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Fluctuations, solar periodicities, and climatic transitions

by

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B E L G I S C H   I N S T I T U U T   V O O R   R U I M T E - A E R O N O M I E

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FOREWORD

This article has been presented at the International Conference "Sun and Climate". It will be published in the proceedings volume edited by the "Centre National d'Etudes Spatiales".

AVANT-PROPOS

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VOORWOORD

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VORWORT

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FLUCTUATIONS, SOLAR PERIODICITIES, AND CLIMATIC TRANSITIONS

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C. NICOLIS

Abstract

The time-dependent properties of a zero-dimensional climatic model, showing bistable behavior and subject both to internal fluctuations and to an external periodic forcing are analyzed. Conditions under which the deterministic response of the system is amplified are found analytically. The results are illustrated by numerical simulations.

Résumé

On étudie le comportement d'un modèle climatique global donnant lieu à deux états stationnaires stables, sous l'influence des fluctuations internes ainsi que d'une perturbation périodique systématique d'origine externe. On détermine les conditions d'amplification de la réponse de ce système, à la fois analytiquement que par simulations numériques.

Samenvatting

De tijdsafhankelijke eigenschappen van een nul-dimensionaal klimatisch model met een bi-stabiel gedrag en onderworpen aan interne fluctuaties en een uitwendige periodieke storing worden geanalyseerd. De voorwaarden onder dewelke het deterministisch antwoord van het systeem wordt versterkt, wordt analytisch bepaald. De resultaten worden geillustreerd met behulp van numerieke simulaties.

Zusammenfassung

Die zeitliche Eigenschaften eines nulldimensioniertes klimatisches Model mit zweibeständigem Betragen und mit innerlichen Schwankungen und aüsserlicher periodischer Kraft sind analysiert. So findet man analytische Bedingungen die zu einer Vergrössung der deterministische Antwort des Systemen leiten. Die Ergebnisse werden durch numerische Simultationen erläuternt.

I. INTRODUCTION

One of the most characteristic features of the climatic system is a very pronounced variability, encountered at widely separated time scales. Thus at a scale smaller than, or of the order of a year, the almost intransitive character of atmospheric processes may cause seemingly erratic variations of temperature or moisture patterns. At the scale of the decade a correlation is often suggested between climatic variations and solar cycles (Pittock, 1978) which themselves display a considerable amount of noise around a mean periodicity. Finally, at longer time scales, of the order of 10^3 to 10^5 years, the interaction between atmosphere, hydrosphere and cryosphere together with the earth's orbital variations appear to have given rise to the glaciation cycles, which are certainly the most dramatic climatic episodes of the Quaternary era (see for instance Imbrie and Imbrie, 1980).

One of the simplest representations of the internal dynamics of climate is given by the zero-dimensional (0-d) and one-dimensional (1-d) energy balance models. As well known such models predict climatic transitions reminiscent of glaciations (North et al, 1980). However, for usually accepted parameter values, the time scale of evolution is far too short (of the order of few years) and cannot possibly explain long term effects associated to glaciations. Moreover the response of such models to an external forcing such as that associated to the earth's orbital variations has also been examined, but found to be very weak and hence incapable of triggering a major climatic change (North and Coakley, 1979). True, if some of the physical mechanisms related to cryospheric dynamics are incorporated in a more detailed manner, one can stretch the time scale and amplify the amplitude of the response (Pollard, 1978). Again, however, very long time responses, specifically with a scale near 10^5 years (the dominant periodicity in glaciation cycles) are not obtained (Ghil, 1980).

Now, in all complex systems -and the earth-atmosphere system is definitely one- there are continuous imbalances between the rates of the various processes going on. Such imbalances are perceived by the system as a stochastic forcing around the deterministic evolution, and are called fluctuations. An individual fluctuation is, typically, a small amplitude event. Yet in a potentially unstable system even small random disturbances associated with fluctuations will sooner or later drive the system to a new regime.

The purpose of the present work is to show that fluctuations provide the long time scale that is missing from the deterministic equations of evolution, and should therefore play an important role in the understanding of glaciation mechanisms. In section 2 the stochastic description is set up. In Section 3 we introduce a simple 0-d model and review the properties of the deterministic response to a periodic variation of incoming solar energy. Sections 4 and 5 are devoted, respectively to the analytic and numerical study of the stochastic response to such a variation. The main result is that the response is considerably amplified when a matching between a characteristic time scale related to fluctuations and the periodicity of the incoming solar energy occurs. The main conclusions are drawn at the end of section 5.

2. STOCHASTIC FORMULATION

Let $\bar{x}$ denote a climatic variable obeying to a closed equation of evolution. A typical example is the surface temperature T averaged over space coordinates. In the absence of fluctuations $\bar{x}$ is supposed to obey to the following dynamics :

$$\frac{d\bar{x}}{dt} = f(\bar{x}, \lambda, t) \equiv f_0(\bar{x}, \lambda) + \varepsilon\, f_1(\bar{x}, \lambda, t) \qquad (2.1)$$

Here f is an appropriate nonlinear rate function, and λ stands for a set of characteristic parameters such as albedo, emissivity and so forth. This function is decomposed into a part f_o corresponding to an autonomous evolution, and to a time-dependent part f_1 describing the effect of some external forcing proportional to ε. Of special interest for our work are cases where the steady-state solutions of the system in the absence of the above time-dependent forcing,

$$f_o(\bar{x}_s, \lambda) = 0 \qquad (2.2)$$

are multiple and see their stability properties change as the parameters λ take different values.

