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FOREWORD

This paper will be published in The Physics of Fluids.

AVANT-PROPOS

Ce travail sera publié dans The Physics of Fluids.

VOORWOORD

Deze tekst zal verschijnen in The Physics of Fluids.

VORWORT

Dieser Text wird in The Physics of Fluids herausgegeben werden.

SIMPLE MODEL FOR AN ION-EXOSPHERE IN AN OPEN MAGNETIC FIELD

by

J. LEMAIRE and M. SCHERER

Summary

A simple model for an ion-exosphere with an open magnetic field is set up. The ions move under the influence of (1) the gravitational field, (2) the monotonic decreasing static magnetic field, and (3) the electrostatic potential due to a small charge separation. Neglecting collisions and particle drift across the magnetic field lines the particles can be classified into four classes : ballistic, escaping, trapped, and incoming particles. For each class the number density, the particle flux, the momentum fluxes, and the energy flux are calculated as a function of the electrostatic potential. Finally, it is shown how this potential can easily be computed by considering two basic physical conditions : (1) the quasineutrality, and (2) the zero current condition.

Résumé

On décrit un modèle cinétique d'exosphère ionique dans lequel les ions et les électrons peuvent s'échapper le long des lignes de force ouvertes d'un champ magnétique dont l'intensité tend asymptotiquement vers une constante à l'infini. En négligent les collisions ioniques et la faible dérive des particules dans la direction perpendiculaire au champ magnétique on peut distinguer quatre classes de particules : celles qui sont piégées, celles qui s'échappent, celles qui sont précipitées et finalement les particules dont la trajectoire est du type ballistique. On donne pour chacune de ces classes la densité, le flux de particules, le flux d'impulsion et d'énergie en fonction du potentiel électrique. Finalement on montre comment ce potentiel électrique peut-être déterminé en fonction de l'altitude, à partir des relations exprimant que le plasma est électriquement neutre et que le courant électrique y est nul.

Samenvatting

Een eenvoudig model voor een ionaire exosfeer in een open magneetveld wordt beschouwd. De beweging der ionen is bepaald door het zwaarteveld, door het monotoon afnemend statisch magneetveld, en door het elektrische veld welke te wijten is aan een kleine scheiding der ladingen. Botsingen en de driftbeweging loodrecht op de veldlijnen van het magneetveld worden verwaarloosd. De deeltjes worden ingedeeld in vier klassen : ballistische, ontsnappende, gevangen en inkomende deeltjes. Voor elke klasse wordt de deeltjes dichtheid, de deeltjes flux, de momenten flux, en de energy flux berekend in functie van de electrostatische potentiaal. Tenslotte wordt getoond hoe deze potentiaal gemakkelijk berekend kan worden door gebruik te maken van twee basis voorwaarden : (1) de quasineutraliteit van het plasma, en (2) de gelijkheid van de flux der negatieve geladen deeltjes aan de flux der positieve geladen deeltjes.

Zusammenfassung

Ein kinetisches Model einer Ionen-Exosphäre worin die Ionen und Elektronen längst der offenen Linien des magnetischen Feldes entfliehen können ist beschrieben. Wenn man die Stösse zwischen den Ionen vernachlässigt können vier verschiedene Teilchenklassen unterscheidet werden : die jenigen die entfliehen, die jenigen die herabschleudern, die jenigen die ins magnetische Feld eingefässelt sind, und die jenigen mit ballistischen Flugbahnen. Für jede dieser Klassen wird die Teilchendichte, der Teilchenfluss der Impuls- und Energiefluss als Funktion des elektrischen Potentiales gegeben. Es wird beschrieben wie man dieses elektrische Potential berechnen kann als Funktion der Höhe.

I. INTRODUCTION

Eviatar, Lenchek, and Singer[1] have defined a simple model for an ion-exosphere of a nonrotating planet. They considered the effects of the superimposed gravitational and static, centered-dipole magnetic field upon the ions and electrons. Moreover, they assumed that the barosphere or collision dominated region in which the charged particles are in hydrostatic equilibrium, is separated from the exosphere by an imaginary spherical surface. On this so-called exobase or baropause the density and temperature are constant. In the upper region or exosphere the collision frequency is so small that collisions can be ignored. Assuming that all charged particles which build up the ion-exosphere, have emerged from the barosphere, Eviatar et al. set up an expression for the variation of the density with altitude.

Recently, this model ion-exosphere, has been generalized by Hartle[2], who considered a more general baropause,consisting of a surface, symmetric about the magnetic equator, over which the density and the temperature can vary. In addition to the species densities, Hartle calculated the particle current densities, the pressures and the temperatures.

Such simplified models are of great importance as they lead to a better understanding of the planetary exosphere where the actual physical problem is of immense complexity. Application of the above described models to the Earth, however, is restrained to exospheric regions with geomagnetic latitude less than 65°. Indeed, since a static dipole magnetic field was used in these model ion-exospheres, the plasma is confined in the closed dipole field regions. Within the polar regions of the Earth, the magnetic field lines do not form closed loops but are open to the magnetospheric tail which is connected with the interplanetary magnetic field. Hence, in the terrestrial exosphere, the charged particles, constrained to move along the magnetic field lines, can escape at higher latitudes. Recent experimental[(3-6)] and theoretical[(7-10)] studies have shown that the peculiar nature of the polar topside ionosphere has to be sought in the open character of the magnetosphere at high latitudes.

The purpose of this paper is to give expressions for the distributions of the number density, the particle flux, the pressure tensor components, and the energy flux in an ion-exosphere with open field lines. In order to simplify the problem we accept the existence of a sharply defined baropause which separates the collision free exosphere from the barosphere, where collisions are so frequent that the velocity distribution function of the charged particles is Maxwellian. Moreover, we assume that along an open magnetic field line, the total field strength is a monotonic decreasing function of distance, which tends to a constant value at infinity. Neglecting the particle drift across magnetic field lines we describe the motion of a charged particle in the exosphere by the well-known nonrelativistic guiding center approximation. In Sec. III it is shown that under these assumptions all particles of the open ion-exosphere can be classified into four classes :
(1) The ballistic particles which emerge from the barosphere, are reflected in the exosphere and cannot escape ; (2) the escaping particles, which leave the barosphere, have sufficient kinetic energy and a proper pitch angle to be lost in the interplanetary medium ; (3) the trapped particles are those with two mirror points in the exosphere ; and finally (4) the incoming particles which come from the interplanetary space and which are reflected in the exosphere or even can enter into the barosphere. The particle velocity distribution function in the exosphere is determined in Sec. IV and the most important moments of these distribution are calculated for each class of particles in Sec. V. Finally, we give

a convenient method to numerically compute the electrostatic potential, and illustrate it with an application to an $(O^+ - H^+ - e)$ exosphere.

II. MOTION OF A CHARGED PARTICLE

The electrostatic potential φ, due to a minute charge separation in the exosphere, satisfies Poisson's equation

$$\vec{\nabla}^2 \varphi = - 4\pi \Sigma_k Q_k n_k \tag{1}$$

where the summation is to be taken over all kinds of particles with density n_k and charge $Q_k = Z_k e$.

Since the righthand side of (1) is very small, we write the electrostatic potential as

$$\varphi(\vec{r}) = \varphi_o(\vec{r})\, r_o/r$$

where r_o is the radial distance of the baropause and $\varphi_o(\vec{r})$ is the solution of

$$\vec{\nabla}^2 \varphi_o - 2 \frac{\vec{r}}{r^2} . \vec{\nabla} \varphi_o = - \frac{4\pi r}{r_o} \Sigma_k Q_k n_k \tag{2}$$

such that $\varphi(\vec{r}) \rightarrow 0$ for $r \rightarrow \infty$

As we assume a static magnetic field in the exosphere, the velocity $v(\vec{r})$ of a particle with mass m and charge Q along a given magnetic field line is obtained from the law of conservation of energy

$$v^2(\vec{r}) + 2\Phi (1+\beta) y = \text{const}, \tag{3}$$

with $y = r_o/r$,

and where we introduced the shorthand notations

$$\beta = \frac{Z\,e}{m\,\Phi}\,\varphi_o(\vec{r}) \tag{4a}$$

for the reduced electric potential energy, and

$$\Phi = -\,MG/r_o \tag{4b}$$

for the gravitational potential at the baropause. In (4b) M denotes the mass of the planet where the atmospheric mass is neglected, and G is the gravitational constant. Moreover, following Eviatar et al[1] we assume that the nonmagnetic forces are weak compared with the magnetic force, so that Alfvén's guiding center approximation will be valid. Using the first adiabatic invariant we get

$$\frac{v^2(\vec{r})\,\sin^2\theta(\vec{r})}{B(\vec{r})} = \text{const}, \tag{5}$$

where $\theta(\vec{r})$ is the pitch angle of the particle, i.e., the angle between the magnetic field $\vec{B}(\vec{r})$ and the velocity vector of the particle. Equations (3) and (5) determine the trajectory of a charged particle in the exosphere.

III. CLASSES OF PARTICLES

In an open ion-exosphere the charged particles, moving along a magnetic field line, can be classified into four classes. First of all there are the particles emerging from the barosphere which do not have enough kinetic energy to escape or, which in compliance with the first adiabatic invariant, have a mirror point in the exosphere. They are the so called ballistic particles. The other group of particles coming from the barosphere are the escaping particles. They are lost in interplanetary space as we assume that the planetary magnetic field lines are interconnected with the interplanetary field. Another consequence of this assumption is that charged particles coming from the interplanetary medium can enter into the exosphere and under certain conditions even penetrate into the barosphere. These are the so called incoming particles. Finally, there exist trapped particles which have two mirror points in the ion-exosphere. They bounce contineously up and down along a magnetic field line. The different regions in phase space, corresponding to those four classes of particles can be determined by means of the basic equations (3) and (5). From (3) we deduce

$$v^2(\vec{r}_o) = v^2(\vec{r}) + R(\vec{r}) = v^2(\infty) - 2\Phi\ (1+\alpha) \tag{6}$$

with

$$R(\vec{r}) = -\ 2\ \Phi\left[1 + \alpha - (1+\beta)y\right] \tag{7}$$

$$\alpha = \frac{Z\ e}{m\ \Phi}\varphi_o(\vec{r}_o) \tag{8}$$

Expressing the first adiabatic invariant (5) at $\vec{r}_o$, $\vec{r}$ and infinity, we obtain, after eleminating the constant and taking into account the relation (6),

