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3 - Avenue Circulaire
B - 1180 BRUXELLES

AERONOMICA ACTA

A - N° 206 - 1979

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B E L G I S C H I N S T I T U U T V O O R R U I M T E - A E R O N O M I E

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FOREWORD

The paper entitled : "Effect of environmental fluctuations on global energy balance of the earth-atmosphere system" will be published in Nature, 1979.

AVANT-PROPOS

L'article intitulé : "Effect of environmental fluctuations on global energy balance of the earth-atmosphere system" sera publié dans Nature, 1979.

VOORWOORD

Het artikel getiteld : "Effect of environmental fluctuations on global energy balance of the earth-atmosphere system" zal verschijnen in het tijdschrift Nature, 1979.

VORWORT

Die Arbeit : "Effect of environmental fluctuations on global energy balance of the earth-atmosphere system" wird in Nature, 1979 herausgegeben werden.

EFFECT OF ENVIRONMENTAL FLUCTUATIONS ON GLOBAL ENERGY BALANCE OF THE EARTH-ATMOSPHERE SYSTEM

by

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Abstract

The effect of environmental fluctuations on the planetary surface temperature in analyzed. Various parameters influencing the energy balance equation are represented as a Gaussian white noise. The resulting stochastic differential equation shows a general tendency for runaway to lower surface temperature values.

Résumé

On étudie l'effet d'un environnement fluctuant sur la température planétaire de surface. Différents paramètres intervenant dans l'équation de bilan d'énergie sont stylisés par un bruit blanc Gaussien. L'équation différentielle stochastique qui en résulte montre une tendance générale de dérive vers des valeurs décroissantes de la température à la surface.

Samenvatting

De invloed van milieufluctuaties op de planetaire oppervlaktetemperatuur wordt onderzocht. Meerdere parameters die optreden in de energiebalansvergelijking worden voorgesteld door een witte ruis van het Gauss-type. De hieruit voortvloeiende stochastische differentiaalvergelijking vertoond de algemene neiging tot een dalen van de oppervlaktetemperatuur.

Zusammenfassung

Die Wirkung einer schwankenden Umgebung auf der planetärischen Grundtemperatur wird analysiert . Verschiedene Parametern aus der Energiebilanzgleichung werden durch ein weisses Gaussrauschen beschrieben. Die daraus folgende stochastische Differentialgleichung zeigt eine Driftbewegung nach kleineren Grundtemperaturen.

In recent years, a lot of effort has been devoted to the development of simple energy-balance climatic models. Although explicit consideration of latitudinal energy transfer[1-4] gives certainly rise to more complete answers it has become increasingly clear that global, "zero dimensional" models may also provide a lot of useful information[5,6]. These models have the following form :

$$C \frac{dT}{dt} = Q\,(1 - a(T)) - \varepsilon\,\sigma\,T^4 \qquad (1)$$

T is the surface temperature, C the thermal inertia coefficient, Q the solar constant, σ the Stefan constant, a(T) the (generally temperature dependent) albedo, and ε the emissivity of the earth-atmosphere system. The variability of the climate system rests, therefore, on various types of change experienced by the solar output, or by such planetary factors as emissivity, albedo, cloudiness and so forth. Now, in addition to some long term trends of the solar constant[7], it has been conjectured recently that the sun is in an almost-intransitive state[8,9]. Hence, it may generate large fluctuations around some mean value of its output, which will be perceived by the earth-atmosphere system as an "external noise" affecting the coefficient Q in eq. (1). One may argue that the very fact that the terrestrial atmosphere is likely to be itself in an almost intransitive state[10] can also generate appreciable fluctuations in factors influencing the albedo and the emissivity. In the absence of precise knowledge of the mechanism of these fluctuations, one will again be tempted to regard them as an "external noise" affecting the coefficients a(T) and ε in eq. (1).

The purpose of this note is to explore the qualitative effect of such environmental fluctuations in the thermal regime, at the level of a zero-dimensional planetary model. Previous analyses of nonlinear systems of chemical and biological interest[11], [12] have shown that external noise can indeed affect dramatically the macroscopic behavior predicted by the deterministic equations of evolution, if coupled to these equations in a multiplicative way.