As discussed in the Introduction, the deterministic description must often be extended to take into account the fluctuations, associated with random imbalances between the various transport and radiative mechanisms involved in the rate function $f(\bar{x}, \lambda, t)$. We denote their effect by a random force F(t) and assume the latter to be x-independent and define a white noise (Wax, 1954) :

$$< F(t) > = 0$$

$$< F(t) F(t') > = q^2 \delta(t - t') \qquad (2.3)$$

Eq. (2.1) is now to be replaced by the stochastic differential equation

$$\frac{dx}{dt} = f(x, \lambda, t) + F(t) \tag{2.4}$$

As well known (see e.g. Arnold, 1973) eqs. (2.3) - (2.4) are equivalent to the following Fokker-Planck equation with nonlinear friction coefficient and constant diffusion coefficient :

$$\frac{\partial P(x,t)}{\partial t} = - \frac{\partial}{\partial x} f(x, \lambda, t) P(x,t) + \frac{q^2}{2} \frac{\partial^2 P(x,t)}{\partial x^2} \tag{2.5}$$

where $P(x,t)$ is the probability density for having the value x of the state variable at time t.

It should be realized that eqs (2.3) defining the properties of the random force are in principle rather restrictive. Nevertheless, we expect them to describe satisfactorily the situation for roughly the same reason as in brownian motion and other problems in statistical mechanics : Namely, because of their local character, the fluctuations of various fluxes are expected to loose rapidly the memory of the state of the system which prevailed when they occurred and, partly as a result of this, to occur independently of each other. Further arguments in essentially the same direction have been developed by Hasselmann (1976).

For a nonlinear function $f(x, \lambda, t)$, the full analysis of eq. (2.5) constitutes an unsolved problem. Let us therefore first focus on the steady-state solution, $\partial P_s/\partial t = 0$, in the absence of the time-dependent forcing terms $\varepsilon = 0$. Integrating once the right hand side with respect to x we get :

$$- J_{P,s}(x) \equiv - f_o(x, \lambda)\, P_s(x) + \frac{q^2}{2} \frac{\partial P_s}{\partial x} = \text{constant} \tag{2.6}$$

Now, in all physically reasonable situations we expect that when x will reach the boundaries of the process (e.g. O and ∞ if x is the temperature), P_s will tend very rapidly to zero. We may therefore set the <u>probability flux</u> $J_P(x)$ zero at the steady state :

$$J_{P,s}(x) = 0 \qquad \text{for all } x \tag{2.7}$$

This is known as generalized detailed balance condition (Haken, 1977) and leads to an exact solution for P_s in the form :

$$P_s(x) = Z^{-1} \exp\left[- \frac{2}{q^2} U_o(x) \right] \tag{2.8}$$

where we defined the kinetic potential $U_o(x)$ for the autonomous part of the evolution :

$$U_o(x) = - \int^x d\xi\, f_o(\xi, \lambda) \tag{2.9}$$

The proportionality constant Z^{-1} is determined from the normalization of P_s

$$\int_D dx\, P_s(x) = 1$$

where D is the domain of variation of x :

$$Z = \int_D dx \exp \left[- \frac{2}{q^2} U_o(x) \right] \tag{2.10}$$

In Nicolis and Nicolis (1980) a detailed analysis of the stationary probability P_s is reported. Therein we were concerned with the solution of the Fokker-Planck equation in the case of an autonomous evolution. Depending on the choice of the parameters describing the system, three different situations were considered :

i) The probability distribution is peaked near the present climate.
ii) The statistical weights of the present climate and of a deep freeze climate are approximately equal.
iii) The probability distribution is peaked around a deep freeze climate.

In each of these three typical situations, the evolution of the probability distribution has been followed for different values of the variance of fluctuations q^2, and for different initial conditions. Special emphasis was put on the characteristic time of passage between present day and deep freeze climates. In this respect a set of parameters was found for which this characteristic time was of the order of a glaciation period. For technical reasons the analysis was limited to a 0-d energy balance model which is briefly outlined in the next section.

3. A SIMPLE ZERO-DIMENSIONAL MODEL. CASE OF PERIODIC FORCING

Suppose that $\bar{x}$ denotes the average surface temperature. The rate function f in eq. (2.1) is then the difference between the solar influx $Q(1 - a(\bar{x}))$ [a being the albedo] and the infrared cooling rate,

$\varepsilon_B \sigma x^4$, [ε_B being the emissivity and σ the Stefan constant]. Eq. (2.1) becomes :

$$\frac{d\bar{x}}{dt} = \frac{1}{C}\left[Q(1 - a(\bar{x})) - \varepsilon_B \sigma \bar{x}^4\right] \qquad (3.1)$$

where C is the thermal inertia coefficient.

