, that below the isopicnic level of 150 km (see eqn. 3.4), the effect of convection is such that the energy $E$ follows the following law

$$
\begin{equation*}
E=E_{u v}\left(\frac{H}{H_{h}}\right)^{I / 2} \propto \frac{\nabla}{\nabla_{h}} \tag{5.47}
\end{equation*}
$$

where he corresponds to $150 \mathrm{~km}, \mathrm{H}<\mathrm{H}_{\mathrm{h}}$ and V is the kinetic energy. The scale height gradient is, according to (5.28),

$$
\begin{equation*}
\beta=\mathrm{E}_{\mathrm{h}} / \mathrm{AH}_{\mathrm{h}}^{1 / 2}=1.05810^{3}\left(\mathrm{E}_{\mathrm{h}} / \mathrm{H}_{\mathrm{h}}^{1 / 2}\right) \tag{5.48}
\end{equation*}
$$

As was said before (see eqn. 3.4), the density remains 2lmost constant at 150 km for any gradient $0.50 \leqslant \beta \leqslant 1.50$. The energy necessary for such gradients varies according to (5.48) from $0.7 \mathrm{erg} \mathrm{cm}^{-2} \mathrm{sec}^{-1}$ to $3.3 \mathrm{erg} \mathrm{cm}^{-2} \mathrm{sec}^{-1}$ (see Fig. 12). The solar energy which must be available for the heating cannot be determined if the absorption cross section is not known. Let us assume for example a mean value of $2 \times 10^{-17} \mathrm{~cm}^{2}$ which corresponds to the most important part of the ultraviolet spoctrum. In such a case, the following ultraviolet energies $\mathbb{E}_{\mathrm{uv}}$ at the top of the earth's atmosphere are necessary.

| $\beta$ | 0.5 | 0.75 | 1.00 | 1.25 | 1.50 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{g}_{\text {uv }}$ | 0.9 | 1.4 | 2.0 | 2.7 | 3.4 erg omi $^{-2} \mathrm{gec}^{-1}$ |

and the corresponding temperatures at 150 km (see Fig. 13) are $\begin{array}{lllll}7 & 725 & 960 & 1190 & 1425\end{array} \quad 1650^{\circ} \mathrm{K}$

Such scale height gradients and temperatures correspond to real parameters since the total kinetic energies which are


Fig. 12

involved in the atmosphere above 150 km are between $10^{4} \mathrm{erg}$ $\mathrm{cm}^{-2}$ and $6 \times 10^{4} \mathrm{erg} \mathrm{cm}{ }^{\infty}$ corresponding to $\mathrm{E}_{\mathrm{uv}}$ from 0.09 erg $\mathrm{cm}^{\infty} \mathrm{sec}^{-1}$ to $3.4 \mathrm{erg} \mathrm{cm}^{-2} \mathrm{sec}^{-1}$ ；respectively。 In other words， the solar energy which is used during one day of 12 hours is of the same order as the kinetic energy of the vertical column above 150 km 。 It is，consequently，certain that the variations in the t＇ermosphere above lookm can be associated with the variations of the solar energy and that the variations of the temperature and its gradient are closely associated with the absorption processes below 200 km ．The temperature at the thermopause depends strongly on its gradient below 200km．But the level of the thermopause must be subject to a diurnal variation according to the laws of cooling after sunset．

## 5．4 Cooling of the atmosphere after sunset

The application of equation（5．23），after sunset，in the following form

$$
\begin{equation*}
\frac{\partial \theta}{\partial t}=\frac{A_{1} T^{1 / 2}}{n} \frac{\partial^{2} \theta}{\partial z^{2}} \tag{5.47}
\end{equation*}
$$

corresponding to an initial atmosphere with a certain vertical gradient leads to the following results．

Let us consider，as an example，an atmosphere giving approximately the vertical distribution of the observed density and corresponding to a heat flow of about $0.3 \mathrm{erg} \mathrm{cm}^{-2} \mathrm{sec}^{-1}$ through atomic oxygen。 Fig． 14 shows the variation of $\mathrm{pH}^{1 / 2}$ during an interval of 24 hours．If the variation is very small at 200 km ，it is of the order of a factor of 10 near 650 km

-53-
in about 12 hours. There is a clear indication that the atmosphere becomes rapidly isothermal at the highest altitudes and the isothermy extends with time to lower levels. It must be pointed out that $\rho H^{1 / 2}$ at highest altitudes decreases rapidly during the first hours, and it is interesting to note that the variation is very small between 12 and 24 hours compared with the variation during the first 12 hours.

