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The oblateness effect on the solar radiation incident at the top of the atmospheres of the outer planets

by

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FOREWORD

The paper entitled "The oblateness effect on the solar radiation incident at the top of the atmospheres of the outer planets" will be published in Icarus, 1982.

AVANT-PROPOS

L'article intitulé "The oblateness effect on the solar radiation incident at the top of the atmospheres of the outer planets" sera publié dans Icarus, 1982.

VOORWOORD

De tekst "The oblateness effect on the solar radiation incident at the top of the atmospheres of the outer planets" zal in het tijdschrift Icarus, 1982 verschijnen.

VORWORT

Der Text "The oblateness effect on the solar radiation incident at the top of the atmospheres of the outer planets" wird in Icarus, 1982 herausgegeben werden.

THE OBLATENESS EFFECT ON THE SOLAR RADIATION INCIDENT AT THE TOP OF THE ATMOSPHERES OF THE OUTER PLANETS

by

E. VAN HEMELRIJCK

Abstract

Calculations of the daily solar radiation incident at the top of the atmospheres of Jupiter, Saturn, Uranus and Neptune, with and without the effect of the oblateness, are presented in a series of figures illustrating the seasonal and latitudinal variation of the ratio of both insolations. It is shown that for parts of the summer, the daily insolation of an oblate planet is increased, the zone of enhanced solar radiation being strongly dependent upon the obliquity whereas the rate of increase is fixed by both the flattening and the obliquity. In winter, the oblateness effect results in a more extensive polar region, the daily solar radiation of an oblate planet being always reduced when compared to a spherical planet. In addition, we also numerically studied the mean daily solar radiation. As already previously stated by Brinkman and Mc Gregor (1979), it is found that in summer the horizon plane is tilted towards the Sun for latitudes less than the subsolar point, but is tilted away from the Sun beyond this latitude. It follows that the mean summer daily insolation is increased between the equator and the sub-solar point, but decreased poleward of the above mentioned limit. In winter, however, the horizon plane is always tilted away from the Sun, causing the mean winter daily insolation to be reduced. The partial gain of the mean summertime insolation being much smaller than the loss during winter season evidently yields a mean annual daily insolation which is decreased at all latitudes.

Résumé

L'insolation diurne au sommet de l'atmosphère des planètes Jupiter, Saturne, Uranus et Neptune est calculée, d'une part, en assimilant ces planètes à des sphères, d'autre part, en tenant compte de leur aplatissement. Les résultats sont présentés dans une série de figures illustrant les variations saisonnières et latitudinales du rapport des deux insolations.

On montre qu'en été l'aplatissement donne lieu à une insolation diurne plus importante dans des régions précises fixées par l'obliquité, le taux de croissance dépendant de l'aplatissement et de l'obliquité. En hiver, l'effet de l'aplatissement implique une région polaire plus étendue, l'insolation diurne d'une planète aplatie s'avérant toujours plus petite comparée à une planète sphérique.

Nous avons également étudié numériquement l'insolation diurne moyenne. Conformément au travail de Brinkman et Mc Gregor (1979) le plan de l'horizon se rapproche du Soleil pour des latitudes comprises entre l'équateur et le point subsolaire, mais s'écarte au-delà de cette dernière latitude. Il en résulte que l'insolation diurne moyenne en été augmente pour des latitudes inférieures à celle du point subsolaire, mais diminue pour des latitudes supérieures. En hiver, le plan de l'horizon s'écarte toujours du Soleil réduisant ainsi l'insolation diurne moyenne. Le gain partiel de l'insolation diurne moyenne en été étant beaucoup plus petit que la perte en hiver, il s'ensuit que l'insolation diurne moyenne annuelle est réduite à toutes les latitudes.

Samenvatting

Berekeningen van de dagelijkse zonnestraling aan de rand van de atmosfeer van de planeten Jupiter, Saturnus, Uranus en Neptunus, met en zonder afplattingseffect, worden voorgesteld in een reeks van figuren die de seizoens- en breedteveranderingen van de verhouding van beide zonnestralingen weergeven.

Er wordt aangetoond dat voor gedeelten van de zomer de dagelijkse zonnestraling van een afgeplatte planeet verhoogt, waarbij het gebied van verhoogde zonnestraling sterk afhankelijk is van de helling van de rotatieas op het baanvlak, terwijl de intensiteitsstijging bepaald wordt door zowel de afplatting als de helling van de rotatieas. In de winter resulteert het afplattingseffect in een uitgestrekter poolgebied en de dagelijkse zonnestraling van een afgeplatte planeet blijkt steeds kleiner te zijn vergeleken met deze van een sferische planeet.

Er werd eveneens een numerieke studie gemaakt van de gemiddelde dagelijkse zonnestraling. Zoals reeds vroeger werd aangetoond door Brinkman en Mc Gregor (1979) hebben we gevonden dat in de zomer het horizonvlak wentelt naar de Zon toe voor breedten kleiner dan het subsolaire punt, maar zich anderzijds van de zon verwijdert boven deze breedte. Hieruit volgt dat de gemiddelde dagelijkse zonnestraling in de zomer verhoogt tussen de evenaar en het subsolaire punt, maar vermindert naar de pool toe. In de winter echter zal het horizonvlak zich steeds van de zon afwentelen waardoor de gemiddelde zonnestraling in deze periode gereduceerd wordt. De gedeeltelijke winst van de gemiddelde zomerse straling is echter heel wat kleiner dan het verlies in het winterseizoen zodat de gemiddelde dagelijkse zonnestraling genomen over een gans jaar kleiner is op alle breedten.

Zusammenfassung

Berechnungen der täglichen Sonnenstrahlung auf den Planeten Jupiter, Saturn, Uranus und Neptun, mit und ohne Effekt der Abplattung, sind vorgestellt in einer Reihe von Abbildungen, die die Jahreszeitlichen- und Breitenvariationen des Verhältnisses der beiden Sonnenstrahlungen darstellen.

Es wirde gefunden dass, für Teilen des Sommers, die tägliche Sonnenstrahlung eines abgeplattenen Planeten zunimmt, wobei das Gebiet der Zuhname abhängig der Schiefe der Ekliptik ist und wobei die Steilheit des Aufstieges ein Funktion der Abplattung und der Schiefe der Ekliptik ist. Im Winter resultiert das Abplattungseffekt in einem augedehnten Polgebiet und es ergibt sich dass die tägliche Sonnenstrahlung auf einem abgeplattenen Planeten immer weniger ist als diese auf einer sphärischen Planeten.

Es wurde auch eine numerische Analyse der mittleren täglichen Sonnenstrahlung durchgeführt. Wie Brinkman und Mc Gregor (1979) haben wird gefunden, dass im Sommer die Horizontebene sich nach der Sonne wälzt für Breiten zwischen dem Äquator und dem Subsolarpunkt, aber dieser Ebene wälzt sich von der Sonne ab für grösseren Breiten. Daraus ergibt sich dass im Sommer die mittlere tägliche Sonnenstrahlung erhöht zwischen dem Äquator und der Subsolarpunkt, aber abnimmt nach dem Pol. Im Winter wälzt die Horizontebene sich immer von der Sonne ab, wodurch die mittlere tägliche Sonnenstrahlung in dieser Periode reduziert wurde. Der teilweise Gewinn der Sonnenstrahlung im Sommer ist abermerklich kleiner als der Verlust während der Wintersaison, so dass die mittlere tägliche Sonnenstrahlung berechnet über ein ganzes Jahr als Funktion der Breite immer kleiner ist.

1. INTRODUCTION

It is well known that the distribution of the solar radiation incident at the top of the atmospheres of the planets of the solar system and its variability with latitude and time (or season) is of interest for various radiation problems. Although the upper-boundary insolation of the Earth's atmosphere and the radiation at its surface have been the subject of several earlier investigations, it should be emphasized that theoretical studies with respect to the other planets are rather scarce and sometimes far from complete. Among the papers published during the last fifteen years we cite those of Soter and Ulrichs (1967), Han-Shou Liu (1968), Murray et al. (1973), Ward (1974), Vorob'yev and Monin (1975), Levine et al. (1977) and Van Hemelrijck and Vercheval (1981).

