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# AERONOMICA ACTA

A - N° 248 - 1982

The oblateness effect on the solar radiation incident  
at the top of the atmosphere of Mars

by

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## FOREWORD

The paper entitled "The oblateness effect on the solar radiation incident at the top of the atmosphere of Mars" has been presented at the symposium "The planet Mars" held at the University of Leeds, England, August 27, 1982 as a part of the European Geophysical Society Meeting. It will be published in the Proceedings (ESA Special Publication SP-185) of the conference.

## AVANT-PROPOS

L'article intitulé "The oblateness effect on the solar radiation incident at the top of the atmosphere of Mars" a été présenté au symposium "The planet Mars" qui s'est tenu à l'Université de Leeds, Angleterre, le 27 août 1982 dans le cadre de la réunion de la "European Geophysical Society". Il sera publié dans les comptes-rendus (ESA Special Publication SP-185) du symposium.

## VOORWOORD

Het artikel getiteld "The oblateness effect on the solar radiation incident at the top of the atmosphere of Mars" werd voorgedragen op het symposium "The planet Mars" dat gehouden werd aan de Universiteit van Leeds, Engeland op 27 augustus 1982 in het kader van de bijeenkomst van de "European Geophysical Society". Het zal gepubliceerd worden in de mededelingen (ESA Special Publication SP-185) van het symposium.

## VORWORT

Der Text "The oblateness effect on the solar radiation incident at the top of the atmosphere of Mars" würde während des Symposiums "The planet Mars" der in Leeds im Rahmen der Sammlung des "European Geophysical Society" am 27 Augustus 1982 gehalten wurden, vorgetragen werden. Dieses Artikel wird in die Proceedings (ESA Special Publication SP-185) des Symposiums herausgegeben werden.

# THE OBLATENESS EFFECT ON THE SOLAR RADIATION INCIDENT

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## AT THE TOP OF THE ATMOSPHERE OF MARS

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by

E. VAN HEMELRIJCK

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### Abstract

Calculations of the daily solar radiation incident at the top of the atmosphere of Mars (the outer planet with the smallest flattening known), with and without the effect of the oblateness, are presented in a figure illustrating the seasonal and latitudinal variation of the ratio of both insolations. It is shown that for parts of the summer, the daily insolation of Mars, assumed as an oblate planet, is slightly increased. In winter, the flattening effect results in a somewhat more extensive polar region, the solar energy input being always reduced (by more than 5 per cent near the poles) when compared to a spherical planet. In addition, we also numerically studied the mean daily solar radiation. It is found that the mean summer daily insolation is scarcely increased between the equator and the subsolar point, but decreased poleward of the above mentioned limit. In winter, however, the mean daily insolation is always reduced, the maximum loss of insolation attaining approximately 2% in the 60-80° latitude interval. The partial gain of the mean summertime insolation being considerably smaller than the reduction during winter season evidently yields a mean annual daily insolation which is decreased, maximally by about 0.5% near mid-latitudes, at all latitudes.

## Résumé

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L'insolation diurne au sommet de l'atmosphère de Mars est calculée, d'une part en assimilant la planète à une sphère, d'autre part, en tenant compte de son aplatissement. Les résultats sont présentés dans une figure, illustrant les variations saisonnières et latitudinales du rapport des deux insolations.

On montre qu'en été, l'aplatissement donne lieu à une insolation légèrement supérieure dans une région symétrique par rapport au solstice d'été. En hiver, l'effet de l'aplatissement implique une région polaire faiblement étendue, l'insolation diurne d'une planète Mars aplatie s'avèrent toujours petite (dans un rapport parfois supérieur à 5% au voisinage des pôles) comparée à une planète sphérique.

Nous avons également étudié numériquement l'insolation diurne moyenne. Il en résulte que l'insolation diurne moyenne en été augmenté légèrement pour des latitudes comprises entre l'équateur et le point subsolaire, mais diminue pour des latitudes supérieures. En hiver, l'insolation diurne moyenne est toujours moindre, la réduction maximale atteignant une valeur approximative de 2% dans un intervalle de latitude allant de 60 à 80°. Le gain d'insolation diurne moyenne en été étant beaucoup plus petit que la perte en hiver, il s'ensuit que l'insolation diurne moyenne annuelle est réduite à toutes les latitudes (de l'ordre de 0.5% au maximum).

## Samenvatting

Berekeningen van de dagelijkse zonnestraling aan de rand van de atmosfeer van Mars (de uitwendige planeet met de kleinste afplatting), met en zonder afplattingseffect, worden voorgesteld in een figuur die de seizoens- en breedteveranderingen van de verhouding van beide zonnestralingen weergeven.

Er wordt aangetoond dat, in de zomer, de dagelijkse zonnestraling van een afgeplatte planeet Mars lichtjes verhoogt in een gebied symmetrisch gelegen ten opzichte van het zomersolstitium. In de winter resulteert het afplattingseffect in een iets uitgestrekter poolgebied en de dagelijkse zonnestraling blijkt steeds kleiner te zijn (met meer dan 5% in de nabijheid van de polen) vergeleken met deze van een sferische planeet.