In the majority of climate models Q is taken to be constant. On the other hand, it is known that the solar output displays very pronounced variability at different time scales. One example is the sunspot cycle which despite an inherent noise, shows an approximate 11 year periodicity. Another example is the slight change in the mean annual influx arising from the variation of the eccentricity of the earth's orbit whose periodicity is about 10^5 years (Berger, 1978). Hereafter we are interested in the effect of such time dependent forcing, in the presence of fluctuations. To simplify the analysis as much as possible we describe the above mentioned nearly periodic variation in the form

$$Q = Q_o(1 + \varepsilon \sin \omega t) \qquad (3.2)$$

The unperturbed solar constant is taken to be $Q_o = 340\ \mathrm{Wm}^{-2}$.

For temperature values near the present-day climate, $a(\bar{x})$ is usually taken to be a roughly linear function of its argument (Cess, 1976; Nicolis, 1980). On the other hand, for very low $\bar{x}$ a must tend to the albedo of ice, a_{ice} whereas for high $\bar{x}$, a should also saturate to some value, a_{hot} descriptive of an ice-free earth. The simplest representation taking these features into account is the zero-dimensional piecewise linear model proposed by Crafoord and Källén (1978) and summarized in Fig. 1. Analytically, we write :

$$
\begin{aligned}
1 - a(\bar{x}) &= 1 - a_{ice} = \gamma_1 , && \bar{x} < T_1 \\
1 - a(\bar{x}) &= 1 - \alpha + \beta\bar{x} = \gamma_0 + \beta\bar{x} , && T_1 < \bar{x} < T_2 \qquad (3.3) \\
1 - a(\bar{x}) &= 1 - a_{hot} = \gamma_2 , && \bar{x} > T_2
\end{aligned}
$$

Using the explicit dependence of the albedo on T as given by eqs. (3.3) in eq. (3.1) we see that in the absence of periodic forcing and for appropriate values of the parameters γ_0, γ_1, γ_2 and β the system may admit <u>three</u> steady state solutions. One of them, denoted hereafter by T_+, corresponds to the present-day climate and is asymptotically stable, provided the parameters γ_0 and β are chosen in such a way that the planetary albedo is 0.30 and the emissivity is ε_B = 0.61. The second solution, denoted by T_-, corresponds to a deep-freeze climate and is also asymptotically stable. A third solution T_0 lies between T_+ and T_- and is unstable.

Before we analyze the stochastic properties of the system defined by eqs. (3.1) to (3.3) we briefly review the main features of the deterministic response. Using the relations (3.3) we first write the energy balance equation in the form (cf. also eq. (2.9)) :

$$
\begin{aligned}
\frac{d\bar{x}}{dt} &= \frac{1}{C} [Q_o(1 - a(\bar{x})) - \varepsilon_B \sigma \bar{x}^4] + \frac{1}{C} Q_o \varepsilon(1 - a(\bar{x})) \sin\omega t \\
&= - U'_o(\bar{x}) + \frac{1}{C} Q_o \varepsilon(1 - a(\bar{x})) \sin\omega t
\end{aligned}
$$

