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**Chemical fluctuations and incoherent scattering theory  
in the terrestrial D-region**

by

**G. KOCKARTS and J. WISEMBERG**

B E L G I S C H I N S T I T U U T V O O R R U I M T E - A E R O N O M I E

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## FOREWORD

The paper "Chemical fluctuations and incoherent scattering theory in the terrestrial D-region" will be published in J. Geophys. Res., 86A, 1981.

## AVANT-PROPOS

L'article "Chemical fluctuations and incoherent scattering theory in the terrestrial D-region" sera publié dans J. Geophys. Res., 86A, 1981.

## VOORWOORD

De tekst "Chemical fluctuations and incoherent scattering theory in the terrestrial D-region" zal in J. Geophys. Res., 86A, 1981 verschijnen.

## VORWORT

Die Arbeit "Chemical fluctuations and incoherent scattering theory in the terrestrial D-region" wird in J. Geophys. Res., 86A, 1981 herausgegeben werden.

# CHEMICAL FLUCTUATIONS AND INCOHERENT SCATTERING THEORY

## IN THE TERRESTRIAL D-REGION

by

G. KOCKARTS and J. WISEMBERG

### Abstract

The continuum theory of incoherently scattered electromagnetic waves is modified in order to include possible effects of thermally induced chemical fluctuations. These fluctuations are taken into account by introducing fluctuating parts in the production and loss terms of the continuity equations. An equivalent ionospheric model is developed for three types of ionized species, i.e. electrons, negative and positive ions. A matrix formulation gives simultaneous access to fluctuating parts of each charged component. Numerical results indicate that chemical fluctuations are important over the height range where negative ions are comparable or greater than the electron concentration. Strong enhancements of the incoherent scatter cross section occur for frequency shifts smaller than approximately 20 Hz. A simple approximation is given for the contribution of chemical fluctuations.

## Résumé

La théorie hydrodynamique de la diffusion incohérente des ondes électromagnétiques est modifiée pour inclure les effets éventuels résultant de fluctuations chimiques induites thermiquement. Ces fluctuations sont prises en compte en introduisant des parties fluctuantes dans les termes de production et de perte des équations de continuité. Un modèle ionosphérique équivalent est établi pour trois types de particules ionisées, c'est-à-dire les électrons les ions positifs et les ions négatifs. Une formulation matricielle conduit à un accès simultané des parties fluctuantes de chaque espèce chargée. Les résultats numériques indiquent que les fluctuations chimiques sont importantes dans le domaine d'altitude où l'abondance des ions négatifs est comparable ou supérieure à celle des électrons. Des accroissements très importants de la section efficace de diffusion incohérente ont lieu pour des décalages en fréquence inférieurs à 20 Hz environ. Une approximation simple est donnée pour représenter l'effet des fluctuations chimiques.

## Samenvatting

De hydrodynamische theorie van de incoherente verstrooiing van electromagnetische golven werd gewijzigd om eventuele effecten van thermisch geïnduceerde chemische fluctuaties op te nemen. Deze schommelingen werden in rekening gebracht door het introduceren van de fluctuerende delen in de productie- en verliestermen van de continuïteitsvergelijkingen. Een equivalent ionosferisch model werd ontwikkeld voor drie types van geïoniseerde deeltjes : elektronen, positieve ionen en negatieve ionen. Een matrix formulering leidt tot een gelijktijdige toegang tot de fluctuerende delen van elk geladen specimen. De numerieke resultaten tonen aan dat de chemische schommelingen belangrijk zijn in het hoogtegebied waar de concentratie van de negatieve ionen vergelijkbaar of hoger is dan deze van de electronen. Zeer belangrijke verhogingen van de incoherente verstrooiingsdoorsnede vinden plaats voor frequentieverschuivingen kleiner dan ongeveer 20 Hz. Een eenvoudige benadering wordt voorgesteld om het effect van de chemische fluctuaties weer te geven.

## Zusammenfassung

Die Kontinuumtheorie der Thomsonstreuung ist verändert, so dass mögliche Effekten der thermischen chemischen Schwankungen einbegriffen sind. Solche Schwankungen sind als schwankende Teile der Teilchenerzeugungen und Verluste in der Kontinuitätsgleichungen angesehen. Ein gleichwertiges ionosphärisches Modell mit drei Typen geladenen Teichen, d.h. Elektronen, positiven und negativen Ionen, wird entwickelt. Eine Matrizenformulation ergibt gleichzeitig die schwankende Teile jedes geladenen Komponenten. Numerische Rechnungen zeigen, dass die chemische Schwankungen wichtig sind über einem Höhebereich wo die negative Ionen vergleichbar oder häufiger als die Elektronen sind. Starke Vergrößerungen der Thomsonstreuungsdurchschnitten erscheinen für Frequenzverschiebungen kleiner als ungefähr 20 Hz. Eine einfache Annäherung der chemischen Schwankungen wird vorgestellt.

## 1. INTRODUCTION

Incoherent or Thomson scattering is a powerful tool for numerous investigations of the upper atmosphere (see Evans, 1969; Bauer, 1975; Alcaydé, 1979; Walker, 1979). Recently, an additional interest was given to the contribution of large radars to middle atmosphere studies between 10 km and 100 km altitude (see Harper and Gordon, 1980). In particular, measurements by Harper (1978) indicate that an ion component of the incoherent scatter spectrum is observable below 80 km altitude where negative ions progressively become dominant.

The continuum theory of Tanenbaum (1968) for a collision-dominated incoherent scatter spectrum has been modified by Mathews (1978) and by Fukuyama and Kofman (1980) to include the effects of negative ions. Hydrodynamic equations used by these authors are essentially identical to the equations adopted by Tanenbaum (1968). The major difference comes from the introduction of a continuity, momentum and energy equation for a negative ion and, following the arguments of Dougherty and Farley (1960), the spectrum is considered as a result of thermal fluctuations of the ambient electrons. Continuity equations are, however, written without production or loss terms.

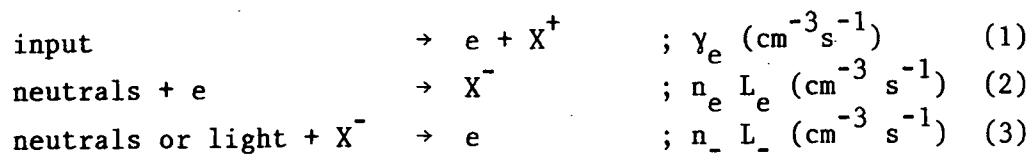
When electron thermal fluctuations are present, it is conceivable that such fluctuations also induce fluctuations of the production and loss terms. In the present paper, these terms are kept in the continuity equations for the three charged species and a matrix formulation is developed to investigate which effect can result on the incoherent scatter cross section. Since production and loss terms depend on physical mechanisms influencing the lower D-region, it is necessary to have a quantitative evaluation for the various production or loss rates which are proportional to charged particle concentrations. An equivalent ionospheric model is, therefore, constructed in section 2 by using the signal flow graph theory developed by Wisenberg and

Kockarts (1980). With such a model, a theoretical expression is deduced for the incoherent scatter cross section in section 3 and this expression reduces to the results of Mathews (1978) and Fukuyama and Kofman (1980) when chemical fluctuations are ignored. An application to the lower D-region is presented in section 4 where it appears that the incoherent scatter cross section is strongly enhanced for low frequency shifts in the height region below 75 km. Since the enhancement essentially depends on the ratio between negative ions and electrons, it can be concluded that mesospheric incoherent scatter observations provide an excellent tool for the determination of the negative ion abundance.

