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FOREWORD

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MODEL OF THE POLAR ION EXOSPHERE

by

J. LEMAIRE and M. SCHERER.

Summary

A model of a polar-ion exosphere in which the geomagnetic field lines are open, is developed. The electrostatic electric field in this region has been calculated, by taking into account two fundemental conditions : (1) quasi-neutrality has to be satisfied everywhere in the exosphere; (2) evaporative fluxes of electrons and ions have to be equal. The density distributions are calculated for different evaporative models in the case of two (O^+ and e) and three (O^+, H^+ and e) constituents. In addition, the mean velocity, pressure and temperature distributions of the several constituents are derived.

Résumé

On a obtenu des modèles de l'exosphère ionique polaire en tenant compte du fait que les lignes de force du champ geomagnétique y sont "ouvertes". Le champ électrostatique dans cette partie de l'ionosphere a été calculé de telle sorte que (1) la condition de quasi-neutralité soit satisfaite en tout point de celle-ci et (2) que les flux d'échappement des électrons et des ions soient égaux. La distribution de densité a été calculée pour différents modèles évaporatifs dans le cas d'une exosphere constituée d'électrons et d'ions O^+. Des modèles à trois constituants (O^+- H^+- e) ont également été donnés. Les distributions radiales de la vitesse moyenne, de la pression et de la temperature sont déterminées analytiquement.

Samenvatting

Gebruik makend van de twee basisvoorwaarden : (1) de quasi-neutraliteit en (2) de gelijkheid der totale ionenflux en electronenflux, werd het elektrostatische veld berekend in de polaire, ionaire exosfeer.
De dichtheidsverdeling in de exosfeer werd bepaald voor verscheidene modellen in het geval van twee (O^+ en e) en drie (O^+, H^+ en e) bestanddelen.
Ten slotte werden nog de gemiddelde snelheid en de druk-en temperatuurverdelingen berekend voor de verschillende soorten deeltjes.

Zusammenfassung

Das elektrostatische Feld in der Polarexosphäre wird festgestellt so dass (1) die Evaporationsflüsse der positieven une negativen geladeten Teilchen gleich sind und (2) dass die Quasineutralitätsgleichung erfüllt ist. Die Dichtverteilung wird für verschiedene Modele mit zwei (O^+ und e) oder drei (O^+, H^+ und e) sorte Teilchen berechnet. Weiter, werden noch die mittlere Geschwindigkeit und die Druck-und Temperaturverteilungen für die verschiedene Bestandteile berechtnet.

1.- INTRODUCTION

Recently, the model ion-exosphere of Eviatar, Lenchek, and Singer [1] for a nonrotating planet has been generalized by Hartle [2], by permetting the density and temperature to vary over the baropause. In both papers the density, pressure and temperature distributions of the thermal particles are calculated in the stable trapping region of a centered-dipole magnetic field, i.e, along closed lines of force. Moreover, they assumed that all the charged particles of the exosphere have emerged from the barosphere, and that the electrostatic polarization potential and field are given by the Pannekoek [3] or Rosseland [4] formula :

$$\Phi_E = - \frac{m^+ - m^-}{2e} \Phi_g(r), \qquad (1)$$

with $\vec{E} = - \vec{\nabla}\Phi_E$ and $\vec{g} = - \vec{\nabla}\Phi_g$

In formula (1) m^- and m^+ are respectively the electron and mean ion masses, $\vec{g}$ is the gravitational acceleration, Φ_E and Φ_g are the electric and gravitational potentials.

In this paper we consider the case of the polar ion-exosphere where the magnetic field lines are open and connected with the magnetotail. It is clear that for such a model not all charged particles are trapped and some can escape if their kinetic energy is high enough. In order to calculate the electric potential Φ_E in the exosphere we use the quasi-neutrality condition and require that the escape fluxes of positive and negative charged particles are equal. We consider three models :

1. an "untrapped-model" (UT) where it is assumed that all exospheric particles emerge from the baropause, and can escape along the open field lines.
2. a "trapped-model" (T) where it is assumed that the exosphere is also populated by trapped particles, which are in thermal equilibrium with those emerging from the barosphere.
3. the well-known barometric model (B) where there is no restriction on the energy values nor on the pitch angles of the exospheric particles.

For each model we determine the density n, the escape flux F, the mean velocity w, the pressure tensor components $p_{\parallel}$, $p_{\perp}$, and the temperatures $T_{\parallel}$ and $T_{\perp}$.

Moreover, in order to simplify the problem we consider that the transition layer separating the barosphere (where the mean free path $\bar{l}$ of the particles is small compared with the electron density scale height H_e) and the exosphere (where $\bar{l} >> H_e$) is reduced to a spherical surface. Experimental electron density profiles in the polar ionosphere [5-8] show that such a spherical surface (the baropause) may be taken about 2000 kilometers above the summer polar cap.

2.- THE GRAVITATIONAL AND ELECTROSTATIC POTENTIALS

It is well known that the gravitational potential Φ_g satisfies Poisson's equation

$$\Delta \Phi_g = 4 \pi G \sum_k n_k m_k, \tag{2}$$

where G is the gravitational constant, and the summation in the right hand side runs over all kinds of particles with mass m_k and density n_k. The solution of (2) is

$$\Phi_g(r) = \Phi(r)\, y, \tag{3}$$

where

$$y = \frac{r_o}{r}; \quad \Phi(r) = - G \frac{M(r)}{r_o}, \tag{4}$$

with M(r) the total mass inside a sphere of radius r.

As the mass density is very small in the atmosphere, equation (2) can in a first approximation be reduced to the Laplace equation. In this case M(r) is the earth's mass and $\Phi(r)$ is a constant equal to $\Phi_g(r_o)$. The gravitational acceleration is then given by

$$g(r) = - \frac{d\Phi_g(r)}{dr} = \Phi_g(r_o) \frac{y^2}{r_o} \quad . \qquad (5)$$

In the exosphere where the collision frequency is very small the electric potential can no longer be given by the Pannekoek's[3] or Rosseland's[4] formula (1), since such an approximation is only valid in the barosphere where the particles subject to collisions are in hydrostatic equilibrium. If the diffusion equilibrium electric potential (1) is used for oxygen ions of mass m_{O^+}, the escape flux of the thermal electrons of mass m_e is at least $(m_{O^+}/ m_e)^{1/2} = 168$ times larger than the ion evaporative flux. Hence, a positively charged layer at the baropause would appear, which would increase the electric field in the exosphere. In any case, this process would necessarily reduce the electron flux and would lead to the stationary state in which the flux of electrons is equal to the flux of the positive ions.

The electrostatic potential Φ_E is a solution of Poisson's equation

$$\Delta\Phi_E = - 4 \pi e \quad \Sigma_k \quad Z_k \, n_k, \qquad (6)$$

where Z_k is an integer : positive and equal to the degree of ionization for the ions and - 1 for the electrons. The summation runs over all kinds of particles with density n_k.

As the baropause is a discountinuity surface separating the barosphere (collision dominated region) and the exosphere (collision free region) it is possible to determine an appropriate constant Φ_2 and to calculate a function $\Phi_1(\vec{r})$, such that the electric potential distribution is given by

$$\Phi_E(\vec{r}) = \left[\Phi_o + \Phi_1(\vec{r}) \, U(r-r_o) \right] \frac{r_o}{r} + \Phi_2 \, U(r-r_o) + \Phi_3 , \quad (7)$$

r_o is the radial distance to the baropause and U(x) is the wellknown Heaviside step-function defined by : $U(x < 0) = 0$, and $U(x > 0) = 1$; Φ_3 is an arbitrary constant ; $\Phi_o + \Phi(\vec{r}) + \Phi_2 + \Phi_3$ is the electric potential at the base of the exosphere and $\Phi_o + \Phi_3$ at the top of the barosphere. In appendix I it is shown that for an isothermal ion-barosphere in hydrostatic equilibrium,

Φ_o is given by

$$\Phi_o = - \frac{\Sigma_j Z_j m_j n_j(r_o)/k\, T_j(r_o)}{\Sigma_j Z_j^2 n_j(r_o)/k\, T_j(r_o)} \cdot \frac{\Phi_g(r_o)}{e} \quad . \tag{8}$$

Finally $\Phi_1(\vec{r})$ is related to the electric space charge in the exosphere. Indeed from (6) and (7) one obtains for $r > r_o$,

$$4 \pi e \Sigma_k Z_k n_k = - \frac{r_o}{r}\left(\Delta \Phi_1 - \frac{2\vec{r}}{r^2} \cdot \vec{\nabla} \Phi_1 \right) \tag{9}$$

As the charge excess, $\Sigma_k Z_k n_k$, although minute, is not strictly zero, Φ_1 is a non-linear function of the radial distance, in the same way as $\Phi(r)$ varies in equation (3) and (4) when $\Sigma_k n_k m_k \neq 0$, i.e, when M(r) is an increasing function of r.

