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Collision frequencies and energy transfer : ions

by P. BANKS

FOREWORD

"Collision Frequencies and Energy Transfer : Ions" is the second of a series of two papers dealing with the general subject of elastic collision processes and energy transfer applied to the constituents of the upper atmosphere. This series will be published in Planetary and Space Science during 1966.

AVANT-PROPOS

"Collision Frequencies and Energy Transfer : Ions" est le second de deux travaux dont le sujet général est l'étude des processus de collision élastique et de transfert d'énergie appliqués aux constituants de l'atmosphère supérieure. Les résultats de ces recherches seront publiés en 1966 dans la revue Planetary and Space Science.

VOORWOORD

"Collision Frequencies and Energy Transfer : Ions" is het tweede in een reeks van twee werken die handelen over het algemeen onderwerp : processen van elastische botsing en energie-overdracht, toegepast op de bestanddelen van de hogere atmosfeer. Deze reeks zal gepubliceerd worden in Planetary and Space Science in de loop van 1966.

VORWORT

"Collision Frequencies and Energy Transfer : Ions" ist die zweite Arbeit einer Serie von zwei Abhandlungen, die sich auf das general Problem der elastischen Stossprozesse und der Energieübertragung in der höheren Atmosphäre bezieht. Diese Arbeit wird im laufenden 1966 in Planetary and Space Science herausgegeben werden.

COLLISION FREQUENCIES AND ENERGY TRANSFER : IONS

by

Peter BANKS

Abstract

An investigation has been made of ion collision frequencies and energy transfer for conditions of gaseous thermal nonequilibrium. Following the methods of a development devoted to electron collisions, the problems of ion-neutral and ion-ion elastic collisions have been considered under the assumption of separate Maxwellian velocity distributions. Expressions for the nonequilibrium momentum transfer cross sections, collision frequencies, and energy transfer rates are given for neutral gases on the basis of a polarization potential and for ions on the basis of the Coulomb interaction. Next, the problem of ion-neutral resonance charge exchange is introduced in its effect upon collision cross sections and ion energy transfer rates. An analysis is made of gathered laboratory data to arrive at suitable expressions for the resonance charge exchange and momentum transfer cross sections for H^+, O^+, He^+, N_2^+, and O_2^+. Using these results, resonance charge exchange and momentum transfer collision frequencies are derived. The resonance ion-neutral energy transfer rates are then presented and compared with expressions used in previous studies of ion temperatures.

Résumé

Nous étudions les fréquences de collision et le transfert d'énergie des ions dans des conditions de gaz en non-équilibre thermique. Suivant les méthodes d'une recherche consacrée aux collisions des électrons, nous considérons les problèmes des collisions élastiques ion-particule neutre et ion-ion, sous l'hypothèse de distributions de vitesses maxwelliennes différentes. Les expressions des sections efficaces de transfert de moment en non-équilibre, les fréquences de collisions et les taux de transfert d'énergie sont donnés pour les gas neutres sur la base d'un potentiel de polarisation et pour les ions sur la base d'une interaction coulombienne. Ensuite, nous considérons le problème de l'échange de charge entre ion et particule neutre, du point de vue de ses effets sur les sections efficaces de collision et sur les taux de transfert d'énergie. Nous analysons ensuite des données de laboratoire pour en tirer des expressions appropriées pour les sections efficaces d'échange de charge par résonance et de transfert d'énergie pour H^+, O^+, He^+, N_2^+ et O_2^+. Les fréquences de collision pour l'échange de charge par resonance et pour le transfert de moment sont déduits de ces résultats. Nous présentons alors les taux de transfert d'énergie par résonance ion-particule neutre et nous les comparons avec des expressions utilisées dans les études antérieures portant sur les températures ioniques.

Samenvatting

In deze tekst worden de botsingsfrekwenties en de energie-overdracht bestudeerd voor ionen van gassen welke niet in thermisch evenwicht zijn. De elastische botsingen tussen ionen en neutrale deeltjes evenals tussen ionen onderling worden bestudeerd volgens de methode welke uiteengezet werd voor de botsingen die de electronen ondergaan in de onderstelling van een verschillende maxwelliaanse snelheidsverdeling. Er worden uitdrukkingen gegeven voor de werkzame doorsnede voor de overdracht van hoeveelheid van beweging ingeval de gassen niet in thermisch evenwicht zijn, evenals voor de botsingsfrekwentie en de mate van energie-overdracht door de ionen. Er wordt een studie gemaakt der gegevens die in verscheidene laboratoria gevonden werden ten einde geschikte uitdrukkingen te bekomen voor de ladingsuitwisseling bij resonantie en de werkzame doorsneden voor de overdracht van hoeveelheid van beweging ingeval van H^+, O^+, He^+, N_2^+ en O_2^+. Gebruik makende van deze resultaten werd de ladingsuitwisseling en botsingsfrekwentie voor de overdracht van hoeveelheid van beweging. bepaald. De mate van energieoverdracht bij ladingsuitwisseling in resonantietoestand tussen ionen en neutrale deeltjes wordt dan berekend en vergeleken met de uitdrukkingen gebruikt in vroegere studies betreffende ionentemperaturen.

Zusammenfassung

Eine Abhandlung der Ionenzusammenstossfrequenz und der Energie-übertragung für nicht thermischem Equilibrium wird ausgeführt. Nach einer Methode, die für Elektronen entwickelt wurde, wird das Problem der Zusammenstösse zwischen Ionen und ungeladenen Teilchen sowie zwischen Ionen im Falle verschiedenen Maxwell - Verteilungsfunktionen analysiert. Ausdrücke für Momentumstreuquerschnitten, Zusammenenstossfrequenzen und Energieübertragungskoeffizienten werden mit Hilfe einer Polarisationspotentiales für ungeladenen Teilchen und mit Hilfe einer Coulomb-Interaktion gegeben. Dann betrachten wir das Problem der Resonanzumladung zwischen Ionen und ungeladenen Teilchen hinsichts des Einflusses auf die Streuquerschnitte und auf die Energieübertragungskoeffizienten. Die Experimentalergebnisse werden analysiert, um Ausdrücke der Umladungsquerschnitte und der Energieübertragungskoeffizienten für H^+, O^+, He^+, N_2^+ und O_2^+ zu erreichen. Daraus folgen Ausdrücke der Zusammenstossfrequenze für Umladung und Momentumübertragung. Endlich werden die Energieübertragungskoeffizienten für Ionen und ungeladenen Teilchen vorgestellt und mann vergleicht diese Ausdrücke mit anderen Formeln, die im Problem der Ionentemperatur gebraucht wurden.

I.- INTRODUCTION

In order to discuss the thermal budget between electrons and ions and neutral gas particles in the ionosphere it is necessary to know all of the energy exchange rates between the different constituents, taking into account their specific temperature dependences under conditions of thermal nonequilibrium. The problem of energy transfer from an electron gas with a Maxwellian velocity distribution as a result of elastic collisions has been discussed in a previous paper (Banks, 1966) using an equation which is of general applicability to problems of elastic energy transfer. In this paper the study of energy transfer is continued with emphasis being placed upon the atmospheric ions. Thus, in Section II we enter into a discussion of the collision frequency and energy transfer equations which are valid for elastic ion-neutral and ion-ion energy transfer.

In contrast to the electron gas problem, there exists for ions a second method of energy loss by means of resonance charge exchange which, while fundamentally elastic in nature, cannot be described by the equation of collisional energy transfer. The problem of resonance charge exchange between ions and their parent neutral gas is discussed in Section III with respect to both collision frequencies and ion energy processes. Following an analysis of laboratory and theoretical cross section data numerical results for the resonance ion-neutral energy transfer rates are obtained and compared with expressions which have been used in different theoretical analyses of ion temperatures.