As we see later, all cases we shall be interested in refer to such multiplicative coupling. In this respect, our analysis differs from the work by Hasselmann[13], Lemke[14] and Fraedrich[5] which is concerned primarily with additive fluctuations. Although such fluctuations are certainly of importance, it is clear that, in general, the multiplicative coupling is a more appropriate representation of the dynamics of a complex system like the earth-atmosphere one.

From the mathematical point of view, under this effect of external noise eq. (1) becomes a stochastic differential equation, to which the Itô calculus can be applied[15]. Let us adopt for the moment a somewhat abstract notation and decompose eq. (1) into two pieces, f(T) and g(T) which are linearly coupled through some parameter λ

$$\frac{dT}{dt} = f(T) + \lambda\, g(T) \tag{2}$$

Let now λ be subject to the environmental fluctuations referred to above. For simplicity, we take the highly idealized picture of a Gaussian white noise*). Hence[15] :

$$\lambda = \bar{\lambda}\,(1 + \xi_t)$$

where $\bar{\lambda}$ is the value of λ appearing in the deterministic description and ξ_t is the derivative of a Wiener process, W_t :

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*) We are fully aware of the possible shortcomings of white noise, related to the possibility that the fluctuating parameters reach negative values. Therefore, in each case one has to check carefully the nature of the boundaries of the stochastic process.

$$\langle \xi_t \rangle = 0 \tag{3}$$

$$\langle \xi_t \, \xi_{t'} \rangle = q^2 \, \delta(t \quad t')$$

Eq. (2) takes then the form. (We choose hereafter the Itô rather than the Stratonovich interpretation[15]) :

$$dT = [f(T) + \bar{\lambda} \, g(T)] \, dt + q \, \bar{\lambda} \, g(T) \, dW_t \tag{4}$$

which is equivalent to a Fokker-Planck equation for the probability distribution P(T,t) :

$$\frac{\partial P}{\partial t} = - \frac{\partial}{\partial T} \left[f(T) + \bar{\lambda} \, g(T) \right] P(T, t) + + \frac{1}{2} q^2 \frac{\partial^2}{\partial T^2} \bar{\lambda}^2 \, g^2(T) \, P(T, t) \tag{5}$$

A macroscopically observable stable temperature clearly corresponds to a maximum of the steady-state distribution $P_{st}(T)$. One can easily show that the latter obeys to the equation :

$$- [f(T_m) + \bar{\lambda} g(T_m)] + \frac{q^2}{2} \frac{d}{dT} [\lambda^2 g^2(T_m)] = 0 \tag{6}$$

We now apply successively this equation to the following situations:

(i) Q is fluctuating around the present-day value of 1.95 cal cm^{-2} min^{-1}. If the planetary albedo, a, is taken to be constant, the noise becomes additive and eq. (6) gives $dg^2/dT = 0$; hence one obtains the usual value of planetary temperature corresponding to present-day conditions. On the other hand, the fluctuations of Q begin to be effective if the planetary albedo depends on T. The simplest non-trivial such dependence is given by the linear function $a(T) = \alpha + \beta T$ (T beeing in degrees Kelvin). Such expressions have been used widely by several authors[5,6,16], at least in the range between the highest temperature that can exist when the whole earth is ice-covered, and the lowest possible temperature when no ice is present. Note that some pathological properties of linear albedo-temperature feedback pointed out by Schneider and Gal-Chen for one-dimensional energy-balance models[17] are not necessarily reproduced in a zero-dimensional planetary model. In the latter, the albedo must depend on the planetary temperature in a continuous fashion at least in a range of temperatures not too remote from present-day conditions.

(ii) ε is fluctuating around some mean value $\bar{\varepsilon}$ corresponding to a surface temperature of 287.6°K in the absence of fluctuations. The planetary albedo is taken to be, successively, equal to a constant value $\bar{a}$ or to a linear function of T. Physically speaking, fluctuations of ε at fixed albedo could be attributed to random modifications of atmospheric composition.