The electric field corresponding to the potential distribution (7) is given by

$$\vec{E}(\vec{r}) = \left[\left[\Phi_o + \Phi_1 U(r-r_o) \right] \frac{r_o}{r^2} - (\Phi_1 + \Phi_2)\, \delta(r-r_o) \right] \frac{\vec{r}}{r} - \frac{r_o}{r} U(r-r_o) \vec{\nabla} \Phi_1 \tag{10}$$

where $\delta(x)$ is the Dirac δ-function.

3. THE EQUATION OF MOTION OF A CHARGED PARTICLE IN THE EXOSPHERE

Following Eviatar et al.[1], the equation of motion can be determined by writing down the law of conservation of the total energy in the static magnetic field and by using the first invariant in the guiding center approximation. $\frac{1}{2} m v^2 + m \phi_g(r) + Ze\Phi_E = \text{constant}$

Hence, we have :

$$v^2(\vec{r}) = v^2(\vec{r_o}) + 2\, \Phi_g(r_o) \left[1 + \alpha\, U(1-y) - (1+\beta) y + \frac{Ze(\Phi_1 y + \Phi_o)}{m\, \Phi_g(r_o)} U(y-1) \right], \tag{11}$$

$$\sin^2 \theta = \eta \frac{v^2(\vec{r_o})}{v^2(\vec{r})} \sin^2 \theta_o, \tag{12}$$

where θ and θ_o are respectively the pitch angles at radial distances r and $r_\bullet$, η is the relative magnetic field intensity along a line of force crossing the baropause at geomagnetic latitude λ_o:

$$\eta = \frac{B(r,\lambda)}{B(r_\bullet,\lambda_\bullet)} \quad . \tag{13}$$

Moreover, we have used the following parameters

$$\alpha = \frac{Z\ e}{m\Phi_g(r_\bullet)} (\Phi_o - \Phi_2), \tag{14}$$

$$\beta = \frac{Z\ e}{m\Phi_g(r_\bullet)} (\Phi_o + \Phi_1), \tag{15}$$

which will be called the first and second reduced electric potential energies of the particle (Z, m).

In order to calculate the density and flux at a given altitude in the exosphere one has to classify the charged particles in several groups. According to their velocity and pitch angle there are trapped, incoming, escaping, and ballistic particles. These different classes of particles are summarized in table $I_{a,b}$ for the two cases encountered in this study.

In the case (a) (table I_a) the velocity v is a decreasing function of the altitude ($1 + \alpha > (1 + \beta)\ y > o$). This occurs for the heavy oxygen ions which are bound to the earth by the gravitational force. In table I_b, on the contrary, the velocity v is increasing with altitude. This is the case of protons accelerated outwards in the exosphere by an electric force which for these particles is larger than the gravitational force. Any ion gas for wich $1 + \alpha < (1 + \beta)\ y < o$ is therefore blown out of the ionosphere by the exospheric electric field.

4.- VELOCITY DISTRIBUTION FUNCTION

In the assumption that above the baropause there are no collisions, Liouville's theorem can be applied to obtain the velocity distribution of the thermal particles in the exosphere [10]. Moreover, since in the barosphere the

collision frequency is expected to be large enough to obtain rapid redistribution of particles, we will assume a Maxwellian velocity distribution function in this region. Hence the velocity distribution in the exosphere is given by :

$$f(\vec{v},r,\lambda) = n(r_o,\lambda_o)\left(\frac{m}{2\pi\ k\ T(r_o,\lambda_o)}\right)^{3/2}\exp\left[-\ q\ -\ \frac{m\ v^2}{2\ k\ T(r_o,\lambda_o)}\right]I(v,\theta), \quad (16)$$

where for $y < 1$,

$$\left.\begin{aligned} q &= \Lambda\ [1 + \alpha - (1 + \beta)\ y], \\ \Lambda &= -\ \frac{m\ \Phi_g(r_o)}{k\ T(r_o,\lambda_o)}. \end{aligned}\right\} \quad (17)$$

Furthermore, in the formula (16), $I(v,\theta)$ is a function wich depends on the population of the different classes of particles at a level r in the exosphere. Under the assumption that the incoming particles which reach the baropause have the same Maxwellian velocity distribution as the barospheric particles, we introduce a parameter ζ such that $[n(r_o,\lambda_o)]_{incoming} = \zeta n(r_o,\lambda_o)$. If $\zeta = 0$ there are no incoming particles in the exosphere, and for $\zeta = 1$ one has a model exosphere in which the incoming particles are in thermal equilibrium with those escaping from the barosphere.

In a quite similar way we introduce a parameter ξ, in order to take into account the particles which are trapped above the baropause, $[n(r_o,\lambda_o)]_{trapped} = \xi\ n(r_o,\lambda_o)$. When $\xi = 0$, we consider that no trapped particles are present above the baropause (UT), and if $\xi = 1$, on the contrary, we assume that trapped particles are in thermal equilibrium with those emerging from the baropause (T). If $\xi = \zeta = 1$, all classes of particles are present and they are in thermal equilibrium, i.e. $I(v,\theta) \equiv 1$; we have then a barometric model (B).

It is easy to show that in the case (a), $I(v,\theta)$ is given by

$$I(v,\theta) = U(v-v_\infty).U(\theta_m - \theta) + U(v_y - v) + U(v-v_y).U(v_\infty - v)[U(\theta_m - \theta) + U(\theta + \theta_m - \pi)]$$

$$+ \zeta\, U(v-v_\infty).U(\theta - \theta_m) \quad + \xi\, U(v_\infty - v).U(v-v_y).U(\theta - \theta_m).U(\pi - \theta_m - \theta), \qquad (18)$$

where v_∞, v_y, and θ_m are defined in Table I_a.

On the other hand, in the case (b), i.e. when $1 + \alpha < (1 + \beta)\, y < 0$, we have

$$I(v,\theta) = U(v-v_z).U(\theta_m - \theta) + \zeta\left[U(v_z - v) + U(v-v_z).U(\theta - \theta_m)\right], \qquad (19)$$

where v_z is defined in Table I_b. If the incoming particles are in thermal equilibrium with the barospheric ones, $\zeta = 1$, hence $I(v,\theta) \equiv 1$ and we have once more a barometric model (B).

5. ESCAPE FLUX

The number of particles flowing each second through a unit surface normal to the magnetic field lines is given by

$$F(r,\lambda) = \int v_{\parallel}\, f(\vec{v},\vec{r})\, d^3v. \qquad (20)$$

In the case (a) where $1 + \alpha > (1 + \beta)\, y > 0$, using the expressions (16) and (18) we obtain

$$F(r,\lambda) = \tfrac{1}{4}\, n(r_o,\lambda_o)\, c_o\, \eta\, (1 - \zeta)[1 + \Lambda\,(1+\alpha)]\exp[-\Lambda(1+\alpha)], \qquad (21)$$

with

$$c_o = \left[\frac{8\, k\, T(r_o,\lambda_o)}{\pi\, m}\right]^{\frac{1}{2}}. \qquad (22)$$

Using the expressions (16) and (19), however, we obtain the evaporative flux in the case (b), where $1 + \alpha < (1 + \beta)y < 0$

$$F(r,\lambda) = \tfrac{1}{4}\, n(r_o,\lambda_o)\, c_o\, \eta\, (1 - \zeta). \qquad (23)$$

It is worthwile to note that F is independant on the ξ-value, i.e. on the population parameter of the trapped particles, and that in both cases (a) and (b), the evaporative flux vanishes in the barometric model, i.e. when $\zeta = 1$.

To avoid any steady electric charge accumulation a necessary condition is given by the equality of the electron flux and the total positive ion-flux :

$$\Sigma_j F_j(r,\lambda) = F_e(r,\lambda). \tag{24}$$

This equality must be satisfied at every point (r,λ) in the exosphere. The condition (24) yields a relation between α_e and all the other α_i and will fix the value of $(\Phi_o - \Phi_2)$.

Moreover as can easily be seen from the definitions (14) and (17), the α_i and Λ_i for each kind of ion are related to α_e and Λ_e by

$$\alpha_i = -\alpha_e \frac{Z_i m_e}{m_i}; \quad \Lambda_i = \Lambda_e \frac{m_i T_e}{m_e T_i}; \quad \Lambda_e = \frac{G M m_e}{r_o k T_e}. \tag{25}$$

Hence for fixed values of $n_i(r_o,\lambda_o)$, $n_e(r_o,\lambda_o)$, $T_i(r_o,\lambda_o)$ and $T_e(r_o,\lambda_o)$ at the baropause, the value of α_e can be calculated by means of (24).