$$\begin{aligned}
\sin^2\theta(\vec{r}_o) &= \sin^2\ \theta_m(\vec{r}_o)\ \sin^2\ \theta(\vec{r}),\\
\sin^2\theta(\vec{r}_o) &= \sin^2\ \theta'_m(\vec{r}_o)\ \sin^2\theta(\infty),\\
\sin^2\theta(\vec{r}) &= \sin^2\ \theta_m(\vec{r})\ \sin^2\theta(\vec{r}_o)\\
\sin^2\theta(\vec{r}) &= \sin^2\ \theta'_m(\vec{r})\ \sin^2\theta(\infty)\\
\sin^2\theta(\infty) &= \sin^2\ \theta_m(\infty)\ \sin^2\theta(\vec{r}_o)\\
\sin^2\theta(\infty) &= \sin^2\ \theta'_m(\infty)\ \sin^2\theta(\vec{r})
\end{aligned} \tag{9}$$

where $\theta_m(\vec{r}_o)$, $\theta'_m(\vec{r}_o)$,..., $\theta'_m\ (\infty)$ are, respectively, defined by

$$\sin^2\ \theta_m(\vec{r}_o) = \eta^{-1}\left[1 - R(\vec{r})/v^2(\vec{r}_o)\right]\quad ,$$

$$\sin^2\ \theta'_m(\vec{r}_o) = a^{-1}\left[1 + 2\Phi\ \ (1+\alpha)/v^2(\vec{r}_o)\right]\quad ,$$

$$\sin^2\ \theta_m(\vec{r}) = \eta\left[1 + R(\vec{r})/v^2(\vec{r})\right]\ , \tag{10}$$

$$\sin^2 \theta'_m(\vec{r}) = \mu[1 + 2\Phi\ (1+\beta)y/v^2(\vec{r})\ ] \ ,$$

$$\sin^2 \theta_m(\infty) = a\left[1 - 2\Phi\ (1+\alpha)/v^2(\infty)\right] \ ,$$

$$\sin^2 \theta'_m(\infty) = \mu^{-1}\left[\ 1 - 2\Phi\ (1+\beta)y/v^2(\infty)\right] \ .$$

In the expressions (10) we used the shorthand notations

$$\begin{aligned} \eta &= B(\vec{r})/B(\vec{r}_o) \leq 1 \ , \\ \mu &= B(\vec{r})/B(\infty) \geq 1 \ , \\ a &= \eta/\mu \ = B(\infty)/B(\vec{r}_o) < 1 \ . \end{aligned} \tag{11}$$

Assuming that along a magnetic field line, the potential energy $\frac{1}{2}\, m\, R(\vec{r})$ is a monotonic function, we have to consider two cases corresponding to the algebraic sign of $R(\vec{r})$.

A. <u>Positive Potential Energy</u>

In this case there exists a potential barrier for a charged particle emerging from the barosphere. The potential energy which is zero at the baropause, reaches a maximum, $- m\,\Phi(1+\alpha)$ at infinity. It is easy to show that in this case $1 + \alpha \geq 0$ and $1 + \beta \geq 0$. Furthermore, we can define the minimal velocity that a charged particle, coming from the barosphere must have at the baropause, in order to reach the level $r \geq r_o$

$$v_a^{\ 2}\ (\vec{r}_o) = R(\vec{r}) \ .$$

As a consequence there exists an escaping velocity given by

$$v_\infty^{\ 2}(\vec{r}_o) = -\ 2\Phi(1+\alpha) \quad ,$$

at the baropause, and by

$$v_\infty^{\ 2}(\vec{r}) = -\ 2\Phi(1+\beta)y \quad ,$$

at the radial distance $r \geq r_o$.
A convenient way to determine the different classes of particles which may occur with a positive potential energy, is to make a detailed study of the six quantities $\theta_m(\vec{r}_o)$, $\theta'_m(\vec{r}_o), \ldots, \theta'_m(\infty)$ definied in (10) as a function of $v(\vec{r}_o)$, $v(\vec{r})$ or $v(\infty)$. This yields

$$\theta_m(\infty) = \theta'_m(\infty) \quad \text{for} \quad v^2(\infty) = v_b^2(\infty) = -2\,\Phi[(1+\alpha)\eta - (1+\beta)y]/(1-\eta),$$

$$\theta_m(\vec{r}) = \theta'_m(\vec{r}) \quad \text{for} \quad v^2(\vec{r}) = v_c^2(\vec{r}) = -2\Phi a\,(1+\alpha)/(1-a) - 2\Phi\,(1+\beta)y \geq 0\,,$$

$$\theta_m(\vec{r}_o) = \theta'_m(\vec{r}_o) \quad \text{for} \quad v^2(\vec{r}_o) = v_d^2(\vec{r}_o) = -2\Phi\,(1+\alpha) - 2\Phi\,(1+\beta)y/(\mu-1) \geq 0.$$

Moreover, using the relation (6), we calculate

$$v_b^2(\vec{r}_o) = R(\vec{r})/(1-\eta) \quad , \quad v_b^2(\vec{r}) = \eta\, v_b^2(\vec{r}_o) \quad ,$$

$$v_c^2(\vec{r}_o) = -2\Phi\,(1+\alpha)/(1-a) \quad , \quad v_c^2(\infty) = a\, v_c^2(\vec{r}_o) \quad ,$$

$$v_d^2(\infty) = -2\Phi\,(1+\beta)y/(\mu-1) \quad , \quad v_d^2(\vec{r}) = \mu\, v_d^2(\infty) \quad .$$

Some straightforward algebraic calculations show that, a priori three possibilities can occur :

(a) $\qquad v_a < v_b < v_\infty < v_c < v_d \quad ,$

which is equivalent with the condition

$$(1+\beta)y - (1+\alpha)\eta > 0 \quad , \tag{12}$$

i.e. $v_b(\infty)$ does not exist in this case.

(b) $\qquad v_a < v_\infty < v_b < v_c < v_d \quad ,$

which is satisfied if

$$(1+\beta)y - (1+\alpha)\eta < 0 \ ,$$
$$(1+\beta)y - \frac{\eta - a}{1 - a}\,(1+\alpha) > 0 \ ; \qquad (13)$$

and (c) $\quad v_a < v_\infty < v_d < v_c < v_b$,

which corresponds to the inequality

$$(1+\beta)y - \frac{\eta - a}{1 - a}\,(1+\alpha) < 0 \ . \qquad (14)$$

The intervals in which $\theta_m(\vec{r}_o)$, $\theta'_m(\vec{r}_o),\ldots,\ \theta'_m(\infty)$ are real functions of $v^2(\vec{r}_o)$, $v^2(\vec{r})$, or $v^2(\infty)$ can easily be determined. For case (a), i.e., under the assumption that condition (12) is satisfied, the results are shown in Figs. 1a, b, and c. From these figures and the relations (9) and (6) we deduce the different classes of particles. They all are summarized in Table I.

In a similar way we can determine the classes of particles for cases (b) and (c). A detailed study of (b) shows that the classes of particles are the same as in (a). Case (c), however, has to be rejected since for the escaping particles the first adiabatic invariant would be violated in the region $[v_d, v_b]$.

B. Negative Potential Energy

The potential energy is now a decreasing function with the minimum $- m\Phi\ (1+\alpha)$ at infinity. Note that in this case $1 + \alpha$ and $1 + \beta$ are both negative. All particles with a negative potential energy are accelerated outwardly, i.e., the electric force is larger than the gravitational force. It is not surprising that in such a case no ballistic nor trapped particles can occur. A particle emerging from the baropause is blown out and therefore has a minimal velocity at the level r, given by

$$v_x^2(\vec{r}) = - R(\vec{r})$$

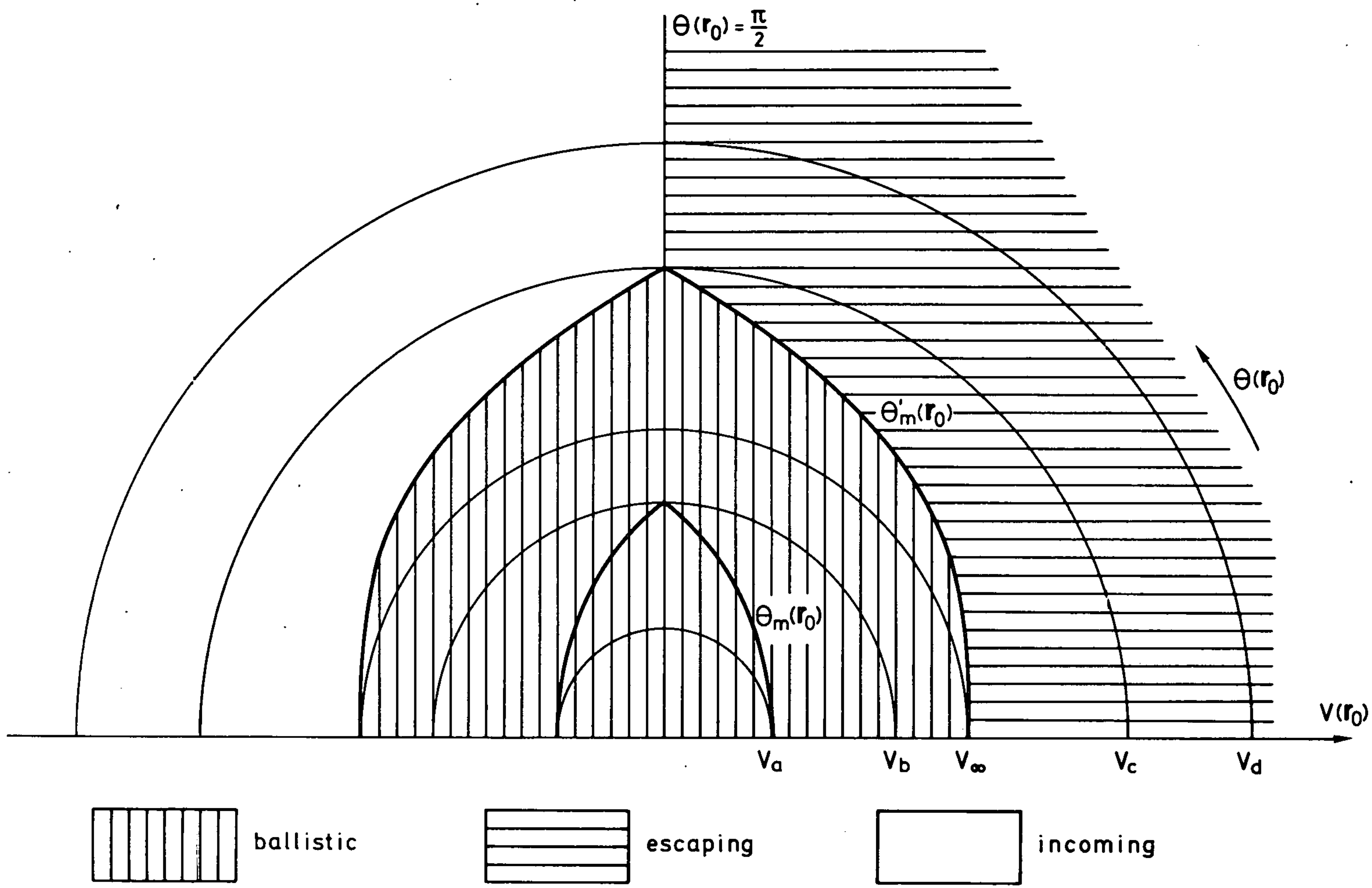

Fig: 1a

Fig. 1a.- Velocity plane at the baropause for particles with a positive potential energy.