Er werd eveneens een numerieke studie gemaakt van de gemiddelde dagelijkse zonnestraling waarbij werd gevonden dat deze in de zomer een weinig verhoogt tussen de evenaar en het subsolaire punt, maar vermindert naar de pool toe. In de winter echter wordt de gemiddelde dagelijkse zonnestraling gereduceerd, maximaal met ongeveer 2% in het geocentrisch breedteinterval  $60-80^\circ$ . De gedeeltelijke winst van de gemiddelde zomerse zonnestraling is echter heel wat kleiner dan het verlies in het winterseizoen zodat de gemiddelde dagelijkse zonnestraling genomen over een gans jaar kleiner is (maximaal met ongeveer 0.5% op gemiddelde breedten) op alle breedten.

## Zusammenfassung

Berechnungen der täglichen Sonnenstrahlung am Rand der Atmosphäre des Planeten Mars (der äusserer Planet mit der geringsten Abplattung), mit und ohne Effekt der Abplattung, sind vorgestellt in einer Abbildung, die die Jahreszeitlichen - und Breitenvariationen des Verhältnisses der beiden Sonnenstrahlungen darstellen.

Es wurde gefunden dass, im Sommer, die tägliche Sonnenstrahlung eines abgeplatteten Planeten Mars ein wenig zunimmt in einem Gebiet, das symmetrisch liegt mit Rücksicht auf dem Sommersolstitium. Im Winter resultiert das Abplattungseffekt in einem ausgedehnten Polgebiet und es ergibt sich, dass die tägliche Sonnenstrahlung auf einem abgeplatteten Planeten Mars immer weniger ist als diese auf einem sphärischen Planeten (bisweilen mit mehr dann 5%).

Es wurde auch eine numerische Analyse der mittleren täglichen Sonnenstrahlung durchgeführt wobei wir gefunden haben, dass im Sommer die mittlere tägliche Sonnenstrahlung erhöht zwischen dem Äquator und der Subsolarpunkt, aber abnimmt nach dem Pol. In Winter wurde die mittlere tägliche Sonnenstrahlung reduziert mit einem Höchstwert von 2% in dem 60-80° Breitenintervall. Die teilweise Gewinn der Sonnenstrahlung im Sommer ist aber merklich kleiner als der Verlust während der Wintersaison, so dass die mittlere tägliche Sonnenstrahlung berechnet über ein ganzes Jahr als Funktion der Breite immer kleiner ist wobei der Höchstwert ungefähr 0.5% beträgt.

## 1. INTRODUCTION

In studies involving solar radiation problems, The instantaneous insolation, defined as the solar heat flux sensed at a given time by a horizontal unit area of the upper boundary of the atmosphere at a given point on the planet and per unit time, constitutes a very important input data. This amount of solar energy and its variability with latitude and time is mainly governed by two factors : on one hand, the solar flux as a function of the orbital distance and, on the other hand, the cosine of the zenith angle of the incident solar radiation.

Considering more particularly the latter parameter, it should be pointed out that for a planet assumed to be spherical, the zenith distance may be expressed as a simple function of latitude, solar declination and local hour angle of the Sun, the radius vector coinciding with the normal to the horizon plane. For an oblate, however, there is an angle (vanishing at the equator and the poles) between the two directions mentioned above. This so-called angle of the vertical is dependent upon the flattening.

Although the oblateness effect on the solar radiation incident at the top of the outer planets Jupiter, Saturn, Uranus and Neptune (Van Hemelrijck, 1981a) and of the Earth (Van Hemelrijck, 1982b) has been treated recently in detail, it should be emphasized that the effect of the flattening on the upper-boundary insolation of Mars, the outer planet having the smallest flattening, has never been studied. This paper, therefore, analyzes this effect.

Our results are presented in the form of a contour map, illustrating the latitudinal and seasonal variation of the ratio of the daily insolation with and without the influence of the oblateness and in a diagram giving the latitudinal variation of the numerical difference of both insolations corresponding to the southern and northern winter

hemisphere respectively. For the sake of completeness we have included a figure showing the percentage difference of the mean (summer, winter and annual) daily insolations as a function of latitude.

## 2. DAILY INSOLATION WITH AND WITHOUT THE OBLATENESS EFFECT

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In this section we only briefly will mention the major expressions needed for the computation of the daily insolation with and without the oblateness effect. For more details see e.g. Ward (1974), Vorob'yev and Monin (1975), Levine et al. (1977), Brinkman and Mc. Gregor (1979), Van Hemelrijck and Vercheval (1981) and Van Hemelrijck (1982a,b,c).

The instantaneous insolation  $I$  at the upper-boundary of a planet can be expressed as :

$$I = S \cos z \quad (1)$$

$$\text{with : } S = S_0 / r_{\odot}^2 \quad (2)$$

$$\text{and : } r_{\odot} = a_{\odot} (1 - e^2) / (1 + e \cos W) \quad (3)$$

where  $z$  is the zenith angle of the incident solar radiation,  $S$  is the solar flux at an heliocentric distance  $r_{\odot}$  and  $S_0$  is the solar constant at the mean Sun-Earth distance of 1AU taken at  $1353 \text{ Wm}^{-2}$  or  $1.94 \text{ cal cm}^{-2}(\text{min})^{-1}$  (Thekaekara, 1973). Furthermore, in expression (3),  $a_{\odot}$ ,  $e$  and  $W$  are respectively the planet's semi-major axis, the eccentricity and the true anomaly which is given by :

$$W = \lambda_{\odot} - \lambda_p \quad (4)$$

where  $\lambda_{\odot}$  and  $\lambda_p$  are the planetocentric longitude of the Sun and the planetocentric longitude of the planet's perihelion. The numerical values of the parameters used for the calculations are listed in Table 1. In this table one can find also the obliquity  $\epsilon$ , the equatorial radius  $a_e$ , the polar radius  $a_p$ , the flattening  $f = (a_e - a_p) / a_e$  and the sidereal

TABLE 1.- Elements of the planetary orbit and dimensions of Mars.