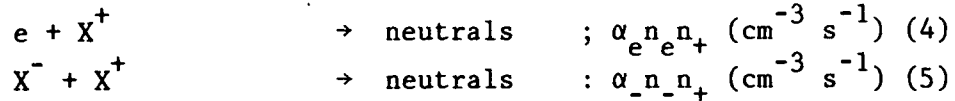
## 2. EQUIVALENT IONOSPHERIC MODEL

More than 100 ion-neutral reactions involved in the construction of a detailed theoretical model of the terrestrial D-region lead to several types of negative and positive ions. Incoherent scatter theory is, however, more tractable if only three types of charged particles are to be considered. It is, therefore, useful to reduce a multicomponent model to an equivalent model which consists of electrons  $e$ , positive ions  $X^+$  and negative ions  $X^-$ . Such a reduction is rather easy when the detailed model is constructed by using a signal flow graph technique as described by Wisenberg and Kockarts (1980).

Five equivalent processes are sufficient to reproduce the total negative, positive and electron concentrations if equivalent or effective reaction rates can be deduced from a multicomponent model in which the external production rate  $\gamma_e$  ( $\text{cm}^{-3} \text{s}^{-1}$ ) for electrons and positive ions is given. These reactions can be schematized as follows







where the concentrations of positive ions  $X^+$  and negative ions  $X^-$  are given by  $n_+$  and  $n_-$ , respectively. The electron concentration  $n_e$  can be affected by attachment processes with an effective loss rate  $L_e$  ( $\text{s}^{-1}$ ) and by electron-ion recombination with an effective rate  $\alpha_e$  ( $\text{cm}^3 \text{s}^{-1}$ ). Negative ions can be lost by collisional detachment, by photodissociation and by photodetachment with an effective loss rate  $L_-$  ( $\text{s}^{-1}$ ) or by ion-ion neutralization with an effective recombination rate  $\alpha_-$  ( $\text{cm}^3 \text{s}^{-1}$ ).

In presence of  $j$  different positive ions with concentrations  $n_j^+$  and  $i$  different negative ions with concentrations  $n_i^-$ , an equivalent model is obtained if effective rate coefficients  $L_e$ ,  $L_-$ ,  $\alpha_e$  and  $\alpha_-$  are available such that  $n_+ = \sum_j n_j^+$  and  $n_- = \sum_i n_i^-$  with  $n_+ = n_e + n_-$ . The effective electron-ion recombination and ion-ion neutralization rates are given respectively by

$$\alpha_e = (\sum_j \alpha_j n_j^+) / \sum_j n_j^+ \quad (6)$$

and

$$\alpha_- = (\sum_i n_i^- \sum_j \alpha_{ij} n_j^+) / (\sum_i n_i^- \sum_j n_j^+) \quad (7)$$

where  $\alpha_j$  is the electron-ion recombination rate of the  $j$ -type positive ion and  $\alpha_{ij}$  is the ion-ion neutralization rate between the  $i$ -type negative ion and the  $j$ -type positive ion. Numerical values for  $L_e$  and  $L_-$  cannot be found in such a simple manner, since one has to take into account the various loops leading from electrons to negative ions and vice versa. Nevertheless, these quantities can be deduced from signal flow graph theory. Wisemberg and Kockarts (1980) have shown that the net electron production rate  $P_e$  resulting from an external input production  $\gamma_e$  associated with numerous chemical processes is given by

$$P_e = \gamma_e T_{(e \leftarrow IN)} \quad (8)$$

where  $T_{(e \leftarrow IN)}$  represents the transmittance from the external input to the steady state electrons. When the net electron production rate (8) is equated to the total production rate resulting from processes (1) and (3), the effective loss rate  $L_-$  for negative ions can be written

$$L_- = \gamma_e (T_{(e \leftarrow IN)} - 1) / (\lambda n_e) \quad (9)$$

where  $\lambda = n_-/n_e$ . The net electron production rate (8) is also equal to the total electron loss rate resulting from processes (2) and (4). The effective electron loss rate  $L_e$  is, therefore, given by

$$L_e = \gamma_e T_{(e \leftarrow IN)} / n_e - \alpha_e n^+ \quad (10)$$

or

$$L_e = \lambda L_- + (\gamma_e / n_e) - \alpha_e n^+ \quad (11)$$

when the transmittance in expression (10) is replaced by its expression resulting from (9). All transmittances were computed in the model of Wisenberg and Kockarts (1980) and particularly  $T_{(e \leftarrow IN)}$ . Using equation (6), (7), (9) and (11), one obtains the numerical values for  $L_e$ ,  $L_-$  and  $\alpha_e$  given in Table 1 which also provides the external input  $\gamma_e$  at 5 km height intervals in the lower ionosphere. The average ion-ion neutralization rate  $\alpha_-$  is not given in Table 1, since all  $\alpha_{ij}$  ion-ion neutralization rates were taken equal to  $6 \times 10^{-8} \text{ cm}^3 \text{ s}^{-1}$  in the Wisenberg and Kockarts model, leading, therefore, to a height independent value  $\alpha_- = 6 \times 10^{-8} \text{ cm}^3 \text{ s}^{-1}$ . Table 2 gives the neutral temperature  $T_n$ , the electron concentration  $n_e$ , the ratio  $\lambda = n_-/n_e$  and the mean molecular masses  $m_+$  and  $m_-$  for positive and negative ions, respectively. The last two quantities are given since they are necessary for computing incoherent scatter cross sections. All values in Tables 1 and 2 have been computed directly from the detailed ionospheric model. With the data of Table 1, it is, however, possible to compute indepen-

TABLE 1 : Effective rate coefficients for the equivalent model.

z(km)	$\gamma_e$ (cm <sup>-3</sup> s <sup>-1</sup> )	$L_e$ (s <sup>-1</sup> )	$L_-$ (s <sup>-1</sup> )	$\alpha_e$ (cm <sup>3</sup> s <sup>-1</sup> )
50	3.20 (-1)*	4.65 (+1)	2.89 (-3)	3.00 (-6)
55	1.73 (-1)	1.38 (+1)	5.79 (-3)	3.00 (-6)
60	9.94 (-2)	3.98 (0)	2.13 (-2)	3.00 (-6)
65	6.19 (-2)	1.08 (0)	8.11 (-2)	2.99 (-6)
70	3.23 (-1)	2.71 (-1)	4.78 (-1)	2.95 (-6)
75	3.11 (0)	5.24 (-2)	2.23 (0)	2.34 (-6)
80	1.11 (1)	1.16 (-2)	4.99 (0)	7.34 (-7)

\* For 3.20 (-1), read  $3.20 \times 10^{-1}$

TABLE 2 : Parameters of the equivalent model (Wisemberg and Kockarts, 1980).

$z$ (km)	$T_n$ (K)	$n_e$ (cm <sup>-3</sup> )	$\lambda = n_-/n_e$	$m_+$ (amu)	$m_-$ (amu)
50	271	1.50 (-1)*	1.53 (4)	55	61
55	261	7.20 (-1)	2.34 (3)	55	61
60	247	6.11 (0)	1.86 (2)	54	61
65	233	3.39 (1)	1.34 (1)	53	61
70	220	2.64 (2)	5.67 (-1)	48	61
75	208	1.14 (3)	2.35 (-2)	42	55
80	198	3.86 (3)	2.32 (-3)	32	39

\* For 1.50 (-1), read  $1.50 \times 10^{-1}$

dently the electron concentrations and  $\lambda$  values of Table 2. When all effective rates are known, it is easily shown that processes (1) to (5) lead to steady state electron concentration given by

$$n_e^2 = \gamma_e / [(1 + \lambda) (\alpha_e + \lambda \alpha_-)] \quad (12)$$

The ratio  $\lambda$  between negative ions and electrons is obtained from expression (11) under the form

$$\lambda = \frac{L_e}{L_-} \left[ 1 + \frac{n_e (1+\lambda) \alpha_e}{L_-} \right]^{-1} \quad (13)$$

This expression is identical to equation (18.12) of Banks and Kockarts (1973) which can be written with present notation

$$\lambda = \frac{L_e}{L_-} \left[ 1 + \frac{\gamma_e}{(1+\lambda) L_- n_e} + \frac{(\alpha_- - \alpha_e) n_e}{L_-} \right]^{-1} \quad (14)$$

When numerical values are introduced in (13) or (14), it appears that an excellent approximation of  $\lambda$  is given by

$$\lambda = L_e / L_- = n_- / n_e \quad (15)$$

Such an expression indicates that the ratio  $\lambda$  is mainly controlled by electron attachment and detachment processes and that recombination processes play a negligible role. This property will be used in the numerical computation of the incoherent scatter cross section which is derived in the following section.