II.- NONRESONANT ION COLLISIONS

The equations describing collisional energy transfer have been discussed previously (Banks, 1966) for the example of two gases with Maxwellian velocity distributions characterized by the temperatures T_1 and T_2 and particle masses m_1 and m_2. The energy transfer rate, dU_1/dt, from the first gas is given by

$$\frac{dU_1}{dt} = -3n_1 \frac{m_1 m_2}{(m_1+m_2)^2} (kT_1 - kT_2) \bar{v}_{12} \tag{1}$$

where n_1 is the first gas number density, k is Boltzmann's constant, and $\bar{v}_{12}$ is the average collision frequency of a single particle of the first type in the second gas. Since the density, masses and temperatures are independent of the interaction forces between the different particles it is seen that it is the collision frequency which must be determined in order for the energy transfer rate between arbitrary types of gases to be known. The result is (see Banks, 1966)

$$\bar{v}_{12} = \frac{4}{3} n_2 \left(\frac{8k}{\pi}\right)^{1/2} \left(\frac{T_1}{m_1} + \frac{T_2}{m_2}\right)^{1/2} \bar{Q}_D \,, \tag{2}$$

where n_2 is the second gas number density and $\bar{Q}_D$ is the average momentum transfer cross section, given in terms of the velocity dependent momentum transfer cross section, q_D, and the interparticle relative velocity, g, as

$$\bar{Q}_D = K^3 \int_0^\infty g^5 q_D \exp(-Kg^2) \, dg \tag{3}$$

$$K = \left[\frac{2kT_1}{m_1} + \frac{2kT_2}{m_2}\right]^{-1} \,.$$

With equations (1), (2), and (3) it is possible to derive accurate expressions for both energy transfer rates and collision frequencies for gases of arbitrary temperature and particle mass once the velocity dependent momentum transfer cross section is known. Thus, the problem of ion elastic energy loss revolves, essentially, about the choice of proper cross sections needed to describe the interaction between the ion gas and the gas with which it is mixed. In the following analysis we will be concerned with two particular situations which are of some practical importance. The first involves the derivation of energy transfer rates and collision frequencies for ion-neutral gas mixtures under conditions where there are only elastic collisions. Next, we consider the problem of ion-ion gas mixtures when each ion species has its own separate Maxwellian velocity distribution.

1. Ion-Neutral Collisions

At low temperatures the most important ion-neutral interaction arises from an induced dipole attraction which has a potential $\varphi = - \alpha e^2/2r^4$. Here, α is the neutral atom polarizability, e is the electric charge, and r is the radial separation. This potential corresponds to the force law acting between Maxwellian molecules and the details of the transport coefficients under conditions of thermal equilibrium are well known. In particular, it is found the collision cross section is proportional to $T^{-1/2}$ and that the collision frequency is independent of the temperature.

For temperatures greater than 300°K, the induced dipole force of attraction is countered by a short range quantum mechanical repulsion. For sufficiently high temperatures, the polarization contribution is negligible and the collision cross section becomes nearly constant, yielding a collision frequency which varies as $T^{1/2}$. Unlike the polarization force, which is independent of the chemical nature of a given ion and depends only upon the atomic polarizability of the neutral gas,

the nature of the repulsive force is linked directly to the details
of the ion and neutral orbital electron structure. Thus, variations
in cross section are to be expected for different ions in the same
neutral gas at elevated temperatures.

Because there exist virtually no data for either collision
frequencies or cross sections at high temperatures, the study of ion-
neutral collisions must be based upon the assumption that the pola-
rization force is of dominating importance. A detailed evaluation of
the ion-neutral interaction potential has been made by Dalgarno et al.
(1958) who terminated the polarization potential at a small atomic
radius and added an elastic sphere repulsive potential to simulate
the quantum repulsion effets. This theoretical cross section was
then compared with reported measurements of ionic mobilities. For
equal ion and neutral temperatures below 300°K the theoretical
results appear to be accurate. Above 300°K, the detailed variation
of the momentum transfer cross section depends upon the specific
ion and neutral gas.

With there being only a few experimental data for tempera-
tures above 500°K, it is necessary to accept the collision equations
developed for the polarization force alone as representing the true
interaction over the entire range of ion and neutral temperatures
discussed here. Accordingly, we take the ion-neutral velocity de-
pendent momentum transfer cross section for singly charged ions to be,
following Dalgarno, et al. (1958),

$$q_D = 2.21 \ \pi \left(\frac{\alpha e^2}{\mu g^2} \right)^{1/2} cm^2 , \tag{4}$$

where α is the neutral gas atomic polarizability, μ is the ion-neutral
reduced mass, e is the electron charge, and g is the ion-neutral par-
ticle relative velocity.

Using equation (3) the average momentum transfer cross section for ion-neutral collisions becomes

$$\bar{Q}_D = \frac{3\sqrt{2}}{16} \, \pi^{3/2} \left(\frac{4.88\alpha e^2}{\mu} \right)^{1/2} \left[\frac{kT_i}{m_i} + \frac{kT_n}{m_n} \right]^{-1/2} \tag{5}$$

or, numerically,

$$\bar{Q}_D = 13.3 \times 10^{-14} \left(\frac{\alpha_o}{\mu_A} \right)^{1/2} \left[\frac{T_i}{A_i} + \frac{T_n}{A_n} \right]^{-1/2} cm^2 \tag{6}$$

where A_i and A_n are the particle masses in atomic mass units (a.m.u.), μ_A is the reduced mass in a.m.u., α_o is the atomic polarizability in units of 10^{-24} cm^3, and the subscripts apply to ions and neutrals, respectively.

For conditions of equal temperatures the mass factors of equation (6) cancel and we have

$$\bar{Q}_D (T_i = T_n) = \frac{3\sqrt{2}}{16} \, \pi^{3/2} \left(\frac{4.88\alpha e^2}{kT_n} \right)^{1/2} \tag{7}$$

which agrees with the value used by Dalgarno, et al. (1958) in their calculations of ion mobility.

Using equations (5) and (2) the average ion-neutral collision frequency for energy transfer is obtained as

$$\bar{\nu}_{in} = 2.21\pi \, n_n \left(\frac{\alpha e^2}{\mu} \right)^{1/2} \tag{8}$$

or,

$$\bar{\nu}_{in} = 2.6 \times 10^{-9} \, n_n \left(\frac{\alpha_o}{\mu_A} \right)^{1/2} sec^{-1}, \tag{9}$$

independent of either gas temperature. Actually, as shown by Morse (1963), although we have derived the present expression for the average momentum transfer cross section on the basis of separate Maxwellian velocity distributions, for the case of Maxwellian molecules the cross section, collision frequency, and energy transfer rate are independent of the actual velocity distribution used.

Table 3-7, taken from Hasted (1964) and Dalgarno(1961a), gives values of the atomic polarizability for the atmospheric gases.

Table 1   Polarizabilities of Neutral Gases

Neutral gas	$\alpha_o (10^{-24} cm^3)$
N_2	1.76
O_2	1.98
H_2	0.81
O	0.77
H	0.67
He	0.21
N	1.13

Equation (8) can be compared with an ion-neutral collision frequency which follows from the work of Chapman and Cowling (1952). In the diffusion of ions across a magnetic field, a collision interval τ appears which can be used to define a collision frequency ν by the relation $\nu = 1/\tau$. The value of τ is related to the ion-neutral diffusion coefficient which, in turn, can be calculated directly for the polarization potential. This process yields an ion-neutral collision frequency, using the cross section of Dalgarno, et al. (1958), given by

$$\bar{\nu}_{in} = \frac{n_i m_i + n_n m_n}{m_i m_n} 2.21\pi (\alpha e^2 \mu)^{1/2} \qquad (10)$$

where n_i is the ion density and n_n is the neutral gas number density. Comparing this result with equation (8) for the energy transfer collision frequency, it is seen that equality follows only if $n_i = n_n$. For other density conditions, in particular where $n_i \ll n_n$, the two collision frequencies are not the same. Dalgarno (1961b) has given ion-neutral collision frequencies based upon equation (10) under the assumption that $n_i \ll n(n)$. For this approximation it follows that his results are smaller than those presented here by the ratio μ/m_i. For the example of O^+ in N_2 this corresponds to a factor of 0.64.

Using the ion-neutral collision frequency given by equation (8), the rate of ion energy transfer in a neutral gas is found from equation (1) to be

$$\frac{dU_i}{dt} = - 3n_i n_n \frac{2.21 \, \pi}{(m_i + m_n)} (\alpha \mu e^2)^{1/2} k(T_i - T_n) \tag{11}$$

or,

$$\frac{dU_i}{dt} = - 6.8 \times 10^{-13} \, n_i n_n \frac{(\mu_A \alpha_o)^{1/2}}{(A_i + A_n)} (T_i - T_n) \text{ ev cm}^{-3} \text{ sec}^{-1} . \tag{12}$$

Appropriate values for the neutral gas polarizabilities are given in Table 1 and have been used to obtain the energy transfer rates listed in Table 2. As a consequence of the particular force law for ion-neutral collisions, the energy transfer rates depend only upon the difference in ion and neutral gas temperatures. In general there is a variation by a factor of 3.6 between the different loss rates considered. Further, except for the collisions $H^+ - He^+$ and $He^- H$, the differences in the rates are determined primarily by the varying mass factors rather than the changes in atomic polarizability.