$a_{\odot}$ (AU)	e	$\lambda_p$ (°)	$\epsilon$ (°)	$a_e$ (km)	$a_p$ (km)	f	T (Earth days)
1.524	0.09339	248	25.20	3397.0	3379.5	0.00515	1.02

period of axial rotation  $T$  (sidereal day). Note that the elements of the planetary orbit and the dimensions of Mars are taken from the Handbook of the British Astronomical Association (1980) and from Levine *et al.* (1977).

For a spherical planet,  $z$  may be expressed as :

$$\cos z = \sin \phi' \sin \delta_{\odot} + \cos \phi' \cos \delta_{\odot} \cos h \quad (5)$$

where  $\phi'$  is the geocentric latitude (which equals the geographic latitude  $\phi$ ),  $\delta_{\odot}$  is the solar declination and  $h$  is the local hour angle of the Sun. Furthermore, the solar declination  $\delta_{\odot}$  can be calculated using the following expression :

$$\sin \delta_{\odot} = \sin \epsilon \sin \lambda_{\odot} \quad (6)$$

The daily insolation  $I_D$  can now be obtained by integrating expression (1) over daytime assumed to be equal to the time that elapses between rising and setting of the Sun and is given by :

$$I_D = (ST/\pi)(h_o \sin \phi' \sin \delta_{\odot} + \sin h_o \cos \phi' \cos \delta_{\odot}) \quad (7)$$

where  $h_o$  is the sunset (or sunrise) hour angle and may be determined from expression (5) by the condition that at sunset (or sunrise)  $\cos z = 0$ . Hence :

$$h_o = \arccos (- \tan \delta_{\odot} \tan \phi') \quad (8)$$

if

$$|\phi'| < \pi/2 - |\delta_{\odot}| .$$

In regions where the Sun does not rise ( $\phi' < -\pi/2 + \delta_{\odot}$  or  $\phi' > \pi/2 + \delta_{\odot}$ ) we have  $h_o = 0$ ; in regions where the Sun remains above the horizon all day ( $\phi' > \pi/2 - \delta_{\odot}$  or  $\phi' < -\pi/2 - \delta_{\odot}$ ) we may put  $h_o = \pi$ .

In the case of an oblate planet there is an angle  $v = \phi - \phi'$ , the so-called angle of the vertical, between the radius vector and the normal to the horizon plane; it vanishes at the equator and the poles while elsewhere  $\phi > \phi'$  numerically. The angle  $v$  is dependent upon the geocentric latitude  $\phi'$  and the flattening  $f$  by the relationship :

$$v = \text{arc tan} [(1-f)^{-2} \tan \phi'] - \phi' \quad (9)$$

Denoting the zenith distance for an oblate planet by  $Z$ , the following relation can easily be obtained by applying the formulas of spherical trigonometry :

$$\cos Z = \cos v \cos z + \sin v (-\tan \phi' \cos z + \sin \delta_{\odot} \sec \phi') \quad (10)$$

The daily insolation of an oblate planet  $I_{DO}$  can now be found by integrating expression (1), within the appropriate time limits, where  $\cos z$  has to be replaced by relation (10) yielding :

$$I_{DO} = (ST/\pi) \{ \cos v (h_{oo} \sin \phi' \sin \delta_{\odot} + \sin h_{oo} \cos \phi' \cos \delta_{\odot}) \\ + \sin v [-\tan \phi' (h_{oo} \sin \phi' \sin \delta_{\odot} + \sin h_{oo} \cos \phi' \cos \delta_{\odot}) \\ + h_{oo} \sin \delta_{\odot} \sec \phi'] \} \quad (11)$$

where  $h_{oo}$ , the local hour angle at sunset (or sunrise) for an oblate planet, is generally slightly different from  $h_o$ . As for a spherical planet,  $h_{oo}$  may be derived from relation (10) by putting  $\cos Z : 0$ . After some rearrangements, and as expected, the expression for  $h_{oo}$  in terms of  $\delta_{\odot}$  and  $\phi$  is found to be similar to formula (8).

Taking into account expressions (7) and (11), the ratio  $I_{DO}/I_D$  at the top of Mars can now easily be determined as a function of geocentric latitude  $\phi'$  and solar longitude  $\lambda_{\odot}$ .