Considering an exosphere in which only O^+ and H^+ ions are present, numerical calculations show that $1 + \alpha_{O+}$ and $1 + \alpha_e$ are positive, i.e. the exospheric oxygen plasma is bounded to the earth, and the evaporative fluxes F_{O+} and F_e are given by formula (21) (case (a)). For the protons, however, $1 + \alpha_{H+}$ is generaly negative. Hence F_{H+} is given by (23) (case (b)) and the protons are all blown out of the exosphere by the large polarization electric field. Therefore condition (24) applied to an exosphere with O^+ and H^+ ions gives the following equation :

$$\frac{n_{O+}(r_o,\lambda_o)}{n_e(r_o,\lambda_o)}\left[\frac{m_e\ T_{O+}(r_o,\lambda_o)}{m_{O+}\ T_e(r_o,\lambda_o)}\right]^{\frac{1}{2}}\left[1+(1+\alpha_{O+})\Lambda_{O+}\right]\exp\left[-(1+\alpha_{O+})\Lambda_{O+}\right]$$

$$+\ \frac{n_{H+}(r_o,\lambda_o)}{n_e(r_o,\lambda_o)}\left[\frac{m_e\ T_{H+}(r_o,\lambda_o)}{m_{H+}\ T_e(r_o,\lambda_o)}\right]^{\frac{1}{2}}=\left[1+(1+\alpha_e)\Lambda_e\right]\exp\left[-(1+\alpha_e)\Lambda_e\right]. \qquad (26)$$

Table II shows the values of $1+\alpha_j$ as a function of the relative concentration $n_{H+}(r_o,\lambda_o)/n_e(r_o,\lambda_o)$ for $r_o=(6371+2000)$km and $T_e(r_o,\lambda_o) = T_{O+}(r_o,\lambda_o)= T_{H+}(r_o,\lambda_o)= 3000°K$.

As can be seen from this table the exospheric electric field depends very strongly on the concentration of the hydrogen ions even if they form only a minor constituent at the baropause. Hence an increase of the number of light ions (i.e. an increase of the total ion-escape-flux) [9] diminishes the absolute value of the exospheric electrostatic potential $|\Phi_2|$.

Moreover, as shown in Table III, which gives the $1+\alpha_j$ values as a function of the exospheric temperature for a constant ion-composition $n_{H+}(r_o,\lambda_o)/n_e(r_o,\lambda_o) = 0.10$, the reduced first electric potential energy for the electrons α_e and the electric potential $|\Phi_2|$ decreases with decreasing temperature.

6. DENSITY DISTRIBUTION

By definition the density distribution of the particles is given by

$$n(r,\lambda) = \int f(\vec{r},\vec{v})\ d^3v. \qquad (27)$$

Taking into account (16) and (18) we find for the case (a), when $1+\alpha > (1+\beta)\ y > o$

$$n(r,\lambda) = n(r_o,\lambda_o)\Big\{(1+\zeta)\ K_2(\infty) + (1-\zeta)\ K_2(V_\infty)$$

$$-\ (1-\eta)^{\frac{1}{2}}\left[(1-\zeta)\ K_2(\infty) + (1+\zeta-2\ \xi)\ K_2(X_\infty)\right]\exp\left(-\frac{\eta q}{1-\eta}\right)\Big\}\exp\ (-q). \qquad (28)$$

where

$$V_\infty^2 = \Lambda(1+\beta)y \quad ; \qquad X_\infty^2 = \Lambda \frac{(1+\beta)y - (1+\alpha)\eta}{1-\eta}. \tag{29}$$

The functions $K_m(x)$ are defined in Appendix B. They can be expressed explicitly in terms of exponential functions and the error function Erf(x).

On the other hand, if $1 + \alpha < (1 + \beta)y < 0$, (case (b)) substitution of formula (16) and (19) in (27) yield

$$n(r,\lambda) = n(r_o,\lambda_o)\Big\{(1+\zeta)\, K_2(\infty) - (1-\zeta)\, K_2\big[(-q)^{\frac{1}{2}}\big]$$

$$- (1-\zeta)(1-\eta)^{\frac{1}{2}}\Big[K_2(\infty) - K_2\big[\big(\frac{-q}{1-\eta}\big)^{\frac{1}{2}}\big]\Big]\exp\Big(-\frac{\eta\, q}{1-\eta}\Big)\Big\}\exp(-q). \tag{30}$$

It may be noted that for $\zeta = \xi = 1$ both formula (28) and (30) reduce to a barometric model (B).

The condition of quasi-neutrality in the exosphere implies that the relation

$$\Sigma_k\, Z_k\, n_k(r,\lambda) \simeq o, \tag{31}$$

is satisfied.

As the density distribution $n_j(r,\lambda)$ of each constituant depends on $\beta_j(r,\lambda)$, the equation (31) enables us to calculate the value of $\beta_e(r,\lambda)$ as the β_j are related to β_e by

$$\beta_j = - \frac{Z_j\, m_e}{m_j}\, \beta_e. \tag{32}$$

For instance, in an $(0^+ - e)$- exosphere, condition (31) yields

$$n_{0+}(r,\, \lambda\, ;\, \alpha_{0+},\, \beta_{0+}) = n_e\,(r,\, \lambda;\, \alpha_e,\, \beta_e) \tag{33}$$

Hence using equations (32) and (33) we can calculate β_e, the reduced second electric potential energy, for the electrons.

In order to maintain the quasi-neutrality, Φ_1 and β_e must be slowly varying functions of the radial distance r.

In fig. 1 we have plotted the value $\beta_e(r)$ as a function of altitude at a geomagnetic latitude of 90°, for an $(O^+ - e)$ - exosphere and for $r_o = (6371 + 2000)$km, $T_e = T_{O+} = 3000°K$. Two models have been considered : The "untrapped" moddel (UT) for which $\zeta = \xi = o$ and the "trapped" model (T) for which $\zeta = 0$, $\xi = 1$.

As β_e is approximately a linear function of r (up to 10.000 km) it can be seen from equation (15) that $\Phi_1(r)$ is also a quasi-linear function of the altitude. Therefore the electric field in an $(O^+ - e)$ - exosphere, given by equation (10), or by

$$\vec{E}(\vec{r}) = \frac{m\, g(r)}{Ze} \left(\beta \frac{\vec{r}}{r} - r \vec{\nabla} \beta\right) \qquad (34)$$

is practically equal to $(m_e - m_{O+})\, \vec{g}/2e$. At higher altitudes however the ratio $eE(r)/(m_e - m_{O+})\, g(r)$ is larger than the Pannekoek-Rosseland's value 0.5, and tends to a constant.

It can also be verified that $d^2\Phi_1/dr^2$ is extremely small so that equation (9) and (31) are consistent to a high degree of accuracy.

7. THE MEAN EXPANSION VELOCITY

The mean velocity w of the particles is parallel to the magnetic field lines and is defined by the relation

$$w(r,\lambda) = F(r,\lambda)/n(r,\lambda). \qquad (35)$$

Moreover the mean mass motion in the exosphere which is given by

$$u(r,\lambda) = \Sigma_k\, n_k\, m_k\, w_k / \Sigma_k\, n_k\, m_k,$$

is equal to zero for $\zeta = 1$. In the trapped and untrapped models, w and u are increasing functions of r. The mean velocities w and u vanish in a barometric model exosphere.

8. PRESSURE TENSORS AND TEMPERATURES

The longitudinal and transverse momentum flux tensors $P_{\parallel}$ and $P_{\perp}$ are respectively defined by

$$\left.\begin{aligned} P_{\parallel} &= m\int v_{\parallel}^2 \, f(\vec{v},\vec{r}) \, d^3v \\ P_{\perp} &= \frac{1}{2} m\int v_{\perp}^2 \, f(\vec{v},\vec{r}) \, d^3v \end{aligned}\right\} \qquad (36)$$

and in the case (a), when $1 + \alpha > (1 + \beta)y > 0$, one finds,

$$P_{\parallel}(r,\lambda) = \frac{2}{3} p(r_o,\lambda_o)\left\{(1 + \zeta).K_4(\infty)+(1 - \zeta). K_4(V_\infty) - [(1 - \zeta).K_4(\infty) + (1 + \zeta - 2\xi).K_4(X_\infty)].(1 - \eta)^{3/2} \exp\left(-\frac{\eta q}{1-\eta}\right)\right\} \exp(-q), \qquad (37)$$

$$P_{\perp}(r,\lambda) = \frac{2}{3} p(r_o,\lambda_o).\left\{(1 + \zeta).K_4(\infty)+(1 - \zeta).K_4(V_\infty)-(1 + \frac{\eta}{2}) (1 - \eta)^{\frac{1}{2}}\right.$$
$$\left.[(1 - \zeta).K_4(\infty)+(1 + \zeta - 2\xi). K_4(X_\infty)]. \exp\left(-\frac{\eta q}{1-\eta}\right) - \frac{3}{2} \eta q (1 - \eta)^{-\frac{1}{2}}\right.$$
$$\left.[(1 - \zeta). K_2(\infty)+(1 + \zeta - 2\xi). K_2(X_\infty)] \exp\left(-\frac{\eta q}{1-\eta}\right)\right\} \exp(-q), \qquad (38)$$

where $p(r_o,\lambda_o) = n(r_o,\lambda_o)\, kT(r_o,\lambda_o)$.

