INSTITUT D'AERONOMIE SPATIALE DE BELGIQUE

3 - Avenue Circulaire
B - 1180 BRUXELLES

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On the molecular scattering in the terrestrial atmosphere : an empirical formula for its calculation in the homosphere

by

Marcel NICOLET

BELGISCH INSTITUUT VOOR RUIMTE-AERONOMIE

3 - Ringlaan
B - 1180 BRUSSEL

FOREWORD

This short notice entitled "On the molecular scattering in the terrestrial atmosphere : An empirical formula for its calculation in the homosphere" was written in consequence of a new determination by Bates of the scattering cross-section of molecules in the atmosphere considering the variation of the depolarization factor with the wavelength. It will be published in Planetary and Space Science, Vol. 32, 1984.

AVANT-PROPOS

Cette courte note intitulée "On the molecular scattering in the terrestrial atmosphere : An empirical formula for its calculation in the homosphere" a été rédigée à la suite d'une nouvelle détermination par Bates de la section efficace de diffusion moléculaire dans l'air tenant compte de la variation du facteur de dépolarisation avec la longueur d'onde. Elle sera publiée dans Planetary and Space Science, Vol. 32, 1984.

VOORWOORD

Dit kort bericht getiteld "On the molecular scattering in the terrestrial atmosphere : An empirical formula for its calculation in the homosphere" werd opgesteld ten gevolge van een nieuwe bepaling door Bates van de werkzame strooingsdoorsnede van moleculen in de atmosfeer, rekening houdend met de verandering van de depolarisatiefactor met de golflengte. Het zal gepubliceerd worden in Planetary and Space Science, Vol. 32, 1984.

VORWORT

Diese Notiz mit dem Titel "On the molecular scattering in the terrestrial atmosphere : An empirical formula for its calculation in the homosphere" wurde aufgesetzt infolge eine neue Bestimmung durch Bates der Streuungsquerschnitt von den Molekülen in der Atmosphäre, Rücksicht nehmend auf der Variation der Depolarisationsfaktor mit der Wellenlänge. Die Notiz wird herausgegeben werden in Planetary and Space Science, Vol. 32, 1984.

ON THE MOLECULAR SCATTERING IN THE TERRESTRIAL ATMOSPHERE : AN EMPIRICAL FORMULA FOR ITS CALCULATION IN THE HOMOSPHERE

Marcel NICOLET
Institut d'Aéronomie Spatiale de Belgique
3, Avenue Circulaire, B-1180 BRUSSELS, Belgium.

Abstract

A recent determination by D.R. BATES of the Rayleigh scattering cross section (σ_{RS}) for air from 0.2 μm to 1 μm leads to a simple empirical formula [λ in μm]

$$\sigma_{RS} = 4.02 \times 10^{-28}/\lambda^{4+x} \text{ cm}^2$$

where $x = 0.389\lambda + 0.09426/\lambda - 0.3228$ for the spectral region $0.2 \mu m < \lambda < 0.55 \mu m$; the accuracy is within ± 0.5%. From the visible at 0.55 μm to the infrared at 1 μm, the same accuracy can be obtained using a constant value, $x = 0.04$. The formula accounts for the degree of depolarization which varies with the wavelength according to the latest determination by Bates.

Résumé

Une nouvelle et récente détermination par Bates de la section efficace de diffusion moléculaire (diffusion Rayleigh, σ_{RS}) dans l'air peut être représentée par une formule empirique simple s'appliquant à toute l'homosphère

$$(2)\ ;\ \sigma_{RS} = 4.02 \times 10^{-28}/\lambda^{4+x}\ \text{cm}^2\ (\lambda \text{ in } \mu\text{m})$$

où

$$(3)\ ;\ x = 0.389\lambda + 0.09426/\lambda - 0.3228$$

qui s'applique à toute la région spectrale $0.2\ \mu m < \lambda < 0.55\ \mu m$, avec une bonne précision. La même exactitude peut être obtenue à partir du maximum du visible jusqu'à 1 µm en utilisant une valeur constante $x = 0.04$. La formule générale (2)-(3) tient compte du degré de dépolarisation qui varie avec la longueur d'onde et peut-être utilisée, suivant la détermination de Bates, dans le domaine des bandes de Schumann-Runge de O_2 entre 0.2 µm et 0.19 µm.