### 3. CONSERVATION EQUATIONS AND INCOHERENT SCATTER CROSS SECTION

According to Dougherty and Farley (1960) the average differential scattering cross section for backscattering can be written as

$$\sigma (\omega_0 \pm \omega) d\omega = r_e^2 L^3 n_e' n_e'^* d\omega \quad (16)$$

where  $\sigma$  is the average power scattered through  $180^\circ$  per unit solid angle, per unit incident power, per unit volume and per unit frequency range. The angular frequency of the incident wave is  $\omega_0$ , the Doppler shift is  $\omega$  and  $r_e = 2.82 \times 10^{-13}$  cm is the classical electron radius. Scattering is assumed to occur in a volume  $L^3$  as a consequence of electron density fluctuations whose ensemble average is given by  $n_e' n_e'^*$ ,  $n_e'^*$  being the complex conjugate of  $n_e'$ . This ensemble average is a function of the Doppler shift  $\omega$  and of a wave number  $k$  which is twice the incident wave number, in the case of backscattering.

Electron density fluctuations in the lower ionosphere can be deduced from conservation equations (Tanenbaum, 1968; Seasholtz and Tanenbaum, 1969; Seasholtz, 1971; Mathews, 1978; Fukuyama and Kofman, 1980). All previous computations implicitly assume that electron thermal fluctuations have no effect on the production and loss terms in the continuity equation. However, the effective electron lifetime becomes shorter in presence of negative ions as it can be seen from the effective loss terms given in Table 1. At 50 km altitude the electron effective lifetime is of the order of  $2 \times 10^{-2}$  s whereas it reaches a value of the order of 100 s at 80 km altitude. In order to investigate the effect of thermally induced chemical fluctuations, we start from the 13-moment approximation of the conservation equation (Schunk, 1975; 1977). When charged particles essentially exchange energy with the ambient neutral components and when magnetic field effects and Joule heating are negligible, the continuity momentum and energy equations can be written for each s-type ionized component as follows :

$$\frac{\partial n_s}{\partial t} + \nabla \cdot (n_s \tilde{u}_s) = P_s - n_s L_s \quad (17)$$

$$\begin{aligned} \frac{\partial \tilde{u}_s}{\partial t} + \tilde{u}_s \cdot \nabla \tilde{u}_s + \frac{1}{n_s m_s} \nabla p_s - \frac{\tilde{F}_s}{m_s} - \frac{q_s}{m_s} \tilde{E} - \frac{\eta_{sn}}{n_s m_s} \left[ \nabla^2 \tilde{u}_s + \frac{1}{3} \nabla(\nabla \cdot \tilde{u}_s) \right] \\ = -v_{sn} (\tilde{u}_s - \tilde{u}_n) \end{aligned} \quad (18)$$

$$\frac{3}{2} \frac{\partial p_s}{\partial t} + \frac{3}{2} \underline{u}_s \cdot \nabla p_s + \frac{5}{2} p_s \nabla \cdot \underline{u}_s - \nabla (\lambda_{sn} \nabla T_s) =$$

$$- \frac{3n_s m_s v_{sn}}{m_s + m_n} K(T_s - T_n) \quad (19)$$

where  $n_s$  is the s-type particle concentration,  $\underline{u}_s$  is its drift velocity,  $P_s$  and  $L_s$  are the production and loss rates, respectively. In the momentum equation  $p_s = n_s K T_s$  is the scalar pressure of the s-type particle with temperature  $T_s$ , mass  $m_s$  and electrical charge  $q_s$ .  $\underline{F}_s$  is any external force acting on the s-type particle,  $\underline{E}$  is the electric field,  $\eta_{sn}$  is the viscosity coefficient of species s in the neutral gas n with drift velocity  $\underline{u}_n$  temperature  $T_n$  and mean molecular mass  $m_n$ . The thermal conductivity  $\lambda_{sn}$  refers to interactions between charged particles and the ambient neutral gas and  $v_{sn}$  is the electron or ion-neutral momentum transfer collision frequency. We follow the technique of Tanenbaum (1968) by assuming that the concentrations  $n_s$ , the pressures  $p_s$  and the temperatures  $T_s$  are composed of static components ( $n_{s0}$ ,  $p_{s0}$ ,  $T_{s0}$ ) and small fluctuating components ( $n'_s$ ,  $p'_s$ ,  $T'_s$ ) proportional to  $e^{i(\omega t - kz)}$ . The conservation equations are only considered along the vertical direction z, the neutral gas is assumed at rest ( $\underline{u}_n = 0$ ) as well as the static components  $\underline{u}_{s0}$  for the charged particles. In addition, the vertical components  $F'_s$ ,  $E$  and  $u'_s$  for the external forces, for the electric field and for the charged particle velocities are all proportional to  $e^{i(\omega t - kz)}$ . With these assumptions it is possible to linearize the conservation equations (17) to (19).

When the production and loss rates resulting from processes (1) to (5) are introduced in the continuity equation (17) one obtains the following matrix relation between the fluctuating concentrations  $n'_s$  and drift speeds  $u'_s$

$$\underline{A} \begin{bmatrix} n'_e/n_{e0} \\ n'_-/n_{-0} \\ n'_+/n_{+0} \end{bmatrix} = \frac{k}{\omega} \begin{bmatrix} u'_e \\ u'_- \\ u'_+ \end{bmatrix} \quad (20)$$

when three type of charged particles are considered, i.e.  $s = e$  for electrons,  $s = -$  for negative ions, and  $s = +$  for positive ions. The matrix  $\underline{\underline{A}}$  given by equation (A.5) in Appendix A depends on the effective loss rates  $L_e$  and  $L_-$  defined for the equivalent ionospheric model in section 2. It should be noted that without production and loss terms in the continuity equations, matrix  $\underline{\underline{A}}$  reduces to a unitary matrix. In such a case, all following results can be easily transformed to obtain the formulation developed by Mathews (1978), and by Fukuyama and Kofman (1980).

The linearization of the three energy equations (19) leads to

$$\begin{bmatrix} p'_e/n_{e0} \\ p'_-/n_{-0} \\ p'_+/n_{+0} \end{bmatrix} = ik \underline{\underline{Q}} \begin{bmatrix} u'_e \\ u'_- \\ u'_+ \end{bmatrix} + \underline{\underline{R}} \begin{bmatrix} n'_e/n_{e0} \\ n'_-/n_{-0} \\ n'_+/n_{+0} \end{bmatrix} \quad (21)$$

where  $\underline{\underline{Q}}$  and  $\underline{\underline{R}}$  are diagonal matrices whose elements are given by equations (A6) and (A7) in Appendix A.

Finally it is shown in Appendix A that the linearization of the three momentum equations (18) combined with expression (21) leads to

$$\underline{\underline{C}} \begin{bmatrix} u'_e \\ u'_- \\ u'_+ \end{bmatrix} + \frac{\omega}{k} \underline{\underline{D}} \begin{bmatrix} n'_e/n_{e0} \\ n'_-/n_{-0} \\ n'_+/n_{+0} \end{bmatrix} = \frac{i \omega}{k^2 K T_e} \underline{\underline{T}} \begin{bmatrix} F'_e \\ F'_- \\ F'_+ \end{bmatrix} \quad (22)$$

where matrices  $\underline{\underline{C}}$ ,  $\underline{\underline{D}}$  and  $\underline{\underline{T}}$  are given in Appendix A by equations (A15), (A14), and (A11), respectively. Combining equation (22) with the continuity equation (20) it is possible to express the concentration fluctuations by

$$\begin{bmatrix} n'_e \\ n'_- \\ n'_+ \end{bmatrix} = \frac{i}{k K T_e} \underline{\underline{X}} (\underline{\underline{C}} \underline{\underline{A}} + \underline{\underline{D}})^{-1} \underline{\underline{T}} \begin{bmatrix} F'_e \\ F'_- \\ F'_+ \end{bmatrix} \quad (23)$$



where  $\tilde{X}$  is a diagonal matrix with real elements  $n_{e0}$ ,  $n_{-0}$ , and  $n_{+0}$ , respectively.