If on the contrary, $1 + \alpha < (1 + \beta)\, y < 0$, we obtain

$$P_{\parallel}(r,\lambda) = \frac{2}{3} p(r_o,\lambda_o)\left\{(1 + \zeta). K_4(\infty)-(1 - \zeta).K_4[(-q)^{\frac{1}{2}}]\right.$$
$$\left. - (1 - \zeta)(1 - \eta)^{3/2}\left[K_4(\infty) - K_4\left[\left(\frac{-q}{1-\eta}\right)^{\frac{1}{2}}\right]\right] \exp\left(-\frac{\eta q}{1-\eta}\right)\right\} \exp(-q), \qquad (39)$$

$$P_{\perp}(r,\lambda)=\frac{2}{3}\,p(r_o,\lambda_o)\,\Big\{(1+\zeta)\,K_4(\infty)-(1-\zeta)K_4\,[(-q)^{\frac{1}{2}}]$$

$$-(1-\zeta)\,(1+\frac{\eta}{2})\,(1-\eta)^{\frac{1}{2}}\Big[K_4(\infty)-K_4[(\frac{-q}{1-\eta})^{\frac{1}{2}}]\Big]\exp\,(-\frac{\eta q}{1-\eta})$$

$$-\frac{3}{2}\,\eta\,q\,(1-\zeta)(1-\eta)^{-\frac{1}{2}}\Big[K_2(\infty)-K_2[(\frac{-q}{1-\eta})^{\frac{1}{2}}]\Big]\exp\,(\frac{-\eta q}{1-\eta})\Big\}\exp\,(-q). \quad (40)$$

Using the results (37) to (40) the longitudinal and transverse pressures can be calculated by means of

$$p_{\parallel} = P_{\parallel} - n\,m\,w^2, \quad (41)$$

$$p_{\perp} = P_{\perp}. \quad (42)$$

The longitudinal and transverse temperatures are defined by

$$k\,T_{\parallel} = \frac{p_{\parallel}}{n},\quad k\,T_{\perp} = \frac{p_{\perp}}{n} \quad (43)$$

and may be calculated from (28), (37), (38), (39), (40), (41) and (42).

9. DISCUSSION

In all our formulas we have not yet specified the function $\eta(r,\lambda)$ which according to (13) is the ratio of the magnetic field strength taken at two different points of a field line. As the dipole configuration is a fairly good approximation up to a radial distance of about four earth's radii [11-12] we have used this geometry to determine $\eta(r,\lambda)$ in the region between 2000 km and 18000 km. Hence

$$\eta(r,\lambda)= y^3\frac{(4-3\cdot\cos^2\lambda)^{\frac{1}{2}}}{(4-3\,\cos^2\lambda_o)^{\frac{1}{2}}}. \quad (44)$$

To obtain the density and mean velocity along the radial direction corresponding to a geomagnetic latitude λ,λ_o has to be defined by $\lambda_o = \arccos\,(y^{\frac{1}{2}}\cos\,\lambda)$.

In Fig. 2 we have plotted the relative radial density distributions in an (O^+ - e)- exosphere at a geomagnetic latitude of 90°. At the baropause which is situated at an altitude of 2000 km, we have assumed that the ion and electron temperatures are both equal to 3000°K. Moreover we considered three different models :

- the untrapped model, equation (28) where $\zeta = \xi = o$, (UT-model)
- the trapped model, equation (28) where $\zeta = o$ and $\xi = 1$ (T-model)
- the barometric model, equation (28) where $\zeta = \xi = 1$. (B-model)

We have also considered an exosphere build-up of electrons and oxygen and hydrogen ions (O^+ - H^+ - e). In this case we calculated numericaly the density and mean velocity for each constituent along a given field line, i.e. for $\lambda = \arccos (y^{-\frac{1}{2}} \cos \lambda_o)$. Note that if λ_o the geomagnetic latitude at the baropause is smaller than the lower limit, λ_{min} [7], ($\lambda_{min} \approx 60°$), the field lines of the magnetosphere are closed and all the particles are trapped. In this case one must use the model ion-exosphere proposed by Eviatar et al. [1] or by Hartle [2].

In Fig. 3 we have plotted the density distributions along a line of force crossing the baropause at the geomagnetic latitude $\lambda_o = 80°$. Furthermore, considering the untrapped and trapped models we made the calculations for the following concentrations : $n_e(r_o,\lambda_o) = 10^3 cm^{-3}$, $n_{O^+}(r_o,\lambda_o) = 9\ 10^2 cm^{-3}$, $n_{H^+}(r_o,\lambda_o) = 10^2 cm^{-3}$, with the temperatures $T_e(r_o,\lambda_o) = T_{O^+}(r_o,\lambda_o) = T_{H^+}(r_o,\lambda_o) =$ 3000° K. From Table II, we immediately find $\alpha_e = 7.991\ 10^3$; $\alpha_{O^+} = - 0.272$; $\alpha_{H^+} = - 4.352$.

To obtain at each level r the appropriate value of β_e such that the quasi-neutrality condition is satisfied, we calculated the solutions of equation (31) by means of an iterative process. The density distributions are respectively given by (28) for the electrons and oxygen ions, and by (30) for the protons.

It can be seen that the O^+ concentration decreases much more rapidly when light ions are present in the exosphere. Note that the trapped particles do not contribute much to increase the O^+ density below an altitude of 6000 km.

For the barospheric conditions we used in this paper, the O^+ ions remain the most abundant constituant up to an altitude of 6300 km.

At large distances the electronic density decreases as r^{-3} due to the dipolar geometry of the magnetic field.

Finally it is worthwile to mention that for boundary conditions at the baropause which are independent of latitude, the density distributions along an open field line do not differ significantly with the choice of the field line ($75° < \lambda_o < 90°$) ; e.g. n_{H+} (17000 km, 75°) is only 4% smaller than n_{H+} (17000 km, 90°).

Fig. 4 shows the mean velocities of O^+, H^+ and e under the same conditions as for Fig. 3. It can be seen that w_{O+} remains quite small (< 20 cm sec^{-1}) comparatively to the mean velocity of the protons which are accelerated by the electric field to supersonic velocities reaching 20 km sec^{-1} at large distances. At an altitude of 3000 km, w_{H+} is equal to 11 km sec^{-1} in both of our"untrapped and trapped models". These results are in good agreement with the 10 km sec^{-1} value reported by Dessler and Cloutier[13].

In both models the fluxes at the baropause are $F_e = 2.0 \times 10^7$ cm^{-2} sec^{-1}, $F_{O+} = 2.4 \times 10^{-1} cm^{-2} sec^{-1}$ and $F_{H+} = 2.0 \times 10^7$ $cm^{-2} sec^{-1}$.

Fig. 5 gives the altitude dependence of β_e for the three components ion-exosphere. Comparison with Fig. 1 shows that an amount of only 10% hydrogen ions at the baropause level reduces by a large factor the value of β_e and the electric potential Φ_1.

In Fig. 6 we compare the intensity of the electric field in two "untrapped models" along the field line, $\lambda_o = 90°$, firstly in the case of a pure O^+ exosphere and secondly for an ($O^+ - H^+ - e$) atmosphere. It can be seen that in the first case the ratio of the electric (eE) and gravitational ($-g\, m_{O+}$) force is, up to 18.000 km, practically equal to 1/2 as in Pannekoek's theory. However a small percentage of light ions at the baropause reduces $|eE/g\, m_{O+}|$ to $m_{H^+}/2m_{O+} = 0{,}03125$ at the high altitudes where H^+ becomes the most abundant constituant. Therefore at large distances, eE, becomes equal to $-\frac{1}{2}\, g\, m_{H^+}$.

CONCLUSION :

The model ion-exosphere which we have proposed in this paper can be used for any magnetic field with open field lines. The electric potential and the corresponding radial electric field in this region have been obtained by requiring quasi-neutrality and equality between the electron and positive ion fluxes i.e. the zero electric current condition. The exospheric potential calculated is different from that of Pannekoek [3] and Rosseland [4]. Using a magnetic dipole field, we have made numerical calculations of the density distributions and mean velocities of the different constituents. The mean expansion velocity of the thermal protons is in our models of the order of 11 km sec^{-1} at an altitude of 3000 km.

Moreover by putting Z = o (leading to $\alpha = \beta = o$) and $\eta(r,\lambda) = y^2$ we recover the well-known neutral exospheric models. Indeed with the above assumption our "untrapped" model with $\zeta = \xi = o$ reduces to the Öpik-Singer [10,14] model where the satellite (or trapped) particles were neglected, and the "trapped" model, where $\zeta = o$ and $\xi = 1$, yields Chamberlain's model [15] where these latter particles contribute to the density in the exosphere.