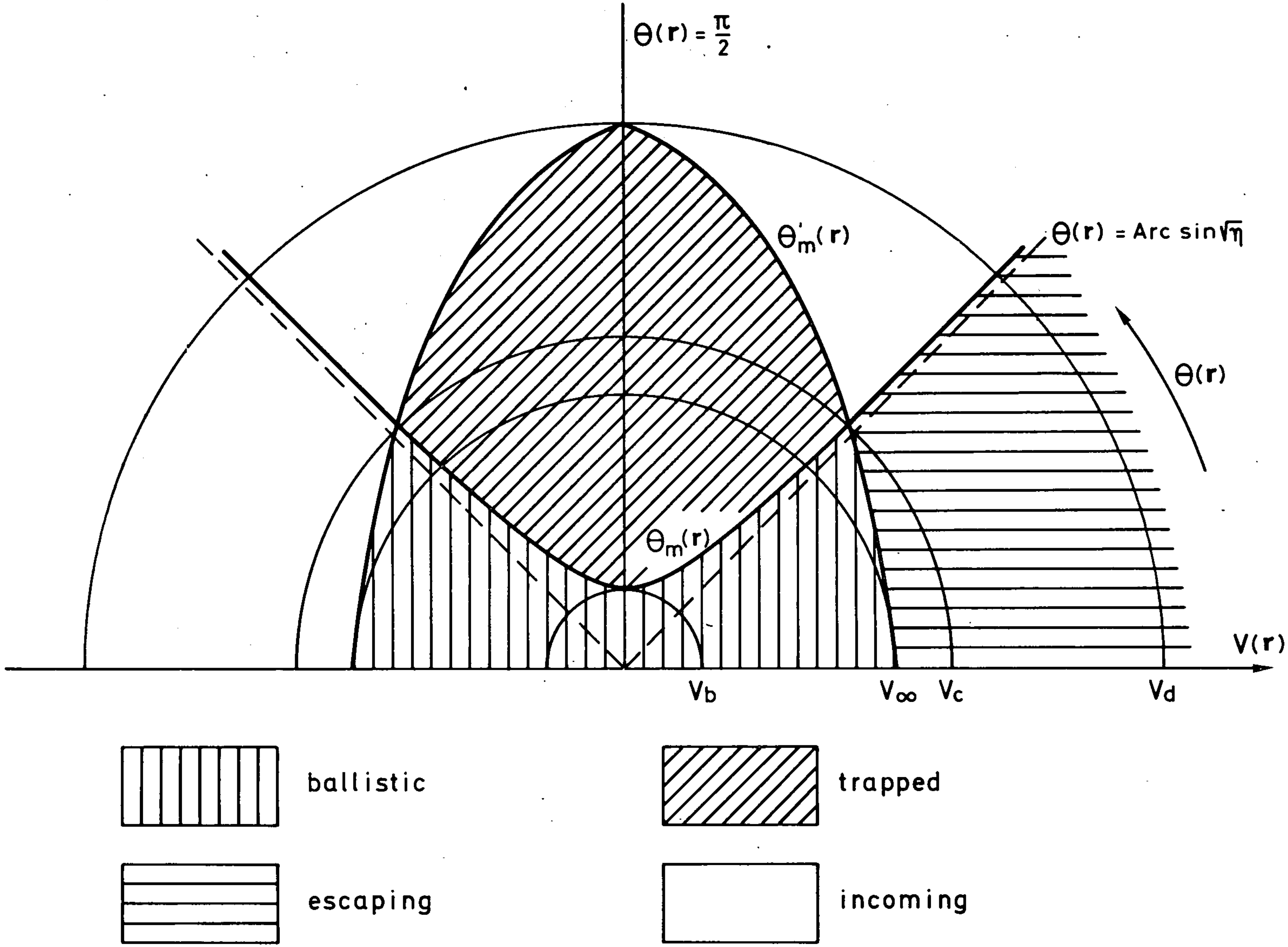

Fig · 1b

Fig. 1b.- Velocity plane at the exospheric level r, for particles with a positive potential energy.

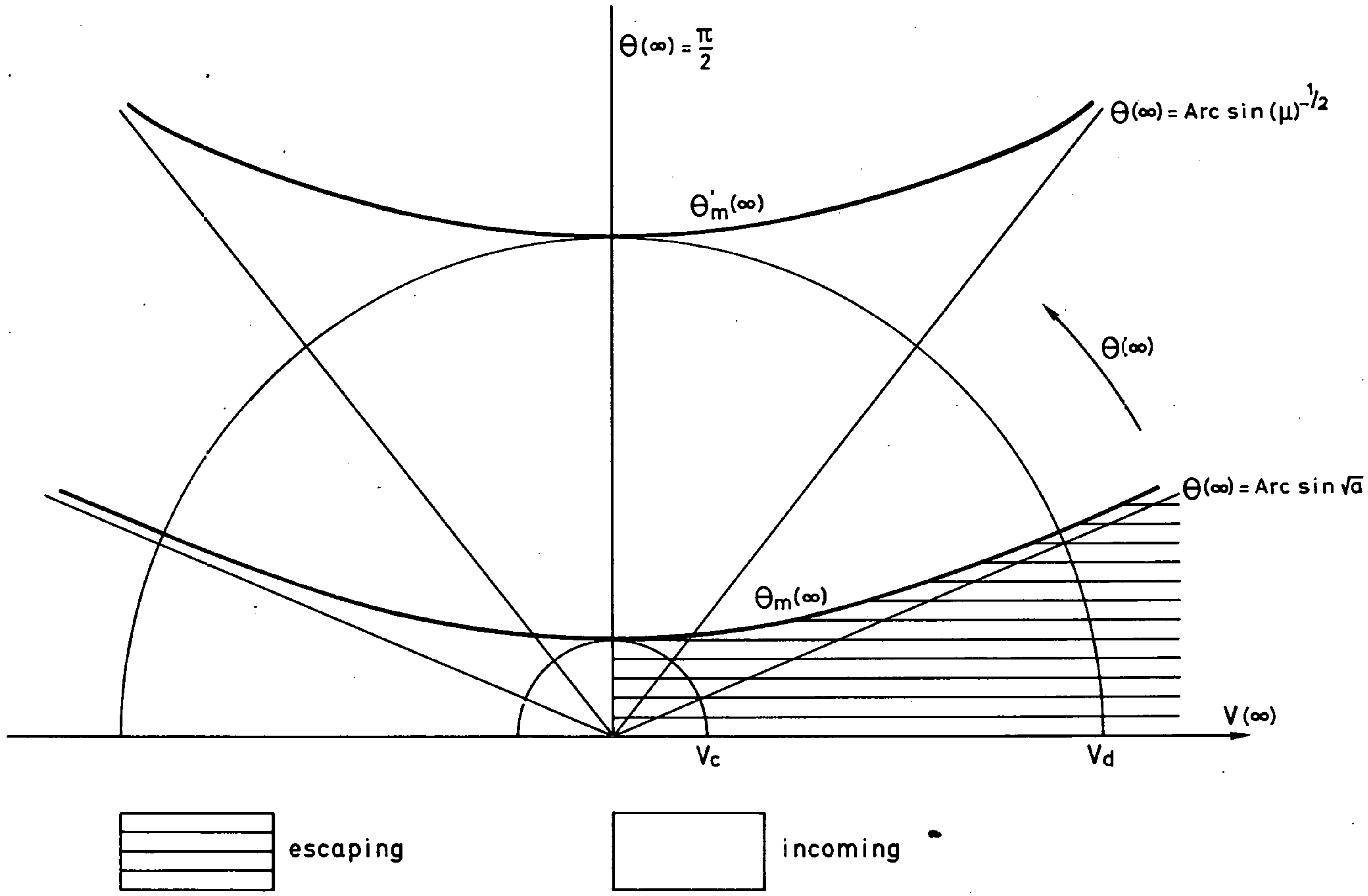

Fig : 1c

Fig. 1c.- Velocity plane at infinity for particles with a positive pontential energy.

Table I. Classes of particles with a Positive Potential energy.

$v(r_o)$	$\theta(r_o)$	$v(r)$	$\theta(r)$	$v(\infty)$	$\theta(\infty)$	Classes of Particles
$[o, v_a]$	$[o, \pi]$	---		---		Ballistic particles, not reaching level r
$[v_a, v_b]$	$[o, \theta_m]$	$[o, v_b]$	$[o, \pi]$	---		Ballistic particles reaching level r
	$[\pi-\theta_m, \pi]$					
	$[\theta_m, \pi-\theta_m]$		---			Ballistic particles not reaching level r
$[v_b, v_\infty]$	$[o, \pi]$	$[v_b, v_\infty]$	$[o, \theta_m]$	---		Ballistic particles reaching level r
			$[\pi-\theta_m, \pi]$			
			$[\theta_m, \pi-\theta_m]$			Trapped particles
$[v_\infty, v_c]$	$[o, \theta'_m]$	$[v_\infty, v_c]$	$[o, \theta'_m]$	$[o, v_c]$	$[o, \frac{\pi}{2}]$	Escaping particles
	$[\pi-\theta'_m, \pi]$		$[\pi-\theta'_m, \pi]$		$[\frac{\pi}{2}, \pi]$	Incoming particles reaching the baropause
	$[\theta'_m, \pi-\theta'_m]$		$[\theta'_m, \theta_m]$		---	Ballistic particles reaching level r
			$[\pi-\theta_m, \pi-\theta'_m]$			
	---		$[\theta_m, \pi-\theta_m]$		---	Trapped particles
$[v_c, v_d]$	$[o, \frac{\pi}{2}]$	$[v_c, v_d]$	$[o, \theta_m]$	$[v_c, v_d]$	$[o, \theta_m]$	Escaping particles
	$[\frac{\pi}{2}, \pi]$		$[\pi-\theta_m, \pi]$		$[\pi-\theta_m, \pi]$	Incoming particles reaching the baropause
			$[\theta_m, \theta'_m]$		$[\theta_m, \pi-\theta_m]$	Incoming particles, not reaching the baropause
			$[\pi-\theta'_m, \pi-\theta_m]$			
			$[\theta'_m, \pi-\theta'_m]$		---	Trapped particles
$[v_d, \infty]$	$[o, \frac{\pi}{2}]$	$[v_d, \infty]$	$[o, \theta_m]$	$[v_d, \infty]$	$[o, \theta_m]$	Escaping particles
	$[\frac{\pi}{2}, \pi]$		$[\pi-\theta_m, \pi]$		$[\pi-\theta_m, \pi]$	Incoming particles reaching the baropause
	---		$[\theta_m, \pi-\theta_m]$		$[\theta_m, \theta'_m]$	Incoming particles, not reaching the baropause
					$[\pi-\theta'_m, \pi-\theta_m]$	
	---		---		$[\theta'_m, \pi-\theta'_m]$	Incoming particles, not reaching level r

On the contrary, a particle coming from the interplanetary space into the exosphere is decelerated, and will reach the level r only if its velocity $v(\infty)$ is not inferior to a minimal velocity $v_y(\infty)$, defined by

$$v_y^2(\infty) = 2\Phi\ (1+\beta)y.$$

Proceeding in the same way as in the former case we can define all classes of particles with a negative potential energy. The results are summarized in Table II.