In the calculations made by the previous authors, the planets are assumed to be spherical. This assumption is valid for the inner planets (Mercury and Venus) and for the outer planets Mars and Pluto, the equatorial radius being equal or nearly equal to the polar radius, but is no more acceptable for the other planets where the flattening is not negligible. The only paper dealing with planetary solar insolation and taking into account the planet's oblateness was, to the best of our knowledge, recently published by Brinkman and Mc Gregor (1979). In this interesting work, the direct insolation pattern of Saturn is calculated as a function of season and latitude, including both the oblateness of the planet and the effect of the shadow of the ring system. It should, however, be noted that the influence of the planet's flattening is only concisely discussed on a qualitative basis.

The main objective of the present paper is, therefore, to analyze in detail the oblateness effect on the solar radiation incident at the top of the atmospheres of the rapidly rotating planets in the solar system Jupiter, Saturn, Uranus and Neptune.

In a first section, we present a general method applicable to those four planets. Then, taking into account their orbital and planetary data, we calculate the daily insolation with and without the effect of the oblateness. Our results, presented in the form of a countour map, give the seasonal and latitudinal variation of the ratio of both insolations. Among the planets discussed in the paper, the obliquity varies from very small (Jupiter) to very large (Uranus), whereas the flattening ranges from approximately 0.021 (Neptune) to 0.1 (Saturn). As expected, the global pattern of the contour maps is strongly dependent on those two parameters.

In addition, the latitudinal variation of the mean daily solar radiations are included in a series of figures demonstrating clearly that the solar radiation of an oblate planet displays a different latitudinal distribution when compared to a spherical planet.

2. CALCULATION OF THE DAILY INSOLATION WITH AND WITHOUT THE OBLATENESS EFFECT

The instantaneous insolation I is defined as the solar heat flux sensed at a given time by a horizontal unit area of the upper-boundary of the atmosphere at a given point on the planet and per unit time. It can be expressed as (see e.g. Ward, 1974; Vorob'yev and Monin, 1975; Levine et al., 1977; Van Hemelrijck and Vercheval, 1981) :

$$I = S \cos z \qquad (1)$$

with :

$$S = S_o / r_\odot^2 \qquad (2)$$

and

$$r_\odot = a_\odot (1 - e^2)/(1 + e\cos W) \qquad (3)$$

z is the zenith angle of the incident solar radiation, S is the solar flux at an heliocentric distance $r_\odot$ and S_o is the solar constant at the mean Sun-Earth distance of 1 AU. For the calculations presented in this paper we have taken a solar constant of 1.94 cal cm^{-2} min^{-1} (or 2.79×10^3 cal cm^{-2} day^{-1}). Furthermore, in expression (3), $a_\odot$, e and W are respectively the planet's semi-major axis, the eccentricity and the true anomaly which is given by :

$$W = \lambda'_\odot - \lambda'_p \qquad (4)$$

where $\lambda'_\odot$ and λ'_p are the planetocentric longitude of the Sun (or solar longitude) and the planetocentric longitude of the planet's perihelion. Table I represents, for the four planets under consideration, the numerical values of the parameters used for the computations. In this table one can find also the obliquity ε, the equatorial radius a_e, the polar radius a_p, the flattening $f = (a_e - a_p)/a_e$ and the sidereal period of axial rotation T (sidereal day). Finally, it has to be mentioned that the orbital and planetary data are taken from the Handbook of the British Astronomical Association (1981) and from Vorob'yev and Monin (1975).

For a spherical planet, the zenith angle z may be written under the following form :

$$\cos z = \sin \varphi' \sin \delta_\odot + \cos \varphi' \cos \delta_\odot \cos h \qquad (5)$$

where φ' is the geocentric latitude (which equals the geographic latitude φ), $\delta_\odot$ is the solar declination and h is the local hour angle of the Sun. Note that in this case the radius vector, characterizing the direction of a surface element, is normal to the horizon plane. Further-

TABLE 1.- Elements of the planetary orbits of Jupiter, Saturn, Uranus and Neptune.

Planet	$a_{\odot}$ (AU)	e	λ'_p (°)	ε (°)	a_e (km)	a_p (km)	f	T (Earth days)
Jupiter	5.2028	0.04847	58.00	3.12	71400	67100	0.06022	0.41
Saturn	9.539	0.05561	279.07	26.73	60000	54000	0.10000	0.44
Uranus	19.18	0.04727	3.02	82.14	26000	24500	0.05769	0.45
Neptune	30.06	0.00859	5.23	29.56	24200	23700	0.02066	0.66

more, the solar declination $\delta_{\odot}$ can be calculated using the following expression :

$$\sin \delta_{\odot} = \sin \varepsilon \sin \lambda'_{\odot} \tag{6}$$

The daily insolation I_D, being defined as the amount of solar radiation incident at the top of a planetary atmosphere over the planet's day, can now be obtained by integrating expression (1) over time during the light time of the day and is given by :

$$I_D = (ST/\pi)\ (h_o \sin \varphi' \sin \delta_{\odot} + \sin h_o \cos \varphi' \cos \delta_{\odot}) \tag{7}$$

where h_o is the local hour angle at sunset (or sunrise) and may be determined from expression (5) by the condition that at sunset (or sunrise) cos z = 0. It follows that :

$$h_o = \arc\cos\ (- \tan \delta_{\odot} \tan \varphi') \tag{8}$$

if

$$|\varphi'| < \pi/2 - |\delta_{\odot}|$$

In regions where the Sun does not rise or more precisely during polar nights ($\varphi' < - \pi/2 + \delta_{\odot}$ or $\varphi' > \pi/2 + \delta_{\odot}$) we have $h_o = 0$. Finally, in regions where the Sun remains above the horizon all day ($\varphi' > \pi/2 - \delta_{\odot}$ or $\varphi' < - \pi/2 - \delta_{\odot}$) we may put $h_o = \pi$.

In the case of an oblate planet there is an angle $v = \varphi - \varphi'$, the so-called angle of the vertical, between the radius vector and the normal to the horizon plane; it vanishes at the equator and the poles, while elsewhere $\varphi > \varphi'$ numerically (note that the maximum value of v is flattening dependent and occurs close to latitude 45°). The angle v is related to the geocentric latitude φ' by the following expression :

$$v = \arctan [(1 - f)^{-2} \tan \varphi'] - \varphi' \qquad (9)$$

Defining Z as the zenith distance for an oblate planet, the following relation can easily be obtained by applying the formulas of spherical trigonometry :

$$\cos Z = \cos v \cos z + \sin v \, (- \tan \varphi' \cos z + \sin \delta_\odot \sec \varphi') \qquad (10)$$

which can also be written under the form :

$$\cos Z = (\cos z \cos \varphi + \sin v \sin \delta_\odot) \sec \varphi' \qquad (11)$$

The daily insolation of an oblate planet I_{DO} can now be obtained by integrating expression (1), within the appropriate time limits, where cos z has to be replaced by relation (10) or (11) yielding :

$$I_{DO} = (ST/\pi) \{\cos v \, (h_{oo} \sin \varphi' \sin \delta_\odot + \sin h_{oo} \cos \varphi' \cos \delta_\odot) +$$

$$\sin v \, [- \tan \varphi' \, (h_{oo} \sin \varphi' \sin \delta_\odot + \sin h_{oo} \cos \varphi' \cos \delta_\odot) +$$

$$h_{oo} \sin \delta_\odot \sec \varphi']\} \qquad (12)$$

or

$$I_{DO} = (ST/\pi)\ [\cos\varphi\ \sec\varphi'\ (h_{oo}\ \sin\varphi'\ \sin\delta_{\odot} + \sin h_{oo}\ \cos\varphi'\ \cos\delta_{\odot}) + h_{oo}\ \sin v\ \sin\delta_{\odot}\ \sec\varphi'] \qquad (13)$$

where h_{oo}, the local hour angle at sunset (or sunrise) for an oblate planet, is generally slightly different from h_o. As for a spherical planet, h_{oo} may be derived from expression (11) by putting $\cos Z = 0$.