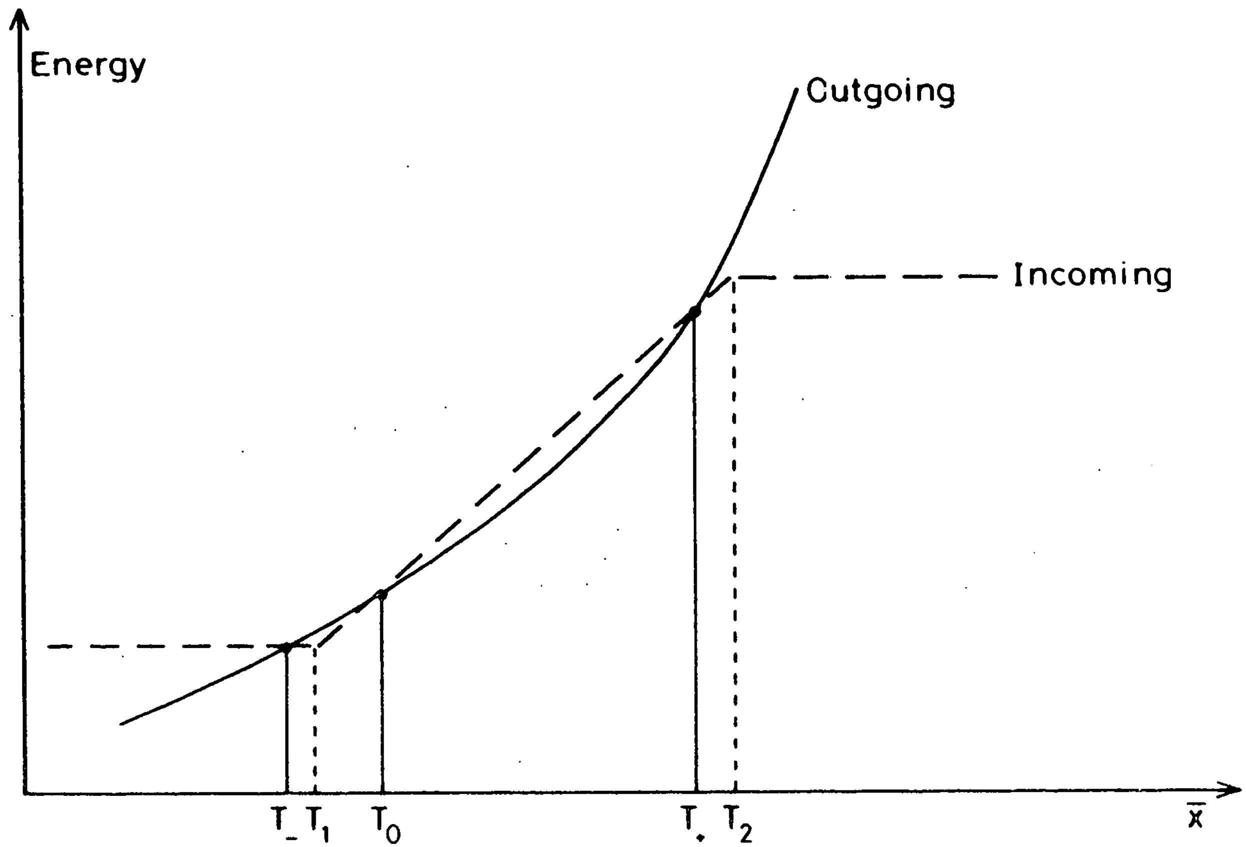

Fig. 1.- Incoming and outgoing radiative energy curves as functions of $\bar{x}$ (global average temperature). Their intersections T_+, T_- and T_0 are the three steady states.

where U'_o denotes the derivative of the potential introduced in section 2 with respect to its argument. As a rule, ε is small. Hence, to a good approximation one may linearize the above equation around the stable states T_+ and T_-. Setting

$$\bar{x}_\pm = T_\pm + \delta T_\pm \tag{3.4}$$

we obtain :

$$\begin{aligned}\frac{d\delta T_\pm}{dt} &= - U''_o(T_\pm)\ \delta T_\pm + \frac{1}{C} Q_o\ \varepsilon(1 - a(T_\pm))\ \sin\omega t \\ &\equiv - U''_{o\pm}\ \delta T_\pm + \frac{1}{C} Q_o\ \varepsilon(1 - a_\pm)\ \sin\omega t\end{aligned} \tag{3.5}$$

In the limit of long times the response around the present-day climate predicted by eq. (3.5) is of the form

$$\delta T_+(t) = \frac{1}{C}\ \frac{1 - a_+}{\omega^2 + (U''_{o+})^2}\ Q_o\ \frac{\varepsilon U''_{o+}}{\cos\theta}\ \sin(\omega t + \theta) \tag{3.6a}$$

where the signal-response phase shift is given by

$$\mathrm{tg}\ \theta = - \frac{\omega}{U''_{o+}} \tag{3.6b}$$

From this expression we see that if $\omega \ll U''_{o+}$ (that is, if the periodicity of the forcing is very long), the phase shift practically vanishes and the amplitude of the response is independent of the thermal inertia coefficient C. As we see later this conclusion changes radically when fluctuations are taken into account.

4. STOCHASTIC RESPONSE

In order to evaluate the stochastic response to the periodic forcing introduced in the preceeding Section, it is necessary to analyze the time-dependent solutions of the Fokker-Planck equation (2.5). Actually, what one is interested in is the long term (time-dependent) regime induced by the forcing, rather than the transient behavior associated with the deterministic time scale $(U''_{o\pm})^{-1}$ featured by eq. (3.5). A detailed description of this analysis is given by Gardiner (1980) and Nicolls (1980). Let us briefly summarize the main results.

Firstly, in the vicinity of the stable deterministic solutions $x_{\pm}(t)$ (which are themselves near the present climate T_+ and the deep freeze climate T_-) it turns out that the distribution function can be represented by two Gaussians peaked on these states, provided that the variance q^2 is sufficiently small. This yields (see Fig. 2) :

$$P(x,t) \cong N_-(t) \frac{1}{(\pi q^2 \sigma_-(t))^{1/2}} \exp\left[- \frac{(x - \bar{x}_-(t))^2}{q^2 \sigma_-(t)} \right]$$

$$+ N_+(t) \frac{1}{(\pi q^2 \sigma_+(t))^{1/2}} \exp\left[- \frac{(x - \bar{x}_+(t))^2}{q^2 \sigma_+(t)} \right] \qquad (4.1)$$

where $\sigma_{\pm}$ are the mean square widths.