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APPENDIX A : Polarization electric field and excess of charge concentration.

In an isothermal ionized atmosphere which is in hydrostatic and diffusion equilibrium the density distributions satisfy the equation

$$k T_j \vec{\nabla} n_j = m_j n_j \vec{g} + Z_j n_j e \vec{E}. \qquad (A\ 1)$$

The electrostatic field, $\vec{E}$, preventing a significant charge separation is given by

$$e \vec{E} = - \mu(r) \vec{g}, \qquad (A\ 2)$$

where

$$\mu(r) = (\Sigma_j Z_j m_J n_j/kT_j)/(\Sigma_j Z_j^2 n_j/kT_j). \qquad (A\ 3)$$

Applying the differential operator $\vec{\nabla}$ to (A.2) and using Poisson's equations (2) and (4) we obtain :

$$4\pi e^2 \Sigma_j Z_j n_j = 4\pi G \mu(r) \Sigma_k m_k n_k - \vec{g}.\vec{\nabla}\left(\frac{\Sigma_j Z_j m_j n_j/kT_j}{\Sigma_j Z_j^2 n_j/kT_j}\right) \qquad (A\ 4)$$

Taking into account (A.1) and (A.2) we can calculate the second term in the right hand side of (A.5). Finally the electric charge density is given by :

$$\Sigma_j Z_j n_j = \frac{G}{e^2} \mu(r) \Sigma_j m_j n_j - \frac{g^2}{4\pi e^2} \sum_j \frac{Z_j n_j}{(kT_j)^2} \cdot \frac{(m_j - Z_j \mu)^2}{\Sigma_i Z_i^2 n_i/kT_i}. \qquad (A\ 5)$$

In the case of a pure hydrogen isothermal atmosphere and with the assumption that $T_{H^+} = T_e$ = constant, the charge excess due to gravitational effects is very small

$$\frac{n_{H^+} - n_e}{n_e} \simeq \frac{G m_{H^+}^2}{2 e^2} \simeq 4\ 10^{-37}$$

In a pure O^+ isothermal atmosphere it would be 256, e.g. $(m_{O^+}/m_{H^+})^2$, times larger but still remain minute. It can be shown that the addition of a third kind of ions in diffusion equilibrium does not change the order of

magnitude of this result and that quasi-neutrality in the barospheric region where equations (A.1) are valid is always satisfied, i.e.

$$\Sigma_k Z_k n_k \simeq 0. \qquad (A.6)$$

In the exosphere where the equation (A1, A2, A3) are not applicable, the charge excess is no longer given by eq. (A4). In this case (9) has to be used. From our models exospheres where $\Phi_1(r)$ has been calculated we have evaluated the excess of charge concentration. It comes out that $\Sigma_k Z_k n_k/n_e$ is negative and smaller than $- 10^{-12}$ except for a small transition layer close to the baropause. Hence, the quasi-neutrality condition (A6) is also satisfied in the exosphere to a very high degree of accuracy.

APPENDIX B : Km (b) - function.

The functions $K_m(b)$ are defined by

$$K_m(b) = \frac{2}{\pi^{\frac{1}{2}}} \int_0^b x^m e^{x^2} \, dx. \qquad \text{(B 1)}$$

Partial integration yields immediately the recurrence formula

$$K_m(b) = \frac{m-1}{2} K_{m-2}(b) - \frac{b^{m-1}}{\pi^{\frac{1}{2}}} \exp(-b^2), \qquad \text{(B 2)}$$

which enables calculation of $K_m(b)$ in terms of the well-known error function, and in terms of exponential functions.

Indeed, straightforward calculations yield

$$K_o(b) = \text{Erf}(b), \qquad \text{(B 3)}$$

$$K_1(b) = \pi^{-\frac{1}{2}}[1 - \exp(-b^2)].$$

Note also that $K_m(\infty) = 1.3.5...(m-1)/2^{m/2}$ if m is even and $K_m(\infty) = 2.4.6...(m-1)/(2^{(m-1)/2}\pi^{\frac{1}{2}})$ if m is odd.

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TABLE I_a.- Classes of particles which have a decreasing velocity with altitude $(1 + \alpha > (1 + \beta)\, y > 0)$*

class	$v(r_o)$	$\theta(r_o)$	$v(r)$	$\theta(r)$	Properties.
a1	$[v_\infty(r_o),\infty]$ (a)	$[o,\frac{\pi}{2}]$	$[v_\infty(r),\infty]$ (e)	$[o,\theta_m(r)]$ (g)	escaping particles.
a2	$[v_y(r_o),v_\infty(r_o)]$ (b)	$[o,\pi]$	$[v_y(r),v_\infty(r)]$ (f)	$[o,\theta_m(r)]\,[\pi-\theta_m(r),\pi]$	ballistic particles reaching level r with $\theta < \theta(r_o)$
a3	$[v_x(r_o),v_y(r_o)]$ (c)	$[o,\theta_m(r_o)]$ (d) $[\pi-\theta_m(r_o),\pi]$	$[o,v_y(r)]$	$[o,\pi]$	ballistic particles reaching level r with $\theta > \theta(r_o)$
a4	$[o,v_x(r_o)]$	$[o,\pi]$	—	—	ballistic particles not reaching level r.
a5	$[v_x(r_o),v_y(r_o)]$	$[\theta_m(r_o),\pi-\theta_m(r_o)]$	—	—	
a6	$[v_\infty(r_o),\infty]$	$[\frac{\pi}{2},\pi]$	$[v_\infty(r),\infty]$	$[\pi-\theta_m(r),\pi]$	incoming particles reaching the baropause.
a7	—	—	$[v_\infty(r),\infty]$	$[\theta_m(r),\pi-\theta_m(r)]$	incoming particles never reaching the baropause.
a8	—	—	$[v_y(r),v_\infty(r)]$	$[\theta_m(r),\pi-\theta_m(r)]$	trapped particles.

(a) $v_\infty^2(r_o) = -2(1+\alpha)\Phi_g(r_o)$; (b) $v_y^2(r_o) = \frac{R}{1-\eta}$; (c) $v_x^2(r_o) = R$; (d) $\sin^2\theta_m(r_o) = \frac{v^2(r_o)-R}{\eta_y v^2(r_o)}$; (e) $v_\infty^2(r) = -2(1+\beta)\, y\, \Phi_g(r_o)$;

(f) $v_y^2(r) = \frac{R\eta}{1-\eta}$; (g) $\sin^2\theta_m(r) = \eta\,\frac{v^2(r)+R}{v^2(r)}$; $R = -2\,[1+\alpha-(1+\beta)y]\,\Phi_g(r_o)$

* $\beta \geq (1+\alpha)\,\frac{\eta}{y} - 1$ for $v_y(r) \leq v_\infty(r)$.

Table I_b.- Classes of particles for which the velocity is an increasing function of altitude $(1 + \alpha < (1 + \beta)\ y < 0)$.

class	$v(r_\circ)$	$\theta(r_\circ)$	$v(r)$	$\theta(r)$	Properties.
b1	$[o,\infty]$	$[o,\frac{\pi}{2}]$	$[v_z(r),\infty]$ (a)	$[o,\theta_m(r)]$ (b)	escaping particles.
b2	$[o,\infty]$	$[\frac{\pi}{2},\pi]$	$[v_z(r),\infty]$	$[\pi-\theta_m(r),\pi]$	incoming particles reaching the baropause.
b3	—	—	$[v_z(r),\infty]$	$[\theta_m(r),\pi-\theta_m(r)]$	incoming particles never reaching the baropause.
b4	—	—	$[o,v_z(r)]$	$[o,\pi]$	

(a) $v_z^2(r) = -R$; (b) $\sin^2\theta_m(r) = \eta \frac{v^2(r) + R}{v^2(r)}$; $R = -2\left[1 + \alpha - (1 + \beta)\ y\right] \Phi_g(r_\circ)$

TABLE II.- Values of $(1 + \alpha_j)$ in a $(O^+ - H^+ - e)$ exosphere, as a function of the ratio $n_{H^+}(r_o,\lambda_o)/n_e(r_o,\lambda_o)$; Baropause altitude is 2000 km and $T_e(r_o,\lambda_o) = T_{O^+}(r_o,\lambda_o) = 3000°K$.

$n_{H^+}(r_o,\lambda_o)/n_e(r_o,\lambda_o)$	$1 + \alpha_e$	$1 + \alpha_{O^+}$	$1 + \alpha_{H^+}$	$n_{O^+}(r_o,\lambda_o)/n_e(r_o,\lambda_o)$
0.000	$1.733\ 10^4$	0.410	- 8.440	1.000
0.001	$1.285\ 10^4$	0.563	- 5.998	0.999
0.010	$1.044\ 10^4$	0.644	- 4.688	0.990
0.100	$7.992\ 10^3$	0.728	- 3.352	0.900
0.500	$6.229\ 10^3$	0.788	- 2.392	0.500
1.000	$5.450\ 10^3$	0.814	- 1.968	0.000

TABLE III.- Values of $(1 + \alpha_j)$ as a function of the exospheric temperatures at the baropause for $n_{H^+}(r_o,\lambda_o)/n_e(r_o,\lambda_o) = 0.10$ and $r_o = 8371$ km.