Samenvatting

Een nieuwe en recente bepaling door Bates van de werkzame strooiïngsdoorsnede van moleculen (Rayleigh strooiïng, σ_{RS}) in de atmosfeer kan door een eenvoudige empirische formule voorgesteld worden die op de gehele homosfeer van toepassing is

$$(2) \; ; \; \sigma_{RS} = 4.02 \times 10^{-28}/\lambda^{4+x} \text{ cm}^2 \; (\lambda \text{ in } \mu\text{m})$$

of

$$(3) \; ; \; x = 0.389\lambda + 0.09426/\lambda - 0.3228$$

die op het gehele spectrale gebied van toepassing is $0.2\ \mu m < \lambda < 0.55\ \mu m$, met een goede nauwkeurigheid. Dezelfde juistheid kan bekomen worden vanaf het maximum van het zichtbare tot 1 µm door een constante waarde $x = 0.04$ te gebruiken. De algemene formule (2) - (3) houdt rekening met de depolarisatiegraad die varieert met de golflengte en misschien, volgens de bepaling van Bates, in het gebied van de Schumann-Runge banden van O_2 tussen 0.2 µm en 0.19 µm.

Zusammenfassung

Eine neue und rezente Bestimmung durch Bates der Streuungsquerschnitt von den Molekülen (Rayleighstreuung, σ_{RS}) in der Atmosphäre kann mit einer einfachen empirischen Formel vorgestellt werden die auf der ganzen Homosphäre Anwendung findet

$$(2)\ ;\ \sigma_{RS} = 4.02 \times 10^{-28}/\lambda^{4+x}\ \mathrm{cm}^2\ (\lambda \text{ in } \mu)$$

oder

$$(3)\ ;\ x = 0.389\lambda + 0.09426/\lambda - 0.3228$$

die auf dem ganzen Spektralgebiet Anwendung findet $0.2\ \mu m < \lambda < 0.55\ \mu m$, mit guter Genauigkeit. Dieselbe Genauigkeit kann bekommen werden vom Maximum der Sichtbare bis 1 µm durch einen konstanten Wert $x = 0.04$ zu benützen. Die allgemeine Formel (2) - (3) rechnet mit dem Depolarisationsgrad der wechselt mit der Wellenlänge und vielleicht, infolge die Determination vom Bates, im Gebiet der Schumann-Runge Banden von O_2 zwischen 0.2 µm und 0.19 µm.

1.- INTRODUCTION

In a recent paper with R.R. Meier and D.E. Anderson (Nicolet et al., 1982) I discussed (Appendix 2) the problem of the numerical value of the molecular scattering cross section σ_{RS} given by Fröhlich and Shaw (1980). The adopted formula was written as follows :

$$\sigma_{RS} = (3.93 \pm 0.05) \times 10^{-28}/\lambda^{4+x} \text{ cm}^2 \qquad (1)$$

assuming a median value of about 1.03 ± 0.01 for the correction factor of King (1923) which corresponds to a possible error of about ± 1% for the molecular scattering optical depth. But, for the direct application to the atmosphere (Meier et al., 1982), we adopted the formula given by Elterman (1968). However, the series of publications by Young (1980, 1981a,b, 1982) on the value of King's correction factor, and also the observations of the direct and the scattered solar flux within the stratosphere (Herman and Mentall, 1982), have led us to reexamine the problem of the radiation field in the stratosphere (cf. Meier and Anderson, 1984), and to make a critical study of the experimental data relating to the depolarisation factors.

2.- NEW DETERMINATION OF RAYLEIGH SCATTERING BY AIR

The detailed reexamination of the problem by Bates (1984) has brought about a clarification of the question. This study makes it possible first to find an accurate value for the molecular scattering cross section, which can be used throughout the homosphere (troposphere, stratosphere and mesosphere) where the mean molecular mass remains constant. It makes it possible also to know the variation of King's correction factor with wavelength; Bates shows that the factor varies from about 1.05 in the visible to 1.08 in the ultraviolet at 0.2 μm. These values are well above the value given by Fröhlich and Shaw (1980).