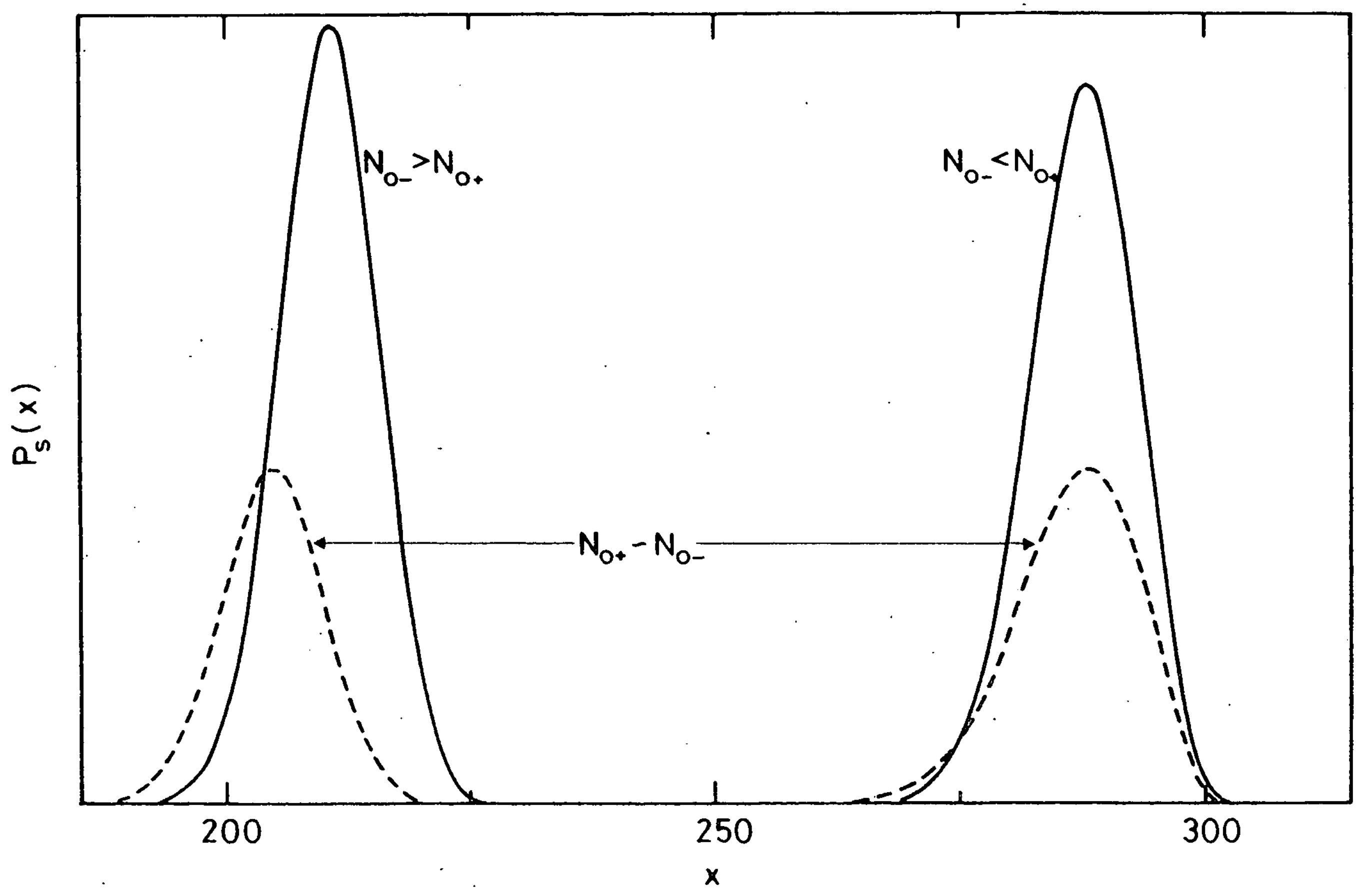

Fig. 2.- Steady-state probability distribution $P_s(x)$ for three representative cases : $N_{o+} < N_{o-}$, $N_{o+} \sim N_{o-}$ and $N_{o+} > N_{o-}$ and for q = 7. Note that, in the second case, $P_s(x)$ is a two-hump distribution. The following parameter values have been used in this latter case : β = 0.0075, a_{ice} = 0.82, a_{hot} = 0.25.

The factors N_- and $N_+ = 1 - N_-$ ensure the normalization of $P(x,t)$ for all t's in the whole domain of x and describe the relative importance of the present day and the deep-freeze climate. If these two climates are equally dominant (see remarks at the end of Section 2), then the steady-state values of N_{o+}, N_{o-} in the absence of time-dependent forcing will be approximately equal,

$$N_{o+} \sim N_{o-} \sim 0.5 \tag{4.2}$$

It can be shown (Nicolis and Nicolis, 1980) that this corresponds to the equality of the part of U_o of the potential that is independent of the time-dependent forcing, evaluated in states T_+ and T_- :

$$U_{o+} \sim U_{o-} \tag{4.3}$$

Starting from a given initial condition of the probability distribution, the main problem of interest is to find the time evolution of the weighting factors N_- and N_+ under the effect of both internal fluctuations and the external forcing term. To simplify matters we will focus attention on the response to a forcing ε which is so small that the deterministic effect, eq. (3.6), is negligible. This allows us to consider that the maxima of the probability distribution remain fixed at T_+ and T_-

$$\bar{x}_{\pm} = T_{\pm} \tag{4.4}$$

In other words only interpeak relaxation of the probability distribution will be considered.