$T_e(r_o,\lambda_o)$	$T_{O^+}(r_o,\lambda_o) = T_{H^+}(r_o,\lambda_o)$	$1 + \alpha_e$	$1 + \alpha_{O^+}$	$1 + \alpha_{H^+}$
1000°K	1000°K	$2.663\ 10^3$	0.909	- 0.450
2000°K	2000°K	$5.327\ 10^3$	0.818	- 1.901
3000°K	3000°K	$7.991\ 10^3$	0.728	- 3.352
4000°K	4000°K	$1.065\ 10^4$	0.637	- 4.803
5000°K	5000°K	$1.332\ 10^4$	0.546	- 6.253
3000°K	2000°K	$8.210\ 10^4$	0.720	- 3.471

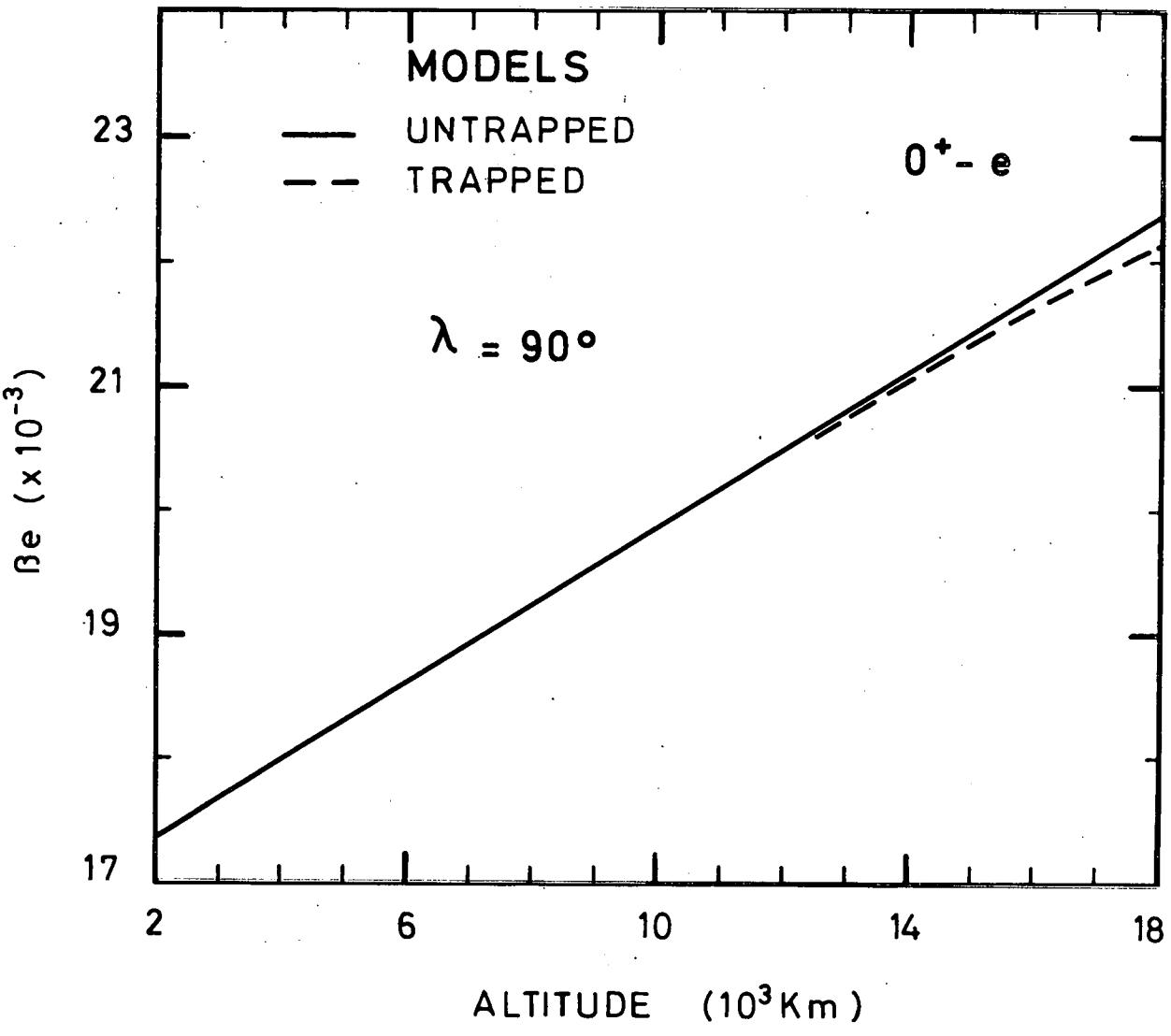

Fig.1.- Values of the second reduced electrical potential β^e in an(0^+- e) exosphere versus altitude above the geomagnetic pole ($\lambda = 90°$) for the "trapped" and "untrapped" models. The altitude of the baropause is 2000 km; $T_e(r_o,\lambda_o) = T_{O+}(r_o,\lambda_o) = 3000°K$; $\alpha_e = 1.733 \times 10^4$ (cf. Table II); $\beta_{O+} = - \beta_e \, m_e/m_{O+}$.

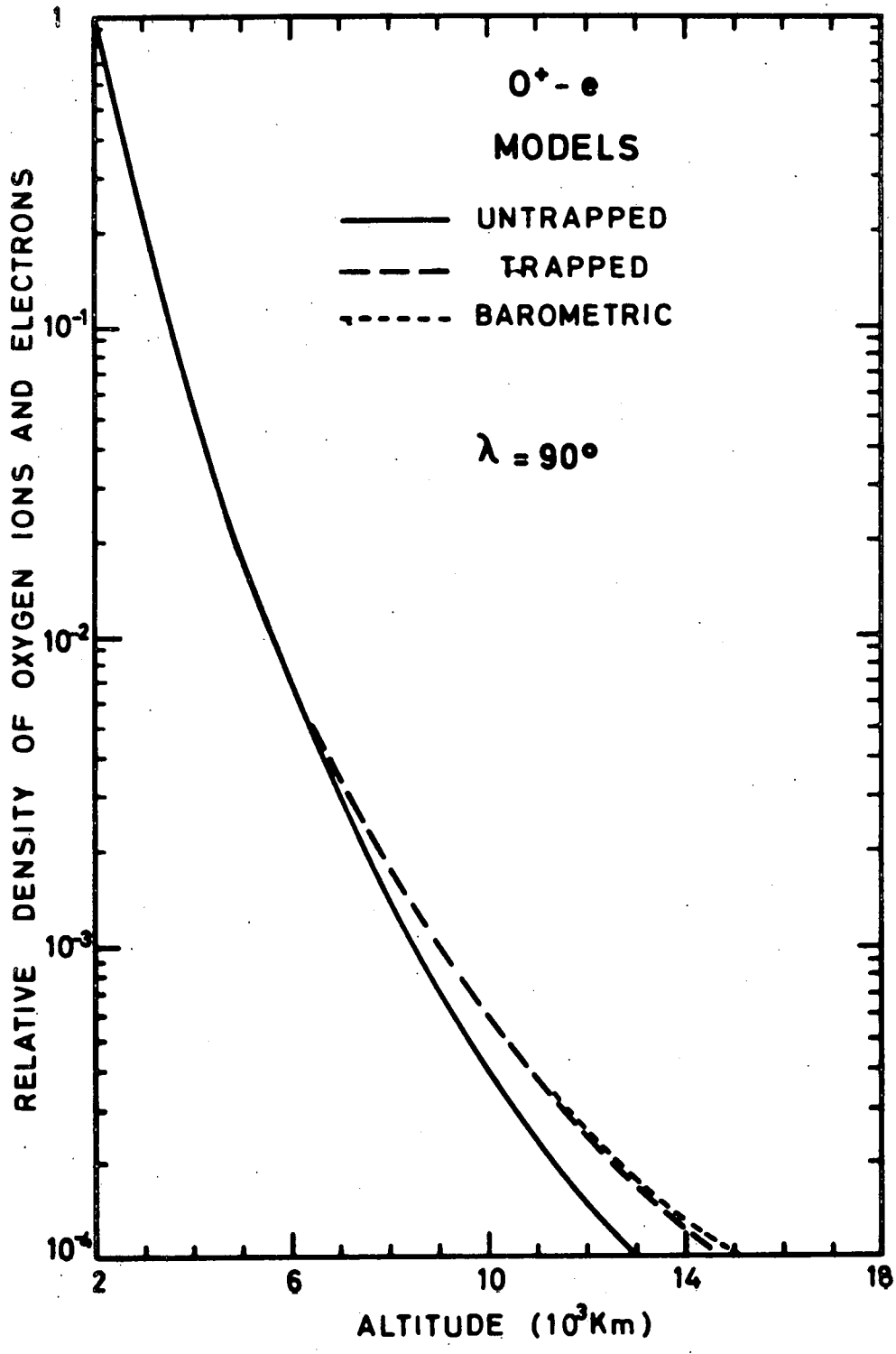

Fig.2.- Density ratio $n_{O+}(r,\lambda)/n_{O+}(r_o,\lambda_o)$ versus altitude along a magnetic field line crossing the baropause at 2000 km and at 90° latitude ($\lambda = \lambda_o = 90°$) for the "trapped", "untrapped" and barometric models. The baropause temperatures are $T_{O+} = T_e = 3000°K$.