Since the incoherent scattering cross section is proportional to the ensemble average of the electron fluctuations, we define a concentration fluctuation matrix  $\tilde{N}$  by

$$\tilde{N} = \begin{bmatrix} n'_e \\ n'_- \\ n'_+ \end{bmatrix} \begin{bmatrix} n_e'^* & n_-'^* & n_+'^* \end{bmatrix} \quad (24)$$

where  $n_e'^*$ ,  $n_-'^*$  and  $n_+'^*$  are complex conjugate quantities. Using relation (23) between concentration fluctuations  $n'_s$  and perturbing forces  $F'_s$ , equation (24) leads to

$$\tilde{N} = (k K T_e)^{-2} \tilde{H} \tilde{F} \tilde{H}^+ \quad (25)$$

where the force matrix  $\tilde{F}$  is defined in a similar way as  $\tilde{N}$  and  $\tilde{H}^+$  is the Hermitian conjugate matrix of  $\tilde{H}$  given by

$$\tilde{H} = \tilde{X} (\tilde{C} \tilde{A} + \tilde{D})^{-1} \tilde{T} \quad (26)$$

By analogy with a theory developed by Barakat (1963) for optical instruments,  $\tilde{F}$  can be considered as an input coherency matrix leading to an output coherency matrix by the action of a transfer function matrix  $\tilde{H}$ . The ensemble average value  $n'_e n_e'^*$  required in expression (16) for the differential cross section is simply the first element of matrix  $\tilde{N}$ .

Computation of the elements of matrix  $\tilde{N}$  requires an analytical expression for  $\tilde{F}$  which can be obtained from the fluctuation - dissipation theorem (see Seasholtz, 1971) relating the applied force matrix  $\tilde{F}$  to an impedance matrix  $\tilde{Z}$  by the relation

$$\underline{\underline{F}} = \frac{K T_e}{2\pi L^3} (\underline{\underline{Z}} + \underline{\underline{Z}}^\dagger) \quad (27)$$

where the impedance matrix  $\underline{\underline{Z}}$  is by definition such that

$$\begin{bmatrix} F'_e \\ F'_- \\ F'_+ \end{bmatrix} = \underline{\underline{Z}} \begin{bmatrix} \Gamma_e \\ \Gamma_- \\ \Gamma_+ \end{bmatrix} \quad (28)$$

$\Gamma_s = n_{s0} u'_s$  is the resulting flux density fluctuation for the s-type species. Comparing equation (28) with (23) and using the continuity equation (20) one obtains

$$\underline{\underline{Z}} = k^2 K T_e \underline{\underline{W}} \quad (29)$$

with

$$\underline{\underline{W}} = (i\omega)^{-1} \underline{\underline{T}}^{-1} (\underline{\underline{C}} + \underline{\underline{D}} \underline{\underline{A}}^{-1}) \underline{\underline{X}}^{-1} \quad (30)$$

where all matrices in  $\underline{\underline{W}}$  have been previously defined. Combining (25), (27) and (30) the concentration fluctuation matrix can be written

$$\underline{\underline{N}} = (2\pi)^{-1} L^{-3} \underline{\underline{H}} (\underline{\underline{W}} + \underline{\underline{W}}^\dagger) \underline{\underline{H}}^\dagger \quad (31)$$

where  $\underline{\underline{W}}^\dagger$  and  $\underline{\underline{H}}^\dagger$  are Hermitian conjugates of  $\underline{\underline{W}}$  and  $\underline{\underline{H}}$ , respectively. The first diagonal term of  $\underline{\underline{N}}$  is the ensemble average  $n'_e n'_e{}^*$  required in the differential scattering cross section given by equation (16) in which the volume term  $L^3$  will be eliminated by the  $L^{-3}$  factor in equation (31). When  $N_{(e,e)}$  is the first diagonal element of  $\underline{\underline{N}}$  computed without the  $L^{-3}$  factor, the differential backscattering cross section (16) is given by

$$\sigma(\omega_0 \pm \omega) d\omega = r_e^2 N_{(e,e)} d\omega \quad (32)$$

The other diagonal elements of  $\underline{\underline{N}}$  represent ensemble average fluctuations of negative and positive ions, respectively. Non-diagonal terms give cross spectral density fluctuations. The matrix formulation previously developed provides a simultaneous access to these quantities.

#### 4. APPLICATION TO THE TERRESTRIAL D-REGION

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The present formulation takes into account chemical fluctuations which are thermally induced on the production and loss terms in the continuity equation. When this equation is written without these terms, matrix  $\tilde{A}$  given by equation (A4) or (A5) reduces to a unitary matrix and the differential backscattering cross section (32) becomes identical to the expression given by Mathews (1978) or by Fukuyama and Kofman (1980). Although our mathematical formalism is different, it is not necessary to repeat here a discussion of the influence of negative ions on mesospheric incoherent scatter spectra in absence of chemical fluctuations.

The general backscattering cross section (32) requires a knowledge of effective loss rates involved in matrix  $\tilde{A}$ , concentrations of ionized species, temperatures, neutral and ionized mean molecular masses, neutral concentrations and collision frequencies. The characteristics for the ionized components are given in Tables 1 and 2. Temperatures of the ionized species are assumed equal to the neutral temperature. The total neutral concentration and mean molecular mass are taken from the US Standard Atmosphere (1976) and the electron-neutral and ion-neutral collision frequencies are computed with the expressions of Banks and Kockarts (1973). All computations are made for an incident frequency  $f_0 = 935$  MHz which corresponds to the incoherent scatter station at Saint-Santin (44.6°N, 2.2°E).

Figure 1 shows the incoherent scatter cross section computed at 60 km, 70 km and 80 km altitude as a function of the frequency shift  $f_0 \pm f$ . Full curves refer to the left hand ordinate which corresponds to a normalized cross section. Dashed curves refer to the right hand ordinate which corresponds to real cross sections expressed in terms of the classical electron radius. The factors  $2\pi$  are introduced by the