The mean (summer, winter and annual) daily insolations, hereafter denoted as  $(\bar{I}_D)_S$ ,  $(\bar{I}_D)_W$  and  $(\bar{I}_D)_A$  (spherical planet) and  $(\bar{I}_{DO})_S$ ,  $(\bar{I}_{DO})_W$  and  $(\bar{I}_{DO})_A$  (oblate planet) respectively, may be found by integrating numerically relation (7) and (11) within the appropriate time limits, yielding the total insolation over a season or a year and by dividing the obtained result by the corresponding length of the season [ $T_S = 381.3$  (summer) and  $T_W = 305.6$  (winter) Earth days] or by the sidereal period of revolution or tropical year ( $T_o = 686.9$  Earth days). Note that  $(\bar{I}_D)_A$  and  $(\bar{I}_{DO})_A$  can also directly be computed from the knowledge of  $(\bar{I}_D)_S$ ,  $(\bar{I}_D)_W$  and  $(\bar{I}_{DO})_S$ ,  $(\bar{I}_{DO})_W$ . Indeed, taking into account the numerical values of  $T_S/T_o$  and  $T_W/T_o$ , the average yearly insolations can, in a very good approximation, be written under the following general form :

$$(\bar{I}_D)_A = 0.56(\bar{I}_D)_S + 0.44(\bar{I}_D)_W \quad (12)$$

and

$$(\bar{I}_{DO})_A = 0.56(\bar{I}_{DO})_S + 0.44(\bar{I}_{DO})_W \quad (13)$$

### 3. DISCUSSION OF THE RATIO DISTRIBUTION OF THE DAILY INSOLATIONS

In two previous works, dealing with the oblateness effect on the solar radiation incident at the top of the atmospheres of the outer planets Jupiter, Saturn, Uranus and Neptune (Van Hemelrijck, 1982a) and of the Earth (Van Hemelrijck, 1982b), we studied qualitatively some characteristic features of the ratio distribution  $I_{DO}/I_D$  in the northern hemisphere both for summer ( $0^\circ < \lambda_\odot < 180^\circ$ ) and winter ( $180^\circ < \lambda_\odot < 360^\circ$ ) period. It should, however, be emphasized that the results presented are also valid for the southern hemisphere and that they are evidently applicable to Mars. The following are the major conclusions that were reached :

- (1) In summer and in the region of permanent sunlight ( $h_{oo} = h_o = \pi$ ) the isocontours  $I_{DO}/I_D$  parallel the lines of constant geocentric

latitude  $\phi'$  and the daily solar radiation  $I_{DO}$  is greater than  $I_D$ . The maximum value of the ratio  $I_{DO}/I_D$  can be expressed by the following relationship :

$$(I_{DO}/I_D)_{\max} = \sin \{ \text{arc tan}[(1-f)^{-2} \cotan \varepsilon] \} / \cos \varepsilon \quad (14)$$

(2) In summer and in the region limited by the seasonal march of the Sun, the solar radiation of an oblate planet ( $I_{DO}$ ) is increased with respect to the insolation of a spherical one ( $I_D$ ).

(3) For latitudes between the subsolar point and the region where the Sun remains above the horizon all day, the ratio  $\cos Z/\cos z$  is decreasing (with  $\cos Z < \cos z$ ), whereas  $h_{oo}/h_o$  is increasing (with  $h_{oo} > h_o$ ). Whether or not the regions mentioned in (1) and (2) are linked depends on the relative effect of those two ratios, both being function of  $f$  and  $\varepsilon$ , and can only be evaluated by computation of the expression  $I_{DO}/I_D$ .

4) In winter, the effect of the flattening results in a more extensive polar region, the insolation is always reduced ( $I_{DO} < I_D$ ) and the curves of constant ratio  $I_{DO}/I_D$  roughly parallel the boundary of the polar night except in the neighborhood of the equinoxes.

Application of expressions (7) and (11) leads to the isocontour map illustrated in Fig. 1, where values of constant ratio distribution  $I_{DO}/I_D$  are given on each curve. Solar declination (lower part) and the region where the Sun does not set (upper part) are indicated by the dashed lines. The area of permanent darkness is shaded and the region of enhanced solar radiation ( $I_{DO} > I_D$ ) is dotted.

From Fig. 1 it can be seen that in the region of permanent sunlight the incoming solar radiation ( $I_{DO}$ ) is increased when compared to