In fact, the tendency to isothermy is very rapid when the density is less than $5 \times 10^{-15} \mathrm{gm} \mathrm{cm}^{-3}$ or concentrations less than $10^{8} \mathrm{~cm}^{-3}$. Equation (5.47) shows that if a temperature gradient corresponding to a heat flow of the order of 0.3 erg $\mathrm{cm}^{-2} \mathrm{sec}^{-1}$ exists in an atomic oxygen atmosphere, it disappears in less than 30 minutes between 450 km and 750 km (Fig. 15). Such a result shows that it is not possible to consider a dark atmosphere with a temperature gradient above $300-350 \mathrm{~km}$ 。 Moreover, the temperature of the isothermal atmosphere decreases In a continuous way following a decrease in the temperature gradient between 200 km and 300 km 。

The absolute value of the decrease, being obviously a function of the initial gradient, the behavior of the variation depends upon the vertical distribution of the ultravioiet radiation absorbed in the sunlit atmosphere. Nevertheless, it is clear that the variation of the temperature of the fisothermal layer between $t=30$ minutes and $t=2$ hours (see Fig. 15 ) is of the same order as that between $t=2$ hours and $t=12$ hours. In other words, after a rapid tendency towards isothermy at highest altitudes, the temperature of the isothermal layer


Fig. 15
decreases according to the rapidity with which the isothermal layer extends downwards and the variation of the temperature below 200 km 。

As a consequence, the heat transport by conduction associated with the gradient of temperature that results from the absorption of solar ultraviolet radiation explains the diurnal variation of the density of the thermosphere deduced from the variation of the acceleration of the satellites. We must also conclude that the diurnal variation of the temperature of the isothermal region is of the order of $500^{\circ} \mathrm{K}$ and that the temperature gradient is certainly subject to a variation down to 150 km 。

Lastly, we must realize that the heating during magnetic storms will have an effect during a time of the order of one day since the effect of the solar electromagnetic radiation leads to a strong diurnal variation. Consequently, any sporadic effect, if energetic enough, will modify the atmospheric structure only during its own life time and its role will be more or less effective if it occurs in a dark or sunlit atmosphere; respectively.

### 5.5 Times of conduction

The application of equation (5.47) to the problem of the cooling of the atmosphere after sunset indicates that the time required for the transport of heat varies greatly with altitude and distance. Actually, the time of conduction is proportional to the concentration, $n$, and to the square of the distance.

In order to provide order of magnitudes for times of conduction, we apply equation (5.47) to very simple examples. Let us consider two infinite regions of the atmosphere where the conditions are such that the temperatures are initially $I_{1}(x \leqslant 0)$ and $T_{2}(x>0)$. The redistribution of the temperature (isotherms) is given by the solution of (5.47)

$$
\begin{equation*}
\theta / \theta_{1}=1-\psi(\mu) \tag{5.48}
\end{equation*}
$$

when $T_{1}$ is kept constant $\left(T \rightarrow T_{1}\right)$ and $x>0$
or

$$
\begin{equation*}
\theta / \theta_{1}=\frac{1}{2}[1+\psi(\mu)] \tag{5.49}
\end{equation*}
$$

if $T \rightarrow \frac{1}{2}\left(T_{1}+T_{2}\right)$ and $-\infty<x<+\infty$. In (5.48) and (5.49),

$$
\begin{equation*}
\psi(\mu)=\frac{2}{\sqrt{2 \pi}} \int_{0}^{\mu} e^{-y^{2}} d y \tag{5.50}
\end{equation*}
$$

and

$$
\begin{equation*}
\mu=\frac{x}{2}\left(\frac{n}{A_{1} T^{1 / 2} t}\right)^{1 / 2} \tag{5.51}
\end{equation*}
$$

where $t$ denotes the time; $A_{1}$ is given by (5.18) or (5.19). By using (5.12), we can write the relation between the temperatures if we adopt a definite value for $\psi$ ( $\mu$ ). Since $\psi(\mu)=0.2$ leads to conditions near to isotherms, such a value is adopted and introduced in (5.48) and (5.49). With (5.48), the solutions ares