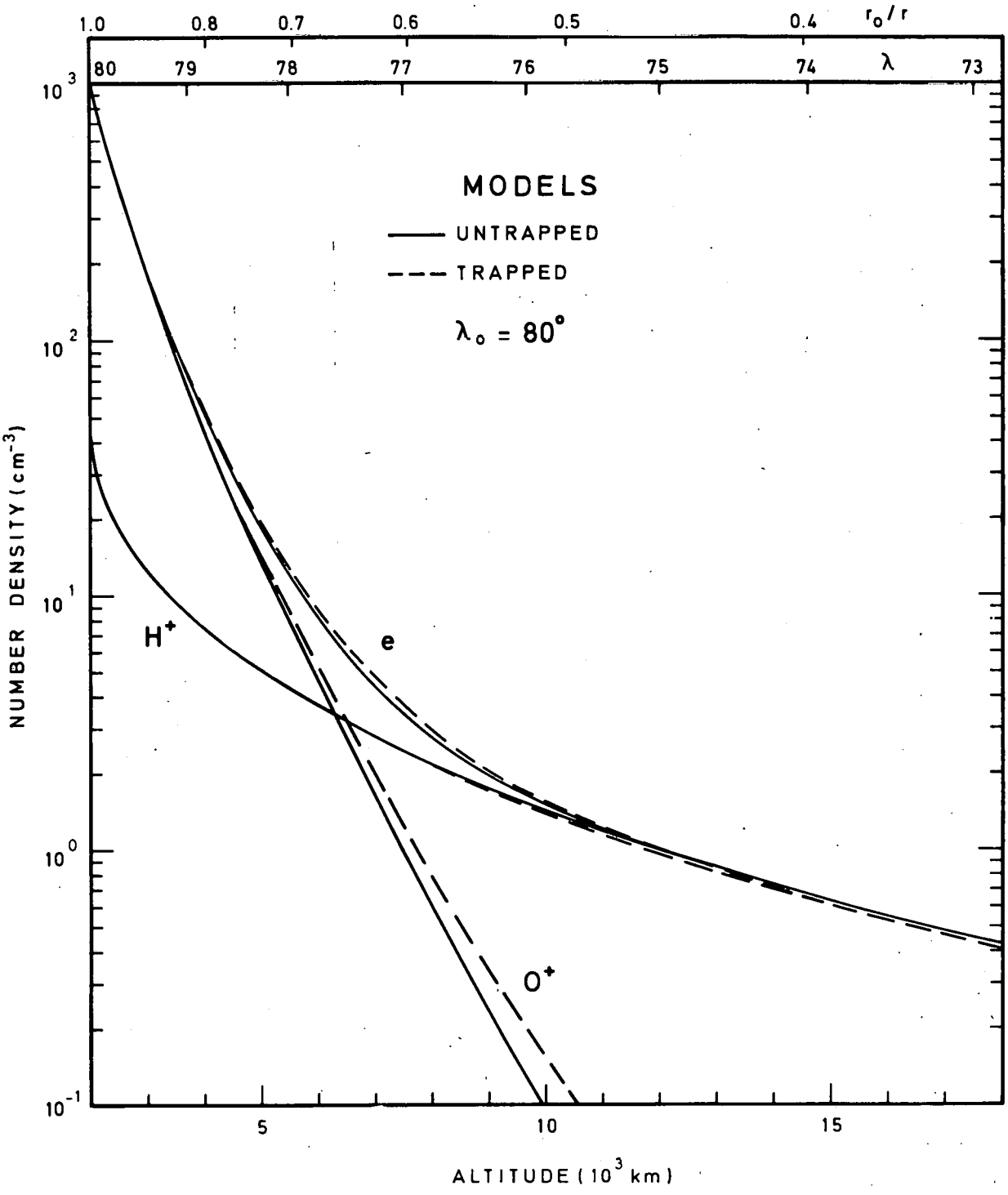

Fig.3.- Density distributions in an (O^+ - H - e)- exosphere versus altitude along a dipole magnetic field line crossing the baropause at 80° latitude. The solid and dashed lines correspond to the "untrapped" and "trapped" models respectively. The baropause temperatures and concentrations are : $T_e = T_{O+} = T_{H+} = 3000°K$; n_e: n_{O+}: n_{H+}= 10:9:1.

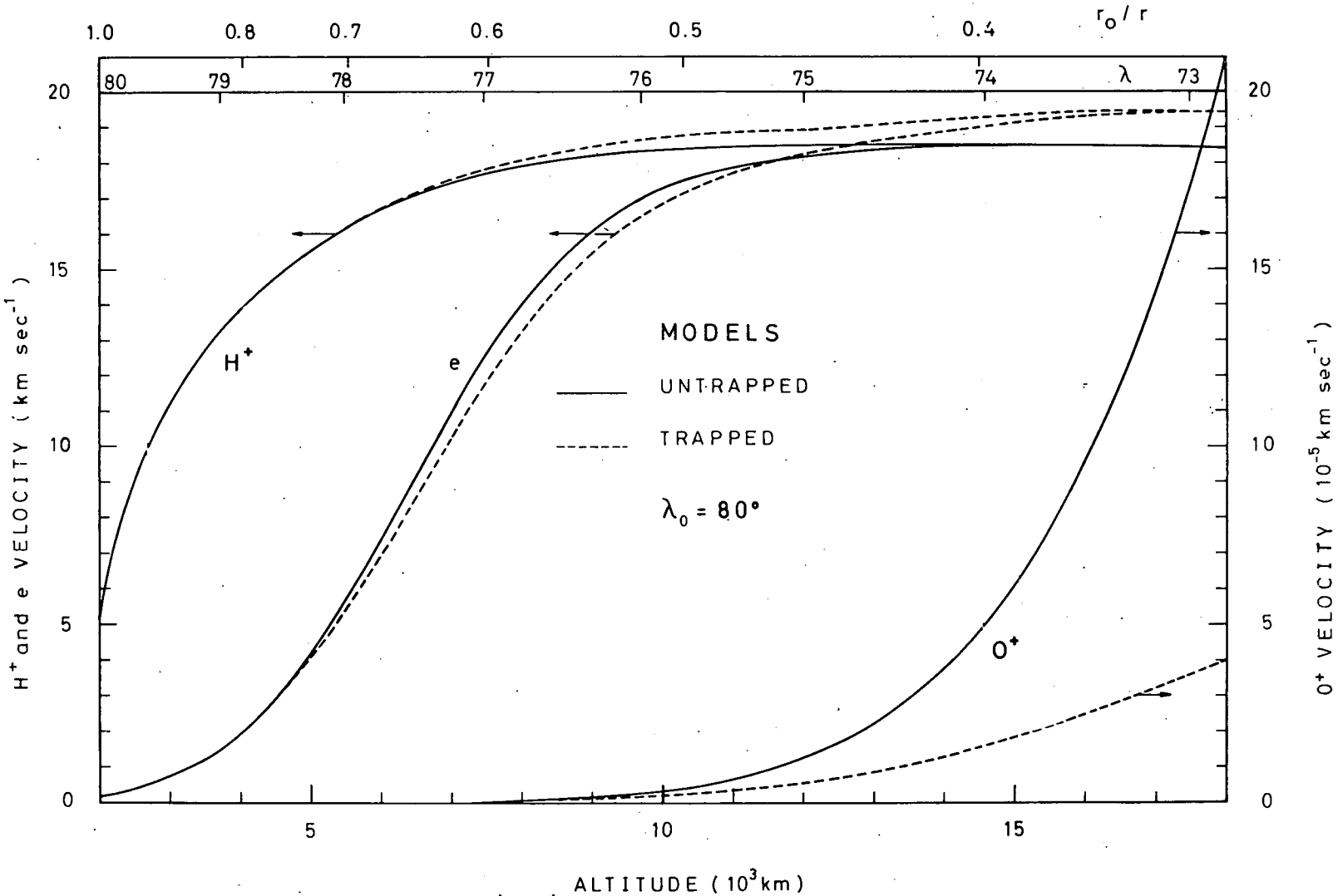

Fig.4.- Mean velocities in an (O^+ - H^+ - e)- exosphere versus altitude, along a dipole magnetic field line crossing the baropause at 80° latitude. The solid and dashed lines correspond to the "untrapped" and "trapped" models respectively. In both cases, the baropause temperatures and concentrations are: $T_e = T_{O+} = T_{H+} = 3000°K$; $n_e : n_{O+} : n_{H+} = 10 : 9 : 1$.