Using a method similar to Kramers' theory of chemical reaction rates we find :

$$\frac{dN_-}{dt} = r(t) - s(t)\, N_- \tag{4.5}$$

with

$$r(t) = \frac{1}{2\pi} \left(- U''(x_o)\, U''(x_-)\right)^{1/2} \exp\left(- \frac{2}{q^2}\ \Delta U_-\right) \tag{4.6a}$$

$$s(t) = \frac{1}{2\pi} \left(- U''(x_o)\, U''(x_+)\right)^{1/2} \exp\left(- \frac{2}{q^2}\ \Delta U_+\right) + r(t) \tag{4.6b}$$

where

$$\Delta U_\pm = U(x_o) - U(x_\pm) > 0 \tag{4.7}$$

This quantity represents the potential barrier that has to be overcome by the system in order to perform a transition from T_+ to T_- or vice versa.

From eq. (4.5) one can immediately see that the coefficient of N_- defines the inverse of the characteristic time associated with a transition of the system. For $q^2 \ll \Delta U_\pm$ (i.e. for small fluctuations) this time will be very long. We have therefore succeeded in identifying the missing long time scale pointed out in the Introduction.

Now, owing to the smallness of the periodic forcing it is legitimate to linearize expressions (4.6) around the autonomous evolution

$$r(t) = r_o + \rho \sin\omega t \tag{4.8a}$$

$$s(t) = s_o + \sigma \sin\omega t \tag{4.8b}$$

This allows us to seek for time dependent solutions of the form

$$N_- = \hat{N}_- \sin(\omega t + \varphi) + N_{o-} \tag{4.9}$$

After some long but straightforward calculations we find the following expressions for the amplitude and the phase shift

$$\hat{N}_- = \frac{1}{\left(1 + \frac{\omega^2}{s_o^2}\right)^{1/2}} \quad \varepsilon \left[\frac{\rho - N_{o-} \sigma}{s_o} \right] \tag{4.10}$$

$$\varphi = - \text{arctg} \frac{\omega}{s_o} \tag{4.11}$$

We have analyzed eq. (4.10) for the three representative cases mentioned in Section 2. We found that for case (i) and case (iii), its numerical value is exponentially small. However, the situation changes if the present climate and a deep-freeze climate are equally probable (case(ii)). In that case, for typical values of the variance of fluctuations, the factor in square brackets is of the order of 10^2. For a forcing amplitude of 0.001 corresponding to the eccentricity variation of the earth's orbit (see e.g. Imbrie and Imbrie, 1980), $\hat{N}_-$ would therefore be conditioned by the inverse of the square root. The latter depends on the ratio of the two inverse characteristic time scales ω and

s_o. For usual values of q^2 and $\Delta U_{\pm}$, s_o is a very small quantity. Therefore, if ω is of the order of 1 (such as the frequency associated with the 11 or 22-year solar cycle), the first factor in eq. (4.10) would be exceedingly small and the stochastic response to this type of forcing would be negligble.

The situation is completely different if ω and s_o are of the same order of magnitude. One then obtains an amplitude of N_- of the order of 0.1, which is quite appreciable compared to the steady-state value $N_{o-} \sim 0.5$ one would obtain in the absence of forcing when the two states T_+ and T_- are equally dominant. Everything happens as if the barrier that has to be overcome for a transition between T_+ and T_- say, (reminiscent of a glaciation) becomes significantly smaller for certain time intervals. The situation is represented in curve (a) of Fig. 3 and Fig. 4.

Similar conclusions have been reached by Benzi et al (1980) on the basis of computer simulations. They refer to this phenomenon as <u>stochastic resonance</u>. As we see however from eq. (4.10) the system does not exhibit a resonance in the usual sense of the term, but rather the ability to amplify the response to a low frequency forcing under certain conditions.

5. NUMERICAL RESULTS - CONCLUDING REMARKS

For the model described by eqs. (3.1) to (3.3) and by Fig. 1 the time dependent Fokker-Planck equation, eq. (2.5), was integrated numerically using a method develloped by Chang and Cooper (1970). First, the steady state probability distribution in the absence of forcing was obtained. And next, the forcing was added and the long time behavior of the probability was determined.