If $T_{2}{ }^{\prime} T_{1}=0.5, T / T_{1}=0.91, \ldots$, if $T_{2} / T_{1}=0.9, T / T_{1}=0.98$. and with (5.49),

$$
\text { if } T_{2} / T_{1}=0.5, T / T_{1}=0.80, \ldots, \text { if } T_{2} / T_{1}=0.9, T / T_{1}=0.95
$$

[^0]related to the concentration（see 5．51）and the distance $x_{0}$ Osing the constants from the formulas（5．17），（5．18）and （5．19），we obtain the times of conduction $t_{\text {sec }}$ in seconds for atomic hydrogen，atomic oxygen and air
\[

$$
\begin{align*}
& t_{s e c}(H)=7.72 \times 10^{-9} n(H) x_{k m}^{2} T^{01 / 2}  \tag{5.52}\\
& t_{s e c}(0)=4041 \times 10^{-8} n(0) x_{k m}^{2} T^{01 / 2}  \tag{5.53}\\
& \mathrm{t}_{\mathrm{sec}}\left(\mathrm{~N}_{2}, \mathrm{O}_{2}\right)=1.46 \times 10^{-\infty} \mathrm{T}_{\mathrm{n}}\left(\mathrm{O}_{2}, \mathrm{~N}_{2}\right) \mathrm{x}_{\mathrm{km}}^{2} \mathrm{~T} \text { 立/2} \tag{5.54}
\end{align*}
$$
\]

For a temperature range of $900^{\circ} \mathrm{K}$ to $2500^{\circ} \mathrm{K}$ ，ioe．for $\bar{T}=$ $(40+10)^{20} \mathrm{~K}$ ，the following times of conduction in seconds for distances $x$ expressed in $k m$ and concentrations in $\mathrm{cm}^{-3}$ are

$$
\begin{array}{ll}
t(H) & =(2 \pm 0.5) \times 10^{-10} x_{\mathrm{km}}^{2} n(H) \\
t(0) & =(1.2 \pm 0.3) \times 10^{-9} x_{\mathrm{km}}^{2} n(0) \\
t\left(0, N_{2}\right) & =(3.8 \pm 10) \times 10^{-9} x_{\mathrm{km}}^{2} n\left(0_{2} \mathrm{~N}_{2}\right) \tag{5.60}
\end{array}
$$

If a horizontal discontinuity exists in the horizontal temperature distribution it cannot be maintained at the highest altitudes．For example，the isothermy should be reached after about 4 hours at a distance of 1000 km in an atmosphere（ $600-700 \mathrm{~km}$ ） with concentration of $10^{7}$ oxygen atoms $\mathrm{cm}^{-3}$ 。 This means that a difference of temperature in latitade can only be maintained during a very short time at high altitudes．Furthermore，a seasonal effect cannot be related to differences of temperatures at high allitudes．All conditions are related to the diurnal variations which are important according to the results of the analysis made in section 504．

If we consider that horizontal layers of a certain thickness $x$ occur with different temperatures $T_{1}$ and $T_{2}$, it is possible to consider the time of conduction which ought to be taken into account to reach isothermy. For example, King-Hele and Walker (1960) in their atmospheric model have introduced a peak in the scale height at 210 km ( $\mathrm{H}=47 \mathrm{~km}$ ) and a minimum at $250 \mathrm{~km}(\mathrm{H}=40 \mathrm{~km})$ o For a molecular nitrogen atmosphere, equation (5.47) shows that

$$
\begin{equation*}
\theta / \theta, \quad \frac{1}{2} \psi(\mu) \tag{5.61}
\end{equation*}
$$

Where $\mu^{2}=\frac{a^{2} n}{A_{1} T^{I / 2} t}$
if a is the thickness of the layer with initial temperature $T_{1}(x<a)$ and initial temperature $T_{2}$ for $-a<x<a$.