Hence :

$$h_{oo} = \text{arc}\cos(-\tan\delta_{\odot}\ \tan\varphi) \qquad (14)$$

As expected, relation (14) is similar to formula (8). Obviously, in areas of permanent darkness ($\varphi < -\pi/2 + \delta_{\odot}$ or $\varphi > \pi/2 + \delta_{\odot}$) and continuous sunlight ($\varphi > \pi/2 - \delta_{\odot}$ or $\varphi < -\pi/2 - \delta_{\odot}$) we have $h_{oo} = 0$ and $h_{oo} = \pi$ respectively.

Taking into account expressions (7) and (12) or (13), the ratio I_{DO}/I_D at the top of the rapidly rotating planets Jupiter, Saturn, Uranus and Neptune can now easily be obtained as a function of geocentric latitude φ' and solar longitude $\lambda'_{\odot}$. Finally, our results are presented in the form of a ratio contour map obtained by an interpolation procedure.

3. SOME QUALITATIVE ASPECTS OF THE RATIO DISTRIBUTION OF THE SOLAR RADIATIONS

It should be pointed out that, considering the complexity of the relations (7) and (12) or (13), a simple analytic expression given φ' as a function of $\lambda'_{\odot}$ and bounding the region in which I_{DO} exceeds I_D, cannot be obtained.

Therefore, in this section, a qualitative study will be given on the behavior of the ratio distribution I_{DO}/I_D for some particular conditions in the northern hemisphere both for summer ($0 < \lambda'_{\odot} < 180°$) and winter ($180° < \lambda'_{\odot} < 360°$) period. It should, however, be emphasized that the results presented are also valid for the southern hemisphere.

3.1. Summer

It is interesting to note that in regions of permanent daylight ($h_o = h_{oo} = \pi$) the isocontours I_{DO}/I_D parallel the lines of constant geocentric latitude and that the solar radiation I_{DO} is always greater than I_D.

Indeed, by putting $h_o = h_{oo} = \pi$ in expressions (12) and (7) we obtain :

$$I_{DO} = ST \sin \delta_{\odot} (\cos v \sin \varphi' + \sin v \cos \varphi') =$$

$$ST \sin \delta_{\odot} \sin (v + \varphi') = ST \sin \delta_{\odot} \sin \varphi \qquad (15)$$

and

stating that $(I_{DO}/I_D)_{max}$ is dependent on the flattening as well as on the obliquity.

Another point of interest regards the length of the day, hereafter defined as the time interval between rising and setting of the Sun. It is clear that the so-defined quantity is proportional to the hour angle at sunset or sunrise.

From (8) and (14), with the condition that $\varphi > \varphi'$ and, in summer, $\delta_\odot > o$ it follows that :

$$(h_{oo})_{SUMMER} > (h_o)_{SUMMER} \tag{19}$$

From expression (19) it can be seen that, in summer, the length of the day of an oblate planet is always greater than the length of the day of a spherical planet except, of course, in the region of permanent sunlight where $h_{oo} = h_o = \pi$.

We can derive an equation governing the time evolution in summer of the function cos Z/cos z at local noon (h = 0) when the Sun is on the meridian plane. From equations (5), by putting h = 0, and (11), where $v = \varphi - \varphi'$, and after some rearrangements we have :

$$\cos Z/\cos z = \cos(\varphi - \delta_\odot)/\cos(\varphi' - \delta_\odot) \tag{20}$$

This expression follows also immediately from the fact that at local noon the zenith distance of the Sun(Z or z) is algebraically equal to the difference between the latitude (φ or φ') and the solar declination $\delta_\odot$.

It can mathematically be proved that, in summer, the iso-contour cos Z/cos z = 1 coïncides closely the curve expressing the solar declination of the Sun ($\delta_{\odot}$) as a function of its planetocentric longitude ($\lambda'_{\odot}$). Moreover, it can be demonstrated that cos Z/cos z > 1for latitudes between the equator and the subsolar point (or $\varphi' < \delta_{\odot}$) and that cos Z/cos z < 1 elsewhere ($\varphi' > \delta_{\odot}$). From (19) and owing to the fact that at local noon cos Z > cos z for $\varphi' < \delta_{\odot}$ it may be concluded that at any time of the day Z < z. In other words, in the region bounded by the equator and the solar declination curve, the horizon plane is tilted towards the Sun (see also Brinkman and Mc Gregor, 1979). Hence, in this area, both the length of the day and cos Z being enhanced, it is obvious that, in this particular region, the solar radiation of an oblate planet is increased with respect to the insolation of a spherical planet. Outside this region, particularly in its neighborhood, it is a priori difficult to predict whether the upper-boundary insolation is increased or decreased, the oblateness effect on the solar radiation depending on the relative importance of the ratio of both the lengths of the day (or h_{oo}/h_o) and the cosines of the zenith distances (cos Z/cos z).

In conclusion we found, on a qualitative basis, two obviously distinguished regions where $I_{DO} > I_D$: the first, near the poles, coinciding with the area of permanent sunlight, the second, in the equatorial region, limited by the seasonal march of the Sun. Note that this two zones are symmetric with respect to the solar longitude $\lambda' = \pi/2$. For latitudes between the subsolar point and the region where the Sun remains above the horizon all day, the ratio cos Z/cos z is decreasing (with cos Z < cos z) whereas h_{oo}/h_o is increasing with the condition that $h_{oo} > h_o$. Whether or how the two above mentioned regions are linked depends, as already stated, on the relative effect of those two ratios and can only be determined by computation of the expression I_{DO}/I_D. In section 4 we will see that the isocontours $I_{DO}/I_D = 1$ coincide remarkably well with the two branches of a hyperbola.

3.2. Winter

During winter season, running from autumnal equinox over winter solstice to vernal equinox and spanning 180°, two main characteristic features are found. On one hand, the length of the day of an oblate planet (h_{oo}) is always smaller than the length of the day of a spherical planet (h_o). This finding can be evaluated by expressions (8) and (14) where the solar declination $\delta_{\odot}$ is reckoned negative. On the other hand, it can be proven that cos Z < cos z over the entire winter season. Indeed, expression (11) can be rewritten as :

$$\cos Z = (\cos \varphi \ \cos z/\cos \varphi') + (\sin v \sin \delta_{\odot}/\cos \varphi') \qquad (21)$$

From the conditions $\sin v > o$, $\cos \varphi' > o$ and $\sin \delta_{\odot} < o$ it follows that the second term on the right hand side of relation (21) is always negative. Hence :

$$\cos Z < (\cos \varphi \ \cos z/\cos \varphi') \qquad (22)$$

The multiplication factor $(\cos \varphi /\cos \varphi')$ being always smaller than unity, finally yields :

$$\cos Z < \cos z \qquad (23)$$

Thus in winter, as already previously stated by Brinkman and Mc Gregor (1979), the horizon plane is tilted away from the Sun, causing both cos Z and the length of the day to be reduced. Consequently, the daily solar radiation of an oblate planet is always reduced when compared to a spherical planet.

4. DISCUSSION OF THE RATIO DISTRIBUTION OF THE SOLAR RADIATIONS

As already mentioned in the introduction and following the method adopted by Levine et al. (1977) in presenting the incident solar radiation in contours of calories per square centimeter per planetary day as a function of geocentric latitude and season, we calculated both I_D and I_{DO}. When comparing our results of I_D with those of the previous authors, a fairly well agreement has to be noticed. As to the computations relative to I_{DO} it is found that, owing to the small flattening of the planets, the global insolation pattern is only slightly different with respect to the solar radiation distribution for a spherical planet. Note that the calculations of I_D and I_{DO} are not included in this paper.

Finally the ratio distribution I_{DO}/I_D is presented in a series of figures, showing very clearly the oblateness effect on the upper-boundary insolation of the atmospheres of Jupiter, Saturn, Uranus and Neptune.

4.1. Jupiter

Application of expressions (7) and (12) or (13) leads to the isopleths illustrated in Fig. 1, where values of constant ratio distribution I_{DO}/I_D are given on each curve.

As for all planets, the effect of the oblateness results in a more extensive polar region (dotted-dashed lines) and, as already demonstrated in section 3.1, in two regions of increased solar radiation (dashed lines). It should be emphasized that the curves $I_D = 0$ and $I_{DO} = 0$ practically coincide (the maximum difference attaining approximately 0.4° at $\lambda'_{\odot} = 270°$) and that the two zones where $I_{DO} > I_D$ are not linked (of which Jupiter is the only case). These two effects are ascribed to the very small obliquity (3.12°) of the planet.