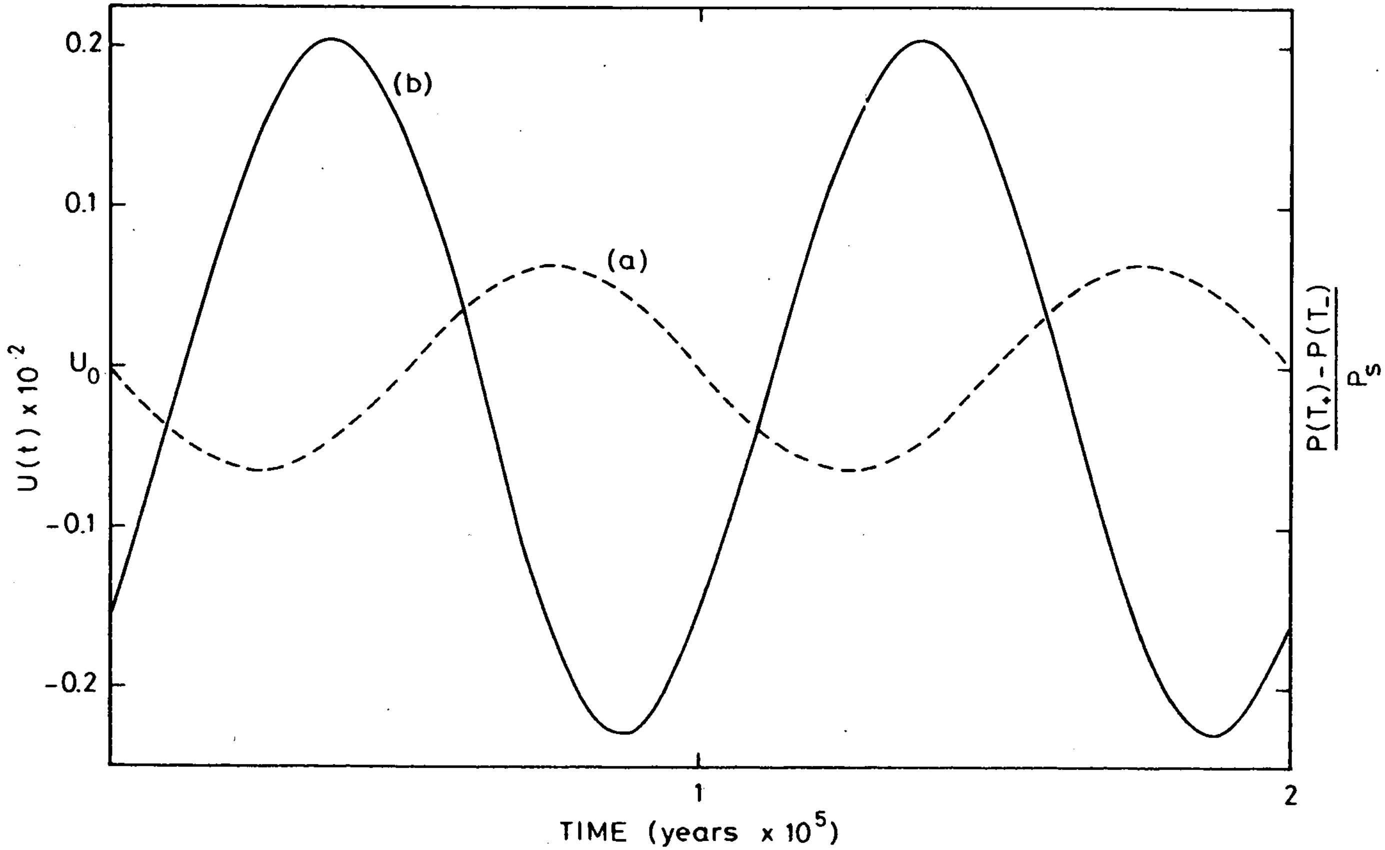

Fig. 3.- Curve (a) : Time dependence of the climatic potential difference $U(t) = U(x_+,t) - U(x_o,t)$, subject to a periodic forcing with $\omega = \frac{2\pi}{10^5}$ and an amplitude $\varepsilon = 0.001$ simulating the variation of the eccentricity of the earth's orbit.

Curve (b) : Time evolution of the difference of the probabilities of the two stable states T_+ and T_-, divided by the steady state probability $P_s \sim P_s(T_+) \sim P_s(T_-)$, in the presence of the forcing represented in curve (a). Here and in Fig. 4 the time scale is normalized in such a way that C = 1. The values of the other parameters are as in Fig. 2.