Therefore the isothermy is reached ( $\mu=0.2$ and $900^{\circ} \mathrm{K} \leqslant$ $T \leq 1600^{\circ} \mathrm{K}$ ) when the time of conduction is seconds, tec, is

$$
\begin{equation*}
t_{s e c}=1.7 \times 10^{-8} a_{\mathrm{km}}^{2} n \tag{5,63}
\end{equation*}
$$

Since the concentration is about $3 \times 10^{9} \mathrm{~cm}^{-3}$ near 250 km and about four times more near 200 km , the temperature discontinuity will disappear in about $2 \times 10^{4}$ seconds between 200 km and 280 km . These results, related to those obtained for the horizontal heat transport, clearly show that discontinuities in the thermospheric temperatures cannot be permanent.
6. Thermospheric Conditions and Atmospheric Models

Heat conduction times which are appropriate to the thermosphere are between 12 hours and 24 hours since the diurnal
variation resulting from the heating by electromagnetic radiation is the more pronounced effect．Any gradient of temperature which is not maintained by an external heat flow has a life time of less than one night above 350 km ，and in a dark atmosphere such an altitude can be considered as being in the neighborhood of the thermopause level．In a sunlit atmosphere its altitude will depend on the solar energy available and the variable tempero ature of the isothermal region will be a function of the ： tempenaturie：gradient between 100 km and 200：kmo． As we have shown previously 2 change of the gradient below 200 km does not greatly change the atmospheric density at 200 km ， but the atmosphere above 300 km is strongly affected．However， a general effect associated to the solar activity（27 day period and sunspot cycle）producing a modification of boundary conditions in the E layer will affect the atmosphere near 200 km with amplified effects with increasing altitudes．

A＂corpuscular＂．effect will also have a detectable life time of the order of one day since it will be involved in the effect of the diurnal variation。 On the other hand， such an effect will be much easier to detect in a dark atmosphere when the temperature reaches its normal minimum。

The diurnal variation has another important effect on the altitude of the conventional exosphere，because differences of several hundreds of km make it impossible to determine relations connected with the outer atmosphere on 2 permanent basis。 It．is， therefore，necessary to take into account diurnal and solar
variations modifying simultaneously the values of the density and the temperature. It should be understood that a diurnal variation of diffusion should exist up to the highest altitudes and that the ratio $N_{2} / 0$ must vary between day and night. On the other hand, the upper part of the ionosphere should be subject to very important diurnal and solar effects. Since the recombination in the $F$ layer depends on reactions related to the temperature, important diurnal variations must occur in the rate coefficients. Above the $F_{2}$ peak there must be a diurnal variation in the scale height associated with the vertical distribution of electrons subject to diffusion. If it is relatively easy to represent atmospheric conditions in a dark atmosphere where the thermopause is at a relatively low altitude, it is very difficult to find an adequate picture of the atmosphere under sunlit conditions until the solar spectrum is known exactly.

An atmospheric model basad solely on heat flow by conduction from outside the atmosphere (Nicolet, 1959a) is an extreme example which cannot be associated with diurnal variation of the density. Such a heating could be easily explained by the Chapman (1957, 1959) process, suggesting that the solar corona extends to the limits of the terrestrial atmosphere. However, it must be considered as a very small fraction in relation to the heating due to the ultraviolet radiation and, in fact, it is negligible in comparison. In the same way, corpuscular and hydromagnetic effects are not important under normal conditions, but disturbed conditions,
associated with magnetic storms，must lead to transient atmospheric conditions in which the temperature of the iso－ thermal layer is higher than that in the nighttime atmosphere。 If it is clear that the scale height increases with height and that its gradient may reaci a peak at a certain altitude，it is also evident that heat conduction does not permit having a temperature gradient increasing with height； even when the temperature increases up to the highest alti－ tudes．For this reason，the atmospheric models such as those of Mikhnevich，Danilin，Repnev and Sokolov（1960），Champion and Minzner（1959），Kallmann（1959），etcig do not represent real physical conditions．

In the Mikhnevich model the temperature gradient increases with the altitude by $1^{\circ} \mathrm{K} \mathrm{km}{ }^{-1}$ at $250 \mathrm{~km}, 2^{\circ}$ at 300 km ， $3^{\circ}$ at 350 km ， $4^{\circ}$ at $425 \mathrm{~km}, 6^{\circ}$ at 450 km and $7^{\circ}$ at 500 km ．Such an increase of the temperature gradient would represent a downward heat flow at 500 km at least ten times greater than the heat flow at 250 km 。 No permanent physical process could explain such a result。