Furthermore, it follows from expression (18) that in the region of permanent sunlight $(I_{DO}/I_D)_{max}$ is negligible small.

An analysis of Fig. 1 reveals that, over near the entire Jovian year, the ratio distribution of both insolations closely parallels the lines of constant geocentric latitude. In summer, it is seen that if φ' increases from about 20 to approximately 60°, the ratio distribution decreases from about 0.98 to 0.92 with a bulge at $\lambda'_{\odot}$ = 90°. In winter, I_{DO}/I_D falls from 0.98 at 20° to 0.85 in the latitude interval 70-80°. At higher latitudes, especially near the region where the Sun does not rise, we have $I_{DO} = 0$ and $I_D \neq 0$. Consequently, in this relatively small zone I_{DO}/I_D drops very rapidly to zero. Furthermore, it is interesting to note that in summer, respectively in winter, the curves are perfectly symmetric with respect to the summer and winter solstices.

In conclusion, it is particularly evident from Fig. 1, that the insolation is reduced, except in two extremely small zones.

4.2. Saturn

The ratio of the solar radiations at the top of Saturn's atmosphere is illustrated in Fig. 2. When comparing Fig. 1 with Fig. 2 it is obvious that the general pattern of the two contour maps is explicitely different.

In summer, the two regions of enhanced solar radiation are joined by curves roughly similar to the two branches of a hyperbola. The effect of the oblateness can clearly be seen to increase the insolation over extensive parts of the summer hemisphere, especially near summer solstice where a rise of the incident solar radiation of the order of 3% has been found. Concerning more particularly the area of continuous sunlight, application of expression (18) leads to a maximum I_{DO}/I_D value of 1.037. Fig. 2 also reveals that in the neighborhood of

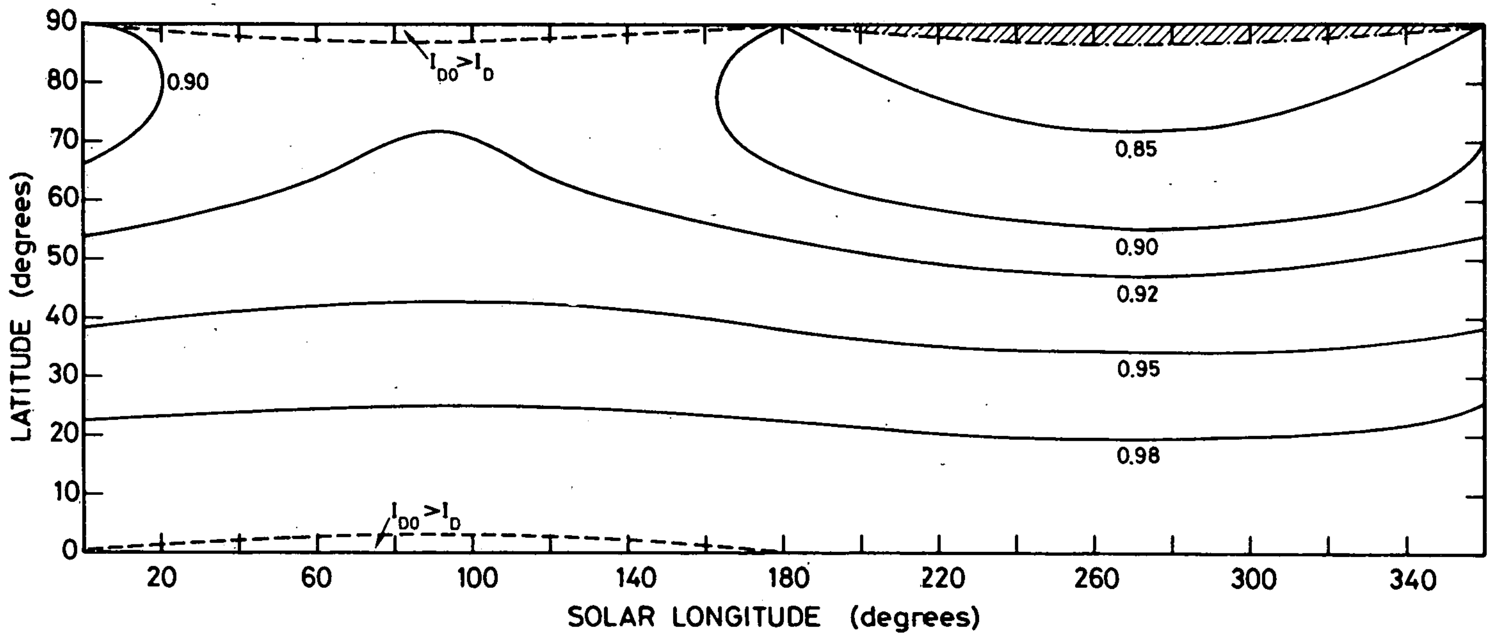


Fig. 1.- Seasonal and latitudinal variation of the ratio of the daily insolation with (I_{DO}) and without (I_D) the oblateness effect at the top of the atmosphere of Jupiter. Solar declination (lower part) and the region where the Sun does not set in the case of a spherical planet (upper part) are represented by the dashed lines. The areas of permanent darkness are shaded and bounded by the dotted-dashed lines. Values of I_{DO}/I_D are given on each curve.

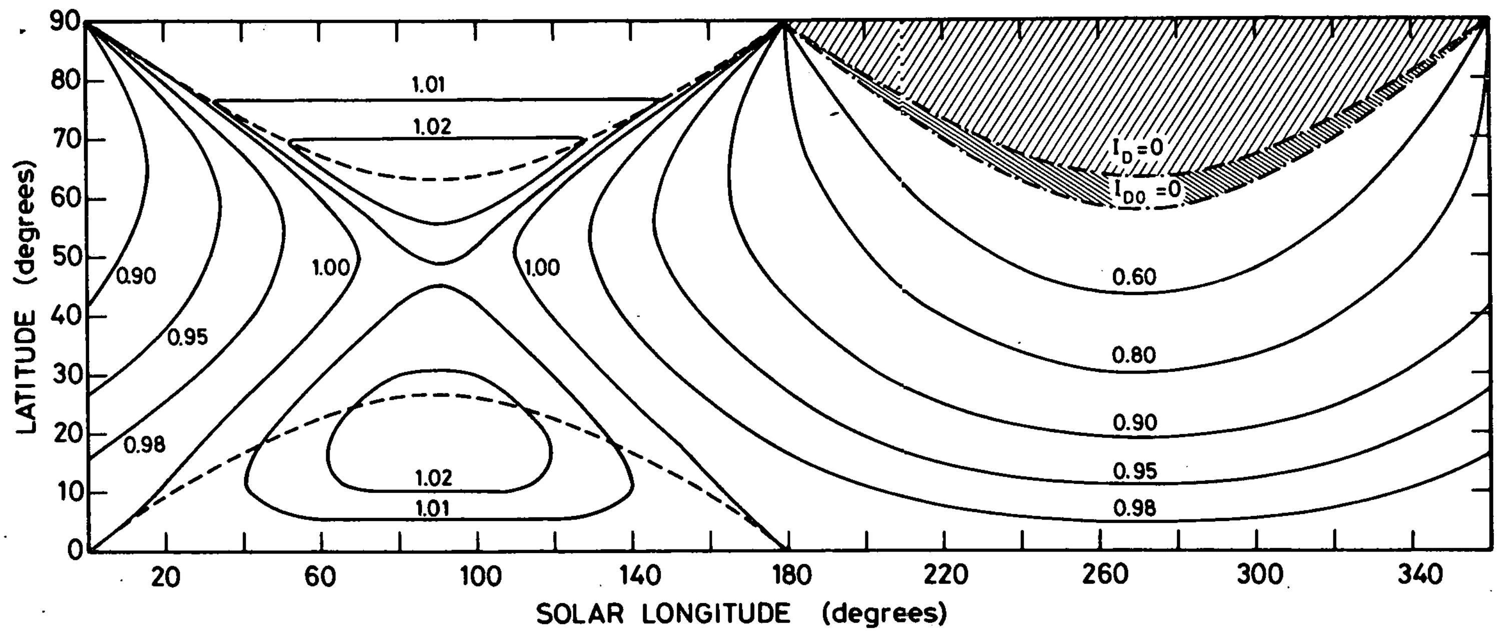


Fig. 2.- Seasonal and latitudinal variation of the ratio of the daily insolation with (I_{DO}) and without (I_D) the oblateness effect at the top of the atmosphere of Saturn. See Fig. 1 for full explanation.

the equinoxes the loss of insolation is of most importance for the mid-latitude regions.