It is also noted that these ion energy loss rates are considerably larger than those found (see Banks, 1966) for elastic electron-neutral collisions. As an example we may consider the O^+ - N_2 rate as, omitting common density and temperature factors, 6.6×10^{-14} ev cm^3 $^\circ$K^{-1}sec^{-1}, while the e-N_2 rate is, at T_e = 1000°K, only 1.55×10^{-16} ev cm^3°K^{-1}sec^{-1}. This difference of a factor of 430 arises from the enhanced ion-neutral cross section and advantageous mass factors in equation (1), mitigated, of course, by the much more rapid average electron velocity.

Table 2   Elastic Ion-Neutral Energy Transfer Rates

Collision Pair	$\dfrac{-dU_i/dt}{n_i n_n (T_i - T_n)}$	(ev cm^3 sec^{-1} $^\circ$K^{-1})
O^+ - N_2	6.6×10^{-14}	
O^+ - O_2	6.5×10^{-14}	
O^+ - He	2.8×10^{-14}	
O^+ - H	3.3×10^{-14}	
H^+ - O	3.5×10^{-14}	
H^+ - N_2	3.1×10^{-14}	
H^+ - He	5.5×10^{-14}	
He^+ - O	5.4×10^{-14}	
He^+ - H	1.0×10^{-13}	
He^+ - N_2	5.3×10^{-14}	

The results given here can be compared with values which have been used in previous studies of ion temperatures in the upper atmosphere. In a recent paper Willmore (1964) has adopted values for the helium and oxygen ion rates the values 3.3×10^{-14} and 3.6×10^{-14}

respectively. In comparing these rates with those listed in Table 2 it appears that Willmore's helium ion cooling rate, which is based upon the diffusion collision frequencies of Dalgarno (1961b), is too low by a factor of 0.63 (see equation 10 and discussion). Likewise, the oxygen ion energy transfer rate is too large by a factor of 1.3.

2. Ion-Ion Collisions

The average momentum transfer cross section for two separate ion gases, each having its own Maxwellian velocity distribution is, Banks (1966),

$$\bar{Q}_D = \frac{\pi}{2} \frac{(Z_1 Z_2 e^2)^2 \ln \Lambda}{\mu^2 \left[\frac{kT_1}{m_1} + \frac{kT_2}{m_2} \right]^2} \tag{13}$$

where $Z_{1,2}$ are the respective ion charges and the parameter Λ is given by

$$\Lambda = \frac{2\mathcal{E}}{Z_1 Z_2 e^2} \lambda_D \tag{14}$$

Here $\mathcal{E}$ is the average energy of relative motion between ions and λ_D, the plasma Debye length, is given by

$$\lambda_D^{-2} = 4\pi e^2 \left[\frac{Z_1 n_1}{kT_1} + \frac{Z_2 n_2}{kT_2} \right] \tag{15}$$

Equation (13) can be evaluated numerically to give

$$\bar{Q}_D = \frac{4.4 \times 10^{-6} \ln \Lambda}{\mu_A^2 \left[\frac{T_1}{A_1} + \frac{T_2}{A_2} \right]^2} \quad cm^2. \tag{16}$$

using the previous notation.

The average ion-ion energy transfer collision frequency is, from equation (3),

$$\bar{\nu}_{12} = \frac{4}{3} \sqrt{2\pi} \; n_2 \; \frac{(Z_1 Z_2 e^2)^2 \; \ln \Lambda}{\mu^2 \left[\dfrac{kT_1}{m_1} + \dfrac{kT_2}{m_2} \right]^{3/2}}$$ (17)

or,

$$\bar{\nu}_{12} = 8.4 \times 10^{-2} \; n_2 \; \frac{(Z_1 Z_2)^2 \; \ln \Lambda}{\mu_A^2 \left[\dfrac{T_1}{A_1} + \dfrac{T_2}{A_2} \right]^{3/2}} \qquad cm^2$$ (18)

To find the average collision frequency of a thermal ion in its own gas we let $T_1 = T_2 = T_i$ and $m_1 = m_2 = m_i$. Thus,

$$\bar{\nu}_{12} = n_i \frac{8}{3} \sqrt{\frac{\pi}{m_i}} \; \frac{z_i^2 \, e^4 \, \ln \Lambda}{(kT_i)^{3/2}} \quad ,$$ (19)

which is a factor of 4/3 larger than the ion collision frequency given by Chapman (1961) for total scattering effects.

The total rate of energy transfer between two ion species having Maxwellian velocity distributions is obtained from equation (1) by using the average ion-ion energy transfer collision frequency given by equation (17). Thus,

$$\frac{dU_1}{dt} = - \; \frac{4 \; \sqrt{2\pi} \; n_1 \, n_2 \; e^4 \; k(T_1 - T_2) \, \ln \Lambda}{m_1 m_2 \left[\dfrac{kT_1}{m_1} + \dfrac{kT_2}{m_2} \right]^{\frac{3}{2}}} \quad ,$$ (20)

or,

$$\frac{dU_1}{dt} = - \frac{2.18 \times 10^{-5} \; n_1 \; n_2 \, (T_1 - T_2) \; \ln \Lambda}{A_1 A_2 \left[\frac{T_1}{A_1} + \frac{T_2}{A_2} \right]^{\frac{3}{2}}} \quad ev \; cm^{-3} \; sec^{-1}, \qquad (21)$$

which agrees with the result Longmire (1963). Thus, the concept of binary collisions permits the exact description of the ion-ion energy transfer rate under the condition that each species has its own Maxwellian velocity distribution.

At this point it is now possible to make a comparison of the elastic and resonant ion-neutral energy transfer rates with the calculated ion-ion values. If we disregard the density and temperature differences it is found from Tables 2 and 11 and equation (21) that the ion-ion rates are generally larger by at least a factor of 10 for ion temperatures near 1000°K. This would imply that the ion gases are more strongly coupled to each other than to the neutral gases. However, for specific problems involving the transfer of energy in dilute gases where the different densities vary widely it is necessary to solve the heat balance equations directly in order to evaluate the assumption of a single ion temperature which is common to all the ion species present.

III.- RESONANT ION-NEUTRAL COLLISIONS

1. Introduction

Charge exchange, exerts a significant influence upon the momentum transfer cross section and is important for both the problems of ion energy transfer and diffusion. For this process, it is found that although there is a transfer of an electric charge between two particles, each tends to retain its original kinetic energy. Thus, even though the identity of the ion has changed, the reaction is still fundamentally elastic in preserving the total kinetic energy. Charge

exchange consequently becomes of particular importance to the ion energy balance since it can provide a rapid means for energetic ions to be transformed into energetic neutral particles. For many of the possible atmospheric ion-neutral collision pairs, it is found that the probability for charge exchange is much larger than that for the normal gas kinetic collisions. Therefore, while an originally energetic ion could cool to the ambient gas temperature by means of multiple elastic collisions, it can lose all of its excess energy in a single charge exchange reaction.

Charge exchange also plays an important part in enhancing the magnitude of the momentum transfer cross section above those values normally associated with high temperature ion-neutral reactions. This action arises from the conversion of very slightly glancing collisions into what appear to be nearly direct impacts with a consequent back-scattering. Under these circumstances there exists a relation between the charge exchange, q_E, and momentum transfer, q_D, cross sections given by Dalgarno (1958) as $q_D = 2q_E$. This relationship is valid only for high temperatures where the ion-neutral polarization effects can be neglected.

An adequate introduction to the details of the quantum theory of resonance charge exchange can be found in Bates (1962). In general, an exact solution to the problem of determining the charge exchange cross section is not possible because of the uncertainties in the form of the energy wave functions for all possible energy states of two colliding particles. Therefore, it is necessary to resort to experimental results to determine specific resonance charge exchange cross sections.