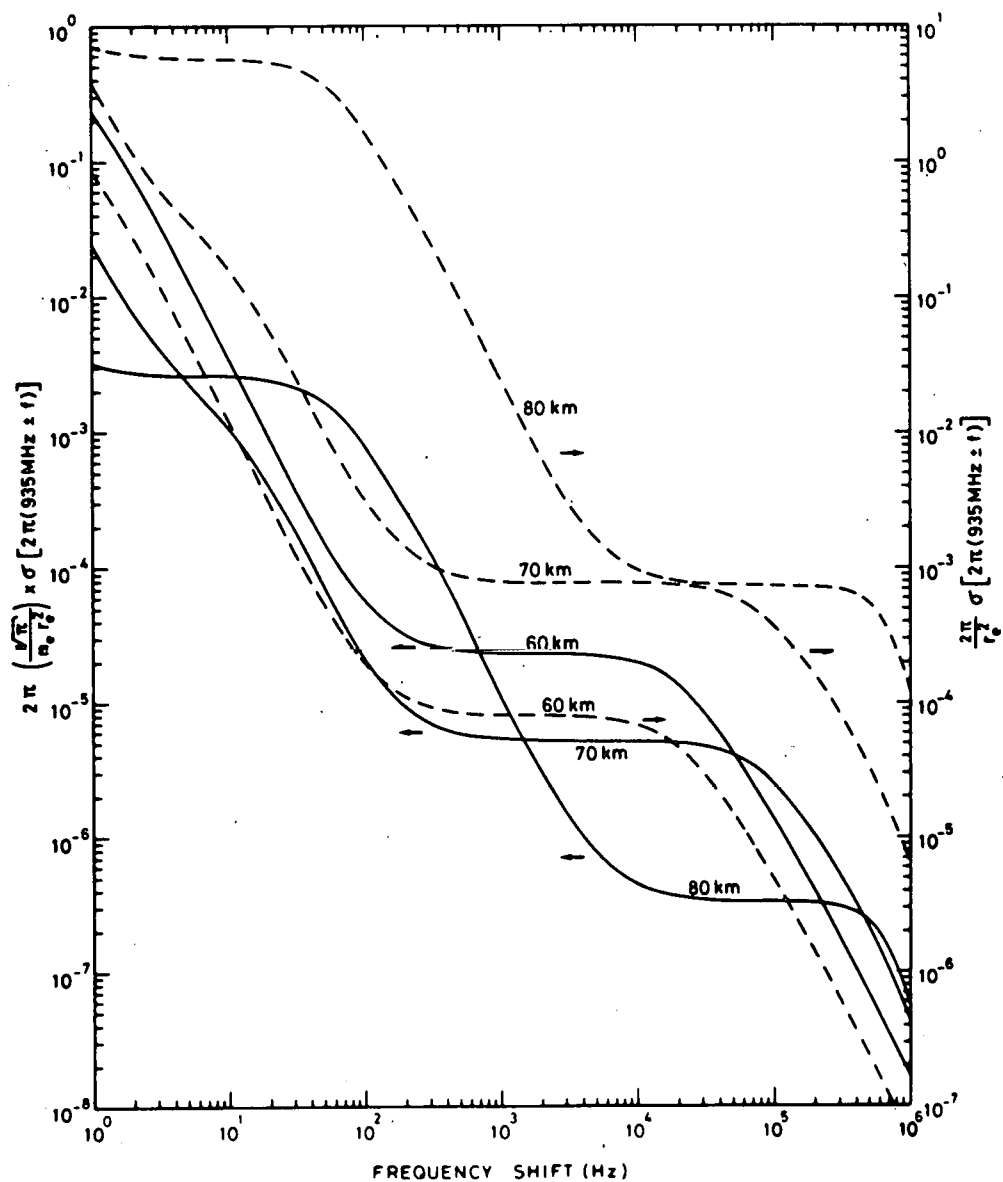


Fig. 1.- Incoherent scatter cross sections as a function of frequency shift for an incident frequency of 935 MHz. Full curves correspond to normalized cross sections (left hand scale). Dashed curves (right hand scale) correspond to cross sections expressed in terms of the classical electron radius  $r_e$ . Calculations are made for three altitudes with the equivalent ionospheric model deduced from detailed computations of Wisenberg and Kockarts (1980).

relation  $\omega = 2\pi f$  between angular and linear frequencies. These cross sections have the usual characteristics with an ionic and an electronic component separated by a plateau. At low frequency shifts, normalized cross sections are, however, larger at 60 km and 70 km altitude than at 80 km where the electron concentration is nevertheless higher. Such an apparently anomalous phenomenon does not appear for the real cross section (dashed curves). Whereas the cross section is almost independent of frequency shifts below 30 Hz at 80 km altitude, a sharp increase still occurs at 60 km and 70 km altitude. Table 2 indicates that negative ions become important below 70 km altitude where the effective loss rates  $L_e$  and  $L_{-}$  begin to play a significant role in the introduction of chemical fluctuations.

The practical importance of thermally induced chemical fluctuations is shown in Figure 2. Total cross sections computed from the full equation (32) are indicated by full lines and cross sections obtained with the theory of Mathews (1978) and Fukuyama and Kofman (1980) are represented by dotted lines. Values are again given for 60 km, 70 km and 80 km altitude, but frequency shifts are limited to  $\pm 50$  Hz in order to give a clear picture of the central portion of the spectrum including zero frequency shift. The contribution of chemical fluctuations is negligible at 80 km, but, at altitudes where  $\lambda$  becomes comparable or greater than one, extremely large differences are obtained between  $\sigma_{\text{Total}}$  and  $\sigma_{\text{MFK}}$ . This is particularly true for small frequency shifts. Depending on the values for  $\lambda$  and  $n_e$ , the present ionic spectrum always converges at some frequency shift towards the spectrum of Mathews (1978) and Fukuyama and Kofman (1980). Chemical fluctuations only enhance incoherent scatter cross sections at small frequency shifts. Interpretation of observed incoherent scatter spectra below 80 km altitude in terms of chemical fluctuations requires a frequency resolution of the order of 1 Hz to 2 Hz in order to identify sharp peaks such as those computed here around 70 km altitude. Narrow spectra have been observed in the mesosphere at Arecibo (Harper, 1978;

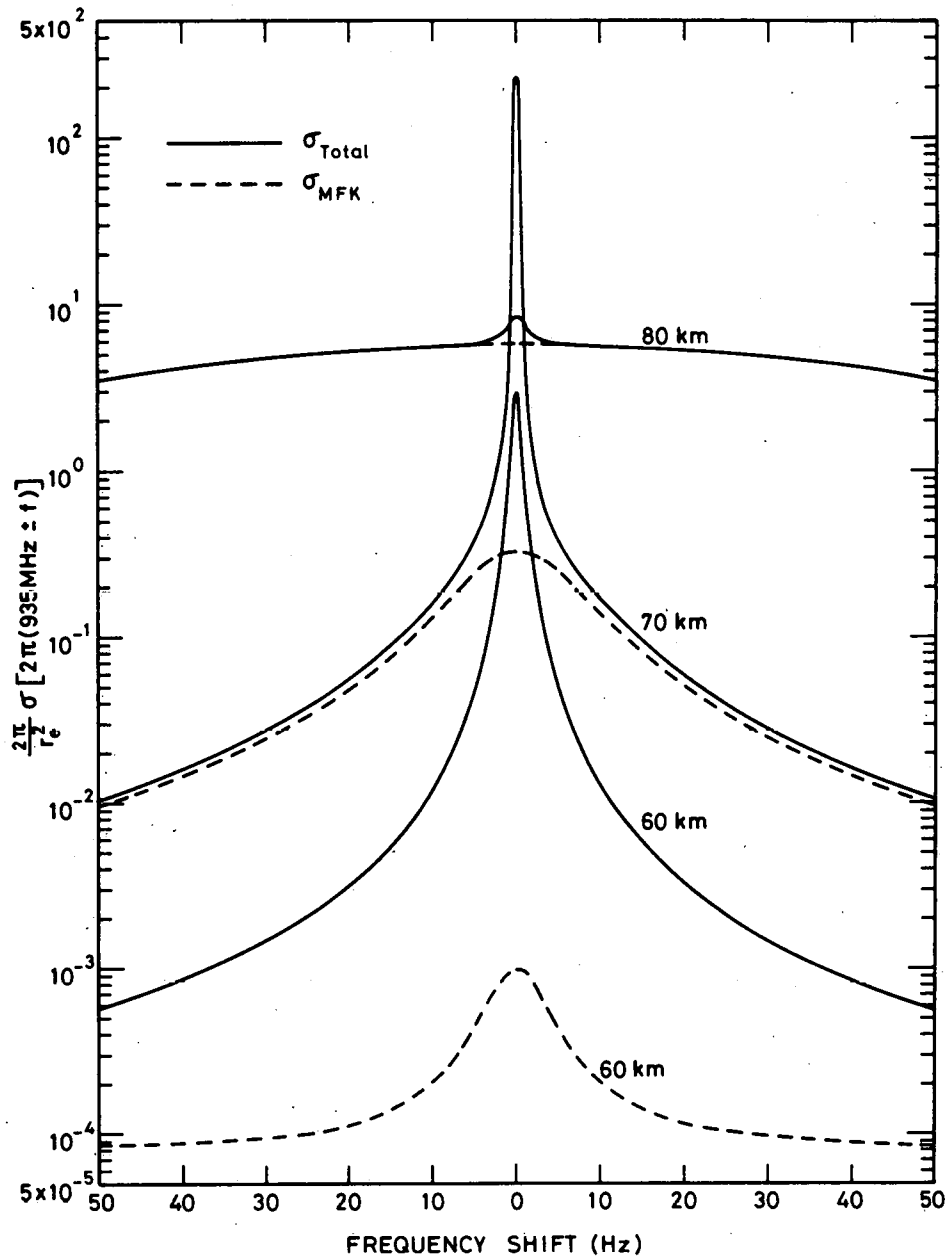


Fig. 2.- Incoherent scatter cross sections at 60, 70 and 80 km altitude as a function of frequency shift. Full curves correspond to present results ( $\sigma_{\text{Total}}$ ) and dashed curves ( $\sigma_{\text{MFK}}$ ) are obtained from expressions given by Mathews (1978) and by Fukuyama and Kofman (1980).