IV. THE VELOCITY DISTRIBUTION

In the barosphere just beneath the baropause, the collisions are still so frequent that the velocity distribution of the charged particles is Maxwellian. To obtain this distribution function in the exosphere we solve the collisionless Boltzmann equation subject to the boundary condition

$$f(\vec{r}_o, \vec{v}) = n_o \left(\frac{m}{2\pi k T_o}\right)^{3/2} \exp\left(-\frac{m}{2kT_o} v^2\right)$$

where n_o and T_o are constants chosen in an appropriate way to obtain a given number density and temperature at the baropause, and k is the Boltzmann constant. Taking into account (6) we obtain

$$f(\vec{r}, \vec{v}) = n_o \left(\frac{m}{2\pi k T_o}\right)^{3/2} \exp\left[-\frac{m}{2kT_o}(v^2 + R)\right] \ ; \quad (r \geq r_o) \quad (15)$$

In order to simplify the calculations we assume that the trapped and incoming particles still have the velocity distribution (15) but multiplied by an appropriate weight factor. Putting this coefficient equal to zero, means that the particles of the class considered do not occur in the model exosphere. If, however, we choose this factor to be 1, the particles are supposed to be in thermal equilibrium with those emerging from the barosphere.

TABLE II.- Classes of particles with a negative potential energy

$v(r_o)$	$\theta(r_o)$	$v(r)$	$\theta(r)$	$v(\infty)$	$\theta(\infty)$	Classes of particles
$[0, \infty]$	$[0, \frac{1}{2}\pi]$	$[v_x, \infty]$	$[0, \theta_m]$	$[v_x, \infty]$	$[0, \theta_m]$	Escaping particles
	$[\frac{1}{2}\pi, \pi]$		$[\pi-\theta_m, \pi]$		$[\pi-\theta_m, \pi]$	Incoming particles reaching the baropause
	...		$[\theta_m, \pi-\theta_m]$		$[\theta_m, \theta_m']$ $[\pi-\theta_m', \pi-\theta_m]$	Incoming particles, not reaching the baropause
	...		...		$[\theta_m', \pi-\theta_m']$	Incoming particles, not reaching level r
...		$[0, v_x]$	$[0, \pi]$	$[v_y, v_z]$	$[0, \theta_m']$ $[\pi-\theta_m', \pi]$	Incoming particles, not reaching the baropause
			...		$[\theta_m', \pi-\theta_m']$	Incoming particles, not reaching level r
...		...		$[0, v_y]$	$[0, \pi]$	Incoming particles, not reaching level r

V. MOMENTS OF THE DISTRIBUTION FUNCTION

Let us first recall the definitions of some macroscopic quantities, which characterize a gas of particles with mass m and velocity distribution $f(\vec{r}, \vec{v})$:

(a) the particle density

$$n(\vec{r}) = \int f(\vec{r}, \vec{v})\, d^3v,$$

(b) the particle flux parallel to the field lines

$$F(\vec{r}) = \int v_{\parallel}\, f(\vec{r}, \vec{v})\, d^3v,$$

(c) the parallel and perpendicular momentum flux

$$P_{\parallel}(\vec{r}) = m \int v_{\parallel}^2\, f(\vec{r},\vec{v})\, d^3v,$$
$$P_{\perp}(\vec{r}) = \frac{1}{2}\, m \int v_{\perp}^2\, f(\vec{r}, \vec{v})\, d^3v,$$

(d) the energy flux parallel to the magnetic field

$$\dot{\varepsilon}(\vec{r}) = \frac{1}{2}\, m \int v^2 v_{\parallel}\, f(\vec{r},\vec{v})\, d^3v.$$

All the integrations are to be taken over the appropriate velocity space. In what follows we will calculate these quantities for the four classes of particles defined previously. We will use the spherical polar coordinates (v, θ, φ) with the polar axis tangent to the magnetic field. In this case θ coincides with the earlier defined pitch angle, and φ varies from 0 to 2π for each class of particles.

A. Ballistic Particles

From Table I we deduce that the ballistic particles have velocities v and pitch angles θ limited by the following inequalities

$$0 \leq v \leq v_b(\vec{r}) \quad , \quad 0 \leq \theta \leq \pi \quad ;$$
$$v_b(\vec{r}) \leq v \leq v_\infty(\vec{r}) \quad , \quad 0 \leq \theta \leq \theta_m(\vec{r}) \; , \quad \text{and } \pi - \theta_m(\vec{r}) \leq \theta < \pi \; ;$$
$$v_\infty(\vec{r}) \leq v \leq v_c(\vec{r}) \quad , \quad \theta'_m(\vec{r}) \leq \theta \leq \theta_m(\vec{r}), \text{ and } \pi - \theta_m(\vec{r}) \leq \theta \leq \pi - \theta'_m(\vec{r}) .$$

Taking into account the explicit form of the velocity distribution (15), after integration over the above defined regions we obtain

$$n^{(B)}(\vec{r}) = 2n_o\, e^{-q}\left\{K_2(V_\infty) - A\, K_2(Xp^{-1/2}) + B\left[W_2(V_\infty\sigma^{-1/2}) - W_2(X\sigma^{-1/2})\right]\right\},$$

$$F^{(B)}(\vec{r}) = 0 \quad , \tag{16}$$

$$p_{\parallel}^{(B)}(\vec{r}) = \frac{4}{3}\, p_o\, e^{-q}\left\{K_4(V_\infty) - p\, A\, K_4(Xp^{-1/2}) + \sigma\, B\left[W_4(V_\infty\sigma^{-1/2}) - W_4(X\sigma^{-1/2})\right]\right\},$$

$$p_{\perp}^{(B)}(\vec{r}) = \frac{4}{3}\, p_o\, e^{-q}\left\{K_4(V_\infty) - A(1+\frac{\eta}{2})\, K_4(Xp^{-1/2}) - B(1+\frac{\mu}{2})\left[W_4(V_\infty\sigma^{-1/2}) - W_4(X\sigma^{-1/2})\right] - \frac{3}{2}\,\eta p^{-1} q\, A\, K_2(Xp^{-1/2}) + \frac{3}{2}\,\mu\sigma^{-1}\, V_\infty^2\, B\left[W_2(V_\infty\sigma^{-1/2}) - W_2(X\sigma^{-1/2})\right]\right\},$$

$$\varepsilon^{(B)}(\vec{r}) = 0 \; .$$

The functions $K_m(z)$ and $W_m(z)$ are defined in the Appendix. Moreover, for convenience we introduced the shorthand notations

$$p_o = n_o k T_o \quad , \quad p = 1 - \eta \quad , \sigma = \mu - 1 \; ,$$
$$q = \Lambda[1 + \alpha - (1+\beta)y\,] \; , \quad \Lambda = -\,m\Phi/kT_o \quad ,$$
$$V_\infty^2 = \Lambda(1+\beta)y \quad , \quad X^2 = \Lambda[(1+\beta)y - \frac{\sigma\, a}{1-a}\,(1+\alpha)] \quad ,$$
$$A = p^{1/2}\, e^{-\eta q/p} \quad , \quad B = \sigma^{1/2}\, e^{-\mu V_\infty^2/\sigma} \; .$$

B. Escaping Particles

Here, we have to make a difference between particles with a positive and a negative potential energy. In the former case, i.e., $1 + \alpha - (1+\beta)y \geq 0$, the velocity space corresponding to this class is defined by

$$v_\infty(\vec{r}) \leq v \leq v_c(\vec{r}) \quad , \quad 0 \leq \theta \leq \theta'_m(\vec{r}) \quad , \quad 0 \leq \varphi < 2\pi$$

$$v_c(\vec{r}) \leq v \leq \infty \quad , \quad 0 \leq \theta \leq \theta_m(\vec{r}) \quad , \quad 0 \leq \varphi < 2\pi$$

Calculating the macroscopic quantities (a)-(d) we obtain

$$n^{(E)}(\vec{r}) = n_o\, e^{-q} \left\{ \frac{1}{2} - K_2(V_\infty) - A\left[\frac{1}{2} - K_2(Xp^{-1/2})\right] - B\left[W_2(V_\infty\sigma^{-1/2}) - W_2(X\sigma^{-1/2})\right]\right\}, \tag{17}$$

$$F^{(E)}(\vec{r}) = \frac{1}{4}\, n_o c_o \eta\, a^{-1}\, e^{-\Lambda(1+\alpha)} \left\{1 + (a-1)\exp\left[-a\Lambda\,(1+\alpha)/(1-a)\right]\right\}, \tag{18}$$

$$p_{\parallel}^{(E)}(\vec{r}) = \frac{2}{3}\, p_o\, e^{-q}\left\{\frac{3}{4} - K_4(V_\infty) - p\,A\left[\frac{3}{4} - K_4(Xp^{-1/2})\right] - \sigma\, B\left[W_4(V_\infty\sigma^{-1/2}) - W_4(X\sigma^{-1/2})\right]\right\},$$

$$p_{\perp}^{(E)}(\vec{r}) = \frac{2}{3}\, p_o\, e^{-q}\left\{\frac{3}{4} - K_4(V_\infty) - A(1 + \frac{\eta}{2})\left[\frac{3}{4} - K_4(Xp^{-1/2})\right] + B(1 + \frac{\mu}{2})\left[W_4(V_\infty\sigma^{-1/2}) - W_4(X\sigma^{-1/2})\right] - \frac{3}{2}\,\eta\, p^{-1}\, q\, A\left[\frac{1}{2} - K_2(Xp^{-1/2})\right] - \frac{3}{2}\,\mu\sigma^{-1}\, V_\infty^2\, B\left[W_2(V_\infty\sigma^{-1/2}) - W_2(X\sigma^{-1/2})\right]\right\},$$

$$\varepsilon^{(E)}(\vec{r}) = \frac{1}{4} p_o c_o \eta a^{-1} e^{-\Lambda(1+\alpha)} \left\{ a \left[V_c^4 + (V_c^2+1)(2+q) \right] \exp\left[- a\Lambda(1+\alpha)/(1-a) \right] \right.$$