2. Resonance Cross Sections - Theory

Since we are interesed in both the charge exchange and momentum transfer cross sections it must be noted that to determine the full temperature variations of the momentum transfer cross section it is not adequate to use the relation $q_D = 2q_E$. For temperatures below a transition region, generally in the neighborhood of 500°K, the momentum transfer cross section is determined largely by the induced dipole polarization force between the ion and the neutral atom. The appropriate cross section for this interaction is given by equation (5) as

$$\bar{Q}_D = \frac{3}{8} \pi^{3/2} \frac{(4.88\alpha e^2)^{1/2}}{(kT_i + kT_n)^{1/2}} . \tag{22}$$

For temperatures above the transition region, the polarization contribution to $\bar{Q}_D$ diminishes rapidly and the effect of the charge exchange enchancement of the momentum transfer cross section becomes dominant. This problem has been discussed by Gosh and Sharma (1964).

It has been shown by Dalgarno (1958) that the velocity dependent charge exchange cross section, q_E, can be expressed as

$$q_E = (A - B\log_{10}\varepsilon)^2 , \tag{23}$$

for particle energies below 500 ev. Here A and B are constants characteristic of a given gas and ε is the kinetic energy of relative motion, measured in electron volts (ev). Using $q_D = 2q_E$, the high temperature velocity dependent momentum transfer cross section can be given as

$$q_D = 2(A - B \ln \varepsilon)^2 . \tag{24}$$

It is important to note that the total cross section for momentum transfer is not equal to the sum of the charge exchange and polarization contributions. Because there exist mutual interaction effects, the polarization potential and charge exchange action combine to form a total cross section whose variation with temperature follows closely the individual cross sections in their own regions of dominance. In the transition region there is a slight enhancement of the cross section to values above those due to either polarization or charge exchange, but this extends only over a limited temperature range. Thus, for temperatures above the transition region, q_D is determined exclusively, to within 1% (Knof, et al., 1964), by charge exchange, while below this point the polarization forces are predominant. This effect illustrated in Figure 1 for the specific example of $N_2^+ - N_2$ collisions. Here the transition region extends , in terms of $\Gamma = T_i + T_n$, from 200° to 700°K and is centered about the point $\Gamma = 340°K$. It is seen that for $\Gamma > 750°K$ there is essentially no contribution to $\overline{Q}_D$ from the polarization.

In order to determine suitable temperature dependent average charge exchange and momentum transfer cross sections, it is necessary to average equations (23) and (24) over the ion and neutral velocity distributions. Thus,

$$\overline{Q}_D = 2\,\overline{Q}_E = \iint f_i f_n \; q_D \; d^3 \vec{v}_i d^3 \vec{v}_n \tag{25}$$

where q_D is given by equation (24), f is the particle velocity distribution function, $d^3\vec{v}$ is a velocity space volume element, and the subscripts refer to ions and neutrals, respectively.

The solution to equation (25) for conditions where $T_i = T_n$ has been given by Mason and Vanderslice (1959) and Sheldon (1964). For situations where the ions and neutrals have separate Maxwellian distributions with $T_i \neq T_n$ it has been possible to generalize these results to obtain
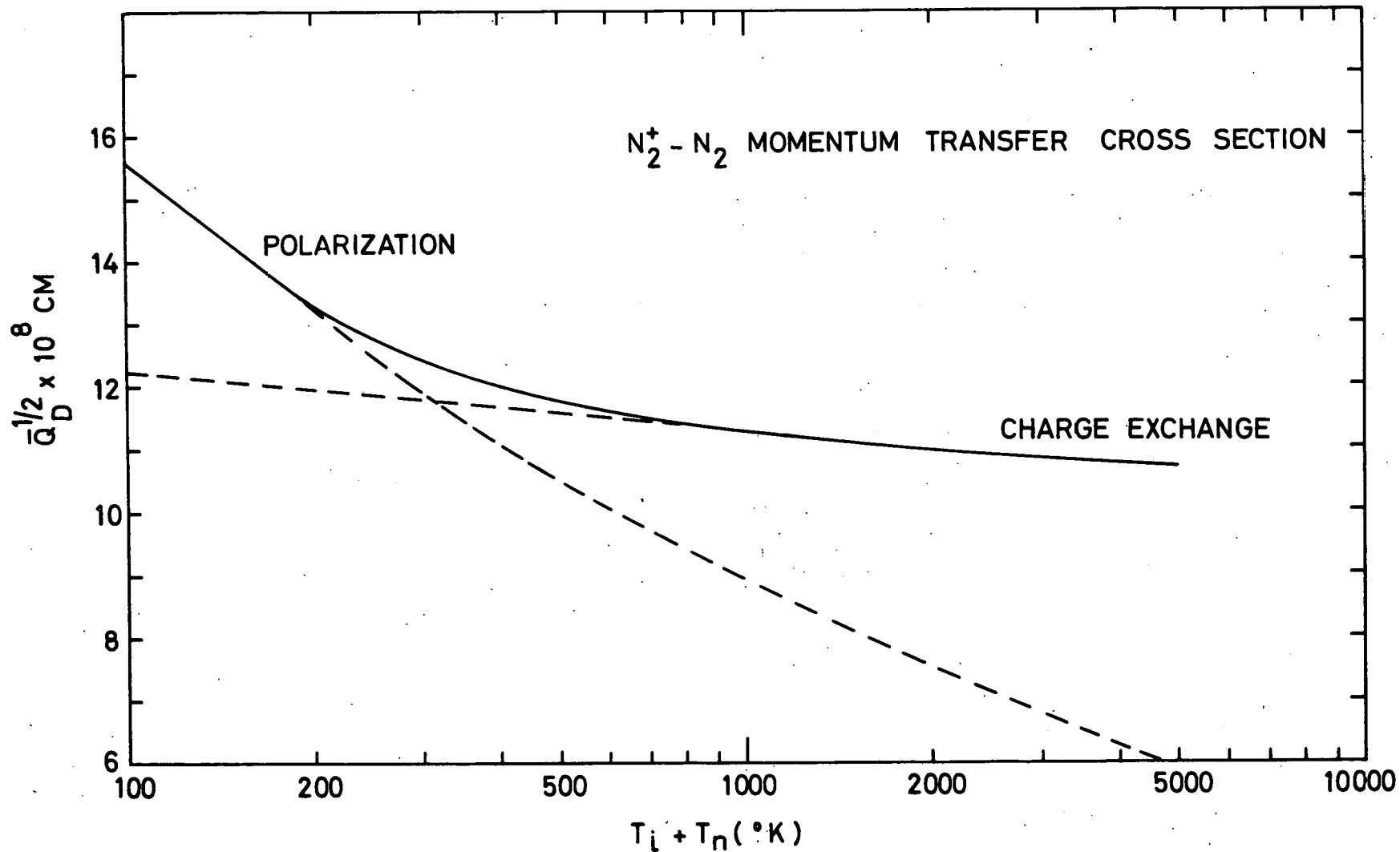
Fig. 1.- N_2^+ - N_2 Momentum Transfer Cross Section. The polarization curve is based upon a molecular
polarizability of 1.76 x 10^{-24} cm^3 while the charge exchange contribution is taken from the
experimental data of Amme and Utterback (1964) extrapolated to thermal energies.

$$\bar{Q}_E = \left[(A + 3.96B) - B \ \mathrm{Log}_{10} \Gamma \right]^2 \times 10^{-16} \ \mathrm{cm}^2 , \qquad (26)$$

where A and B, measured in units of 10^{-8} cm, are the coefficients appearing in the equation

$$q_E = (A - B \ \mathrm{Log}_{10} \mathcal{E})^2 . \qquad (27)$$

Following equation (2), we now define a resonance ion-neutral collision frequency, $\bar{\nu}_R$, as

$$\bar{\nu}_R = \frac{4}{3} \ n_n \left(\frac{8k}{\pi m} \right)^{1/2} (T_i + T)^{1/2} \ \bar{Q}_D , \qquad (28)$$

with m the ion or neutral particle mass. In a similar manner we define the average collision frequency for charge exchange, ν_E, as

$$\bar{\nu}_E = \frac{4}{3} \ n_n \left(\frac{8k}{\pi m} \right)^{1/2} (T_i + T)^{1/2} \ \bar{Q}_E , \qquad (29)$$

the factor of 4/3 appearing since we desire to use this equation in an energy transfer equation which is analogous to equation (1), the mass factor being omitted. From equations (28) and (29), we have

$$\bar{\nu}_E = \frac{1}{2} \ \bar{\nu}_R , \qquad (30)$$

for the temperature regime where polarization effects are negligible.

3. Resonance Ion-Neutral Energy Transfer

The derivation of the energy transfer rate of an ion gas under resonance conditions requires the use of the ion continuity equation with the charge exchange collision frequency. Thus, in the present section, the general equation of collisional energy loss does not apply.