Ganguly, 1980). Values for the parameter  $\lambda$  have been deduced by Ganguly et al. (1979), but it is not obvious from published spectra that frequency resolution was sufficient to extract an effect related to chemical fluctuations.

Interpretation of experimental spectra is easier when a simple analytical formula is available for the incoherent scatter cross section. Using the cross section  $\sigma_{\text{MFK}}$  given by Mathews (1978) and Fukuyama and Kofman (1980), we approximate the present cross section  $\sigma_{\text{T}}$  by

$$\sigma_{\text{T}} = \sigma_{\text{C}} + \sigma_{\text{MFK}} \quad (33)$$

where  $\sigma_{\text{C}}$  represents the contribution resulting from chemical fluctuations. It turns out that an excellent approximation for  $\sigma_{\text{C}}$  is simply given by

$$\sigma_{\text{C}} = \frac{n_e r_e^2}{\pi} \frac{L_e}{\omega^2 + (L_e + L_-)^2} \quad (34)$$

This approximation is also valid at zero frequency shift as it can be seen on Figure 3 where the total cross section at  $f = 0$  is shown as a function of height. The full curve corresponds to results obtained with our detailed formulation,  $\sigma_{\text{MFK}}$  corresponds to the results given by Mathews, Fukuyama and Kofman. When negative ions are completely ignored, one obtains curve labeled  $\sigma_{+}$ . The dotted-dashed curve  $\sigma_{\text{C}}$  is obtained with expression (34) for  $\omega = 0$ . It appears that our approximation for chemical fluctuations fits the detailed calculation below 75 km altitude where negative ions become important. The frequency shift dependence given by the approximation (34) is able to reproduce the spectra of Figure 2 as long as chemical fluctuations are important. Therefore, a measured spectrum with high resolution in frequency shift, should give immediately an accurate value of  $\lambda = L_e/L_-$ , since the effective loss rates are directly proportional to the width of the ionic component.

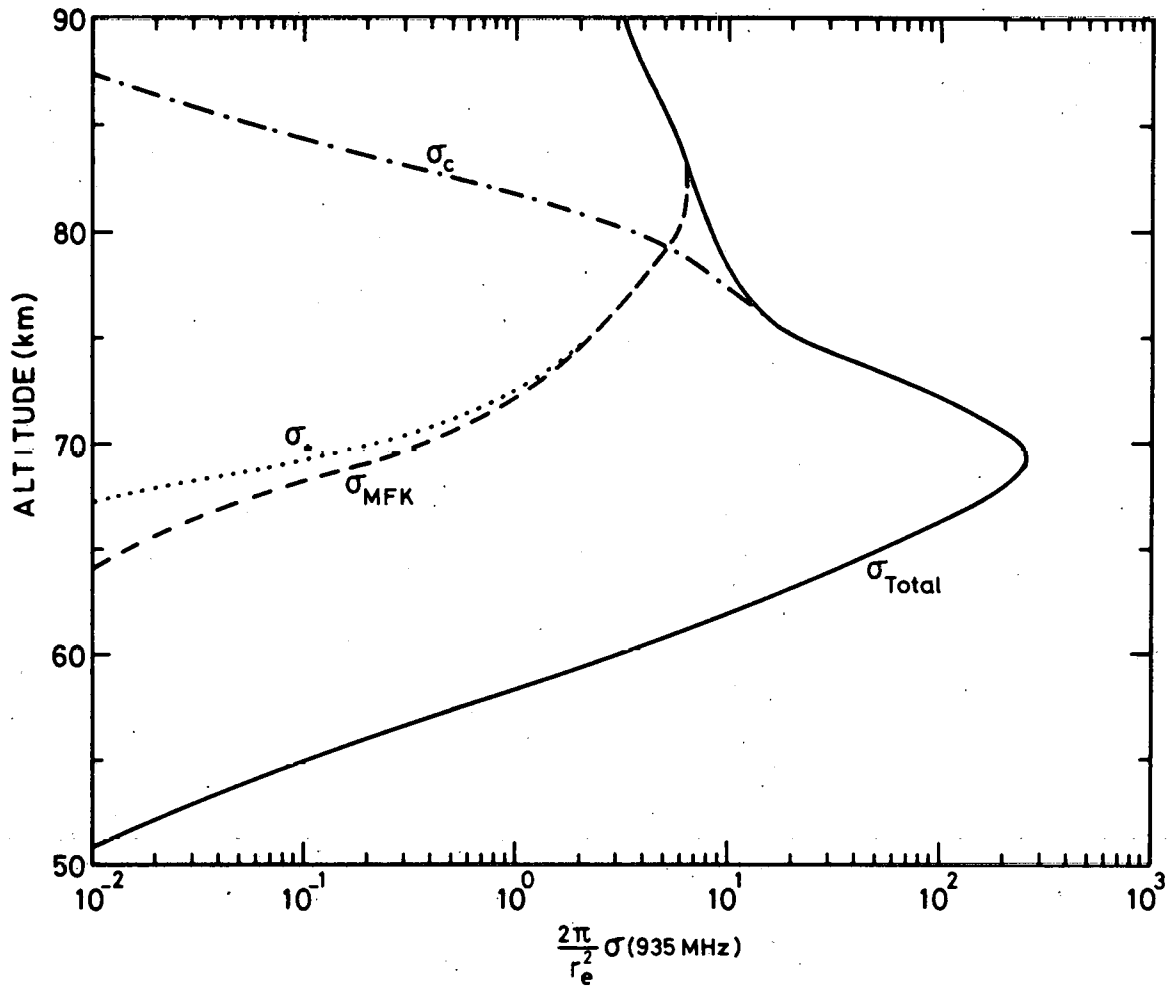


Fig. 3.- Vertical distribution of incoherent scatter cross section for zero frequency shift.  $\sigma_{\text{Total}}$  and  $\sigma_{\text{MFK}}$  have same meaning as in Fig. 2.  $\sigma_+$  is obtained in absence of negative ions and  $\sigma_c$  represents chemical fluctuations approximated given by expression (34).



The total scattered power  $P_{\text{MFK}}$  per unit volume has been derived by Mathews (1978) and by Fukuyama and Kofman (1980) in absence of chemical fluctuations. Furthermore, Fukuyama and Kofman (1980) also deduced an approximate expression  $P_{\text{iMFK}}$  for the ion component. Since the chemical part  $\sigma_{\text{C}}$  given by (34) is easily integrable, we approximate the present total power  $P_{\text{T}}$  by

$$P_{\text{T}} = P_{\text{iC}} + P_{\text{MFK}} \quad (36)$$

with

$$P_{\text{iC}} = n_{\text{e}} L_{\text{e}} / (L_{\text{e}} + L_{-}) \quad (37)$$

This total power is shown as a function of height in Figure 4 where  $P_{\text{MFK}}$  is given for comparison. When chemical fluctuations are neglected, ionic powers with and without negative ions are indicated by  $P_{\text{iMFK}}$  and  $P_{\text{i+}}$ , respectively. The chemical part  $P_{\text{iC}}$  becomes almost identical to  $P_{\text{MFK}}$  below 65 km in the present ionospheric model. Below this altitude the total power is approximately a factor of two higher than the value obtained without chemical fluctuations. It appears again that chemical fluctuations play a significant role in the height range where  $\lambda$  becomes comparable or greater than one.