$$\left. + 2 + V_\infty^2 - \left[V_c^4 + (V_c^2+1)(2-V_\infty^2) \right] \exp\left[- a\Lambda(1+\alpha)/(1-a) \right] \right\}, \quad (19)$$

with

$$c_o = (8kT_o/\pi m)^{1/2} \quad ,$$

$$V_c^2 = \Lambda \left[(1+\beta)y + a(1+\alpha)/(1-a) \right] .$$

To calculate these quantities for escaping particles with a negative potential energy, the integrations are to be taken over the region

$$v_x(\vec{r}) \leq v < \infty \quad , \quad 0 \leq \theta \leq \theta_m(\vec{r}) \quad , \quad 0 \leq \varphi < 2\pi \quad ,$$

which leads to the Formulas

$$n^{(E)}(\vec{r}) = n_o e^{-q} \left\{ \frac{1}{2} - K_2(V_x) - A \left[\frac{1}{2} - K_2(V_x p^{-1/2}) \right] \right\} , \quad (20)$$

$$F^{(E)}(\vec{r}) = \frac{1}{4} n_o c_o \eta \quad , \quad (21)$$

$$p_{\parallel}^{(E)}(\vec{r}) = \frac{2}{3} p_o e^{-q} \left\{ \frac{3}{4} - K_4(V_x) - pA \left[\frac{3}{4} - K_4(V_x p^{-1/2}) \right] \right\} ,$$

$$p_{\perp}^{(E)}(\vec{r}) = \frac{2}{3} p_o e^{-q} \left\{ \frac{3}{4} - K_4(V_x) - A(1+ \frac{\eta}{2}) \left[\frac{3}{4} - K_4(V_x p^{-1/2}) \right] \right.$$

$$\left. - \frac{3}{2} \eta p^{-1} q A \left[\frac{1}{2} - K_2(V_x p^{-1/2}) \right] \right\}$$

$$\varepsilon^{(E)}(\vec{r}) = \frac{1}{4} p_o c_o \eta (2-q) \quad (22)$$

where we used the notation

$$V_x = (-q)^{1/2}$$

C. Trapped Particles

The velocity space for this class of particles is given in Table I. From the definitions of the macroscopic quantities (a) - (d) we now obtain

$$n^{(T)}(\vec{r}) = 2\xi\, n_o\, e^{-q}\Big[A\, K_2(Xp^{-1/2}) + B\, W_2(X\sigma^{-1/2})\Big] \quad , \qquad (23)$$

$$F^{(T)}(\vec{r}) = 0 \quad ,$$

$$p_{\parallel}^{(T)}(\vec{r}) = \frac{4}{3}\,\xi\, p_o\, e^{-q}\Big[p\, A\, K_4(Xp^{-1/2}) + \sigma B\, W_4(X\sigma^{-1/2})\Big] \quad ,$$

$$p_{\perp}^{(T)}(\vec{r}) = \frac{4}{3}\,\xi\, p_o\, e^{-q}\Big[A(1+\frac{\eta}{2})\, K_4(Xp^{-1/2}) - B(1+\frac{\mu}{2})\, W_4(X\sigma^{-1/2})$$
$$+ \frac{3}{2}\,\eta\, p^{-1}\, q\, A\, K_2(X\sigma^{-1/2}) + \frac{3}{2}\,\mu\sigma^{-1}\, V_\infty^2\, B\, W_2(X\sigma^{-1/2})\Big] \quad ,$$

$$\varepsilon^{(T)}(\vec{r}) = 0 \quad ,$$

where ξ is a weightfactor introduced in the distribution function, as mentionned in Sec. IV.

D. Incoming Particles

Proceeding in a way similar to that for the escaping particles, the density, the particle, momentum, and energy fluxes for particles with a positive potential energy are found to be given by the formulas

$$n^{(I)}(\vec{r}) = \zeta n_o\, e^{-q}\Big\{\frac{1}{2} - K_2(V_\infty) + A\Big[\frac{1}{2} - K_2(Xp^{-1/2})\Big] - B\Big[W_2(V_\infty\sigma^{-1/2})$$
$$+ W_2(X\sigma^{-1/2})\Big]\Big\} \quad , \qquad (24)$$

$$F^{(I)}(\vec{r}) = -\,\zeta\, F^{(E)}(\vec{r})$$

$$p_{\parallel}^{(I)}(\vec{r}) = \frac{2}{3}\,\zeta p_o\, e^{-q}\Big\{\frac{3}{4} - K_4(V_\infty) + pA\Big[\frac{3}{4} - K_4(Xp^{-1/2})\Big] - \sigma\, B\Big[W_4(V_\infty\sigma^{-1/2})$$
$$+ W_4(X\sigma^{-1/2})\Big]\Big\} \quad ,$$

$$p_{\perp}^{(I)}(\vec{r}) = \frac{2}{3}\zeta p_o\, e^{-q}\left\{\frac{3}{4} - K_4(V_\infty) + A(1+\frac{\eta}{2})\left[\frac{3}{4} - K_4(Xp^{-1/2})\right]\right.$$
$$+ B(1+\frac{\mu}{2})\left[W_4(V_\infty\sigma^{-1/2}) + W_4(X\sigma^{-1/2})\right]$$
$$+ \frac{3}{2}\eta p^{-1}q\, A\left[\frac{1}{2} - K_2(Xp^{-1/2})\right] - \frac{3}{2}\mu\sigma^{-1}V_\infty^2\, B\left[W_2(V_\infty\sigma^{-1/2})\right.$$
$$\left.\left. + W_2(X\sigma^{-1/2})\right]\right\} ,$$

$$\varepsilon^{(I)}(\vec{r}) = -\zeta\varepsilon^{(E)}(\vec{r}) ,$$

where $F^{(E)}$ and $\varepsilon^{(E)}$ are given by (18) and (19), respectively.
On the other hand for the incoming particles with a negative potential energy, these quantities can be calculated through the expressions

$$n^{(I)}(\vec{r}) = \zeta n_o\, e^{-q}\left\{\frac{1}{2} + K_2(V_x) + A\left[\frac{1}{2} - K_2(V_x p^{-1/2})\right]\right\} , \qquad (25)$$

$$F^{(I)}(\vec{r}) = -\zeta\, F^{(E)}(\vec{r}) ,$$

$$p^{(I)}(\vec{r}) = \frac{2}{3}\zeta\, p_o\, e^{-q}\left\{\frac{3}{4} + K_4(V_x) + pA\left[\frac{3}{4} - K_4(V_x p^{-1/2})\right]\right\} ,$$

$$p^{(I)}(\vec{r}) = \frac{2}{3}\zeta p_o\, e^{-q}\left\{\frac{3}{4} + K_4(V_x) + A(1+\frac{\eta}{2})\left[\frac{3}{4} - K_4(V_x p^{-1/2})\right]\right.$$
$$\left. + \frac{3}{2}\eta p^{-1}q\, A\left[\frac{1}{2} - K_2(V_x p^{-1/2})\right]\right\} ,$$

$$\varepsilon^{(I)}(\vec{r}) = -\zeta\, \varepsilon^{(E)}(\vec{r}) ,$$

where $F^{(E)}$ and $\varepsilon^{(E)}$ are now defined in (21) and (22).

VI. EXOSPHERIC MODEL

Let us now consider an exosphere in which there are different kinds of charged particles with density $n_j(r_o)$ and temperature $T_j(r_o)$ at the baropause. If the particles of species j have a positive potential energy, their total density distribution is given by

$$n_j(\vec{r}) = n_j^{(B)}(\vec{r}) + n_j^{(E)}(\vec{r}) + n_j^{(T)}(\vec{r}) + n_j^{(I)}(\vec{r})$$

where the four partial densities on the right hand side are, respectively, given by the expressions (16), (17), (23), and (24). If on the contrary we are dealing with particles which have a negative potential energy, we obtain

$$n_j(\vec{r}) = n_j^{(E)}(\vec{r}) + n_j^{(I)}(\vec{r}),$$

where the escaping and incoming particle densities, now are given by (20) and (25). The other three macroscopic quantities, defined in Sec. V, can be obtained in a similar way. In all those expressions, α_j and $\beta_j(r)$ still remain unknown parameters, which in principle can be calculated by solving the differential equation (2). This, however, is an enormously complex problem, since the right hand side of (2) is a very complicated function of $\varphi_o(r)$. In a recent paper Lemaire and Scherer[11] pointed out that in practice this difficult task can be avoided, except in a small transition region close to the baropause, by using two fundamental physical conditions : (1) the quasineutrality condition

$$n_e(\vec{r}) = \sum_{\text{ions}} Z_i\, n_i(\vec{r}) \quad , \tag{26}$$

and (2) the zero current condition

$$F_e(\vec{r}) = \sum_{\text{ions}} Z_i F_i(\vec{r}) \quad . \tag{27}$$

Indeed, from the definitions (4a) and (8) if follows that for each kind of ion the values α_i, and $\beta_i(\vec{r})$ are related to the values α_e and $\beta_e(\vec{r})$ of the electrons through the relations

$$\alpha_i = - \alpha_e\, Z_i\, m_e/m_i \quad ,$$

$$\beta_i(\vec{r}) = - \beta_e(\vec{r})\, Z_i\, m_e/m_i \quad .$$

Moreover, from the results of Sec. V it follows that the particle escape fluxes are only a function of the α-values and not of the β's. Hence, we can determine α_e by using condition (27). After substitution of these values in the explicit expressions of the particle densities, the quasi-neutrality condition (26) yields a transcendental equation in $\beta_e(\vec{r})$ which can be solved numerically for each altitude.

As an example we have considered the case of an $(O^+ - H^+ - e)$ exosphere, since recent measurements indicate that oxygen and hydrogen ions are the predominant constituents in the terrestrial polar ionosphere[4,12]. Lemaire and Scherer[10] have shown that the baropause level can be taken at about 2000 km above the polar cap for ion and electron temperatures of 3000°K. Hence, in the case of the Earth, the value a corresponding with the ratio of the magnetic field strength at infinity to the magnetic field strength at the baropause is very small ($\sim 10^{-4}$), so that we consider the case of a vanishing magnetic field at infinity. This simplifies the numerical calculations very much. Since the dipole configuration is a fairly good approximation for the terrestrial magnetic field up to a radial distance of about four Earth radii[13], the parameter η which is defined in (11), can be calculated by means of the formula

$$\eta(r,\lambda) = y^3(4 - 3\cos^2\lambda)^{1/2}\,(4 - 3\cos^2\lambda_o)^{-1/2} \quad ,$$

where λ_o and λ are, respectively, the geomagnetic latitude at the baropause and at the radial distance r. Along a magnetic field line we have the relation

$$y \cos^2\lambda = \cos^2\lambda_o \,.$$

At the baropause level we assume the following concentrations

$$n_e(r_o) = 10^3\ \mathrm{cm}^{-3} \quad , \quad n_{O^+}(r_o) = 9 \times 10^2\ \mathrm{cm}^{-3}, \quad \text{and } n_{H^+}(r_o) = 10^2\ \mathrm{cm}^{-3}$$

Moreover, we neglect the class of incoming particles ($\zeta = 0$) and suppose that the trapped particles are in thermal equilibrium with those emerging from the barosphere ($\xi = 1$).