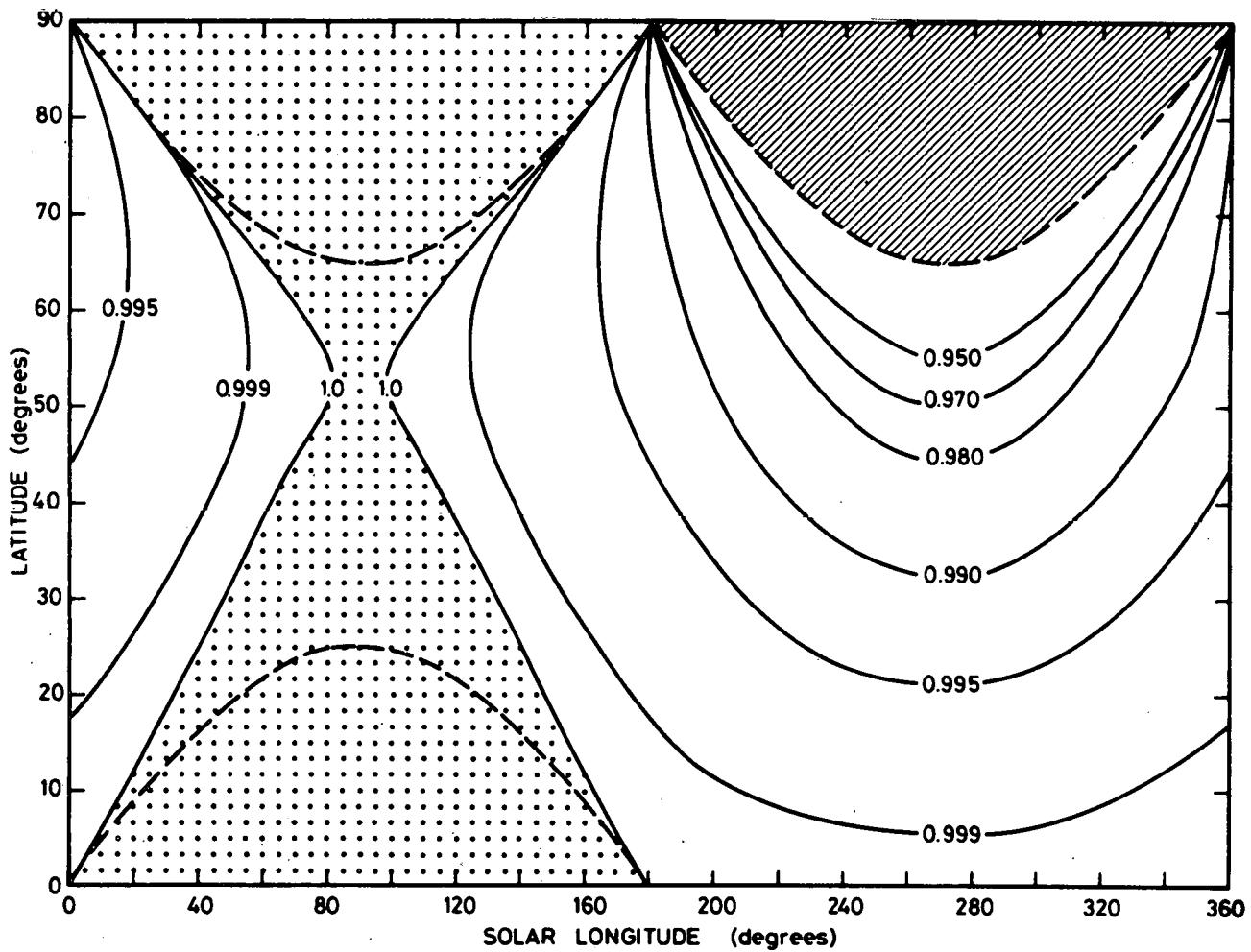


Fig. 1.- Seasonal and latitudinal variation of the ratio  $I_{D0}/I_D$  of the daily insolation with ( $I_{D0}$ ) and without the oblateness effect at the top of the atmosphere of Mars. Solar declination (lower part) and the region where the Sun does not set (upper part) are indicated by the dashed lines. The area of permanent darkness is shaded, whereas the region of enhanced solar radiation ( $I_{D0} > I_D$ ) is dotted. Values of constant ratio distribution  $I_{D0}/I_D$  are given on each curve.

$I_D$ . Furthermore, it follows from expression (14) that in the region considered  $(I_{DO}/I_D)_{max} = 1.0019 (\approx 0.2\%)$ . This extremely small gain of insolation is particularly due to the even small value of the flattening. For comparaison, it is instructive to note that  $(I_{DO}/I_D)_{max}$  is equal to 1.0010, 1.0003, 1.037, 1.124 and 1.010 for the Earth, Jupiter, Saturn, Uranus and Neptune respectively. The effect of parallelism between the isocontours  $I_{DO}/I_D$  and the lines of constant geocentric latitude  $\phi'$  is not illustrated in Fig. 1. The reason for this non-representation lies in the fact that the numerical value of  $I_{DO}/I_D$  is negligible small (1.0012 and 1.0003 at  $\phi' = 70^\circ$  and  $80^\circ$  respectively).

It can mathematically be proved that, in summer, in the region bounded by the equator and the solar declination curve, both the length of the day and the cosine of the zenith distance are enhanced by the effect of the flattening. Hence, it follows that in this particular region, the solar energy input at the top of the atmosphere of Mars, assumed as an oblate, is increased with respect to the insolation of a spherical planet.

Fig. 1 also reveals that the two zones where  $I_{DO} > I_D$  are linked by two curves coinciding remarkably well with the two branches of an hyperbola. Outside the region of increased solar radiation the loss of insolation depends upon the solar longitude. For example, if, at  $\phi' = 45^\circ$ ,  $\lambda$  increases from 0 to approximately  $50^\circ$ , the ratio  $I_{DO}/I_D$  increases from 0.995 ( $\approx 0.5\%$ ) to about 0.999 ( $\approx 0.1\%$ ). For the sake of clearness it has to be pointed out that, in summer, the increase or decrease in solar energy is significantly small.