In the Champion and Minaner model，the temperature difference by steps of 100 km from 200 km to 700 km changes successively from $19^{\circ} \mathrm{K}$ to $57^{\circ} \mathrm{K}, 96^{\circ} \mathrm{K}, 115^{\circ} \mathrm{K}$ and $121^{\circ} \mathrm{K}$ ．Such an increase of the temperature gradient with height associ－ ated with an isothermy between 210 and 260 km would 7 ead also to a downard heat flow；maximum at the highest levels and disappearing near 250 km 。 Hence the high temperatures near 700 kn cannot result from the vertical distribution deduced
by Champion and Minzner（1959）。
In summary，all atmospheric models involving an increase with height，of the gradient of temperature cannot be accepted． Such models lead to the ratio of heat flows $E$ and $E_{0}$ ，

$$
\begin{equation*}
\frac{E}{E_{0}}=\frac{A T^{I / 2} d T / d z}{A_{0} T_{0}^{I / 2} d T T_{0} / d z} \tag{5.64}
\end{equation*}
$$

in which $E$ is for ${ }^{2}$ height $z$ and $E_{0}$ is for a height $z=0$ ． Since $A T^{1 / 2}<A_{0} T_{0}^{1 / 2}$ and $E \leqslant E_{0}$ ，if there is no radiation loss，$d T / d z$ must be less than $d T T_{0} / d z$ ．On the other hand， since the conduction of heat is a very rapid process at highest altitudes there is no way to avoid conclusions obtained from （5．64）．

Considering now that the tendency of the atmosphere after sunset is to attain isothermy，a nighttime model of the atmosphere must be represented at the highest levels by a quasi－isothermal atmosphere in which the constituents are subject to diffusion。 In Section 2.1 （see Figo 2），it has been shown that an isothermal atmosphere subject to diffusion is not far from 2 quasi－isothermal atmosphere agreeing with nighttime satellite data。 Furthermore，in Section 3，it has been found that the density（expression 3.4 ）at 150 km is not sensitive to the temperature gradient when constant boundary conditions are assumed at 120 km 。 For these two reasons it can be assumed that atmospheric models for nighttime conditions at the highest altitudes must be considered in such a way that the temperature is nearly constant and that diffusion affects
the atmospheric scale height.
In such circumstances, an important parameter is the ratio of the densities of molecules and atoms. We start from conditions given by $(3.4)$, i.e. at 150 km ,
total density: $P=1.5 \times 10^{-12} \mathrm{gm} \mathrm{cm} \mathrm{cm}^{-3}$
temperature : $725^{\circ} \mathrm{K} \leqslant \mathrm{T} \leqslant 1650^{\circ} \mathrm{K}$
concentrations: $n\left(N_{2}\right)=2.6 \times 10^{10} \mathrm{~cm}^{-3}$
$n(0)=3.3 \times 10^{9} \mathrm{~cm}^{-3}$
$n\left(O_{2}\right)=5.0 \times 10^{9} \mathrm{~cm}^{-3}$
leading to the approximate ratios

$$
\begin{equation*}
n(M): n\left(N_{2}\right) \quad n\left(0_{2}\right): n(0)=1: 0.76: 0.14: 0.10 \tag{5.65d}
\end{equation*}
$$

The effect of diffusion is to lead, at a certain altitude, to the following relation between the densities

$$
\begin{equation*}
\rho\left(\mathrm{O}_{2}, \mathrm{~N}_{2}\right)=\rho(0) \tag{5.66}
\end{equation*}
$$

corresponding to a certain total density $\rho$. Computation shows that for the conditions $(5.65),(5.66)$ is obtained where the total density has a definite value, i。e。

$$
\begin{equation*}
\rho=(4.45 \pm 0.05) \times 10^{-15} \mathrm{gm} \mathrm{~cm}^{-3} \tag{5.67}
\end{equation*}
$$

This important relation (5.67) leads to the immediate conclusion that the temperature of the isothermal atmosphere can be found when the altitude of $\rho\left(\mathrm{O}_{2}, \mathrm{~N}_{2}\right)=\rho(0)$ is known.