We plotted the macroscopic quantities, which are defined in Sec. V, only along one line of force intersecting the baropause at the geomagnetic latitude $\lambda_o = 80°$, because the results do not differ significantly with choice of the field line ($75° \leq \lambda_o \leq 90°$). Figure 2 illustrates the reduced potential energy

$$q = \frac{m\,R}{2k\,T_o} = \Lambda\left[1 + \alpha - (1+\beta)y\right]$$

versus altitude. The electrons and oxygen ions have a monotonic increasing potential energy which tends to a positive constant value at infinity, given by $-(1 + \alpha_e)\,\Phi\,m_e = 1.94\,\mathrm{eV}$ and $-(1 + \alpha_{O^+})\,\Phi m_{O^+} = 5.94\,\mathrm{eV}$, respectively. Therefore, the electrons and O^+ particles emerging from the barosphere are decelerated. The protons on the contrary have a negative monotonic decreasing potential energy tending to $-(1 + \alpha_{H^+})\,\Phi m_{H^+} = -1.45\ \mathrm{eV}$, and consequently, they are blown out. The small polarisation electric field which accelerates the ions outwardly and retains the thermal electrons is shown in Fig. 3, with the electric potential $\varphi_o(r)$.

Once that the α and β values have been computed we can calculate the number density, the particle flux, the momentum fluxes, and the energy flux by means of the formulas determined in Sec. V. Figure 4 gives the ion number densities which show a difference in the scale height of the O^+ and H^+ ions. As a consequence the oxygen ion, which is predominant at the baropause level becomes a minor constituent above 5500 km. Since all the H^+ ions are blown out, their total number density is given by the escaping proton density. As far as the oxygen ions are concerned the situation is quite different. Up to an altitude of about 17 500 km the ballistic O^+ ions form the major class ; at higher altitudes, however, the trapped particles become more important. The number density of the escaping oxygen ions is not plotted, since it varies from only $1.5 \times 10^{-7}\ \mathrm{cm}^{-3}$ at 2500 km to $6.2 \times 10^{-9}\ \mathrm{cm}^{-3}$ at 20 000 km.

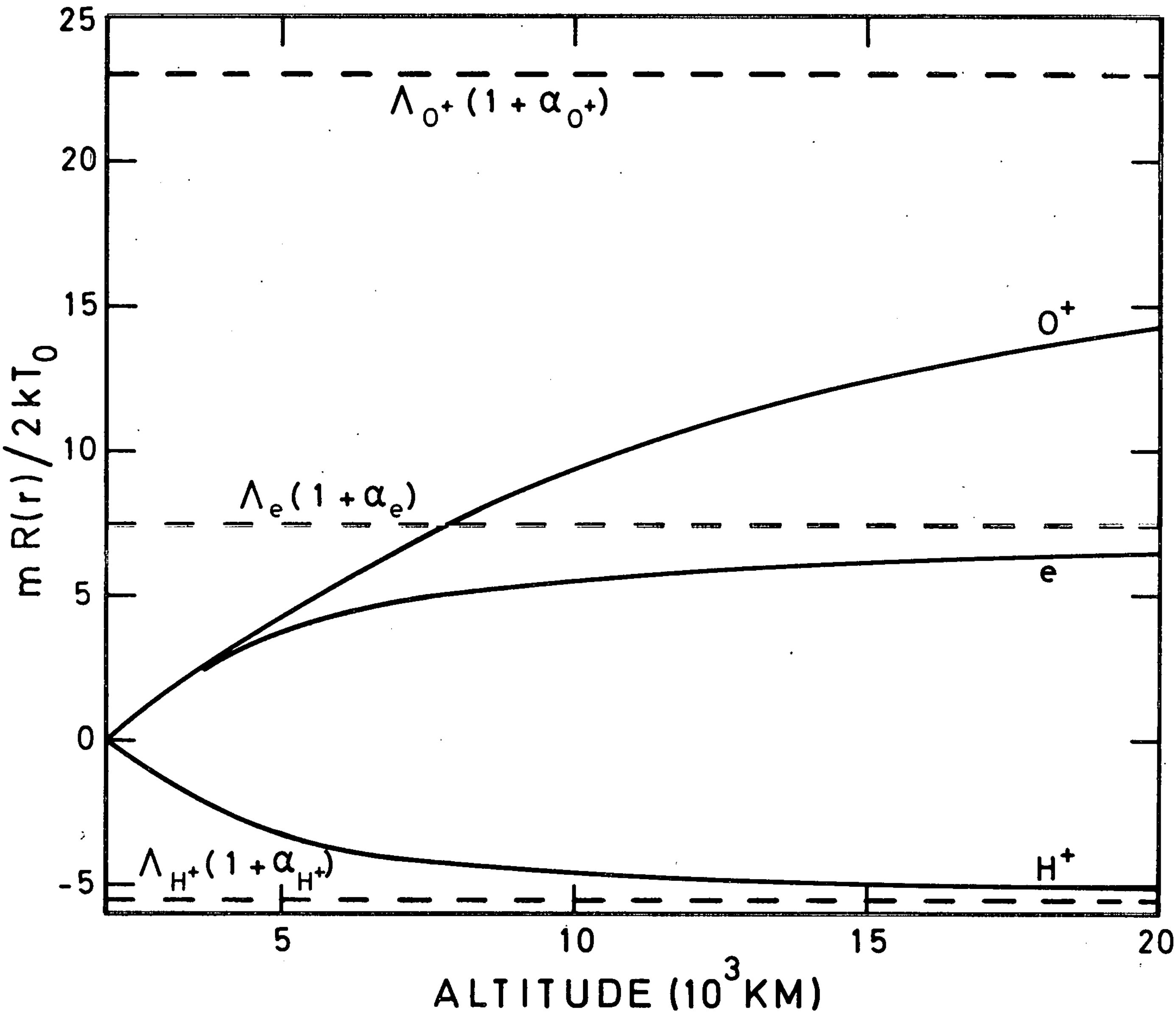

Fig. 2.- Reduced potential energies versus altitude along a magnetic field line crossing the baropause (h_o = 2000 km) at 80° latitude. The baropause temperature is 3000°K for each species and the concentrations are $n_e(r_o) = 10^3$ cm^{-3}, $n_{O^+}(r_o) = 9 \times 10^2$ cm^{-3}, and $n_{H^+}(r_o) = 10^2$ cm^{-3}. The dashed lines correspond with the horizontal asymptotes.

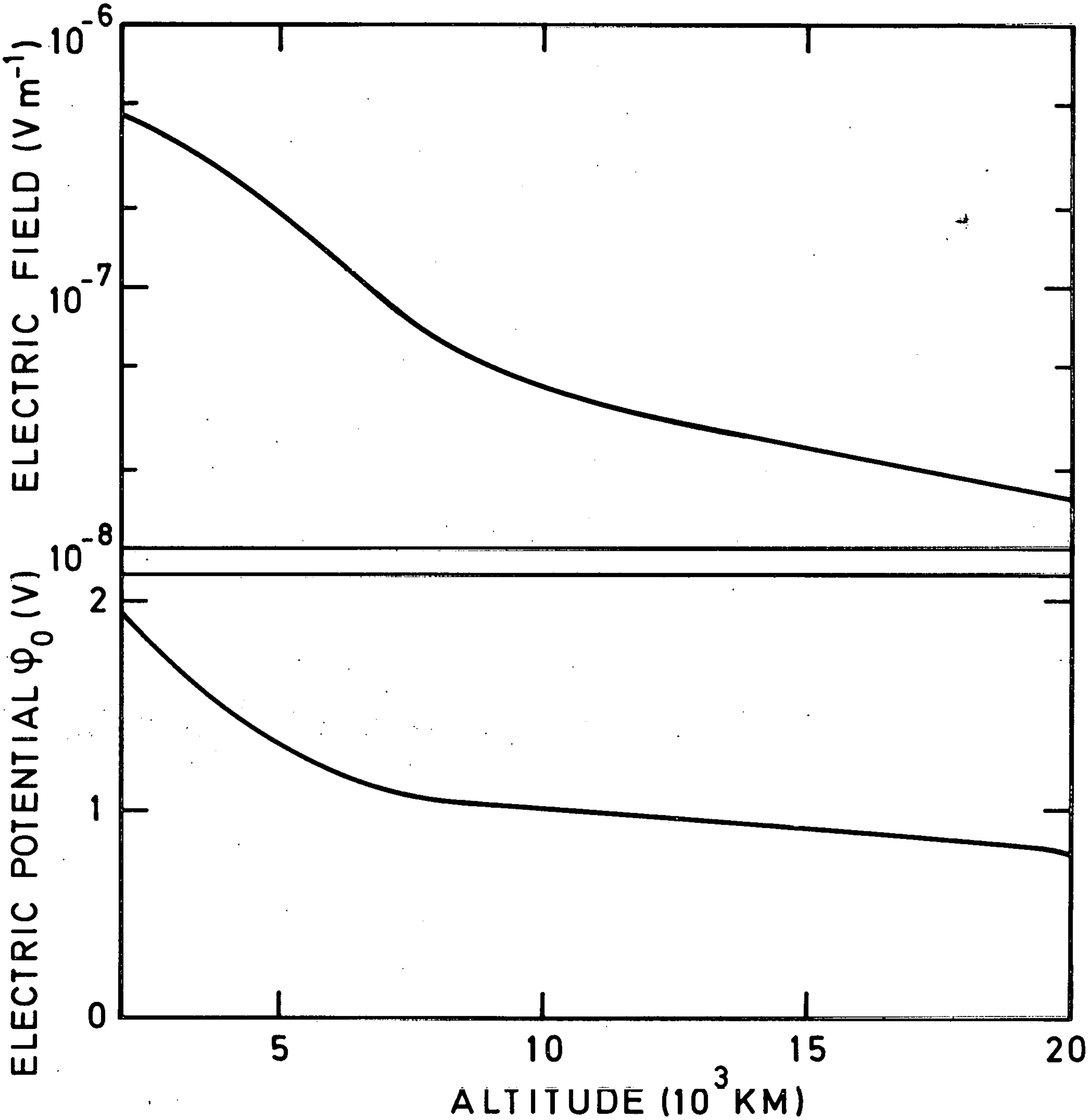

Fig. 3.- Electric potential and field versus altitude along a magnetic field line crossing the baropause (h_o = 2000 km) at 80° latitude. The baropause temperature is 3000°K for each species and the concentrations are $n_e(r_o) = 10^3$ cm^{-3}, $n_O+(r_o) = 9 \times 10^2$ cm^{-3}, and $n_H{}^+(r_o) = 10^2$ cm^{-3}.