## 5. CONCLUSION

The continuum theory of incoherent backscattering is extended to include possible effects of production and loss terms in the continuity equations. Chemical reactions can be considered as probabilistic processes which induce additional fluctuation of the particle concentrations. Such fluctuations are superimposed to the classical thermal fluctuation which results from collisions between particles without any change of the identity of the particles. When production and loss terms are considered in the lower D region it appears that the switching reactions between negative ions and electrons are sufficiently fast to induce a fluctuation of the electron concentration which is capable to modify the

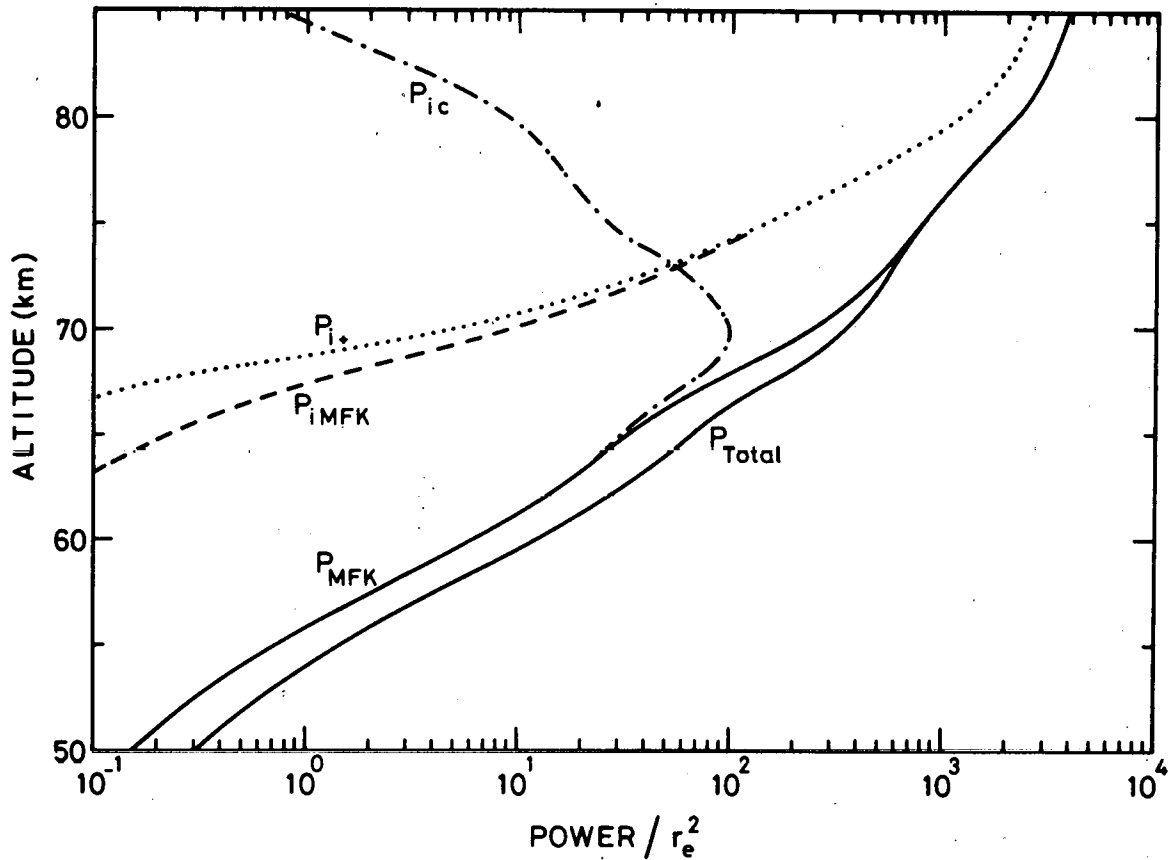


Fig. 4.- Vertical distribution of total power  $P_{Total}$ . Results of Mathews (1978) and Fukuyama and Kofman (1980) are indicated by full curve  $P_{MFK}$ . The corresponding ion component is labelled  $P_{iMFK}$ , whereas  $P_{i+}$  gives the contribution of positive ions. Effect of chemical fluctuations is represented by curve  $P_{ic}$  which becomes identical to the total power  $P_{MFK}$  below 65 km altitude.

classical incoherent scatter spectrum based on pure thermal fluctuations. The ion component of the backscattering cross section is strongly enhanced for frequency shifts below 20 Hz at altitudes where the ratio  $\lambda$  between negative ions and electrons is greater than one. As a consequence, high resolution measurements should provide a direct determination of the ratio  $\lambda$  since the shape, width and amplitude of the backscattering spectra influenced by chemical fluctuations essentially depends on the electron and negative ion loss rates  $L_e$  and  $L_-$ . These two quantities allow a direct determination of  $\lambda$  with expression (15) which is very accurate over the height range where negative ions play a significant role in the lower D-region. Furthermore, the total power of the incoherent scatter spectra is increased by a factor of two below 70 km altitude for the ionospheric model adopted in the present calculations. The ionic power is, however, increased by a much larger factor which may reached 2 orders of magnitude at 65 km. These facts should make the experimental detection easier when sufficient frequency resolution is available.

## APPENDIX A : Matrix expressions

The equivalent ionospheric model described in section 2 leads to the following one dimensional continuity equations

$$\frac{\partial n_e}{\partial t} + \frac{\partial}{\partial z} (n_e u_e) = \gamma_e + n_- L_- - n_e L_e - n_e n_+ \alpha_e \quad (A1)$$

$$\frac{\partial n_-}{\partial t} + \frac{\partial}{\partial z} (n_- u_-) = n_e L_e - n_- L_- - n_- n_+ \alpha_- \quad (A2)$$

and

$$\frac{\partial n_+}{\partial t} + \frac{\partial}{\partial z} (n_+ u_+) = \gamma_e - n_+ n_e \alpha_e - n_+ n_- \alpha_- \quad (A3)$$

where the various production and loss terms are obtained from equations (1) to (5) in section 2. Concentrations  $n_s$  are assumed to have a steady state component  $n_{s0}$  and a fluctuating component  $n'_s$  proportional to  $\exp[i(\omega t - kz)]$ . The continuous component of  $u_s$  is assumed to be zero, but the fluctuating part is also proportional to  $\exp[i(\omega t - kz)]$  (Tanenbaum, 1968). Substituting  $n_s$  and  $u_s$  by their expressions in (A1) to (A3) and neglecting second order fluctuations, one obtains the matrix equation (20) where  $\tilde{A}$  is given by

$$\tilde{A} = \omega^{-1} \begin{bmatrix} \omega - i(L_e + \alpha_e n_{+0}) & iL_- \lambda & -i\alpha_e n_{+0} \\ iL_e / \lambda & \omega - i(L_- + \alpha_- n_{+0}) & -i\alpha_- n_{+0} \\ -i\alpha_e n_{e0} & -i\alpha_- n_{+0} & \omega - i(\alpha_e n_{e0} + \alpha_- n_{-0}) \end{bmatrix} \quad (A4)$$

with  $\lambda = n_{-0}/n_{e0}$ . It has been shown in section 2 that all losses resulting from recombination processes are negligible compared to the effective loss rates  $L_e$  or  $L_-$ . Although matrix  $\tilde{A}$  can be used in its form (A4), it is easier to adopt the approximate expression

$$\tilde{A} = \omega^{-1} \begin{bmatrix} \omega - iL_e & iL_e & 0 \\ iL_- & \omega - iL_- & 0 \\ 0 & 0 & \omega \end{bmatrix} \quad (A5)$$

where  $\lambda$  has been replaced by expression (15).