Finally, Bates' detailed results, which he kindly made available to me prior to publication, can lead to the determination of a simple expression which permits the rapid calculation of the atmospheric optical thickness attributable to molecular scattering. The simplicity of a formula makes it possible to determine quickly, and as a function of wavelength, the importance of the effects due to the various components of atmospheric absorption. The most important of these arise from oxygen at wavelengths less than 240 nm, and from ozone throughout the ultraviolet and also in the visible (Chappuis bands). The absorption in these two cases must be compared with the effect of molecular diffusion in different spectral regions.

3.- EMPIRICAL FORMULA

Between 0.55 μm in the visible and the shorter ultraviolet wavelengths, the data provided by Bates (1984) can be represented by the expression

$$\sigma_{RS} = 4.02 \times 10^{-28} / \lambda^{4+x} \qquad (2)$$

where

$$x = 0.389\lambda + 0.09426/\lambda - 0.3228 \qquad (3)$$

and λ is expressed in μm. The value of x increases from about 0.04 at 0.55 μm to 0.23 at 0.2 μm. In practive, the accuracy of the simple formula is better than ± 0.5%. Hence it can be assumed that, for atmospheric applications, the molecular scattering optical depth is known to an accuracy of better than ± 1%; this leads to a determination of the optical thickness better than that resulting from oxygen and ozone. It should also result in a better understanding of the radiation field in the stratosphere.

Between the visible ($\lambda > 0.55$ μm) and the infrared regions, the required accuracy can be obtained by adopting the constant value : $x = 0.04$. Thus for $0.55 < \lambda < 1$ μm, the simple expression

$$\sigma_{RS} = 4.02 \times 10^{-28}/\lambda^{4.04} \ \mathrm{cm}^2 \qquad (4)$$

can be used for making comparisons with the absorption of the Chappuis bands of ozone.

The importance of the new determination of molecular scattering cross section that has been made by Bates can be clearly seen in the study of the atmospheric transmittance at 0.2 μm. At this wavelength, the O_2 absorption cross section may reach a low value of about 6×10^{-24} cm^2 (Nicolet, 1983), while that for ozone is near its minimum value, and hence molecular scattering must play an important role. The molecular scattering optical depth may be as great as 30% of the O_2 optical depth near 0.2 μm. Near 0.225 μm, its effect is greater than 50% of the O_2 optical depth when the theoretical absorption cross section of the O_2 Herzberg continuum is reduced by a factor of 2.

If it is permissible to consider the Herzberg continuum in the spectral region of the (2-0) to (5-0) Schumann-Runge bands of O_2 (at wavelengths greater than 0.19 μm), where the rotational lines do not play any role in the atmospheric absorption, the contribution of molecular scattering in the absorption is of the order of 30% of that of molecular oxygen.

When new experimental values of the absorption cross sections of O_2 and O_3 become available, it will be possible to determine with better accuracy the relative importance of the direct attenuated solar flux and of the multiply-scattered flux at various levels in the stratosphere and mesosphere.

ACKNOWLEDGEMENTS

I am particularly grateful to Dr. Andrew Young, whose personal communications have allowed me to "find my way about in the (Rayleigh scattering) jungle". It is also a great pleasure for me too to thank Professor Sir David Bates for agreeing to apply his wide experience to the rapid resolution of the problem of molecular scattering, which poses so many difficulties in studies of the radiation field in the atmosphere.

TABLE I : Molecular Scattering Cross Section. Formulas (2) and (3).

Wavelength (μm)	x	Cross section (cm^2)	Cross section (O_2) (cm^2)
0.200	0.226	3.60 (- 25)	1.73 (- 24)
.201	.224	3.51	1.69
.202	.222	3.43	1.65
.203	.221	3.35	1.61
.204	.219	3.27	1.57
.205	.217	3.19	1.53
.206	.215	3.12	1.50
.207	.213	3.05	1.46
.208	.211	2.98	1.43
.209	.210	2.91	1.40
.210	.208	2.84	1.37
.211	.206	2.78	1.33
.212	.204	2.72	1.30
.213	.203	2.66	1.28
.214	.201	2.60	1.25
.215	.199	2.54	1.22
.216	.198	2.49	1.19
.217	.196	2.43	1.17
.218	.194	2.38	1.14
.219	.193	2.33	1.12
.220	.191	2.28	1.09
.221	.190	2.23	1.07
.222	.188	2.19	1.05
.223	.187	2.14	1.03
.224	.185	2.10	1.01
.225	.184	2.05	9.85 (- 25)

TABLE I : Continued.