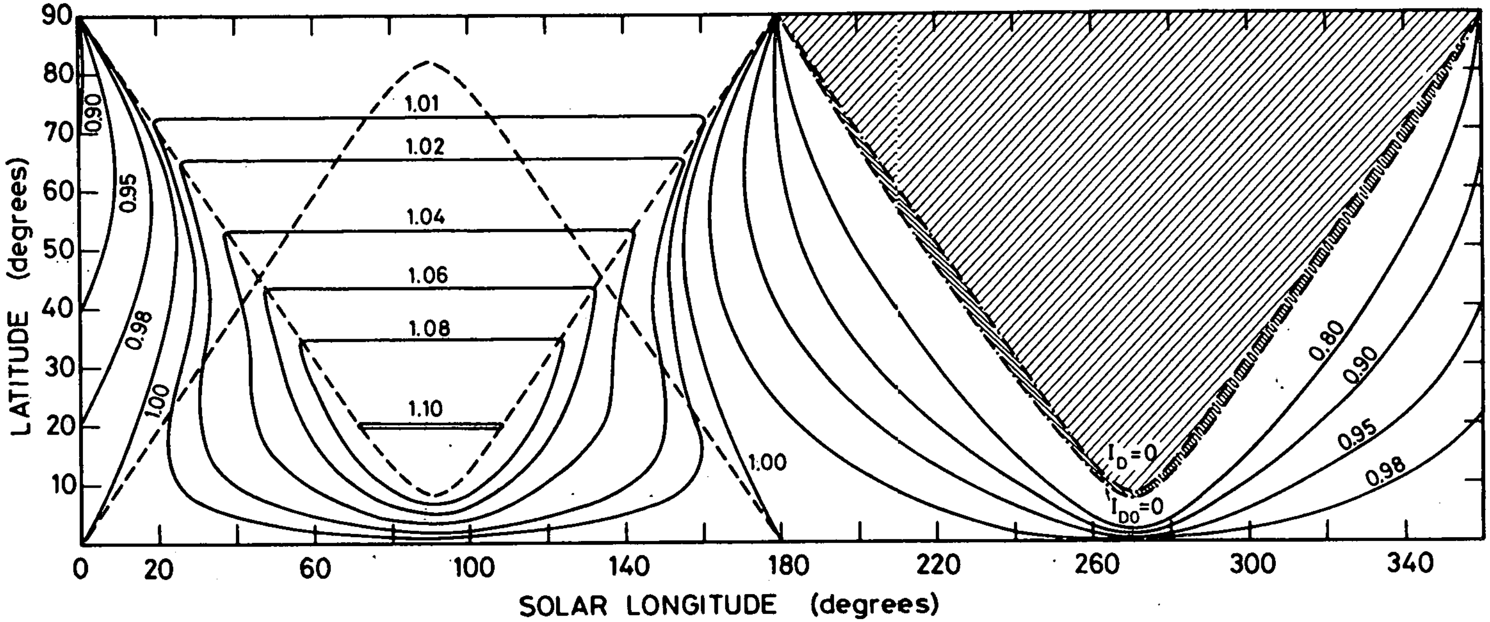
In winter, as already stated previously, the effect of the flattening results in a more extensive polar region; the larger flattening gives rise to a maximum difference, occuring at winter solstice, of approximately 5° as shown in the figure. The solar radiation significantly decreases in passing from equator latitudes to mid- and polar latitudes. For example, if, at $\lambda'_{\odot} = 270°$, φ' increases from 10 to 40°, the ratio I_{DO}/I_D decreases from 0.95 to about 0.60. Moreover, at the same solar longitude and for $\varphi' = 50°$ the solar radiation is reduced by more than a factor of 2.

4.3. Uranus

At $\varepsilon > 45°$ (of which Uranus is the only case), the Arctic circle, bounding the polar region in which there are days without sunset, lies inside of the tropical zone (in which there are days on which the Sun reaches the zenith). As a consequence, the very large obliquity of Uranus (82.14°) results in an increased solar radiation over, roughly speaking, the entire summer season (Fig. 3).

Introducing the numerical values for a_e, a_p and ε (Table I) expression (18) yields : $(I_{DO}/I_D)_{max} = 1.124$. This value approaches very closely the maximum theoretical value equal to $(1 - f)^{-2}$ (= 1.128). It follows that the obliquity and the oblateness of Uranus causes near the equator a gain of insolation of approximately 12% decreasing systematically to about 1% at a geocentric latitude of 70°.

In summer, an important ratio distribution difference exists in that Uranus displays only one maximum whereas for the other planets two maxima are found (Contrast Fig. 3 with Figs. 2 and 4). Fig. 3 also clearly illustrates that in the region where the Sun remains above the horizon all day, the ratio I_{DO}/I_D is independent on the solar longitude $\lambda'_{\odot}$.



<u>Fig. 3.-</u> Seasonal and latitudinal variation of the ratio of the daily insolation with (I_{DO}) and without (I_D) the oblateness effect at the top of the atmosphere of Uranus. See Fig. 1 for full explanation.

In winter, as for all planets, the insolation is reduced, the rate of change being extremely rapid near winter solstice but less sensitive near the equinoxes. Finally, it is worthwhile to note that the maximum difference of the Arctic circles $I_D = 0$ and $I_{DO} = 0$ occurs at $\lambda'_{\odot} = 225°$ and $\lambda'_{\odot} = 315°$ with a value of about 3.4°; at winter solstice this difference is rather small ($\sim$ 0.9°).

4.4. Neptune

The ratio pattern of Neptune is plotted in Fig. 4. Comparison of Fig. 2 with Fig. 4 illustrates some obvious geometric similarities, especially in summer, the two branches of the hyperbola of both planets roughly coinciding. This is due to the fact that the obliquities are approximately equal. On the other hand, a striking difference exists as to the maximum value of the ratio of both insolations (1.010 for Neptune and 1.037 for Saturn). This is easily explained by observing that the flattening of Neptune (0.02066) is much smaller than the value for Saturn (0.1).

In winter, the solar radiation very slowly decreases with increasing geocentric latitude φ'. This effect is also ascribed to the small value of f. Another point about the curves is that the isocontours $I_{DO} = 0$ and $I_D = 0$ practically coincide, the maximum difference being only of the order of 1°.

In conclusion, the sensitivity of the curves to changes in the obliquity and the flattening is clearly illustrated in Figs. 1-4.

5. DISCUSSION OF CALCULATION OF THE MEAN DAILY INSOLATIONS

In the previous sections we discussed the effect of the oblateness on the daily insolation. Here we examine the influence of the flattening on the mean (annual, summer and winter) daily insolations. It should, however, be emphasized that a detailed physical analysis of the