(iii) a and ε are both following the fluctuations of cloudiness, n. The parameterizations of a and ε in terms of n are taken from Cess[16] and Schneider[18], and

$$n \doteq 0.5 \ (1 + \xi_t) \qquad (7)$$

Let us now describe the results of some representative numerical simulations referring to the above three situations.

Figure 1 depicts T_m as a function of q, the square root of the variance of Q. As q increases from 0.005 to 0.05 the most probable temperature decreases from about 287.6°K to 287°K. For higher q the decrease of T_m is more pronounced.

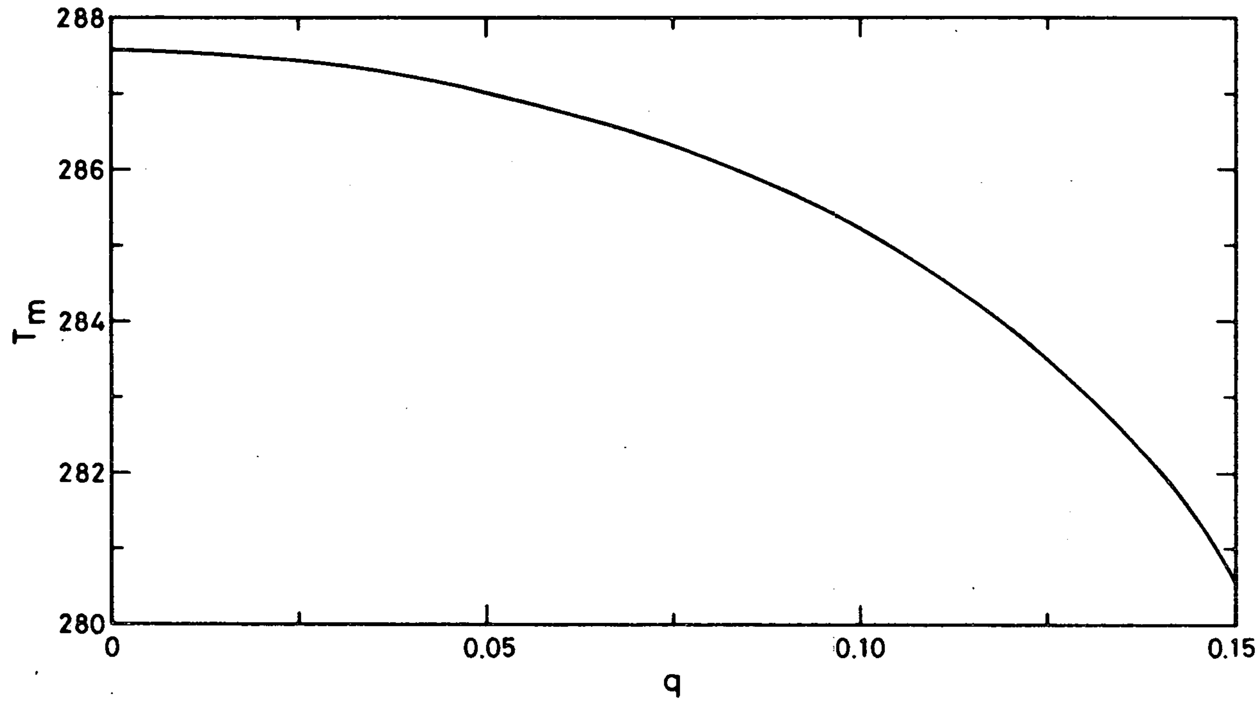

Fig. 1.- Dependence of most probable temperature T_m on variance q. Q fluctuates around the mean value of 1.95 cal cm^{-2} min^{-1}. The thermal inertia coefficient used in the simulation is $C = 7.5$ Kcal cm^{-2} °K^{-1}. The emissivity ε is kept at the value 0.6, and the albedo is $a = \alpha + \beta T$ where $\alpha = 2.7888$, $\beta = - 0.0086$.

Finally, for $q \geqslant 0.16$, T_m drops to unphysically low values. The precise numbers are of no importance here, as for such ranges of temperature the model should not be valid anyway, and additional feedback mechanisms could take over. The main point to retain therefore from this figure is the tendency for runaway to lower temperature values.