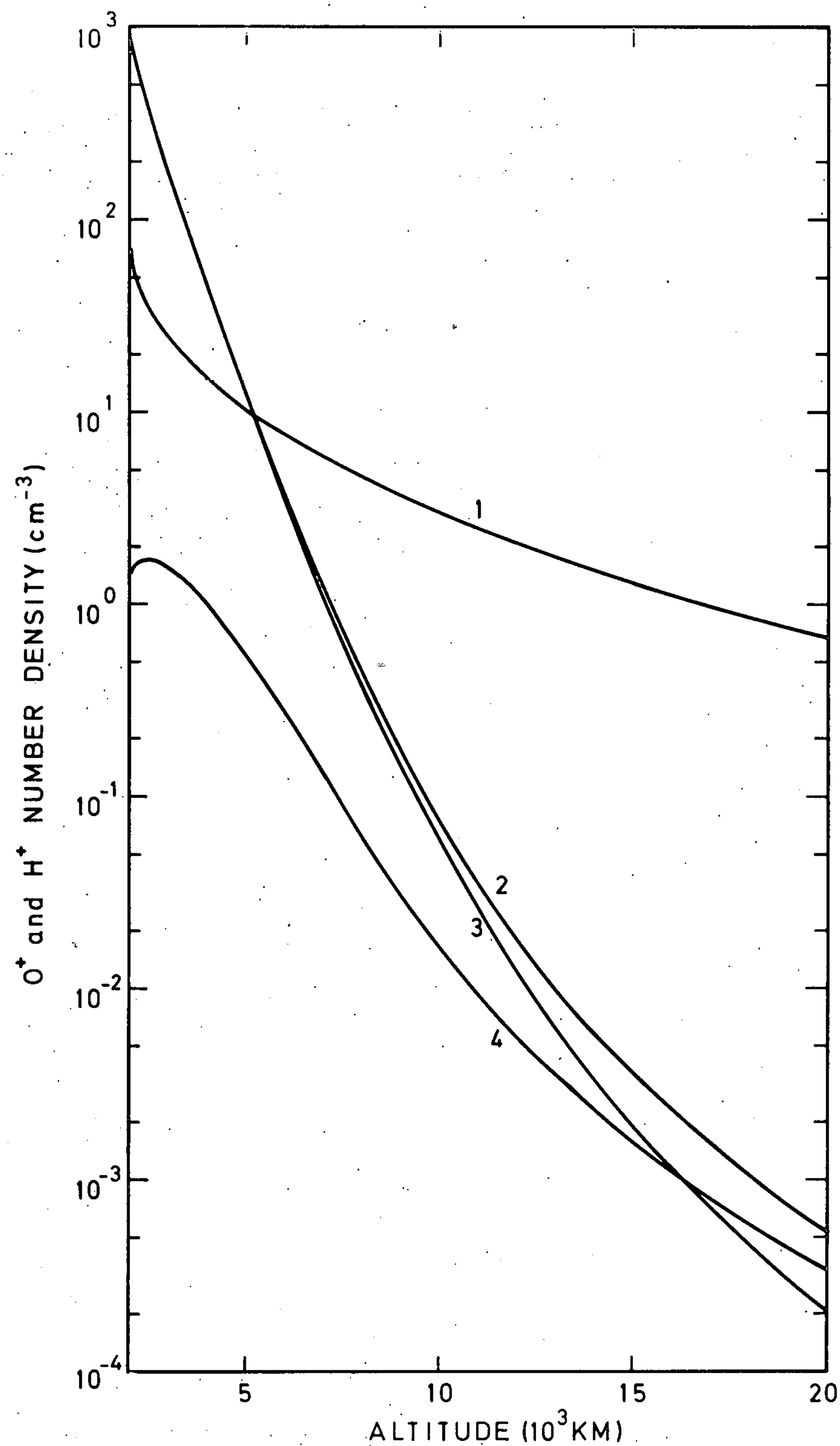

Fig. 4.- Density distributions versus altitude along a magnetic field line crossing the baropause (h_0 = 2000 km) at 80° latitude : (1) H^+ density, (2) total O^+ density, (3) ballistic O^+ density, and (4) trapped O^+ density. The baropause temperature is 3000°K for each species and the concentrations are $n_e(r_o) = 10^3$ cm^{-3}, $n_{O^+}(r_o) = 9 \times 10^2$ cm^{-3}, and $n_{H^+}(r_o) = 10^2 cm^{-3}$.

We can also determine for each species, the bulk velocity

$$w(\vec{r}) = F(\vec{r})/n(\vec{r})$$

which is illustrated in Fig. 5. The H^+ flow rapidly becomes supersonic. Such behavior is in agreement with the results obtained by Banks and Holzer(9) and is known as the polar wind. At large distances the proton and electron bulk velocities tend to a common constant value ($\sim$ 18.6 km sec^{-1}). It is worthwhile to note, however, that the oxygen ion flow velocity is much smaller and of the order of only 6 cm sec^{-1} at 20 000 km.

The transverse and longitudinal pressures, defined by

$$p_{\perp}(\vec{r}) = P\,(\vec{r})\ ,$$

$$p_{\parallel}(\vec{r}) = m\int (v_{\parallel} - w)^2\ f(\vec{r},\ \vec{v})d^3v$$
$$= P_{\parallel}(\vec{r}) - m\ w(\vec{r})\ F(\vec{r})\ ,$$

allow to calculate : (1) the parallel and perpendicular temperatures

$$T_{\parallel}(\vec{r}) = p_{\parallel}(\vec{r})/k\ n(\vec{r})\ ,$$
$$T_{\perp}(\vec{r}) = p_{\perp}(\vec{r})/k\ n(\vec{r})\ ;$$

(2) the average temperature

$$\langle T(\vec{r})\rangle = \frac{1}{3}\left[\,T_{\parallel}(\vec{r}) + 2\ T_{\perp}(\vec{r})\right]\ ;$$

and (3) the temperature anisotropy $T_{\parallel}(\vec{r})/T_{\perp}(\vec{r})$.

Figure 6 gives the mean temperature and anisotropy for each constituant as a function of the altitude. Owing to the small escape flux of the oxygen ions, the O^+ temperature does not change significantly and remains isotropic. The electron temperature decreases slowly and

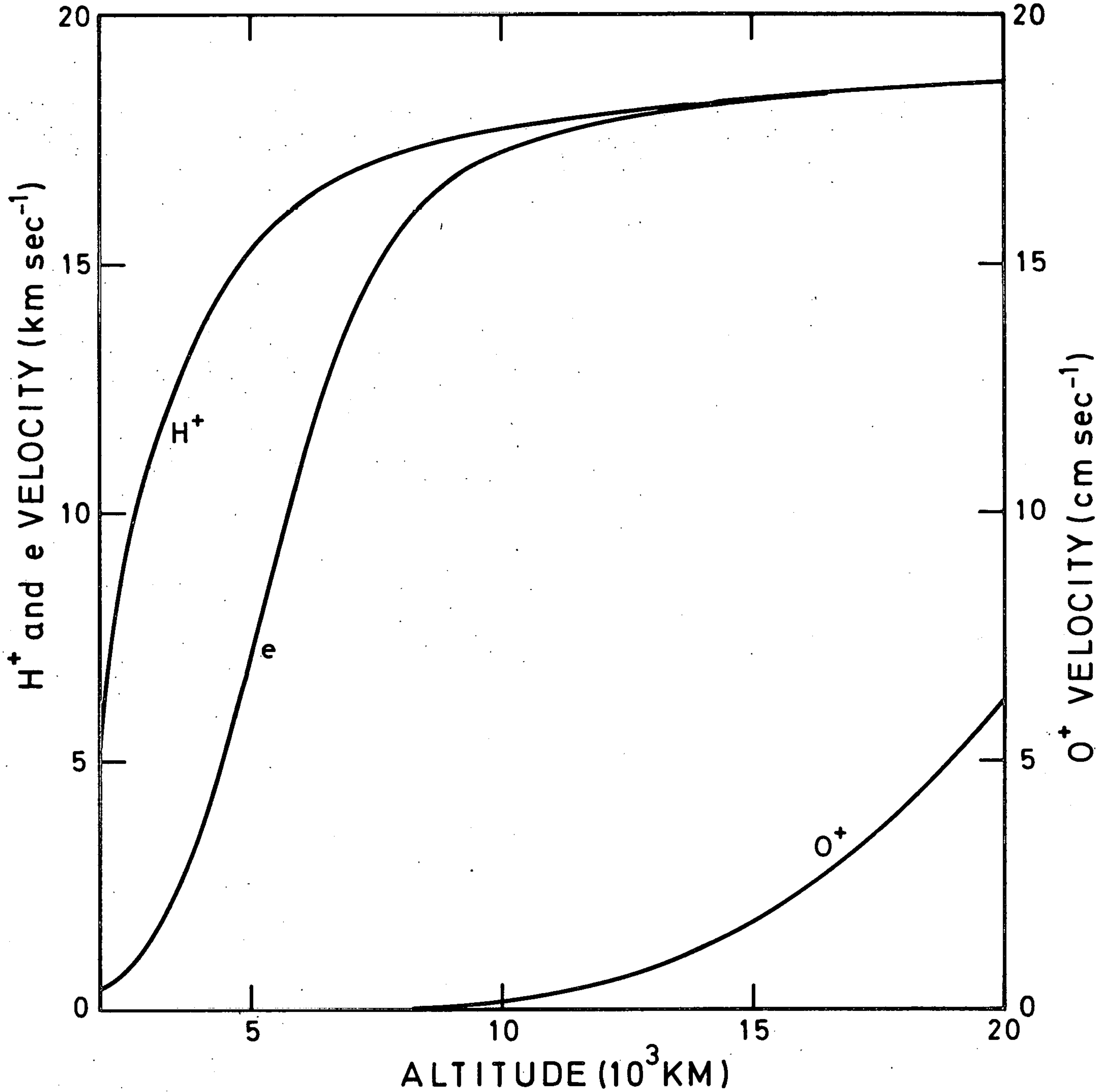

Fig. 5.- Bulk velocities versus altitude along a magnetic field line crossing the baropause (h_o = 2000 km) at 80° latitude. The baropause temperature is 3000°K for each species and the concentrations are $n_e(r_o) = 10^3$ cm^{-3}, $n_{H^+}(r_o) = 10^2$ cm^{-3}, and $n_{O^+}(r_o) = 9 \times 10^2$ cm^{-3}

