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The solar radiation incident at the top of the atmosphere of Uranus and Neptune

by

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FOREWORD

The paper entitled "The solar radiation incident at the top of the atmosphere of Uranus and Neptunus" will be published in "Earth, Moon, and Planets", 1988.

AVANT-PROPOS

L'article intitulé "The solar radiation incident at the top of the atmosphere of Uranus and Neptune" sera publié dans la revue "Earth, Moon, and Planets", 1988.

VOORWOORD

Het artikel "The solar radiation incident at the top of the atmosphere of Uranus and Neptune" zal gepubliceerd worden in het tijdschrift "Earth, Moon, and Planets", 1988.

VORWORT

Der Artikel "The solar radiation incident at the top of the atmosphere of Uranus and Neptune" wird herausgegeben in "Earth, Moon, and Planets", 1988.

THE SOLAR RADIATION INCIDENT AT THE TOP OF THE ATMOSPHERE OF URANUS AND NEPTUNE

by

E. VAN HEMELRIJCK

Abstract

The latitudinal and seasonal variation of the direct solar radiation incident at the top of the atmosphere of Uranus and Neptune has been recalculated based on updating values for the period of axial rotation and the oblateness. Values for the solar radiation are given in Watt per square meter instead of the unit used in earlier papers (calories per square centimeter per planetary day). The solar radiation averaged over a season and a year as a function of planetocentric latitude has also been reviewed. In addition, attention is made to the ratio of the solar radiation incident on an oblate planet to that incident on a spherical planet.

Résumé

Les variations saisonnières et latitudinales de l'insolation directe au sommet de l'atmosphère d'Uranus et de Neptune ont été re-calculées basées sur des valeurs récentes de la période de rotation sidérale et de l'aplatissement. L'insolation est exprimée en Watt par mètre carré au lieu de l'unité utilisée dans des publications antérieures (calories par centimètre carré par jour planétaire). L'insolation moyenne saisonnière et annuelle a également été revue ainsi que l'effet de l'aplatissement sur l'insolation.

Samenvatting

De breedte- en seizoensveranderingen van de directe zonnestraling invallend aan de rand van de atmosfeer van de planeten Uranus en Neptunus werden herrekend gesteund op recente waarden voor de siderische rotatieperiode en de afplatting. De zonnestraling werd uitgedrukt in Watt per vierkante meter in plaats van de eenheid die in vroegere publicaties werd gebruikt (calorieën per vierkante centimeter per planeetdag). De gemiddelde seizoens- en jaarlijkse zonnestralingen werden eveneens herzien. Het effect van de afplatting op de zonnestraling werd ook besproken.

Zusammenfassung

Die Jahreszeitlichen- und Breitenvariationen der direkten Sonnenstrahlung am Rand der Atmosphäre des Planeten Uranus und Neptun wurden herrechnet gemäss rezenten Werten für die Axialrotationperiode und die Abplattung. Die Sonnenstrahlung ist formuliert in Watt je Quadratmeter statt der Einheit gebraucht in früheren Veröffentlichungen (Kalories je Quadratzentimeter je Planettag). Die mittlere jahreszeitlichen- und jährlichen Sonnenstrahlungen wurden auch besprochen wie auch der Effekt der Abplattung auf der Sonnenstrahlung.

1. INTRODUCTION

The solar radiation incident at the top of the atmosphere of the outer planets (Jupiter, Saturn, Uranus, Neptune and Pluto) has been calculated by several investigators (Vorob'yev and Monin, 1975; Levine et al., 1977; Brinkman and McGregor, 1979; Van Hemelrijck, 1982a,b,c, 1985, 1986; Beebe et al., 1986).

In the first two papers the seasonal variation of the solar energy outside the atmosphere as a function of latitude is calculated for the so-called Jovian or giant planets and the planets are assumed to be spherical. Brinkman and McGregor (1979) and Van Hemelrijck (1987) represented only the Saturnian upper-boundary insolation but included both the oblateness effect and the effect of the ring system. Corrections due to the flattening of the outer planets (excluding Pluto) were studied in detail by Van Hemelrijck (1982a), whereas an attempt has been made to compute the insolation at Pluto (Van Hemelrijck, 1982b,c, 1985). Very recently, calculations have been carried out by Beebe et al. (1986) defining the extent of variation in solar radiation incident at the top of the Jovian atmosphere for a selected set of planetocentric latitudes.