The energy loss rate of an ion gas follows from consideration of the energy balance of ion production and loss. Thus,

$$\frac{dU_i}{dt} = - \left\{ E_i n_i \bar{v}_E - E_n n_i \bar{v}_E \right\}$$ (31)

where E_i and E_n are the respective ion and neutral particle energies, U_i is total ion gas energy per unit volume, and $\bar{v}_E$ is the charge exchange collision frequency defined by equation (29). If we apply the relations $E_i = 3kT_i/2$ and $E_n = 3kT_n/2$, which apply for gases with Maxwellian velocity distributions, we obtain

$$\frac{dU_i}{dt} = - \frac{3}{2} n_i k \bar{v}_E (T_i - T_n) ,$$ (32)

or,

$$\frac{dU_i}{dt} = - 1.3 \times 10^{-4} n_i (T_i - T_n) \bar{v}_E \quad ev \ cm^{-3} \ sec^{-1} .$$ (33)

To apply the preceding results to obtain accurate values for the resonance ion-neutral collision frequencies and energy transfer rates it is necessary to know the charge exchange and momentum transfer cross sections for each ion. Thus, in the following section an analysis is made of the experimental and theoretical results for each atmospheric ion in order to arrive at satisfactory values for $\bar{Q}_D$ and $\bar{Q}_E$. These quantities are then used to obtain adequate expressions for the appropriate collision frequencies and the ion energy loss rates.

4. Resonance Experimental Cross Sections

A. Atomic Hydrogen

The charge exchange reaction $H^+ + H \rightarrow H + H^+$ has been studied extensively because of its theoretical simplicity. Dalgarno and Yadev (1953) made accurate calculations using perturbed stationary state wave functions. Dalgarno (1958), in calculating the mobility of ions in parent gases, was able to reproduce essentially the same

results and compare them with experiment. Gurnee and Magee (1957).
applied the semi-classical impact parameter method to obtain values
of the charge exchange cross section at two ion velocities. Rapp and
Ortenburger (1960) found their theoretical approach gave values in
substantial agreement with those of Dalgarno and Yadev. Later, ex-
tensions were made by Rapp and Francis (1962).

Experimental data have been obtained by Fite, et al. (1960)
by using beam techniques with ion energies as low as 400 ev. The data
were later extended by Fite, et al. (1962) to a lower limit of 20 ev,
giving close agreement with the previous results. The values of the
various and theoretical measurements are listed in Table 3.

For the present purposes, the charge exchange cross section given
by Dalgarno and Yadev (1953) appears to represent the best compromise
between experiment and theory. Thus, using equation (26)

Table 3   Atomic Hydrogen Charge Exchange

$$H^+ + H \rightarrow H + H^+$$

$$q_E^{1/2} = A - B \, \text{Log}_{10} \, \epsilon$$

Source	$A \times 10^8$	$B \times 10^8$
Dalgarno (1958)	6.93	0.82
Gurnee and Magee (1957)	6.78	0.91
Rapp and Francis (1962)	6.05	0.88
Fite, et al. (1962)	7.60	1.06

we obtain for the hydrogen ion charge exchange cross section

$$\overline{Q}_E(H^+) = [10.2 - 0.82 \, \text{Log}_{10} \, \Gamma]^2 \times 10^{-16} \, \text{cm}^2 \quad , \tag{34}$$

while the average momentum transfer cross section corresponding to the high temperature limit is

$$\overline{Q}_D(H^+) = [14.4 - 1.16 \, Log_{10} \, \Gamma]^2 \times 10^{-16} \, cm^2. \tag{35}$$

Here, as in the subsequent equations, $\Gamma = T_i + T_n$.

To determine the extent of the temperature transition region between the polarization and charge exchange dominance of the momentum transfer cross section, equations (35) and (22) have been examined. It is found that the temperature transition point where the two asymptotic effects are equal lies near $\Gamma = 100°K$. Thus, for the case of atomic hydrogen, there is essentially no error involved in assuming that the charge exchange contribution to $\overline{Q}_D$ is dominant at all combined temperatures above 100°K.

B. Atomic Oxygen

The symmetrical charge exchange reaction $O^+ + O \rightarrow O + O^+$ has been studied by Dalgarno (1958) using a semi-classical approximation for the scattering phase shifts. Because there was considerable uncertainty in the approximation used to express the interaction energy of the ion and atom as a function of the radial separation, accuracy was claimed only to within a factor of two. Knof, et al. (1964) have made the most detailed study of the oxygen problem by taking explicit account of twelve possible electronic states of the O_2^+ pseudo-molecule. Their results are listed in Table 4, along with those of others authors.

Table 4   Atomic Oxygen Charge Exchange

$$O^+ + O \rightarrow O + O^+$$

$$q_E^{1/2} = A - B \ Log_{10} \ \varepsilon$$
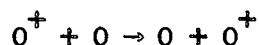
Source	$A \times 10^8$	$B \times 10^8$
Knof, et al. (1964)	5.59	0.48
Dalgarno (1958)	9.38	0.57
(see also Dalgarno ; 1964)		
Stebbings, et al. (1964)	5.88	0.57
Rapp and Francis (1962)	6.16	0.76

Experimentally, there is good agreement between the values given by
Knof, et al. and the values obtained by Stebbings, et al. (1964). These
latter results extend down to 40 ev and, according to Knof, et al.,
the average deviation between the predicted and measured values is
only 5.5%. Therefore, the charge exchange cross section of Knof, et al.
will be used here.

Using equation (26) the average charge exchange cross section
is

$$\bar{Q}_E(O^+) = [7.47 - 0.475 \ Log_{10} \varepsilon] \times 10^{-16} \ cm^2. \tag{36}$$

The transition region between polarization and charge exchange
dominance has been investigated. Figure 2 shows the momentum transfer cross
section and the extension of the asymptotic polarization and charge exchange
contributions. The transition is found to take place near $\Gamma = 470°K$.
It follows from figure 2 that the maximum error arising from the omis-
sion of the polarization effect is 11% at $\Gamma = 470°K$ and diminishes
rapidly at higher temperatures. Thus, for the computation of the
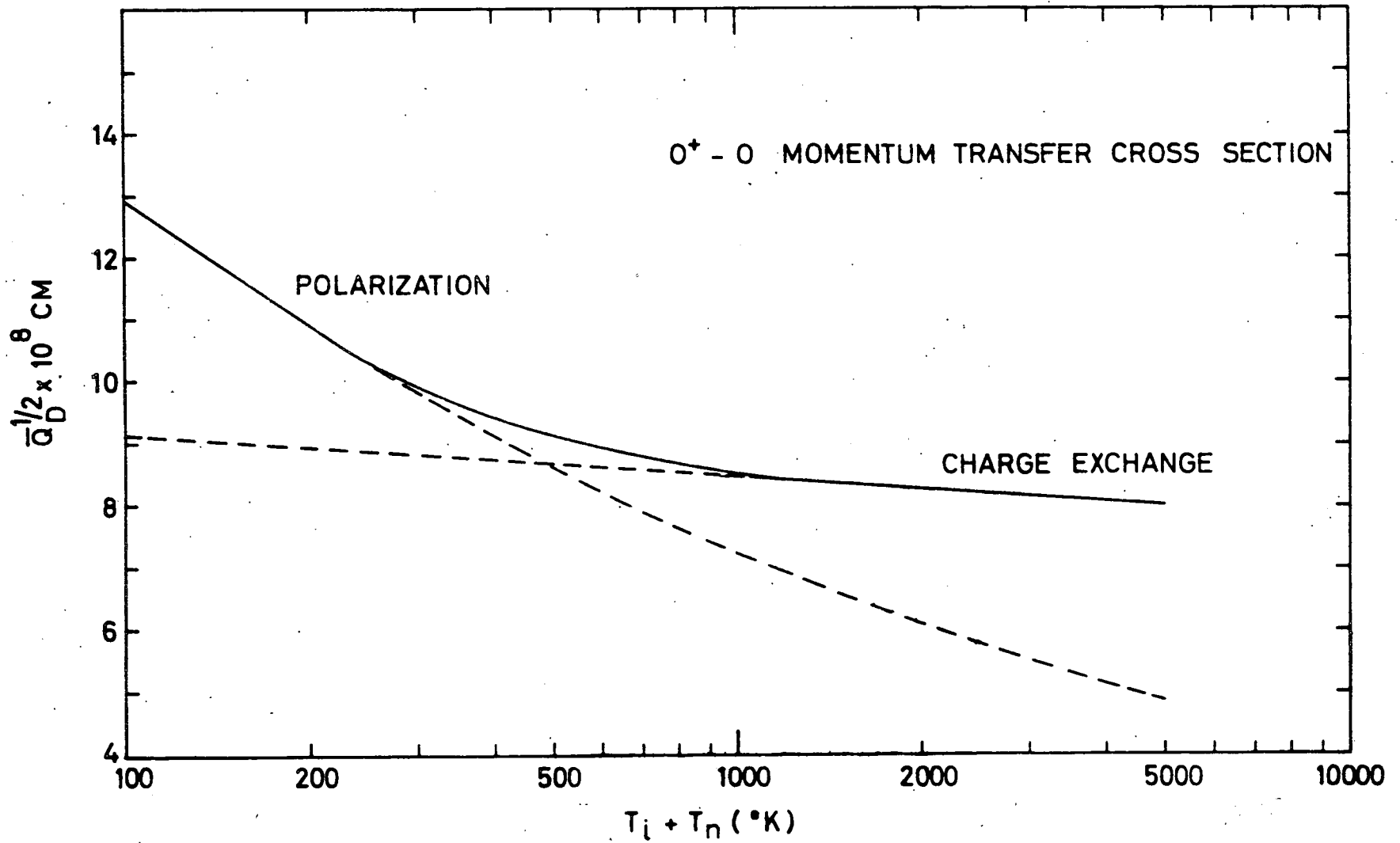momentum transfer cross section, it is possible to segment the true curve