As for all planets (Van Hemelrijck 1982a,b) the polar region is extended. It should, however, be emphasized that the Artic circles  $I_{DO} = 0$  and  $I_D = 0$  practically coincide, the maximum difference attaining scarcely  $0.23^\circ$  at a solar longitude of  $270^\circ$ . This value is higher than the

one for the Earth ( $0^{\circ}14$ ) but is considerably lower than those for the other planets varying from about 0.4 (Jupiter) to approximately  $5^{\circ}$  (Saturn). It is clear that this finding is ascribed to the extremely small flattening.

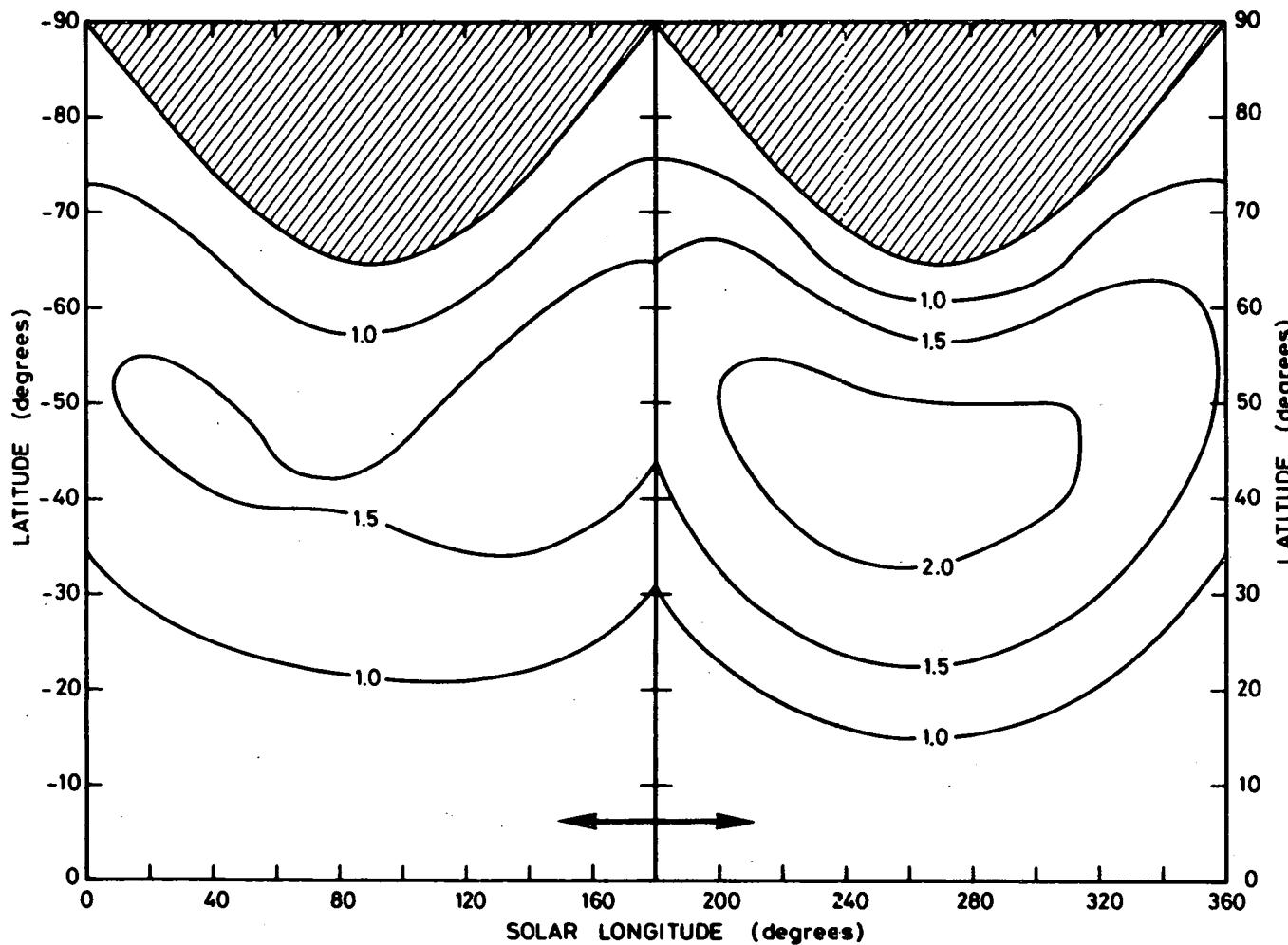
Fig. 1 shows that in winter, as stated earlier, the insolation  $I_{DO}$  is always reduced when compared to  $I_D$ . The incoming solar energy slowly decreases in passing from equator latitudes to mid-latitudes; it is found that at winter solstice and if  $\phi'$  increases from 10 to about  $55^{\circ}$  the ratio  $I_{DO}/I_D$  decreases from 0.999 ( $\approx 0.1\%$ ) to approximately 0.950 ( $\approx 5\%$ ). At higher latitudes, particularly near the region where the Sun does not rise,  $I_{DO}/I_D$  drops very rapidly to zero. Although the loss of insolation is not very striking it follows from Fig. 1 that for parts of the winter the incident solar radiation is decreased by more than 5% through the flattening.