Although the orbital and planetary elements needed for the determination of the solar radiation reaching the top of the atmosphere of Jupiter and Saturn are known with sufficient accuracy, some of the parameters for the three most distant members of the solar system are more difficult to measure and are poorly determined. The rotation periods of Uranus and Neptune are not yet sufficiently known and their geometrical flattening is still questionable (Hughes, 1979; Davies et al., 1980; Franklin et al., 1980; Elliot et al., 1980; Beebe, 1983; Belton and Terrile, 1984; French, 1984; Podolak et al., 1985; Cruikshank, 1985; French et al., 1985). For Pluto, measurements of the angle between the planet's spin axis and its orbit normal (or obliquity) vary greatly (Anderson and Fix, 1973; Golitsyn, 1979; The Handbook of the British

Astronomical Association, 1980). However, in a paper published by Dobrovolskis and Harris (1983), the authors investigated the history of Pluto's obliquity by numerical integration using analytic approximations of Williams and Benson (1971).

In this work, we recalculated the solar radiation at Uranus and Neptune using for the rotation period and the oblateness those values which seem to be in best accord with the observations and which are less or more accepted by the planetary science community.

In a first section we briefly summarize some expressions needed for the determination of the upper - boundary insolation. Then, taking into account their orbital and planetary data we calculate the daily insolation with (I_{DO}) and without (I_D) the effect of the oblateness. The results are presented in two contour maps showing the incident solar radiation in Watt per square meter as a function of latitude and solar longitude and in two other ones giving the seasonal and latitudinal variation of the ratio of both insolations (I_{DO}/I_D). In addition, the latitudinal dependency of the mean annual [$(\bar{I}_{DO})_A$, $(\bar{I}_D)_A$], summer [$(\bar{I}_{DO})_S$, $(\bar{I}_D)_S$] and winter [$(\bar{I}_{DO})_W$, $(\bar{I}_D)_W$] daily insolations are included in two plots, whereas the percent differences 100 $(\bar{I}_{DO} - \bar{I}_D)/\bar{I}_D$ of the mean daily insolations are illustrated in two graphs.

In our calculations and for the northern hemisphere, the summer season is arbitrary defined as running from vernal equinox to autumnal equinox and spanning 180°; thus the planetocentric longitudes of the Sun equal to 180° and 360° respectively mark the beginning and the end of the winter period. In the southern hemisphere, the solar longitude intervals (0-180°) and (180-360°) divide the year into astronomical winter and summer respectively.

2. CALCULATION OF THE SOLAR RADIATION

The daily insolation for a spherical planet may be expressed as (see e.g. Ward, 1974; Vorob'yev and Monin, 1975; Levine et al., 1977; Van Hemelrijck, 1982a,b,c, 1983, 1985, 1987) :

$$I_D = [S_o T(1+e\cos W)^2/\pi a_\odot^2 (1-e^2)^2] (h_o \sin\phi' \sin\delta_\odot + \sin h_o \cos\phi' \cos\delta_\odot) \quad (1)$$

where S_o is the solar constant at the mean Sun-Earth distance of 1AU taken at 1368 W m^{-2} (Wilson et al., 1981; Wilson, 1982), T is the rotation period, e is the eccentricity, $a_\odot$ is the semi-major axis, h_o is the local hour angle at sunset or sunrise, ϕ' is the planetocentric latitude, $\delta_\odot$ is the solar declination or subsolar latitude and W is the true anomaly which is given by :

$$W = \lambda_\odot - \lambda_P \quad (2)$$

where $\lambda_\odot$ and λ_P are respectively the planetocentric longitude of the Sun (called solar longitude in the Figures) and the planetocentric longitude of the planet's perihelion. Furthermore, h_o and $\delta_\odot$ may be obtained from standard spherical trigonometric relationships and depend upon ε, called the obliquity or axial tilt of the planet.

For an oblate planet, characterized by a flattening factor $f = (r_e - r_p)/r_e$ where r_e and r_p are respectively the equatorial and the polar radius, there is an angle v (the so-called angle of the vertical) between the radius vector and the normal to the horizon plane. This angle is also equal to the mathematical difference between the planetographic latitude (ϕ) and the planetocentric latitude (ϕ'). In terms of the latter, v can be written as :

$$v = \text{atan} [(1 - f)^{-2} \tan \phi'] - \phi' \quad (3)$$

The daily insolation of an oblate planet may be expressed as (see e.g. Van Hemelrijck, 1982a, 1987) :

$$I_{DO} = [S_o T(1 + e \cos W)^2/\pi a_{\odot}^2 (1 - e^2)^2]$$

$$\times \{\cos v(h_{oo} \sin \phi' \sin \delta_{\odot} + \sin h_{oo} \cos \phi' \cos \delta_{\odot})$$

$$+ \sin v[- \tan \phi' (h_{oo} \sin \phi' \sin \delta_{\odot} + \sin h_{oo} \cos \phi' \cos \delta_{\odot})$$

$$+ h_{oo} \sin \delta_{\odot} \sec \phi']\} \quad (4)$$

where h_{oo}, the local hour angle at rising or setting of the Sun for an oblate planet is, in general, slightly different from h_o.