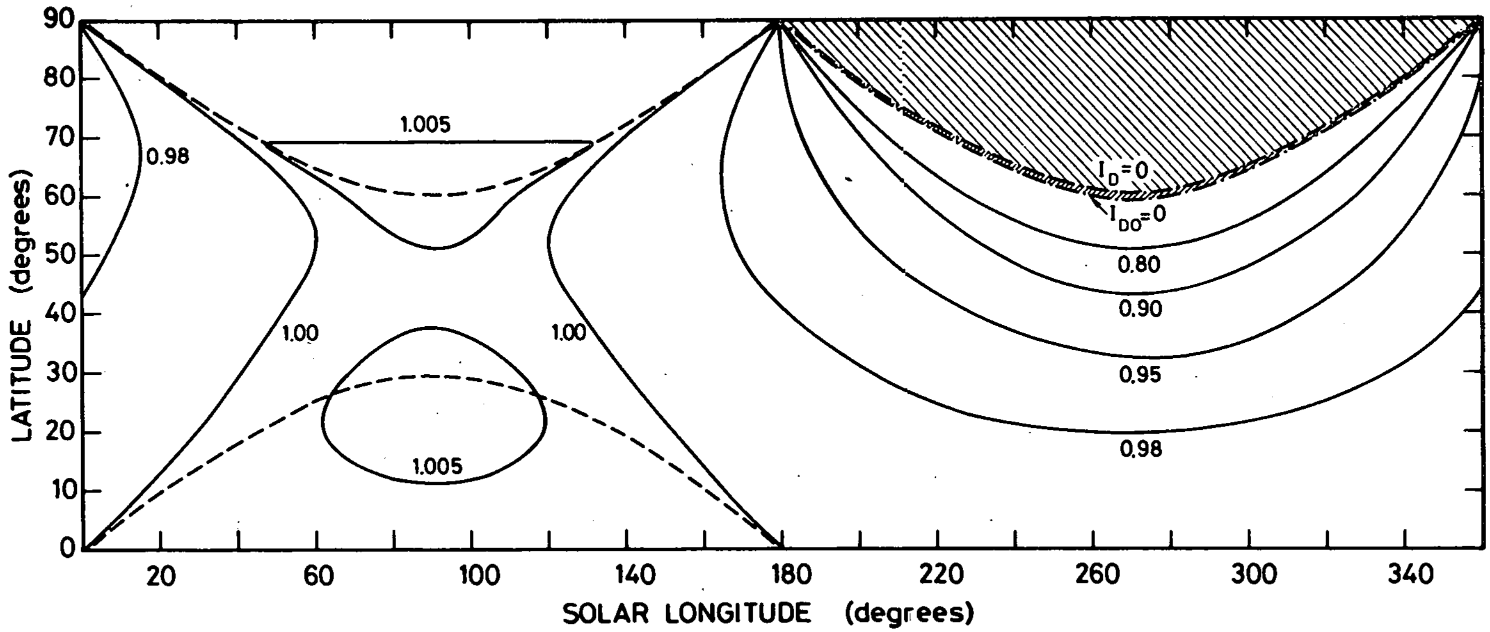


Fig. 4.- Seasonal and latitudinal variation of the ratio of the daily insolation with (I_{DO}) and without (I_D) the oblateness effect at the top of the atmosphere of Neptune. See Fig. 1 for full explanation.

insolation distribution curves is beyond the scope of the present work (see e.g. Levine et al., 1977).

5.1. Jupiter

The oblateness dependence on the mean daily insolations as a function of geocentric latitude is given in Fig. 5. Curves corresponding to a spherical planet are indicated by the full lines, whereas calculations related to an oblate planet are shown by the dashed lines. It has to be pointed out that the mean annual daily insolation distribution on Jupiter, assumed as a spherical planet, is in reasonable agreement with the results of Levine et al. (1977), this conclusion being also valid for the other planets discussed in this work.

Concerning more particularly the mean summertime insolations, hereafter denoted as $(\bar{I}_D)_S$ and $(\bar{I}_{DO})_S$, the importance of the oblateness effect is obviously evident from Fig. 5. Indeed, for latitudes between 60 and 80° as much as 8% of the mean summer daily insolation is lost through the flattening effect. For latitudes less than 60° this effect is of decreasing significance. Another interesting phenomenon (not visible on Fig. 5 due to the insufficiency of the scale adopted for the ordinate) is that for latitudes between the equator and the subsolar point ($\sim$ 3°), $(\bar{I}_{DO})_S > (\bar{I}_D)_S$ (see also Brinkman and Mc Gregor, 1979). However, owing to the small obliquity of Jupiter, the increase of insolation is practically inexistent.

In winter, as already stated previously, the daily insolation of an oblate planet I_{DO} is always reduced when compared to a spherical planet I_D. Consequently, $(\bar{I}_{DO})_W < (\bar{I}_D)_W$ at any latitude, the difference ranging from about 6% at 40° to approximately 20% at polar region latitudes.

Finally, it is obvious that the mean annual daily insolations $(\bar{I}_D)_A$ and $(\bar{I}_{DO})_A$ can be found by the following expressions :

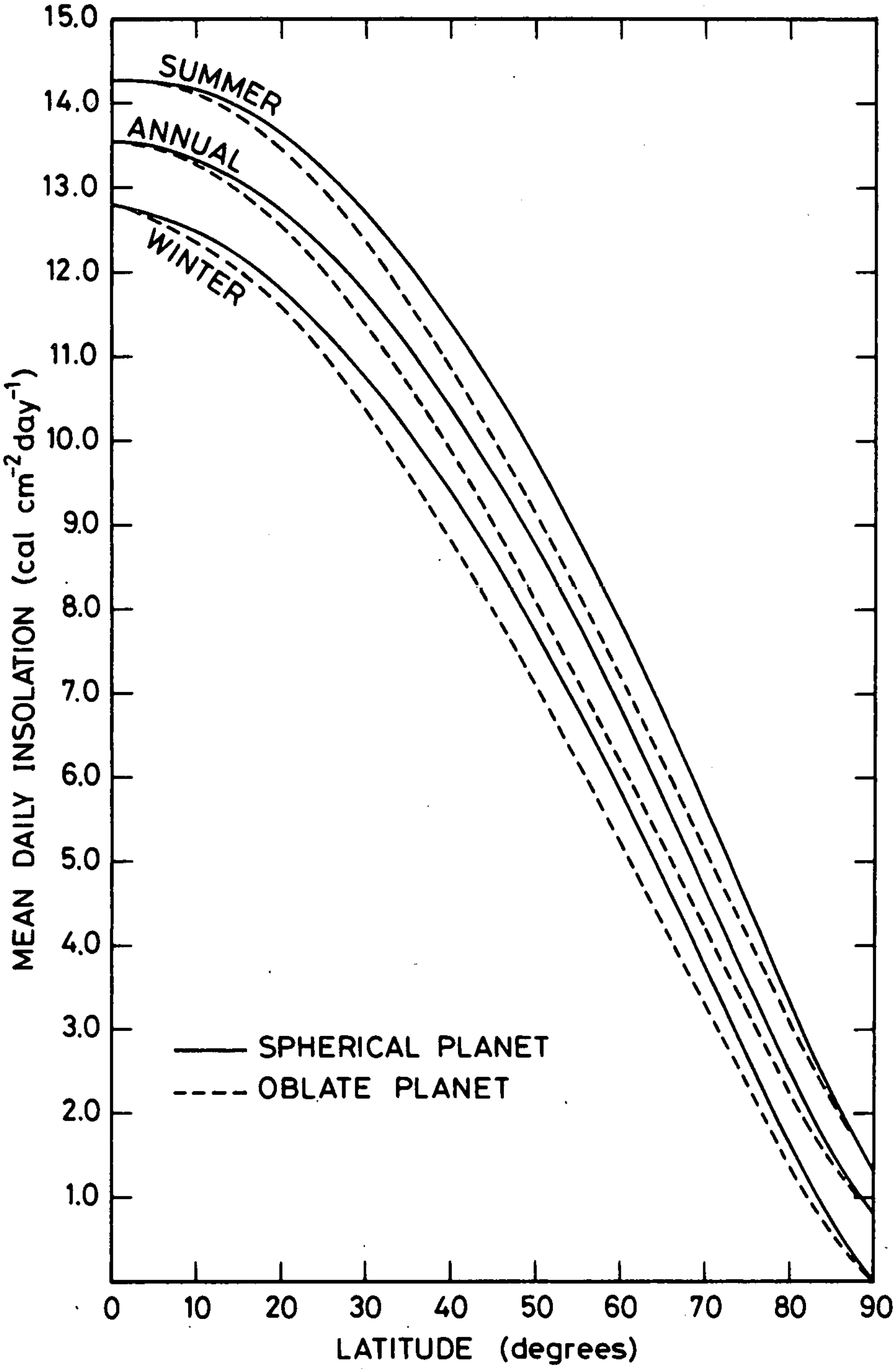


Fig. 5.- Latitudinal variation of the mean daily insolations at the top of the atmosphere of Jupiter.

$$(\bar{I}_D)_A = [(\bar{I}_D)_S \, T_S + (\bar{I}_D)_W \, T_W]/T_0 \quad (24)$$

and

$$(\bar{I}_{DO})_A = [(\bar{I}_{DO})_S \, T_S + (\bar{I}_{DO})_W \, T_W]/T_0 \quad (25)$$

where T_S, T_W and T_O are respectively the length of the northern summer, the length of the northern winter and the sidereal period of revolution (tropical year). Application of expressions (24) and (25) leads to a maximum difference as much as 10% near the pole.