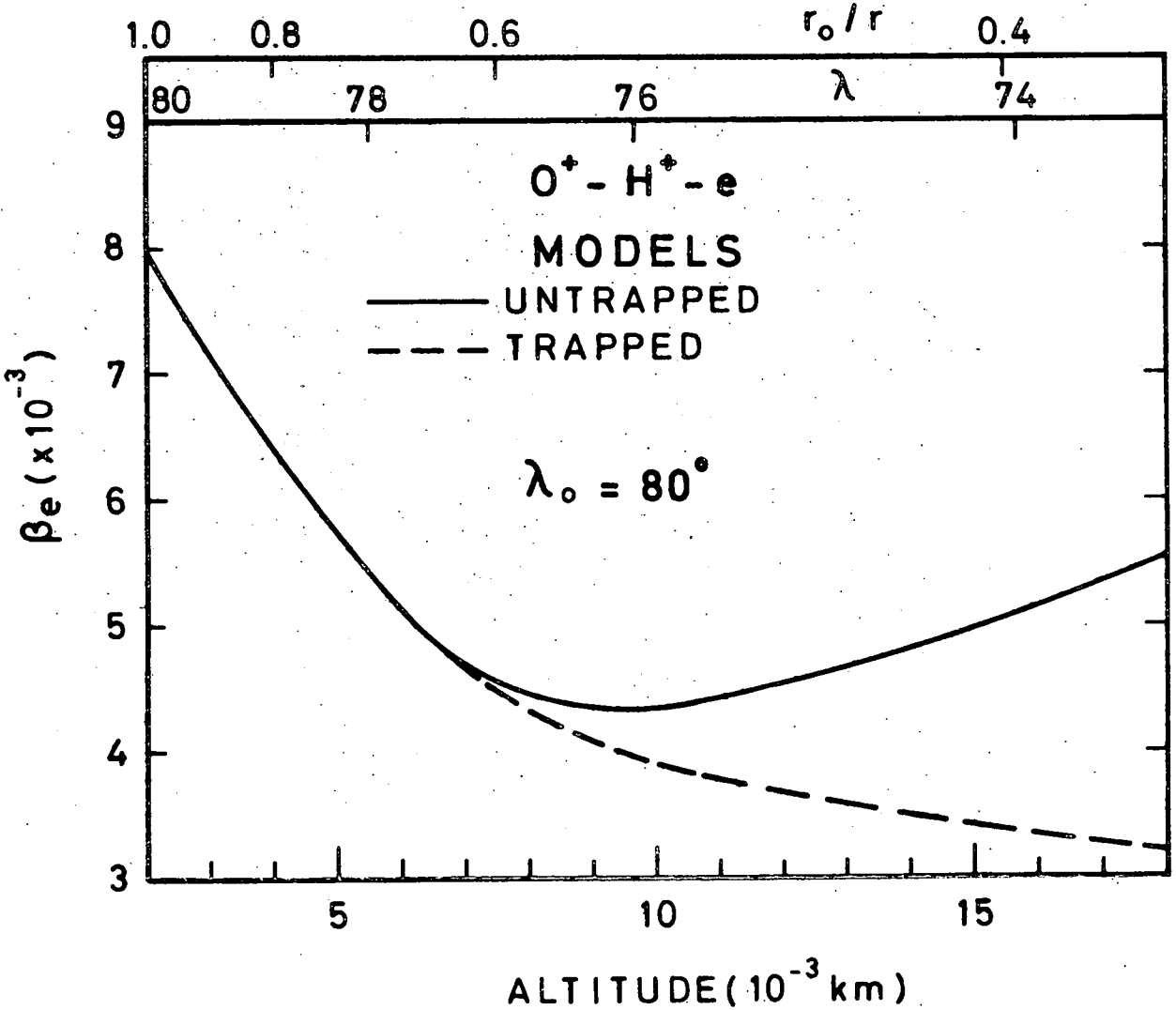

Fig.5.- Altitude distributions of β_e in an ($O^+ - H^+ - e$)- exosphere, for the "trapped" and "untrapped" models under the same conditions as in Fig. 3 and 4.

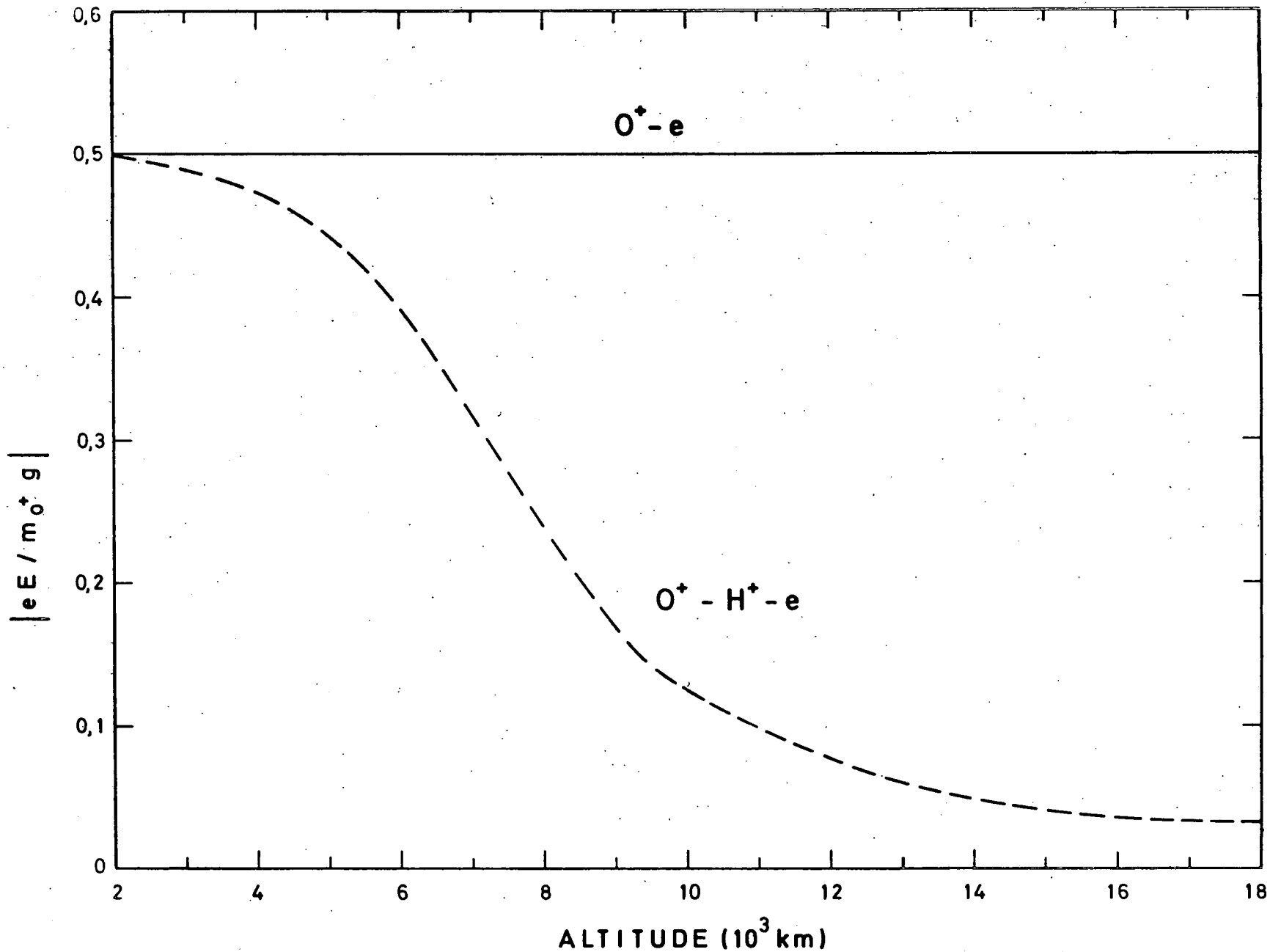

Fig.6.- Ratio of the electric force (eE) and gravitational force (g m_{O^+}) acting upon an O^+ ion versus altitude above the geomagnetic pole ($\lambda = \lambda_o = 90°$) for two "untrapped" models : ($O^+ - e$) and $O^+ - H^+ - e$) for the same temperature and concentration conditions as in Fig. 1 - 5.