Another point about the curves is that they roughly parallel the limit of the polar night except, of course, in the vicinity of the equinoxes. Finally, it is also suitable to remark that in summer, respectively in winter, the curves of constant ratio  $I_{DO}/I_D$  are perfectly symmetric with respect to the summer and winter solstices.

#### 4. DISCUSSION OF THE NUMERICAL DIFFERENCE DISTRIBUTION OF THE DAILY INSOLATIONS

In Fig. 2 we have plotted the numerical difference [ in  $\text{cal cm}^{-2}$  ( $\text{day})^{-1}$ ] of the daily solar radiation, without ( $I_D$ ) and with ( $I_{DO}$ ) the oblateness effect, as a function of latitude corresponding to the southern and northern winter hemisphere respectively.

Although the isocontours representing the ratio distribution of both insolations ( $I_{DO}/I_D$ ) are perfectly identical in both hemispheres, it follows from the north-south seasonal asymmetry in the daily insolation



**Fig. 2.-** Latitudinal variation of the numerical difference ( $I_D - I_{D0}$ ) of the daily insolations corresponding to the southern (left) and northern (right) winter hemisphere. The areas of permanent darkness are shaded. Values of constant numerical difference distribution, in  $\text{cal cm}^{-2} (\text{day})^{-1}$ , are given on each curve.

produced by the eccentricity of the orbit that the absolute loss of solar radiation as a function of latitude and solar longitude is different in both hemispheres. Fig. 2 clearly indicates that the decrease of solar energy in the northern winter hemisphere is obviously higher than in the southern one reaching a maximum value as much as  $2 \text{ cal cm}^{-2} (\text{day})^{-1}$  at midlatitudes; in the southern hemisphere the maximum loss of insolation is about  $1.5 \text{ cal cm}^{-2} (\text{day})^{-1}$ .

## 5. DISCUSSION OF THE MEAN DAILY INSOLATIONS

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In Fig. 3 the influence of the flattening is plotted in terms of the percentage difference  $[100(\bar{I}_{DO} - \bar{I}_D)/\bar{I}_D]$  as a function of geocentric latitude. As in section 2, the bars over symbols signify seasonal or annual averages.

Concerning more particularly the mean summertime insolation, the influence of the flattening, although very small, is obviously evident from Fig. 3. Indeed, in the latitude interval  $(50-60^\circ)$  it can be seen that about 0.2% of the mean summertime insolation is lost through the oblateness effect. Outside this interval, the effect is of decreasing significance. Another interesting phenomenon is that for latitudes between the equator and the subsolar point, the mean daily summer insolation of Mars, assumed as an oblate planet, is increased (Van Hemelrijck, 1982a,b; Brinkman and McGregor, 1979). However, owing to the small flattening, the rise of insolation is practically negligible, reaching only a maximum value of about 0.05% at a latitude of approximately  $10^\circ$ .

During winter season, as already stated previously, the daily insolation  $I_{DO}$  is always reduced with respect to  $I_D$ ; consequently  $(\bar{I}_{DO})_W < (\bar{I}_D)_W$  at any latitude. Fig. 3 illustrates that in winter the loss of insolation is of most importance between  $45$  and  $85^\circ$  where a percentage difference as much as 1% has been found with a maximum value near 2% in the  $60-80^\circ$  latitude interval.