5.2. Saturn

For the case of Saturn, we neglected the shadow effect of the ring system (Brinkman and McGregor, 1979).

In summer, the increase of insolation for latitudes less than the subsolar point (∿ 27°) is clearly demonstrated in the upper part of Fig. 6, the maximum difference occuring at a latitude of about 10° and reaching a value of the order of 1%. Beyond the subsolar point $(\bar{I}_{DO})_S < (\bar{I}_D)_S$, the maximum loss of insolation being found at mid-latitudes and amounts approximately 3%.

The difference between the mean winter insolations increases with increasing latitude and may attain values of about 30% and more at polar region latitudes.

As to the mean annual daily insolation, the maximum difference is equal to about 10% at $\varphi' \sim 50°$.

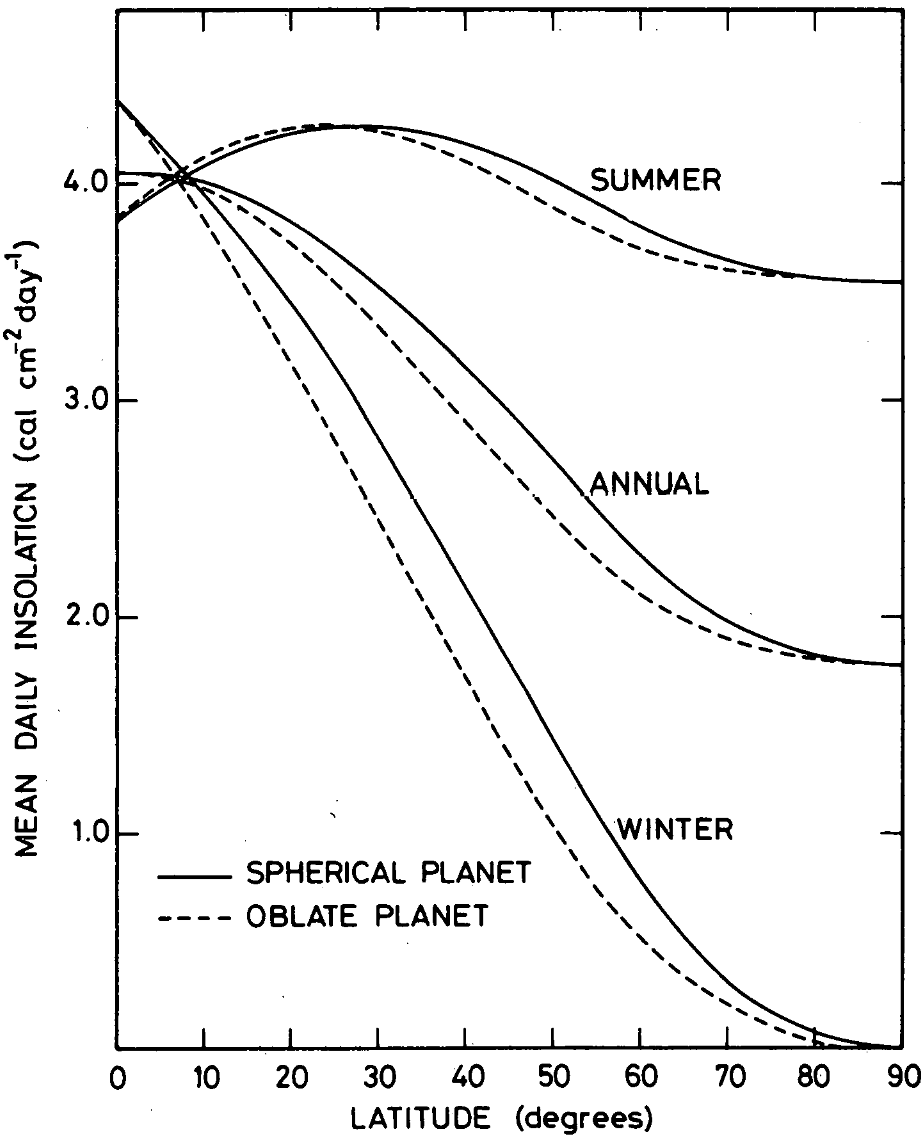


Fig. 6.- Latitudinal variation of the mean daily insolations at the top of the atmosphere of Saturn.

5.3. Uranus

The very large obliquity of Uranus results in an increased mean summertime insolation over practically the entire northern hemisphere except at latitudes between the subsolar point (∿ 82°) and the north pole where it is found that $(\bar{I}_D)_S$ is scarcely above $(\bar{I}_{DO})_S$. The gain of insolation is of most importance for latitudes between 20 and 30° with values of about 5% (Fig. 7).

In winter, especially at high latitudes, as much as 20% of the mean winter daily insolation is lost through the oblateness effect. Furthermore, it is obvious that the mean annual daily insolation will be reduced by an amount determined by the two opposite summer and winter effects. It follows that the decrease of insolation is much smaller than in winter season taking a maximum value approaching 2.5% at latitudes near 30-40°.

5.4. Neptune

The mean daily solar radiation incident on Neptune is given in Fig. 8.

When comparing Fig. 6 with Fig. 8 one agreement as well as one striking difference are noticed. On one hand, the shape of the curves is qualitatively similar, the numerical value of the obliquity of Neptune being close to the value for Saturn. On the other hand, their exists a discrepancy as to the maximum differences between the curves with and without the oblateness effect, the flattening of Neptune (∿ 0.02) being much smaller, by a factor of about 5, than the one for Saturn (0.1). So, in summer, we only obtained + 0.3% and - 0.5% at about 20 and 50° respectively. In winter, the solar energy is reduced by 8% near 70°. Finally, the mean annual daily insolation is increased by approximately 1.5% in the latitude region 40-60°. It is interesting to note that the above mentioned maxima occur at practically the same geocentric latitudes as those for Saturn.