Fig. 2.- O^+ - O Momentum Transfer Cross Section. The polarization curve is based upon an atomic polarizability of 0.77×10^{-24} cm^3. The charge exchange region is based upon the theoretical work of Knof, et al. (1964) expanded to include the effects of differing ion and neutral gas temperatures.

of Figure 2 into two portions : one for charge exchange and the other polarization. Hence,

$$\Gamma > 470°K \qquad \overline{Q}_D(O^+) = \left[10.5 - 0.67 \ Log_{10}\Gamma\right]^2 \times 10^{-16} cm^2$$

$$\Gamma < 470°K \qquad \overline{Q}_D(O^+) = \frac{1.66 \times 10^{-13}}{\Gamma^{1/2}} \ cm^2 \qquad (37)$$

C. Atomic Nitrogen

Two derivations of the nitrogen charge transfer cross section have been made. Gurnee and Magee (1957) and Knof, et al (1964) obtained the expression

$$q_E(N^+) = \left[5.53 - 0.46 \ Log_{10} \ \varepsilon\right]^2 \times 10^{-16} \ cm^2, \qquad (38)$$

which is accepted for the present calculations.

The average charge exchange cross section is, from equation (26),

$$\overline{Q}_E(N^+) = \left[7.33 - 0.455 \ Log_{10} \ \Gamma\right]^2 \times 10^{-16} \ cm^2. \qquad (39)$$

Applying equations (22) and (26), it is possible to find the asymptotic charge exchange and polarization contributions to $\overline{Q}_D$. However, because the atomic polarizability of atomic nitrogen is only slightly larger than that for atomic oxygen and because the two exchange cross sections are almost identical, it is not necessary to present the data graphically. For atomic nitrogen it is found that the transition temperature corresponds to $\Gamma = 550°K$ and that the error at this point in neglecting the polarization contribution is 10%. Thus, for the present purposes, $\overline{Q}_D$ can be taken as
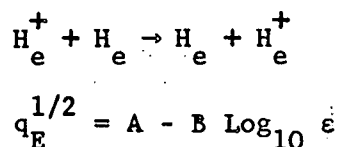
$$\Gamma > 550°K \qquad \overline{Q}_D(N^+) = \left[10.3 - 0.645 \ Log_{10} \ \Gamma\right]^2 \times 10^{-16} \ cm^2$$

$$\Gamma < 550°K \quad \bar{Q}_D(N^+) = \frac{2.0 \times 10^{-13}}{\Gamma^{1/2}} \quad cm^2. \tag{40}$$

D. Helium

Like atomic hydrogen, helium has been the object of intensive theoretical and experimental study. A paper by Rapp and Francis (1962) summarizes the results of 21 experiments and 4 theoretical studies.

Table 5   Helium Charge Exchange

$$H_e^+ + H_e \rightarrow H_e + H_e^+$$

$$q_E^{1/2} = A - B \, Log_{10} \, \epsilon$$

Source	$A \times 10^8$	$B \times 10^8$
Cramer and Simons (1957)	5.25	0.74
Dalgarno (1958)	5.39	0.62
Rapp and Francis (1962)	4.51	0.68
Gurnee and Magee (1957)	5.02	0.64
Hasted (1951)	5.14	0.55
Chanin and Biondi (1957)	4.93	0.82

Dalgarno (1958) calculated both ion mobility and charge transfer cross sections and compared them with the experimental mobility data of Chanin and Biondi (1957). Agreement was found to be within 20% at the higher temperatures. Rapp and Francis (1962) applied an empirical two state model which gives good accord with the experimental data for energies above 200 ev. Cramer and Simons (1957) were able to measure the charge exchange cross section at energies down to 16 ev. These data agree well with the earlier results of Hasted (1951) and Dillon, et al (1955). The correspondence is very good between the various authors with a spread of 15% at 1000°K, if the results of Rapp and Francis are disregarded. The data of Cramer and Simons listed in Table 5 are used here as representing the charge exchange cross section.

The average charge exchange cross section for helium may
be taken as

$$\overline{Q}_E(He^+) = [8.17 - 0.74 \, Log_{10} \, \Gamma]^2 \times 10^{-16} \, cm^2. \tag{41}$$

For the momentum transfer cross section it is found that the transition
temperature lies below $\Gamma = 200°K$ and virtually no error is involved in
neglecting the polarization effect. Consequently, we take

$$\overline{Q}_E(He^+) = [11.6 - 1.04 \, Log_{10} \, \Gamma] \times 10^{-16} \, cm^2 \quad . \tag{42}$$

E. Molecular Oxygen

For the reaction $O_2^+ + O_2 \rightarrow O_2 + O_2^+$ there appear to be no
theoretically derived values for the charge exchange cross section.
However, several molecular beam experiments have been conducted with
energies below 100 ev giving results which can be extrapolated to
thermal energies.

Gosh and Sheridan (1957) obtained cross section data for ion
energies below 100 ev which agree well with the later measurements of
Stebbings, et al. (1963). The earlier work of Potter (1954) does
not appear to be accurate in comparison, since the cross sections given
are a factor of two less than the above authors. Amme and Utter-
back (1964) repeated the experiment of Stebbings, et al. while making
a close determination of the dependence of the cross sections upon
the degree of ionic excitation. At high electron impact energies
there was a significant decrease in the charge transfer cross section,
indicating that the resonance behavior was easily destroyed by small energy
defects between the excited ions and the neutral gas. Table 6 summarizes
the various results.

For the present calculations the data of Amme and Utterback (1964) are accepted. The charge exchange cross section is

$$\bar{Q}_E(O_2^+) = [7.51 - 5.4 \log_{10}\Gamma] \times 10^{-16} \text{ cm}^2 \quad . \tag{43}$$

The relative importance of the polarization and charge exchange effects in determining $\bar{Q}_D$ can be seen in Figure 3 where the transition