The following results leading to a thermoaisobaric relation", have been obtained:

| Altitude in km |  |
| :---: | :---: |
| of $\rho\left(\mathrm{O}_{2}, \mathrm{~N}_{2}\right)=\rho(0)$ | Temperature $\left({ }^{\mathrm{O}} \mathrm{K}\right)$ |
| 300 | 725 |
| 350 | 958 |
| 400 | 1191 |
| 450 | 1424 |
| 500 | 1657 |

Fig. 16 shows the thermo-isobaric relation, i.e. the relationship between the temperature of the isothermal layer and the altitude of the constant density given by (5.67) corresponding to the same densities for atoms and molecules. The concentrations are approximately as follows:
$n\left(N_{2}\right)=4.4 \times 10^{7} \mathrm{~cm}^{-3} ; n\left(0_{2}\right)=3.3 \times 10^{6} \mathrm{~cm}^{-3} ; n(0)=8.6 \times 10^{7} \mathrm{~cm}^{-3}(5.68)$
The vertical distribution of the scale height is
shown in Fig. 17 for various isothermal atmospheres. An increase of H from 50 km at 250 km to 100 km at 1000 km is given for a temperature of the order of $1400^{\circ} \mathrm{K}$ 。 Such an increase results only from the decrease of the molecular mass with altitude. It can also be seen that the fluctuations of the scale height deduced by King $H$ Hele and Walker (1960b) are anomalous variations of the scale haights in an isothermal atmosphere around $1200^{\circ} \mathrm{K}$; not too different from the temperature $T=1250^{\circ} \mathrm{K}$ deduced (Fig. 16) from the thermowisobaric relation.

Fig. 18 shows the vertical distribution of the density for which the constant density defined by (5.67) corresponds to the equality of the density of molecules and atoms. As was


Fig. 16


Fig. 17
shown before, (Fig. 2), the curves leading to (5.66) at 400 km and 450 km are not far from the nighttime densities deduced by Kingohele and Walker (1960b) and Jacchia (1960); respectively。 It is clear that the preceding determination of nighttime conditions of the isothermal layer does not give a complete answer since it depends on boundary conditions for diffusion and temperature. The atmospheric model is consiso tent in the use of all physical parameters; howeverg it represents only a guide to the study of atmospheric behavior. In order to show how small differences in physical conditions can modify the conclusions, we assume instead of (5.66),

$$
\begin{equation*}
p\left(0_{2}, 0\right)=\rho\left(N_{2}\right) \tag{5.69}
\end{equation*}
$$

corresponding again for all temperatures to a constant density. The expression (5.69) leads to a different distribution of the density, even if the concentration of atomic oxygen, $n(0)=$ $8.5 \times 10^{7} \mathrm{~cm}^{-3}$, remains the same at the thermooisobaric level.. The vertical distribution of density for various temperatures are shown in Fig。 18 where it is interesting to compare the results represented by the dotted curves with the data shown by continuous curves.

Computation shows that the condition (5.69) is obtained Where the total density has the definite value

$$
p=(4.75+0.25) \times 10^{-15} \mathrm{gm} \mathrm{~cm}^{-3}
$$

This thermo-isobaric relation leads to the following results:


Fif 18

$$
-70=
$$

Altitude in km
of $\rho\left(N_{2}\right)=\rho\left(0_{2}, 0\right)$
Temperature $\left({ }^{\circ} \mathrm{K}\right)$

| 350 | 1050 |
| :--- | ---: |
| 400 | 1285 |
| 450 | 1515 |
| 500 | 1745 |
| 550 | 1975 |

The main conclusions are given in the following table :