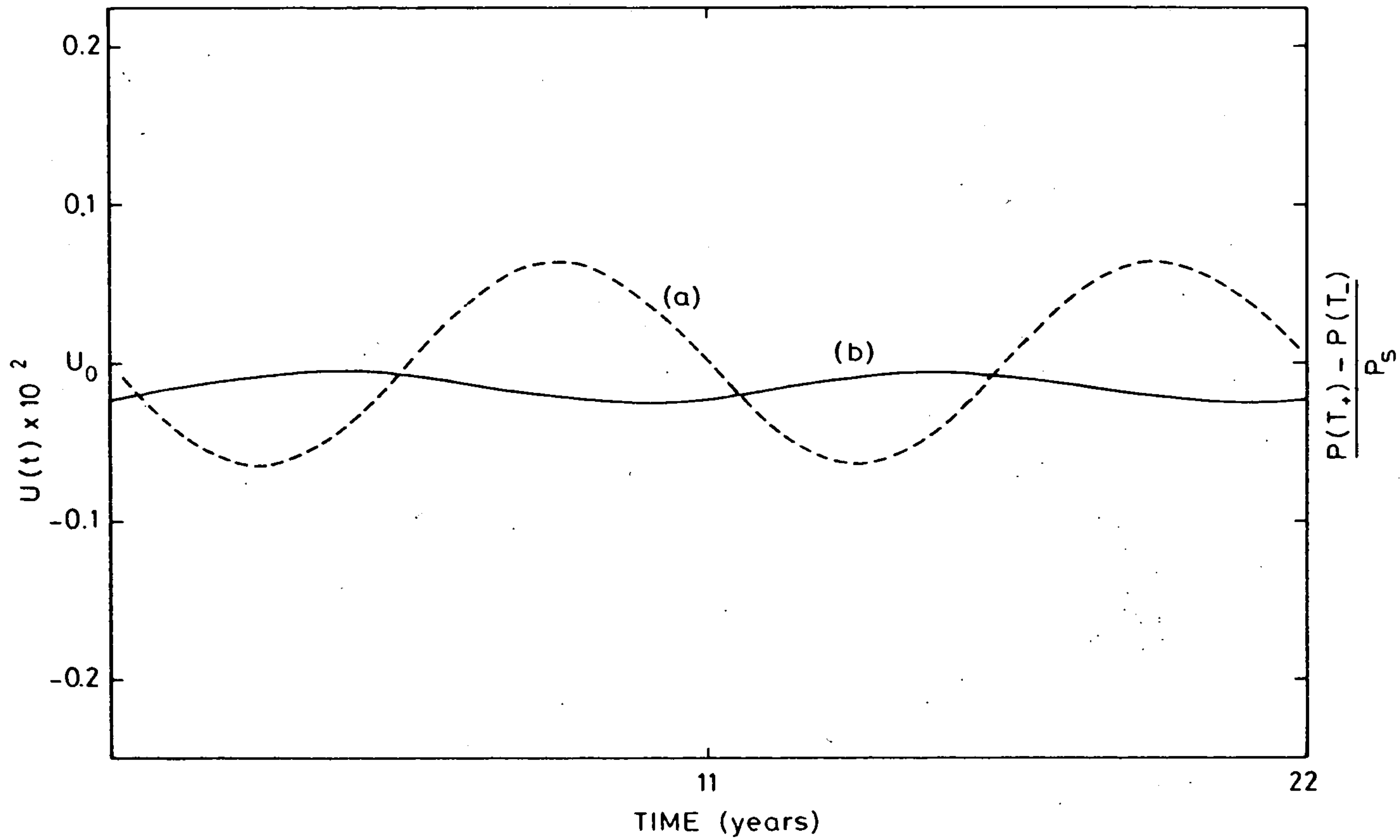

Fig. 4.- Curve (a) : Time dependence of the climatic potential difference $U(t) = U(x_+,t) - U(x_o,t)$, subject to a periodic forcing with frequency $\omega = \frac{2\pi}{11}$ and an amplitude $\varepsilon = 0.001$.

Curve (b) : Time evolution of the difference of the probabilities of the two stable states T_+ and T_-, divided by the steady state probability $P_s \sim P_s(T_+) \sim P_s(T_-)$, in the presence of the forcing represented in curve (a).

Curve (b) of Fig. 3 gives the main result in the case of a long periodicity simulating the 100,000 year variation of eccentricity. We start with a steady-state solution in the absence of forcing such that $P_s(T_+) \sim P_s(T_-)$, and choose the variance q^2 such that $\omega = s_o$ (see eq. 4.10). The presence of forcing introduces then a rather dramatic variation of $(P(T_+)-P(T_-))$, of the order of 20% compared to the steady state value. This reflects the fact that the passage over the barrier becomes easier during certain time intervals. Note that there is a considerable time lag between forcing (curve (a)) and response (curve (b)), in quantitative agreement with eq. (4.11).

Curve (b) of Fig. 4 gives the stochastic response to the 11-year cycle. We see that $P(T_+) - P(T_-)$ is now practically negligible in agreement with the analytical expression, eq. (4.10).

In summary, in this paper we performed a stochastic analysis of a simple 0-d energy balance model showing bistable behavior, in the presence of a periodic forcing. The amplitude of the forcing was so small that the deterministic response was negligible. Yet in the presence of fluctuations, the amplitude of the response could change dramatically, depending on two basic quantities : i) the properties of the climatic potential and ii) a characteristic time scale related to the variance of fluctuations. Under certain conditions the passage over the potential barrier is facilitated and the shape of the probability distribution changes periodically, favoring one of the stable states during certain time intervals. An attempt was made to relate these results to the 100,000 yr periodicity in glaciation cycles.

The work we reported can be extended in many directions. It would be interesting for instance to consider the effect of fluctuations which couple to the system in a multiplicative way through such parameters as Q and ε_B. Similarly, we can relax the hypothesis of purely periodic variation of the solar influx and analyze the effect of a random forcing around some mean periodicity. Finally, we could use more sophisticated climate models taking spatial effects into account. This latter extention is particularly interesting in view of the local character of the fluctuations.

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