Wavelength	x	Cross section	Cross section $(O_2)^{(*)}$
0.226	0.182	2.01 (- 25)	9.65 (- 25)
.227	.181	1.97	9.45
.228	.179	1.93	9.26
.229	.178	1.89	9.07
.230	.176	1.85	8.89
.231	.175	1.82	8.72
.232	.174	1.78	8.54
.233	.172	1.74	8.37
.234	.171	1.71	8.21
.235	.170	1.68	8.05
.236	.168	1.64	7.89
.237	.167	1.61	7.74
.238	.166	1.58	7.59
.239	.165	1.55	7.45
.240	.163	1.52	7.31
.241	.162	1.49	7.17
.242	.161	1.47	7.03
.243	.160	1.44	6.90
.244	.158	1.41	6.77
.245	.157	1.38	6.65
.246	.156	1.36	6.53
.247	.155	1.33	6.41
.248	.154	1.31	6.29
.249	.153	1.29	6.18
.250	.151	1.26	6.06
.251	.150	1.24	5.95
.252	.149	1.22	5.85
.253	.148	1.20	5.74

TABLE I : Continued.

Wavelength	x	Cross section	Cross section $(O_2)^{(*)}$
0.254	0.147	1.18 (- 25)	5.64 (- 25)
.255	.146	1.15	5.54
.256	.145	1.13	5.45
.257	.144	1.11	5.35
.258	.143	1.10	5.26
.259	.142	1.08	5.17
.260	.141	1.06	5.08
.261	.140	1.04	4.99
.262	.139	1.02	4.91
.263	.138	1.01	4.82
.264	.137	9.88 (- 26)	4.74
.265	.136	9.72	4.66
.266	.135	9.55	4.59
.267	.134	9.40	4.51
.268	.133	9.24	4.44
.269	.132	9.09	4.36
.270	.131	8.94	4.29
.271	.130	8.79	4.22
.272	.130	8.65	4.15
.273	.129	8.51	4.09
.274	.128	8.37	4.02
.275	.127	8.24	3.95
.276	.126	8.11	3.89
.277	.125	7.98	3.83
.278	.124	7.85	3.77
.279	.124	7.73	3.71
.280	.123	7.61	3.65

TABLE I : Continued.

Wavelength	x	Cross section	Cross section $(O_2)^{(*)}$
0.281	0.122	7.49 (- 26)	3.60 (- 25)
.282	.121	7.37	3.54
.283	.120	7.26	3.48
.284	.120	7.15	3.43
.285	.119	7.04	3.38
.286	.118	6.93	3.33
.287	.117	6.83	3.28
.288	.117	6.72	3.23
.289	.116	6.62	3.18
.290	.115	6.52	3.13
.291	.114	6.42	3.08
.292	.114	6.33	3.04
.293	.113	6.23	2.99
.294	.112	6.14	2.95
.295	.111	6.05	2.90
.296	.111	5.96	2.86
.297	.110	5.88	2.82
.298	.109	5.79	2.78
.299	.109	5.71	2.74
.300	.108	5.62	2.70
.301	.107	5.54	2.66
.302	.107	5.46	2.62
.303	.106	5.39	2.59
.304	.106	5.31	2.55
.305	.105	5.24	2.51
.306	.104	5.16	2.48
.307	.104	5.09	2.44
.308	.103	5.02	2.41

TABLE I : Continued.

Wavelength	x	Cross section	Cross section $(O_2)^{(*)}$
0.309	0.102	4.95 (- 26)	2.38 (- 25)
.310	.102	4.88	2.34
.311	.101	4.81	2.31
.312	.101	4.75	2.28
.313	.100	4.68	2.25
.314	.0995	4.62	2.22
.315	.0990	4.55	2.19
.316	.0984	4.49	2.16
.317	.0979	4.43	2.13
.318	.0973	4.37	2.10
.319	.0968	4.31	2.07
.320	.0962	4.26	2.04
.321	.0957	4.20	2.02
.322	.0952	4.14	1.99
.323	.0947	4.09	1.96
.324	.0942	4.04	1.94
.325	.0937	3.98	1.91
.326	.0932	3.93	1.89
.327	.0927	3.88	1.86
.328	.0922	3.83	1.84
.329	.0917	3.78	1.81
.330	.0912	3.73	1.79
.331	.0907	3.68	1.77
.332	.0903	3.64	1.75
.333	.0898	3.59	1.72
.334	.0893	3.55	1.70
.335	.0889	3.50	1.68
.336	.0884	3.46	1.66

TABLE I : Continued.