Density at
220 km
$10^{-13} \mathrm{gm} \mathrm{cm}^{-3}$

Isothermal Atmosphere

$$
\left({ }^{0} \mathrm{~K}\right) \quad\left({ }^{\circ} \mathrm{K}\right)
$$


(1) 0.32 805
(2) $0.85 \pm 0.5$

725
1050
(3) $1.7+0.1$

958 1285
(4) $2.5 \pm 0.1$ 1190 1515
(5) $3.2 \pm 0.1$
(6) $3.9 \pm 0.1$

1424
2745
1657
1975

the conditions of a dark atmosphere．From the preceding table and from satellite data，the following data may be taken as a guide（see preceding table） $\rho(220 \mathrm{~km})=(2.5 \pm 0.7) \times 10^{-13}$ for $960^{\circ} \mathrm{K} \leqslant \mathrm{T} \leqslant 1425^{\circ} \mathrm{K}$ leading to $P=405 \times 10^{-15} \mathrm{gm} \mathrm{cm}{ }^{-3}$ for $350 \mathrm{~km} \leqslant \mathrm{z} \leqslant 450 \mathrm{~km}$ and $\rho=$ $8.5 \times 10^{-17} \mathrm{gm} \mathrm{cm}^{-3}$ for $550 \mathrm{~km} \leqslant \mathrm{z} \leqslant 750 \mathrm{~km}$ 。 The altitudes for $\rho=405 \times 10^{-15} \mathrm{gm} \mathrm{cm}^{-3}$ and $\rho=8.5 \times 10^{\infty 17} \mathrm{gm} \mathrm{cm}^{-3}$ are related directly to the temperature。 Furthermore，the cone centrations are given by $(5.68)$ where $\rho=405 \times 10^{\infty 15} \mathrm{gm} \mathrm{cm}{ }^{-3}$ and $n(0)=23 n\left(N_{2}\right)$ where $\rho=8.5 \times 10^{\infty 15} \mathrm{gm} \mathrm{cm}^{-3}$ 。Finally， the mean molecular masses are approximately $M=20.33$ at

$$
\rho=405 \times 10^{-15} \mathrm{gm} \mathrm{~cm}^{-3} \text { and } M=16.50 \text { at } \rho=8.5 \times 10^{-17} \mathrm{gm} \mathrm{~cm}^{-3}
$$

These various values will serve as a guide in the analysis of nighttime conditions in the ionosphere，of air－ glow and auroras keeping in mind that in a normal dark atmom sphere during maximum sunspot conditions the temperature of the isothermal atmosphere is between $1200^{\circ} \mathrm{K}$ and $1400^{\circ} \mathrm{K}$ 。

The problem of the sunlit．atmospheres must be studied under various conditions depending on the ultraviolet solar radiation available and on the vertical distribution of its absorption．

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## Legends of Figures

Fig． 1 Density －altitude relation above 150 km from various determinations．The average curve represented by Groves ${ }^{\circ}$ data shows that the height variation of the density can be only explained by an increase of the scale height with altitude。

Fig． $2-\rho H^{1 / 2}$ proportional to orbital acceleration of satellites in an isothermal atmosphere subject to diffusion．The density $\infty$ altitude relations deduced from observational data can be followed．It is possible to fit the satellite observations of density，corresponding to a nighttime atmosphere by an isothermal atmosphere。

Fig． 3 －Variations of the scale height with altitude。 The variation of the scale height is associated with a variation of the mean molecular mass depending on diffusion．In this figure the scale height－altitude relations must be associated with the density distributions shown in Fig。2。 It is possible to deduce from observational data of $\rho^{H^{1 / 2}}$ inconsistent values of the scale height．

Fig。 4 ＝Orbital acceleration of Vanguard $I(1958 \beta 2) .-d P / d t$ os $f^{1 / 2}$（Briॄgs，1959）。Perigee latitudes $(\varphi)$ and
angular distance of the perigee from the sub－solar point（ $\mathbb{Y}$ ）（Jacchia\＆1959d）．The solar activity is represented by daily values of solar radiation at 10.7 cm（Ottawa，Covington）and the magnetic activity by the daily $K$ values deduced from the threewhourly indices（Bartels）。

Fig． 5 －Orbital acceleration of Vanguard II（1959 $\alpha$ ）。 $\mathrm{dP} / \mathrm{dt} \propto \rho^{1 / 2}$（Jacchia，1960）。 Other symbols；see Fig． 4

Fig． 6 －Orbital acceleration of Sputnik III（1958 S 2）。a $\mathrm{dP} / \mathrm{dt} \propto \rho \mathrm{H}^{1 / 2}$（Kozai，1960） Other symbols，see Fig． 4

Fig． 7 －Crbital acceleration of Sputnik III（1958 S 1）－－ $\mathrm{dP} / \mathrm{dt} \propto \rho^{\mathrm{H}^{1 / 2}}$（Jacchia，1959c） Other symbols，see Fig． 4