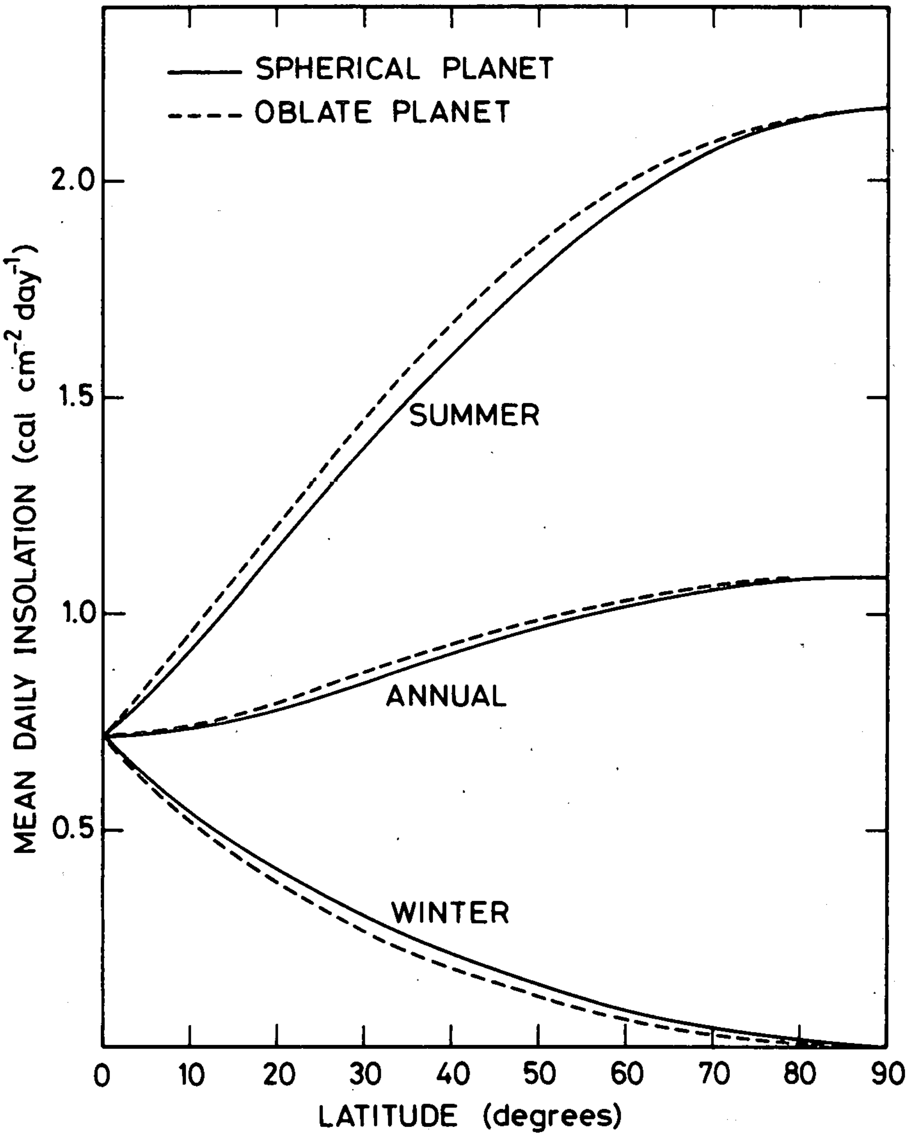


Fig. 7.- Latitudinal variation of the mean daily insolations at the top of the atmosphere of Uranus.

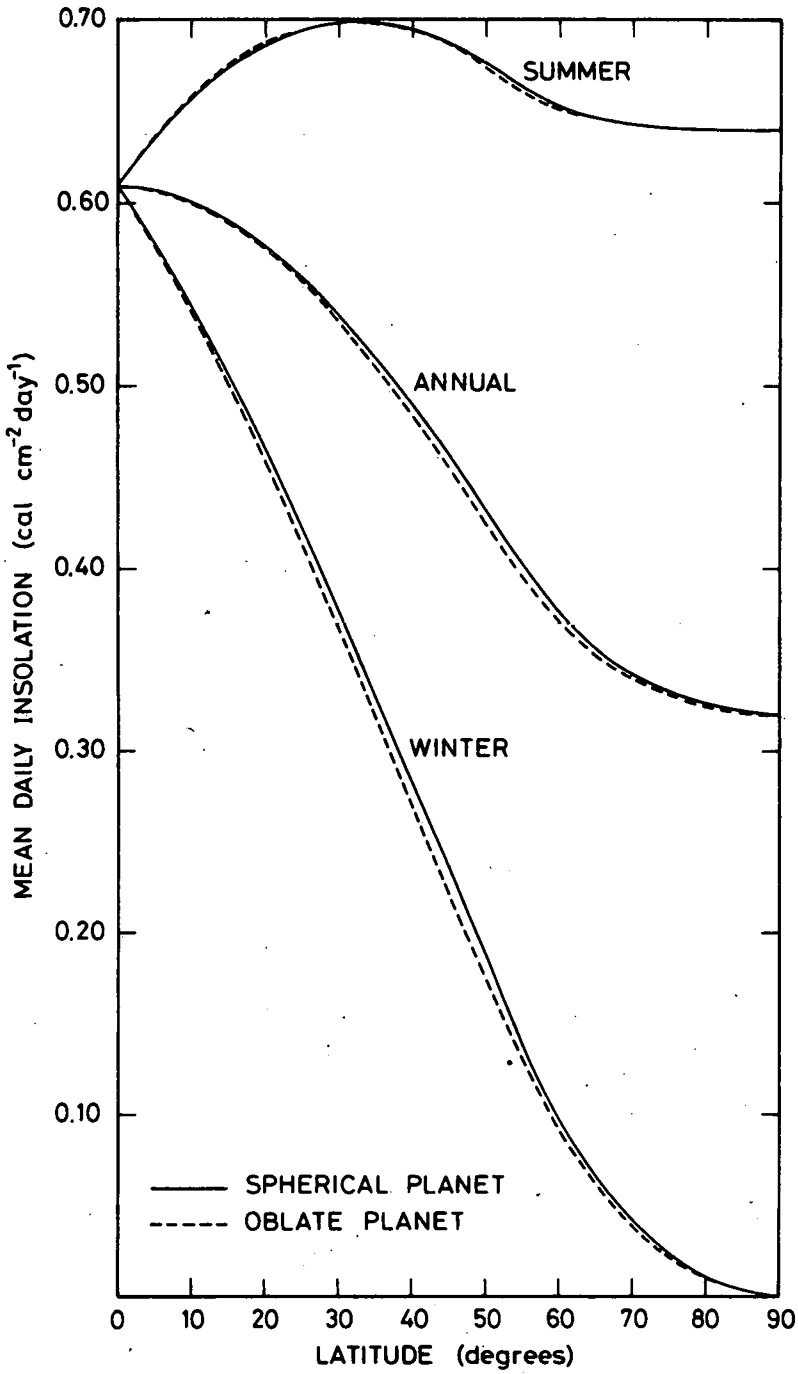


Fig. 8.- Latitudinal variation of the mean daily insolations at the top of the atmosphere of Neptune.

6. CONCLUSIONS

In the preceeding sections we have investigated the influence of the oblateness on the solar radiation incident on Jupiter, Saturn, Uranus and Neptune. As a result of this study, it follows that the flattening causes significant variations in both the planetary-wide distribution and the intensity of the daily insolation.

In summer, the daily insolation is increased over periods strongly dependent upon the obliquity. This phenomenon is particularly evident from Figs. 1-4. Indeed, it can be seen that the very small obliquity of Jupiter ($\varepsilon = 3.12°$) results in two extremely small zones of enhanced solar radiation whereas for Uranus ($\varepsilon = 82.14°$) the region where $I_{DO} > I_D$ extends over almost all of the summer hemisphere. Furthermore, the obliquities of Saturn and Neptune being only slightly different, it follows that the two areas of increased solar energy nearly coincide.

According to expression (18), the maximum value of I_{DO}/I_D in the region of permanent sunlight is a function of both the obliquity and the flattening. This ratio is equal to 1.00033, 1.037, 1.124 and 1.010 for Jupiter, Saturn, Uranus and Neptune respectively. Although for Jupiter the gain of insolation is negligible small, it should be emphasized that for the other planets and for relatively brief periods the oblateness effect plays an obvious role.

In winter, the effect of the flattening results in a more extensive polar region, the maximum difference of the Arctic circles, varying from about 0.4 (Jupiter) to approximately 5° (Saturn). The insolation is always reduced and the curves of constant ratio I_{DO}/I_D roughly parallel the boundary of the polar night except in the neighborhood of the equinoxes. At winter solstice, the rate of change of I_{DO}/I_D with geocentric latitude is very sensitive both to the obliquity and the flattening. When comparing Jupiter with Uranus, having nearly the same flattening but obliquities representing the extreme values, it can be seen that for Jupiter a loss of insolation of 15% occurs over the latitude interval 70-80° whereas for Uranus the above mentioned decrease is already reached in the vicinity of the equator. On the other hand, comparison of the two planets (Saturn and Neptune) with approximately the same obliquity reveals that the gradient of I_{DO}/I_D is strongly dependent upon the flattening. More precisely, for latitudes between the equator and 40°, I_{DO}/I_D falls from unity to 0.6 (Saturn) whereas it drops only to 0.9 in the case of Neptune. It has to be noted that near the equinoxes the loss of insolation is of decreasing importance.

Finally, we also have studied the latitudinal variation of the mean daily insolations. It is found that for latitudes equatorward of the subsolar point, the mean summer daily insolation of an oblate planet is increased when compared to a spherical planet. Although for Jupiter, the gain of insolation is extremely small, the maximum increase is equal to 1,5 and 0.3% for Saturn, Uranus and Neptune respectively.

The mean wintertime insolation however is always decreased, the maximum loss of insolation being much higher than the mean summer-time increase. For example, at polar region latitudes as much as 20 and 30% of the mean winter daily insolation is lost through the oblateness effect for Jupiter and Saturn respectively. The partial gain of the mean summertime insolation being much smaller than the loss of solar energy during winterseason evidently results in a mean annual daily insolation which is reduced over the entire latitude region.

In conclusion, we believe that the effect of the oblateness must be taken into account in studies related to the radiation and energy budget and the dynamical behavior of the planets discussed in this paper.

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