In the linearized energy equation (21),  $\tilde{Q}$  and  $\tilde{R}$  are diagonal matrices whose elements are given by

$$Q_{(s,s)} = (5/3) m_s V_s^2 / (i\omega + \sigma_s) \quad (A6)$$

and

$$R_{(s,s)} = (3/5) \sigma_s Q_{(s,s)} \quad (A7)$$

with  $V_s^2 = K T_{s0} / m_s$  and  $\sigma_s = 2m_s v_{sn} / (m_s + m_n) + (5/2) k^2 V_s^2 / (c_s v_{sn})$  when the thermal conductivity  $\lambda_s$  in equation (21) is taken from Tanenbaum (1968),  $c_s$  being a numerical constant between 1 and 2.

Maxwell's equation relating the displacement current to the current density is used in the same way as Tanenbaum (1968) for expressing the electric field in the momentum equation (18). After linearization one obtains the following matrix equation

$$(\tilde{G} + \tilde{J}) \begin{bmatrix} u'_e \\ u'_- \\ u'_+ \end{bmatrix} + \frac{\omega}{k} \tilde{V}^{-1} \begin{bmatrix} p'_e / n_{e0} \\ p'_- / n_{-0} \\ p'_+ / n_{+0} \end{bmatrix} = \frac{i\omega}{k^2 K T_{e0}} \tilde{T} \begin{bmatrix} F'_e \\ F'_- \\ F'_+ \end{bmatrix} \quad (A8)$$

where  $\tilde{G}$ ,  $\tilde{V}$  and  $\tilde{T}$  are diagonal matrices and  $\tilde{J}$  represents the coupling resulting from the electric field. The diagonal elements of  $\tilde{V}$  and  $\tilde{G}$  are given by

$$V_{(s,s)} = m_s V_s^2 \quad (A9)$$

and

$$G_{(s,s)} = -\omega^2 k^{-2} V_s^{-2} + i\omega [4 (3 d_s v_{sn})^{-1} + v_{sn} k^{-2} V_s^{-2}] \quad (A10)$$

where the viscosity  $\eta_s$  in  $\underline{G}$  has been replaced by Tanebaum's (1968) expression with a numerical constant  $d_s$  between 1 and 2. The temperature matrix  $\underline{T}$  is given in terms of non-fluctuating parts  $T_{s0}$  by

$$\underline{T} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & T_{e0}/T_{-0} & 0 \\ 0 & 0 & T_{e0}/T_{+0} \end{bmatrix} \quad (\text{A11})$$

The electric field coupling matrix for different temperatures of the ionized components is given by

$$\underline{J} = \begin{bmatrix} \beta_e^2 & \lambda \beta_e^2 & -(1+\lambda) \beta_e^2 \\ \lambda^{-1} \beta_-^2 & \beta_-^2 & -\lambda^{-1}(1+\lambda) \beta_-^2 \\ -(1+\lambda)^{-1} \beta_-^2 & -\lambda(1+\lambda)^{-1} \beta_+^2 & \beta_+^2 \end{bmatrix} \quad (\text{A12})$$

with  $\beta_s = (k \lambda_{sD})^{-1}$ . The Debye length for each species is defined by  $\lambda_{sD} = [\epsilon_0 K T_{s0} / (n_{s0} e^2)]^{1/2}$  where  $\epsilon_0$  is the free space permittivity and  $e$  is the electron charge.

When equation (21) is introduced in the momentum equation (A8), one obtains

$$\underline{C} \begin{bmatrix} u'_e \\ u'_- \\ u'_+ \end{bmatrix} + \frac{\omega}{k} \underline{D} \begin{bmatrix} n'_e/n_{e0} \\ n'_-/n_{-0} \\ n'_+/n_{+0} \end{bmatrix} = \frac{i\omega}{k^2 K T_{e0}} \underline{T} \begin{bmatrix} F'_e \\ F'_- \\ F'_+ \end{bmatrix} \quad (\text{A13})$$

where  $\underline{D} = \underline{V}^{-1} \underline{R}$  is a diagonal matrix whose elements are obtained from (A7) and (A9) such as

$$D_{(s,s)} = \sigma_s / (i\omega + \sigma_s) \quad (\text{A14})$$

Matrix  $\underline{C}$  is a combination of previously defined matrices. Its expression can be written

$$\underline{\underline{C}} = \underline{\underline{G}} + \underline{\underline{J}} + i\omega \underline{\underline{V}}^{-1} \underline{\underline{Q}} \quad (\text{A15})$$

where  $\underline{\underline{G}}$  is given by (A10),  $\underline{\underline{J}}$  by (A12),  $\underline{\underline{V}}^{-1}$  by the inverse of (A9) and  $\underline{\underline{Q}}$  by (A6), respectively.

## REFERENCES

- Alcaydé, D., Incoherent scatter data related to thermospheric modelling, Space Res., 19, 211-229, 1979.
- Banks, P.M. and G. Kockarts, Aeronomy, Part A and Part B, Academic, New York, 1973.
- Barakat, R., Theory of the coherency matrix for light of arbitrary spectral bandwidth, J. Opt. Soc. Am., 53, 317-323, 1963.
- Bauer, P., Theory of waves incoherently scattered, Phil. Trans. Roy. Soc. Lond., A 280, 167-191, 1975.
- Doughterty, J.P., and D.T. Farley, A theory of incoherent scattering of radio waves by a plasma, Proc. Roy. Soc. London, A 259, 79-99, 1960.
- Evans, J.V., Theory and practice of ionosphere study by Thomson scatter radar, Proc. IEEE, 57, 496-530, 1969.
- Fukuyama, K., and W. Kofman, Incoherent scattering of an electromagnetic wave in the mesosphere : a theoretical consideration, J. Geomag. Geoelectr., 32, 67-81, 1980.
- Ganguly, S., Incoherent scatter observations of mesospheric dynamics at Arecibo, Geophys. Res. Lett., 7, 369-372, 1980.
- Ganguly, S., J.D. Mathews, and C.A. Tepley, Thomson scatter radar detection of D-region negative ions at Arecibo, Geophys. Res. Lett., 6, 89-92, 1979.
- Harper, R.M., Preliminary measurements of the ion component of the incoherent scatter spectrum in the 60-90 km region over Arecibo, Geophys. Res. Lett., 5, 784-786, 1978.
- Harper, R.M., and W.E. Gordon, A review of radar studies of the middle atmosphere, Radio Sci., 15, 195-211, 1980.
- Mathews, J.D., The effect of negative ions on collision-dominated Thomson scattering, J. Geophys. Res., 83, 505-512, 1978.
- Seasholtz, R.G., Effect of collisions on Thomson scattering in a magnetic field with unequal electron and ion temperature and electron drift, J. Geophys. Res., 76, 1793-1802, 1971.



- Seasholtz, R.G., and B.S. Tanenbaum, Effect of collisions on Thomson scattering with unequal electron and ion temperatures, J. Geophys. Res., 74, 2271-2275, 1969.
- Schunk, R.W., Transport equations for aeronomy, Planet. Space Sci., 23, 437-483, 1975.
- Schunk, R.W., Mathematical structure of transport equations for multi-species flows, Rev. Geophys. Space Phys., 15, 429-445, 1977.
- Tanenbaum, B.S., Continuum theory of Thomson scattering, Phys. Rev., 171, 215-221, 1968.
- Walker, J.C.G., Radar measurement of the upper atmosphere, Science, 206, 180-189, 1979.
- Wisemberg, J., and G. Kockarts, Negative ion chemistry in the terrestrial D-region and signal flow graph theory, J. Geophys. Res., 85, 4642-4652, 1980.