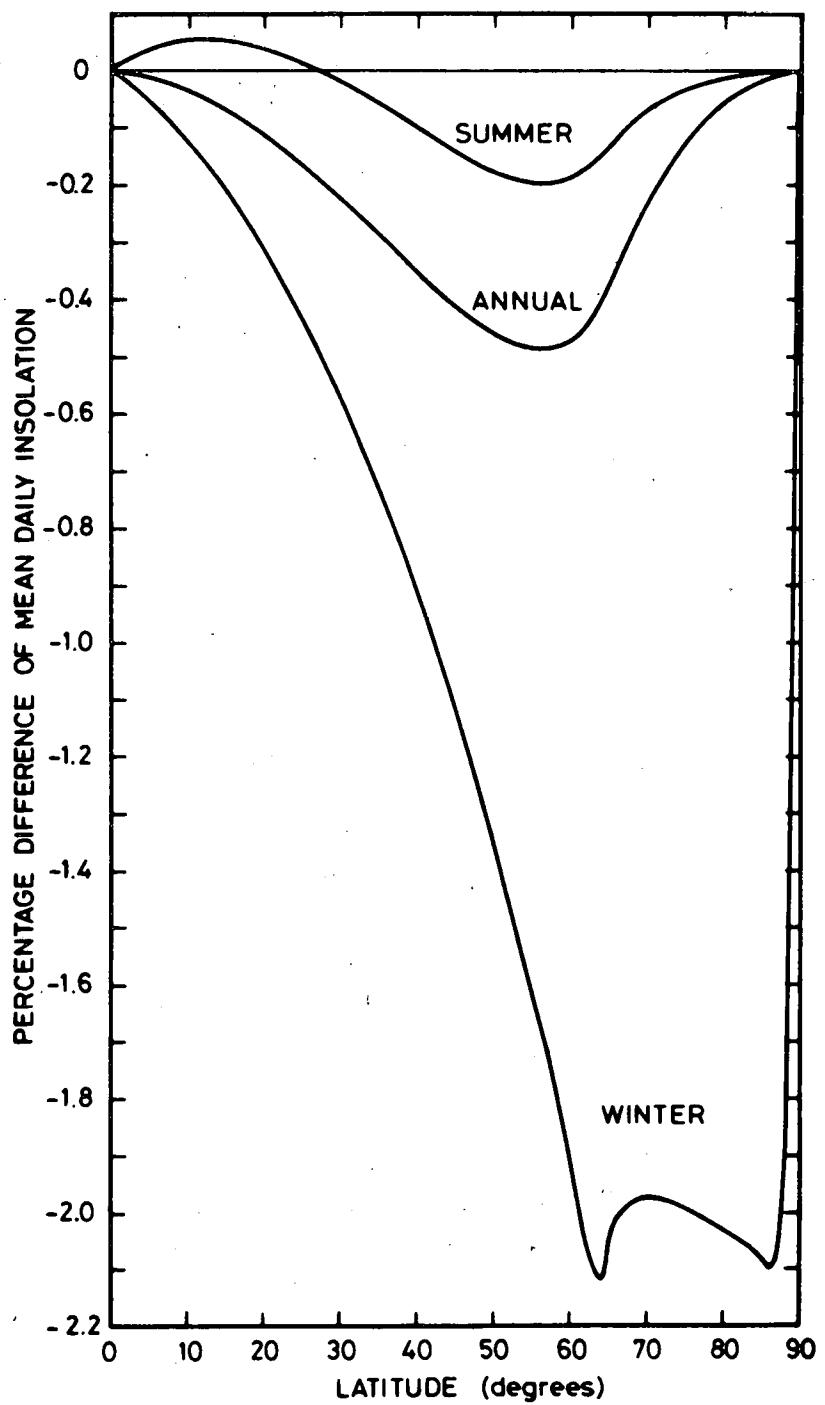


Fig. 3.- Latitudinal variation of the percentage difference  $[100(\bar{I}_{D0} - \bar{I}_D)/\bar{I}_D]$  of the mean (summer, winter and annual) daily insolations. The bars over symbols signify seasonal and annual averages.

The partial gain of the mean summertime insolation equatorward of the subsolar point being considerably lower than the corresponding loss of insolation in winter evidently results in a mean annual daily insolation which is reduced over the entire latitude interval as shown in Fig. 3. It can be seen that the daily insolation averaged over a one year cycle is decreased by about 0.5% at mid-latitudes.

## 6. CONCLUDING REMARKS

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In the present paper, we have investigated the influence of the flattening on the solar energy input at the top of the atmosphere of Mars. As a result of this study we can draw the general conclusion that the effect of the oblateness causes non-negligible, although relatively small, variations in both the planetary-wide distribution and the intensity of the daily solar radiation.

For parts of the summer, the daily insolation of Mars, assumed as an oblate planet, is slightly increased. The maximum increase of the incoming solar radiation, occurring at a geocentric latitude of about  $65^\circ$  near summer solstice is approximately equal to 0.2%. Outside the dotted zone of Fig. 1, the direct solar radiation  $I_{DO}$  is decreased when compared to  $I_D$ . For example, in the neighborhood of the equinoxes the loss of insolation ranges from 0.1 to 0.5% (except at equator latitudes), whereas elsewhere the oblateness effect is either unimportant.

In winter, the influence of the flattening causes the polar region to enlarge over an extremely small distance of about 15 km at winter solstice. The insolation is always reduced, the rate of decrease depending to a large extent on the geocentric latitude. For comparison, at  $\lambda_\odot = 270^\circ$ , the loss of solar energy amounts to about 0.1, 0.5, 1.0, 2.0, 3.0 and 5.0% respectively at latitudes of approximately 5, 20, 30, 45, 50 and  $55^\circ$ . Moreover, in the relatively small area limited by the isocontour  $I_{DO}/I_D = 0.950$  and the region of permanent darkness the

effect of the flattening plays a more significant role and the decrease of incident solar radiation is correspondingly greater. Furthermore, it is particularly evident from Fig. 1 that the curves of constant ratio  $I_{DO}/I_D$  roughly parallel the Arctic Circle bounding the polar region in which there are days without sunrise.

Finally, we also have studied the latitudinal variation of the percentage difference of the mean daily insolations. It is found that for latitudes equatorward of the subsolar point, the mean summer daily insolation of an oblate Mars is increased when compared to a spherical one, the maximum rise being extremely small ( $\approx 0.05\%$ ). At higher latitudes, there is a loss of insolation which is of most importance at mid-latitude regions ( $\approx 0.2\%$ ).

In winter, the horizon plane is always tilted away from the Sun causing both the cosine of the zenith angle and the length of the day to be reduced. Consequently, the daily insolation as well as the mean daily insolation are reduced, the latter decreasing maximally by about 2% at high latitudes.

Despite of the partial gain of the mean summertime insolation near the equator, the effect of the flattening can clearly be seen to reduce the mean daily insolation over the entire year.

In conclusion, we believe that the effect of the oblateness, although very small except in the vicinity of the polar night, has to be taken into account in studies related to theoretical models for the calculation of solar global insolation on Mars.

#### ACKNOWLEDGEMENTS

The author is grateful to J. Schmitz and F. Vandreck for the realisation of the three illustrations.

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