Wavelength	x	Cross section	Cross section (O_2)(*)
0.337	0.0880	3.41 (- 26)	1.64 (- 25)
.338	.0876	3.37	1.62
.339	.0871	3.33	1.60
.340	.0867	3.29	1.58
.341	.0863	3.25	1.56
.342	.0859	3.21	1.54
.343	.0854	3.17	1.52
.344	.0850	3.13	1.50
.345	.0846	3.09	1.48
.346	.0842	3.05	1.46
.347	.0838	3.01	1.45
.348	.0834	2.98	1.43
.349	.0830	2.94	1.41
.350	.0827	2.91	1.40
.351	.0823	2.87	1.38
.352	.0819	2.84	1.36
.353	.0815	2.80	1.35
.354	.0812	2.77	1.33
.355	.0808	2.74	1.31
.356	.0805	2.71	1.30
.357	.0801	2.67	1.28
.358	.0798	2.64	1.27
.359	.0794	2.61	1.25
.360	.0791	2.58	1.24
.361	.0787	2.55	1.22
.362	.0784	2.52	1.21
.363	.0781	2.49	1.20
.364	.0778	2.46	1.18

TABLE I : Continued.

Wavelength	x	Cross section	Cross section $(O_2)^{(*)}$
0.365	0.0774	2.44 (- 26)	1.17 (- 25)
.366	.0771	2.41	1.16
.367	.0768	2.38	1.14
.368	.0765	2.35	1.13
.369	.0762	2.33	1.12
.370	.0759	2.30	1.10
.371	.0756	2.28	1.09
.372	.0753	2.25	1.08
.373	.0750	2.23	1.07
.374	.0747	2.20	1.06
.375	.0744	2.18	1.04
.376	.0742	2.15	1.03
.377	.0739	2.13	1.02
.378	.0736	2.10	1.01
.379	.0733	2.08	9.99 (- 26)
.380	.0731	2.06	9.88
.381	.0728	2.04	9.77
.382	.0726	2.01	9.67
.383	.0723	1.99	9.56
.384	.0720	1.97	9.46
.385	.0718	1.95	9.36
.386	.0716	1.93	9.26
.387	.0713	1.91	9.16
.388	.0711	1.89	9.06
.389	.0708	1.87	8.96
.390	.0706	1.85	8.87
.391	.0704	1.83	8.78
.392	.0701	1.81	8.68

TABLE I : Continued.

Wavelength	x	Cross section	Cross section $(O_2)^{(*)}$
0.393	0.0699	1.79 (- 26)	8.59 (- 26)
.394	.0697	1.77	8.50
.395	.0695	1.75	8.41
.396	.0693	1.73	8.33
.397	.0691	1.72	8.24
.398	.0689	1.70	8.15
.399	.0687	1.68	8.07
.400	.0685	1.66	7.99

(*) The total homospheric concentration $n(M) = 4.8\ n(O_2)$, the O_2 concentration, the molecular scattering cross sections of the fourth column is related to the O_2 concentration. It leads to an immediate comparison of the role played in the atmospheric optical depth with the O_2 absorption cross section in the Herzberg continuum.

TABLE II : Molecular Scattering Cross Section. Formulas (2) and (3). Spectral region of the (2-0) to (5-0) Schumann-Runge bands of O_2.

Wavelength (μm)	x(*)	Cross Section (cm^2)	Cross Section (O_2) (cm^2)
0.200	0.226	3.60 (- 25)(**)	1.73 (- 24)
.199	.228	3.69	1.77
.198	.230	3.78	1.81
.197	.232	3.87	1.86
.196	.234	3.97	1.91
.195	.236	4.07	1.95 (***)
.194	.239	4.18	2.00
.193	.241	4.28	2.06
.192	.243	4.39	2.11
.191	.245	4.51	2.16
.190	.247	4.63	2.22

(*) The x value leads to the possibility for a simple interpolation at all wavelengths.

(**) In good agreement with an extension of his published calculation (Bates, private communication).

(***) Value corresponding to 20% - 30% of the O_2 absorption cross section peak in the Herzberg continuum.

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