Table 6   Molecular Oxygen Charge Exchange

$$O_2^+ + O_2 \rightarrow O_2 + O_2^+$$

$$q_E^{1/2} = A - B \, \text{Log}_{10}\,\epsilon$$
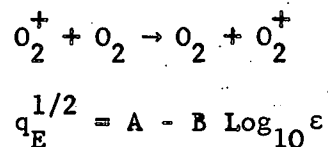
Source	$A \times 10^8$	$B \times 10^8$
Stebbings, et al. (1963)	5.32	0.65
Amme and Utterback (1964)	5.37	0.54
Potter (1954)	3.82	0.36

temperature is 1600°K. This relatively high temperature appears to be a consequence of the small charge exchange cross section for molecular oxygen. The maximum error associated with the neglect of the polarization effect at the transition temperature is 10%. Thus, we may approximate the momentum transfer cross section by

$$\Gamma > 1600°K \quad \bar{Q}_D(O_2^+) = [10.6 - 0.765 \, \text{Log}_{10} \, \Gamma]^2 \times 10^{-16} \text{ cm}^2$$

$$\tag{44}$$

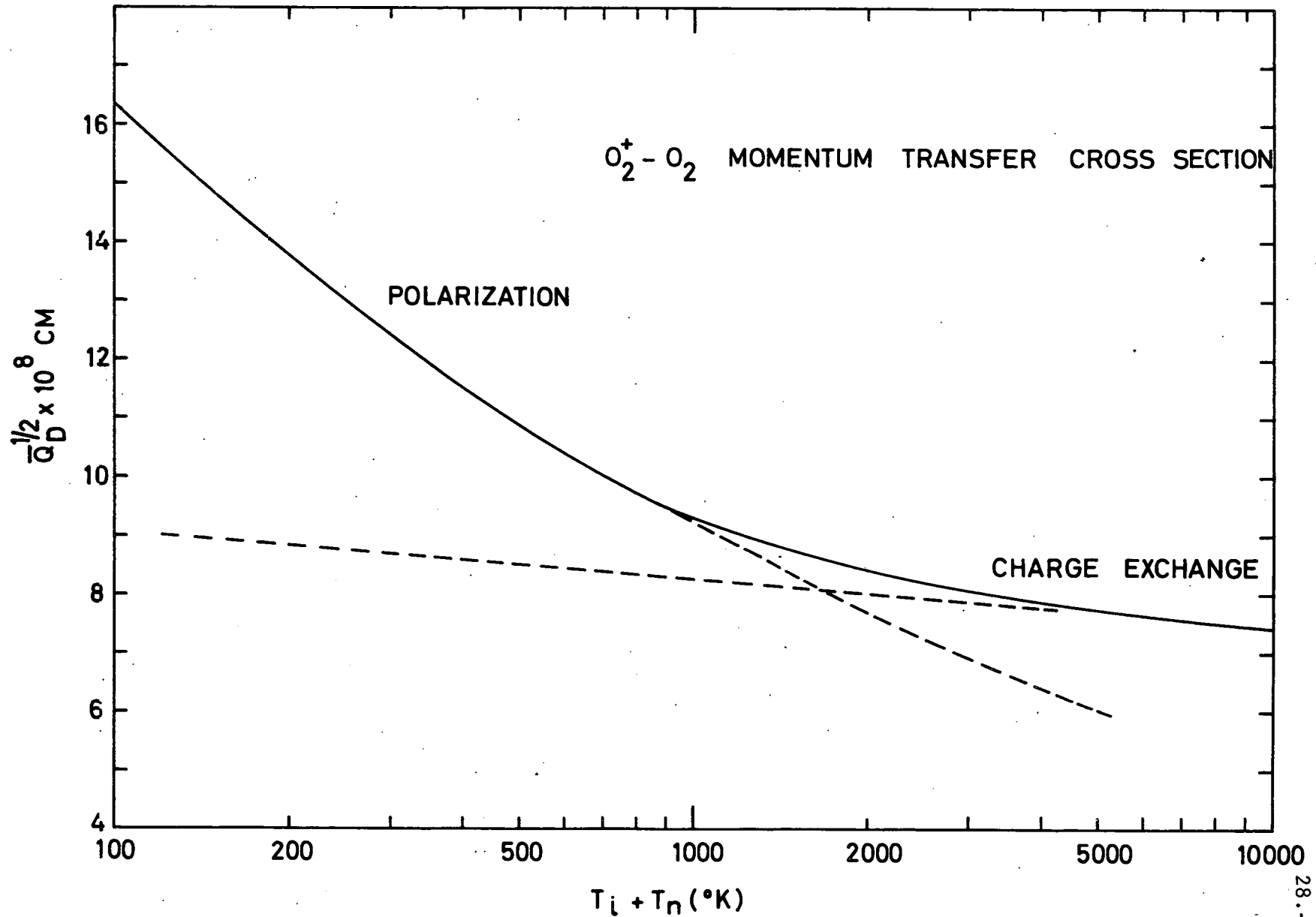$$\Gamma < 1600°K \quad \bar{Q}_D(O_2^+) = \frac{2.7 \times 10^{-13}}{\Gamma^{1/2}} \text{ cm}^2 \quad .$$

Fig. 3.- $O_2^+ - O_2$ Momentum Transfer Cross Section. The polarization curve is based upon a molecular ——— polarizability of 1.98 x 10^{-24} cm^3. The charge exchange curve is based upon the experimental data of Amme and Utterback (1964) extrapolated to thermal energies.

F. Molecular Nitrogen

Theoretical data for charge exchange in N_2 have been given by Gurnee and Magee (1957) and are listed in Table 7.

Table 7   Molecular Nitrogen Charge Exchange

$$N_2^+ + N_2 \rightarrow N_2 + N_2^+$$

$$q_E^{1/2} = A - B \, Log_{10} \, \varepsilon$$
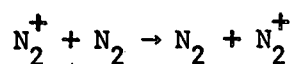
Source	$A \times 10^8$	$B \times 10^8$
Amme and Utterback (1964)	7.36	0.68
Stebbings, et al. (1963)	6.18	0.43
Gurnee and Magee (1957)	8.28	0.58
Potter (1954)	6.05	1.26

A number of experimental studies have been conducted, but the problem of ionic excitation has been responsible for inconsistent values. Stebbings, et al. (1963) have made extensive measurements of the charge transfer cross section and have analyzed many of the previous experimental studies in an effort to determine the sources of error. Amme and Utterback (1964) have recently been able to show that the work of Stebbings, et al. leads to cross sections which are too low as a consequence of excited molecular ions which are able to destroy the resonance behavior of the charge exchange process. They repeated the experiment using low energy electrons to produce the nitrogen ions found a considerable increase in the charge exchange cross section.

Using the results of Amme and Utterback (1964) , the charge exchange cross section at thermal energies is taken as

$$\overline{Q}_E(N_2^+) = [10.1 - 0.68 \, \text{Log}_{10} \, \Gamma]^2 \times 10^{-16} \, \text{cm}^2 \, . \qquad (45)$$

The behavior of the momentum transfer cross section is shown in Figure 1 where a transition temperature of $\Gamma = 340°K$ is indicated. The maximum error involved in neglecting polarization is found to be 9% at this point. Accepting this uncertainty, the momentum transfer cross section is

$$\Gamma \geqslant 340°K \qquad \overline{Q}_D(N_2^+) = [14.2 - 0.92 \, \text{Log}_{10}\Gamma]^2 \times 10^{-16} \, \text{cm}^2 \quad (46)$$

5. Resonance Ion-Neutral Collision Frequencies

With the derivation of the appropriate cross sections equations (28) and (29) can now be used to find the charge exchange and momentum transfer collision frequencies. The former are of direct importance to the problem of resonance ion-neutral energy transfer while the latter can be applied to the study of ion diffusion. The results for the charge exchange collision frequencies of the atmospheric ions are listed in Table 8. In order to compare the various values it is convenient to adopt the approximation that $\Gamma = 3000°K$ in the logarithm. This process leads to errors of less than 10% for $1200 < \Gamma < 4000°K$. Table 9 lists these reduced charge exchange collision frequencies and shows that there is an order of magnitude difference between the extremum values.

Table 8   Average Ion-Neutral Charge Exchange Collision Frequencies

Species	$\nu_E \, (\text{sec}^{-1})$
H^+, H	$9.5 \times 10^{-13} \, n(H) \, \Gamma^{1/2} \, [14.4 - 1.15 \, \text{Log}_{10} \, \Gamma]^2$
O^+, O	$2.3 \times 10^{-13} \, n(O) \, \Gamma^{1/2} \, [10.5 - 0.61 \, \text{Log}_{10} \, \Gamma]^2$
N^+, N	$2.6 \times 10^{-13} \, n(N) \, \Gamma^{1/2} \, [10.3 - 0.64 \, \text{Log}_{10} \, \Gamma]^2$
He^+, He	$4.4 \times 10^{-13} \, n(He) \, \Gamma^{1/2} \, [11.6 - 1.04 \, \text{Log}_{10} \, \Gamma]^2$
O_2^+, O_2	$1.7 \times 10^{-13} \, n(O_2) \, \Gamma^{1/2} \, [10.6 - 0.76 \, \text{Log}_{10} \, \Gamma]^2$
N_2^+, N_2	$1.8 \times 10^{-13} \, n(N_2) \, \Gamma^{1/2} \, [14.2 - 0.96 \, \text{Log}_{10} \, \Gamma]^2$

Table 9   Reduced Charge Exchange Collision Frequencies

$$\text{Log}_{10} \, \Gamma = \text{Log}_{10} \, (3000°K)$$

Species	$\bar{\nu}_E$ (sec^{-1})
H^+ - H	1.6×10^{-10} n(H) $\Gamma^{1/2}$
O^+ - O	2.1×10^{-11} n(O) $\Gamma^{1/2}$
N^+ - N	2.3×10^{-11} n(N) $\Gamma^{1/2}$
He^+ - He	4.4×10^{-11} n(He) $\Gamma^{1/2}$
O_2^+ - O_2	1.5×10^{-11} n(O_2) $\Gamma^{1/2}$
N_2^+ - N_2	2.9×10^{-11} n(N_2) $\Gamma^{1/2}$

The resonance ion neutral collision frequencies for momentum transfer have been obtained and are listed in Table 10. For values of Γ below the transition temperature it has been assumed that only the polarization effect is present. Hence, a given gas can be characterized by two different functional forms for $\bar{\nu}_R$. These results indicate significant differences exist between the different ions but that for atoms with similar electronic structures, such as O and N, nearly the same values are found.