In curve (b) of Figure 2 we give the analogous result in terms of the variance of ε for a linear albedo-temperature feedback (note that in this case eq. (6) is an equation of seventh degree in T_m, i.e. of a degree higher than that of the deterministic equation). Again, for $q \geqslant 0.16$ there is a similar tendency for runaway to extremely low values of T_m. Curve (a) describes the situation in the constant albedo case. We see that the same tendency to lower T is observed, although there appears to be no runaway effect.

The effect of fluctuating cloudiness is rather different as described in Figure 3 (here again, the equation for T_m is of seventh degree). As q increases from 0 to 0.30, the temperature remains substancially the same, varying by less than one tenth of a degree. It appears that the fluctuations affect in opposite ways the input and output terms in eq. (4), thus giving a small overall effect. A second reason of the smallness of the effect may be that in the parameterization of eq. (7) cloudiness is taken to be temperature-independent.

In conclusion, the thermal regime in a fluctuating environment appears to be established at temperatures that may differ substantially from those corresponding to a constant environment (actually, there appears to be always a tendency toward decreasing values). Eventually, in the presence of giant fluctuations a runaway regime may be reached in which there cannot be a stationary solution in a physically reasonable temperature range. These unexpected phenomena are all due to the multiplicative coupling between fluctuating parameters and the temperature[11]. In the case of additive noise[5,13,14] the effect