Fig． 8 －Orbital acceleration of Sputnik II（1957 ß）。 $\mathrm{dP} / \mathrm{\alpha i} \propto \rho^{\mathrm{H}^{1 / 2}}$（Jacchia 1959a） Other symbols，see Fig． 4

Fig． 9 －Density－altitude relations above 120 km for various gradients of temperature．The height variation of the density shows that any solution for an inadequately short range of altitudes is arbitrary even if the
beginning of the diffusion is introduced at 120 km or 150 km . Boundary conditions being fixed at 120 $\mathrm{km}(T$ and $\rho$ constant) 。 variation of density at 200 km is small.

Fig. 10 - Comparison between solar fluxes at 10.7 cm (Ottawa) and at 20 cm (Berlin) in 1958. While the difference between maximum and minimum corresponds to a factor of the order of 5 at Berling it is only of the order of a factor 2 at Ottawa. The real variation cannot be more than a factor of 2 and the flux ratio of $20 \mathrm{~cm} / 10 \mathrm{~cm}$ cannot vary from 1.5 to 0.4

Fig. 11 - Comparison between solar fluxes at 10.7 cm (Ottawa) and at 20 cm (Berlin) in 1959. The flux ratio $20 \mathrm{~cm} / 10$ cm varies only of about $+20 \%$ and such a small variation indicates that 1958 results on 20 cm cannot be accepted.

Fig. 12 - Heat flow and scale height with constant scale height gradient. The effect of thermal conductivity at 150 km is shown by a curve relating the scale height gradient and the energy necessary to maintain a certain gradient. Assuming an absorption crosssection of the order of $2 \times 10^{-17} \mathrm{~cm}^{2}$ the ultraviolet energy necessary at the top of earth ${ }^{\text {n }}$ s atmosphere $\mathrm{E}_{\mathrm{uv}}$ is deduced.

Fig． 13 －Temperature at 150 km and ultraviolet energy available at tine top of the earth ${ }^{8}$ s atmosphere．From the abscissae representing the ultraviolet energy，may be deduced the temperature at 150 km if the absorption cross section is assumed。

Fig． 14 －Effect of cooling by conduction。 If one adopts a certain rate of conduction of energy，i．e． 0.3 erg $\mathrm{cm}^{\mathbf{- 2}} \mathrm{sec}^{-1}$ it can be seen that thermospheric densities will decrease rapidly at highest altitudes．This figure shows a variation of a factor of 10 at 650 km in 12 hours for $\rho H^{1 / 2}$ 。

Fig． 15 －Effect of conduction on the temperature．The temperature of the isothermal layer decreases rapidly after sunset and the variation is more important during the first six hours than during the following 18 hours．However，at lowest altitudes the temperature decreases almost at the same rate。

Fig． 16 －Temperature－altitude relation for a total density $\rho=4.5 \times 10^{-15} \mathrm{gm} \mathrm{cm}^{-3}$ where $\rho\left(\mathrm{o}_{2}, \mathrm{~N}_{2}\right)=\rho(0)$, in an isothermal atmosphere．The knowledge of the altitude of the isobaric level leads to a determination of the temperature and therefore of the vertical distribution of the density。

Fig. 17 Sc: $=$ height - altitude relations for various isothermal atmospheres. Scale height values deduced by Jacchia (1960), and King-Hele and Walker (1960 b) are shown with their different vertical distribution.

Fig. 18 - Densities in isothermal atmospheres. The absolute values of density are determined by conditions $\rho\left(\mathrm{O}_{2} ; \mathrm{N}_{2}\right)=\rho(0)$ at an isobaric level $\rho=4.5 \times 10^{-15}$ $\mathrm{gm} \mathrm{cm}{ }^{-3}$ and $\rho\left(\mathrm{O}_{2} \circ 0\right)=\rho\left(\mathrm{N}_{2}\right)$ at an isobaric level $4.75 \times 10^{-15} \mathrm{gm} \mathrm{cm}^{-3}$ leading to densities at 220 km corresponding to observed values and to remarkable differences at highest altitudes.


[^0]:    It is, therefore, possible to define a time of conduction