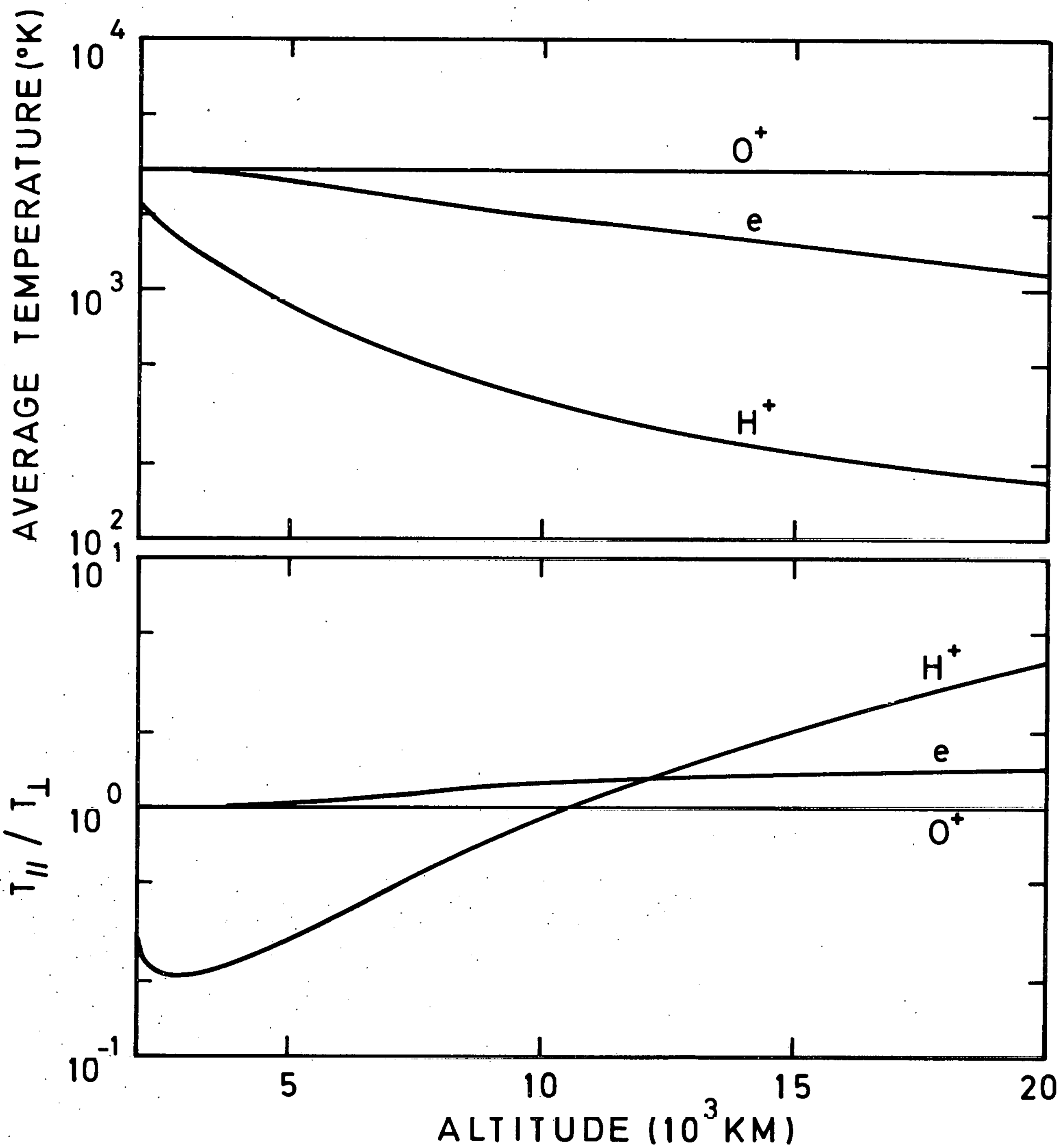

Fig. 6.- Average temperatures and temperature anisotropies versus altitude along a magnetic field line crossing the baropause (h_o = 2000 km) at 80° latitude. The baropause temperature $T(r_o)$ = 3000°K for each species, and the concentrations are $n_e(r_o) = 10^3$ cm^{-3}, $n_{O^+}(r_o)$ = 9×10^2 cm^{-3}, and $n_{H^+}(r_o) = 10^2$ cm^{-3}.

becomes slightly anisotropic ($T_{\parallel}/T_{\perp} = 1.4$ at 20 000 km). For the proton temperature, however, there occurs a relatively large discontinuity in the parallel temperature and a fortiori in the average temperature. This is due to the assumption that there exists a sharply defined baropause which separates the collision dominated barosphere from the collision free exosphere. In a more realistic model, however, one should consider a transition region with nonvanishing thickness.

Finally, in Fig. 7 we plotted the conduction flux which is defined by

$$C(\vec{r}) = \frac{1}{2} m \int (\vec{v} - \vec{w})^2 \, (v_{\parallel} - w) \, f(\vec{r}, \vec{v}) \, d^3v$$

$$= \varepsilon(\vec{r}) + w(\vec{r}) \left[m w(\vec{r}) \, F(\vec{r}) - \frac{3}{2} P_{\parallel}(\vec{r}) - P_{\perp}(\vec{r}) \right] .$$

Note that for the electrons the conduction flux is at least two orders of magnitude larger than for the hydrogen ions and that the O^+ conduction flux is negligible compared with the proton conduction flux.

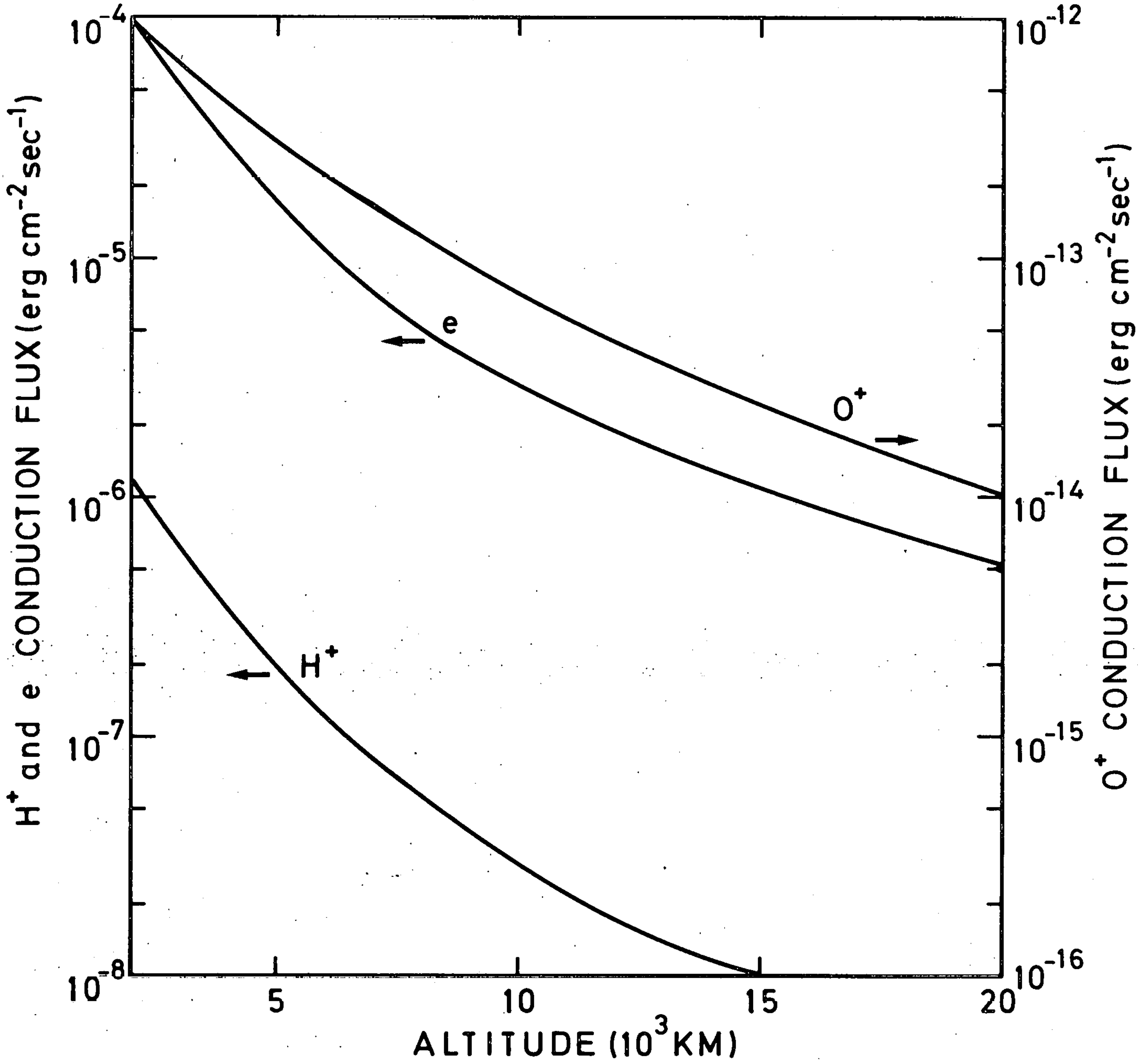

Fig. 7.- Conduction fluxes versus altitude along a magnetic field line crossing the baropause (h_o = 2000 km) at 80° latitude. The baropause temperature is 3000°K for each species and the concentrations are $n_e(r_o) = 10^3$ cm^{-3}, $n_{O^+}(r_o) = 9 \times 10^2$ cm^{-3}, and $n_{H^+}(r_o) = 10^2$ cm^{-3}.

APPENDIX

The functions $K_m(z)$ and $W_m(z)$ are respectively defined by

$$K_m(z) = \frac{2}{\sqrt{\pi}} \int_o^z dt\ e^{-t^2}\ t^m \quad ,$$

$$W_m(z) = \frac{2}{\sqrt{\pi}} \int_o^z dt\ e^{t^2}\ t^m \quad .$$

The function $K_m(z)$ may be expressed in terms of the well-known error function and in terms of exponential functions. Indeed, partial integration yields the recurrence formula

$$K_m(z) = \frac{1}{2}(m-1)\ K_{m-2}(z) - \pi^{-1/2}\ z^{m-1}\ \exp(-z^2) \quad ,$$

where straightforward calculations lead to the results

$$K_o(z) = \mathrm{erf}(z) \quad ,$$

$$K_1(z) = \pi^{-1/2}[1 - \exp(-z^2)] .$$

In a similar way it is possible to show that

$$W_m(z) = \pi^{-1/2}\ z^{m-1}\ \exp(z^2) - \frac{1}{2}(m-1)\ W_{m-2}(z) \quad ,$$

$$W_o(z) = \frac{2}{\sqrt{\pi}} \exp(z^2)\ D(z)$$

$$W_1(z) = \pi^{-1/2}[\exp(z^2) - 1]$$

where D(z) is Dawson's integral[14].

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