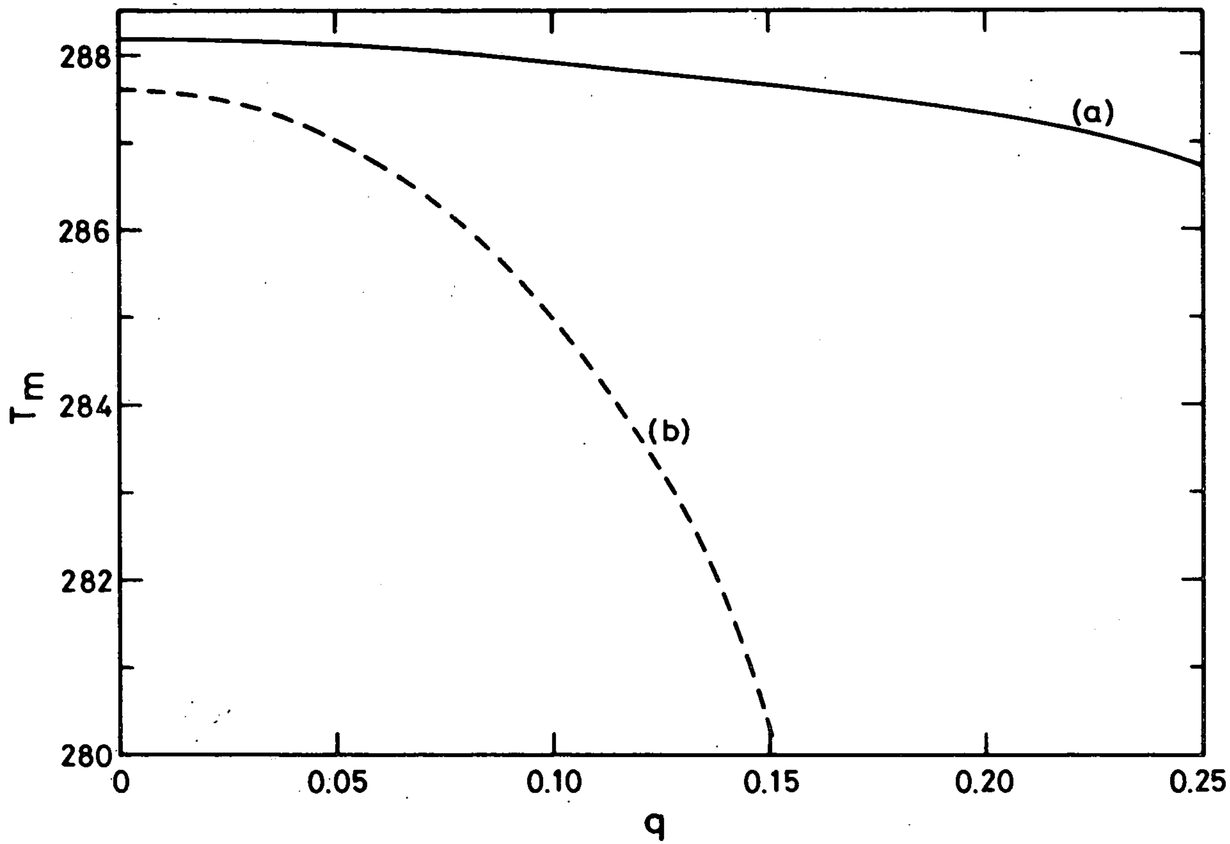

Fig. 2.- T_m versus q when ε fluctuates around the value of 0.6. The thermal inertia coefficient is as in Fig. 1. Curve (a) : the albedo is taken to be constant, $\bar{a}$ = 0.31. Curve (b) : the albedo is discribed by a = α + βT where α = 2.788, β = - 0.0086.

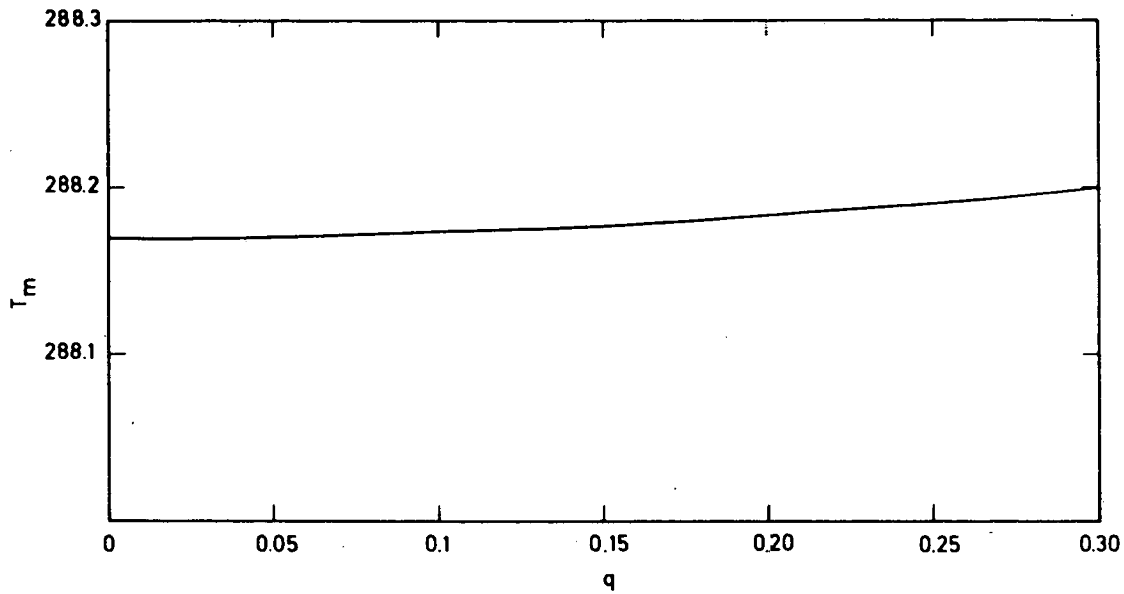

Fig. 3.- Effect of fluctuating cloudiness using the following parameterizations. For the albedo (see ref. 16) : $a = 0.18 + 0.26\ n$. For the emissivity, (see ref. 18, for an effective cloud height of about 6.5 km) : $\varepsilon = 0.71 - 0.22\ n$.

would vanish, as the function g^2 in eq. (6) would be constant. One would thus recover the solutions of the deterministic rate equation . It would be interesting to extend the present investigation to more realistic types of noise and to conditions prevailing in other planets, where a runaway regime is believed to exist.

ACKNOWLEDGMENTS

We acknowledge a fruitful discussion with Dr. W. Horsthemke.

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