Table 10   Average Ion-Neutral Resonance Momentum Transfer Collision Frequencies

Species		$\bar{\nu}_R$ (sec^{-1})
H^+, H	$\Gamma < 100°K$	1.9×10^{-12} n(H)$\Gamma^{1/2}$ $(14.4-1.15 \, \text{Log}_{10}\Gamma)^2$
O^+, O	$\Gamma > 470°K$	4.7×10^{-13} n(O)$\Gamma^{1/2}$ $(10.5-0.67 \, \text{Log}_{10}\Gamma)^2$
	$\Gamma < 470°K$	8.0×10^{-10} n(O)
N^+, N	$\Gamma > 550°K$	5.2×10^{-13} n(N)$\Gamma^{1/2}$ $(10.3-0.64 \, \text{Log}_{10}\Gamma)^2$
	$\Gamma < 550°K$	1.0×10^{-9} n(N)
He^+, He	$\Gamma > 100°K$	8.7×10^{-13}n(He)$\Gamma^{1/2}$ $(11.6-1.04 \, \text{Log}_{10}\Gamma)^2$

Species	$\bar{\nu}_R$ (sec^{-1})		
O_2^+, O_2	$\Gamma > 1600°K$	3.4×10^{-13}	$n(O_2)\Gamma^{1/2}$ $(10.6 - 0.76 \, Log_{10}\Gamma)^2$
	$\Gamma < 1600°K$	9.1×10^{-10}	$n(O_2)$
N_2^+, N_2	$\Gamma > 340°K$	3.6×10^{-13}	$n(N_2)\Gamma^{1/2}$ $(14.2 - 0.96 \, Log_{10}\Gamma)^2$

6. Resonance Ion-Neutral Energy Transfer

The charge exchange collision frequencies given in Table 8 have been used to derive the ion energy loss rates listed in Table 11. A simplification has again been made, however, by noting that the collision cross sections for resonance charge exchange vary only slightly for the normal range of ionospheric ion and neutral temperatures. Thus, the average value $\Gamma = 3000°K$ has been chosen for the logarithmic term of the cross section.

Table 11   Resonance Ion-Neutral Energy Transfer Rates

$$(ev \; cm^{-3} \; sec^{-1})$$

H $$\frac{dU_i}{dt} = -1.4 \times 10^{-14} \, n(H^+) \, n(H) \, (T_i + T)^{1/2} \, (T_i - T)$$

O $$\frac{dU_i}{dt} = -2.1 \times 10^{-15} \, n(O^+) \, n(O) \, (T_i + T)^{1/2} \, (T_i - T)$$

N $$\frac{dU_i}{dt} = -2.1 \times 10^{-15} \, n(N^+) \, n(N) \, (T_i + T)^{1/2} \, (T_i - T)$$

He $$\frac{dU_i}{dt} = -4.0 \times 10^{-15} \, n(He^+) \, n(He) \, (T_i + T)^{1/2} \, (T_i - T)$$

O_2 $$\frac{dU_i}{dt} = -1.4 \times 10^{-15} \, n(O_2^+) \, n(O_2) \, (T_i + T)^{1/2} \, (T_i - T)$$

N_2 $$\frac{dU_i}{dt} = -2.7 \times 10^{-15} \, n(N_2^+) \, n(N_2) \, (T_i + T)^{1/2} \, (T_i - T)$$

These results can be compared with values adopted in several studies of charged particle temperatures. Hanson (1963) has used an oxygen ion energy loss rate of

$$\frac{dU(O^+ - O)}{dt} = -1.82 \times 10^{-13} \, n(O) \, n(O^+)(T_i - T_n) \text{ ev cm}^{-3} \text{ sec}^{-1},$$

(47)

based upon the collision frequencies of Dalgarno (1961b). Analysis of this equation indicates that it was derived for a temperature of 300°K and, further, that it was based upon the momentum transfer cross section rather than the charge exchange cross section. However, because Dalgarno's momentum transfer cross section is too large by a factor of 2.7 - 3.0 (Knof, et al. 1964), the final result is larger than the present value by the factors 2.2 - 1.4 over the temperature range $1600 < \Gamma < 3600°K$.

Another calculation of the oxygen ion cooling rate has been made by Dalgarno (1963). He finds

$$\frac{dU_i(O^+ - O)}{dt} = -4.47 \times 10^{-14} \, n(O) \, n(O^+)(T_i - T) \text{ ev cm}^{-3} \text{ sec}^{-1}.$$

(48)

Comparing this equation with that obtained from Table 11, it is seen that the present energy transfer rate is larger by the factor $4.7 \times 10^{-2} \Gamma^{1/2}$ which, for temperatures in the range $1600° < \Gamma < 3600°K$, varies between 1.9 - 2.8.

Willmore (1964) has calculated the effect of helium resonance charge exchange upon the cooling of helium ions. His result is

$$\frac{dU_i(He^+ - He)}{dt} = -1.19 \times 10^{-13} \, n(He^+) \, n(He)(T_i - T) \text{ ev cm}^{-3} \text{ sec}^{-1}.$$

(49)

From Table 11, the corresponding equation from the present data is larger than equation (49) by the factor $3.4 \times 10^{-2} \, \Gamma$. For temperatures between $1600^\circ K < \Gamma < 3600^\circ K$, this factor varies between 1.4 and 2.0. At higher temperatures an increasing divergence will be noted.

Further calculations of resonance ion-neutral energy transfer rates are not known to have been made.

IV.- SUMMARY

The primary purpose of this paper has been to present a consistent development of the problems of ion energy transfer and collision frequencies. Knowledge of these factors is needed for any detailed study of the thermal behavior of ions in a dilute, partially ionized plasma. In the first section emphasis was placed upon ion-neutral and ion-ion collisions where resonance charge exchange is not an important factor. Using a general equation of energy transfer, adequate expressions for collision frequencies and energy transfer rates were obtained. For ion-neutral collisions it was necessary to use a momentum transfer cross section appropriate to an ion-neutral polarization interaction. Thus, while the present results are probably accurate for temperatures below $300^\circ K$, the true behavior at high temperatures must remain in doubt until adequate experimental studies have been made.

The ion-ion collision frequency and energy transfer rate have been computed directly from the coulomb potential. Under conditions of equivalent particle densities the ion-ion energy transfer rate is found to be an order of magnitude larger than the listed ion-neutral rates.

The problem of resonance collisions between an ion and its parent neutral particle has been discussed. Appropriate equations have

been developed for the thermal nonequilibrium collision frequencies
for both charge exchange and momentum transfer. It was shown that
ion energy transfer rates for charge exchange must dominate at tem-
peratures above the polarization-charge exchange transition region.
For all of the ions, with the exception of O_2, there is little error
involved in ignoring the polarization contribution to $\bar{Q}_D$ in the
atmosphere.

In general, both the nonresonant and resonant ion-neutral
energy transfer rates show only a small dependence upon the gas
temperatures. The resonance behavior leads to rates which are
somewhat larger than those found for elastic collisions.

Since the ion-neutral energy transfer rates are generally
much larger than electron-neutral and, in a similar fashion, electron-
ion energy transfer is much slower than ion-ion energy exchange, for
an external source giving heat to electrons in a weakly ionized plas-
ma there will be a complex set of relations needed to arrive at
equilibrium values of the electron, ion and neutral gas temperatures.

ACKNOWLEDGEMENTS

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