Finally, the mean summer, winter or annual daily solar radiations may be found by integrating relation (1) or (4) within the appropriate time limits, yielding the total amount of solar energy received over a season or a year, and by dividing the obtained result by the corresponding length of the summer (T_S) or winter (T_W) or tropical year (T_o). For the calculation of T_S or T_W we refer e.g. to Van Hemelrijck (1982c).

As already mentioned in the introduction, the period of rotation (T) and the planetary flattening (f) are not easy quantities to determine especially in the case of the outer planets. Table I (Uranus) and Table II (Neptune) represent some values of the period of rotation reported in the literature over the last 15 years.

The Tables clearly demonstrate that the discrepancy between the various determinations of the rotation rates of Uranus and Neptune present a major problem. However, in the planetary science community there seems to be a consensus that the rotation period of Uranus is about 16 hr and that of Neptune is of the order of 18 hr (Podolak et al., 1985) since those values seem to be in reasonable agreement with recent analysis of the observations.

TABLE I.- Rotation period of Uranus.

ROTATION PERIOD (HR)	REFERENCE
24(+ 3/- 3)	Hayes and Belton (1977)
24	Slavsky and Smith (1981)
23.923 (+ 0.003/- 0.003)	Smith and Slavsky (1979)
23(+ 5/- 2)	Trafton (1977)
16.6(+ 0.5/- 0.5)	Franklin et al. (1980)
16.4	O'Meara (1984)
16.2	O'Meara (1984)
16.16(+ 0.33/- 0.33)	Brown and Goody (1980)
16.0	O'Meara (1984)
16(+ 1/- 1)	French (1984)
15.57(+ 0.80/- 0.80)	Brown and Goody (1977)
15.5	Elliot et al. (1981)
15.0(+ 4.0/- 2.6)	Munch and Hippelein (1980)
15-17	Belton and Terrile (1984)
13.0(+ 1.3/- 1.3)	Trauger et al. (1978)
12.8(+ 1.7/- 1.7)	Elliot et al. (1980)
10.8	Newburn and Gulkis (1973)

TABLE II.- Rotation period of Neptune.

ROTATION PERIOD (HR)	REFERENCE
22(+ 4/- 4)	Hayes and Belton (1977)
19.583(+ 0.005/- 0.005)	Cruikshank (1978)
18.56	Belton et al. (1981)
18.44(+ 0.01/- 0.01)	Slavsky and Smith (1978)
18.432	Smith and Slavsky (1978)
18.29	Belton et al. (1981)
18.173(+ 0.005/- 0.005)	Cruikshank (1978)
17.83(+ 0.1/- 0.1)	Podolak et al. (1985)
17.73(+ 0.1/- 0.1)	Brown et al. (1981)
17.73	Belton et al. (1981)
17.7 - 18.6	Belton and Terrile (1984)
15.8	Newburn and Gulkis (1973)
15.4(+ 3/- 3)	Belton et al. (1980)
15(about)	Kovalevsky and Link (1969)
15(about)	Freeman and Lynga (1970)
15(about)	Hubbard et al. (1985)
11.2(+ 1.8/- 1.8)	Munch and Hippelein (1980)

In the Report of the IAU Working Group on Cartograhic Coordinates and Rotational Elements of the Planets and Satellites (Davies et al., 1980), the recommended value for the geometric oblateness or flattening (also called ellipticity) of Uranus was taken from Dollfus (1970) and equals 0.030; other measurements cited in the same paper are 0.033 (Elliot et al., 1980), 0.022 (Franklin et al., 1980) and 0.010 (Danielson et al., 1972). According to Franklin et al. (1980) a correction has to be made to the value deduced by Danielson et al. (1972). After applying the correction the new value amounts to 0.020. Hildebrand et al. (1985) and Orton et al. (1986) used a value f = 0.024 which was given by Elliot et al. (1981), whereas the calculations made by Wagener et al. (1986) are based on an oblateness factor equal to 0.022 (French, 1984).

In the IAU report the flattening for Neptune (0.0259) came from analysis of occultation observations by Freeman and Lynga (1970). Finally, Hildebrand et al. (1985), Wagener et al. (1986) and Orton et al. (1986) used an ellipticity f = 0.021 given by Elliot (1979).

In our earlier paper (Van Hemelrijck, 1982a) dealing with the oblateness effect on the solar radiation incident at the top of the atmosphere of the outer planets (excluding Pluto) the planetary and orbital data for Uranus and Neptune were based on Vorob'yev and Monin (1975), Levine et al. (1977) and the Handbook of the British Astronomical Association (1981). In the present paper the same values are used except, of course, for the rotational period (T) and for the oblateness (f). For Uranus and Neptune we used T = 0.66 (Earth days, corresponding to 16 hr) and T = 0.75 (18 hr) respectively. For the oblateness of Uranus two values were taken, i.e. f = 0.022 and 0.033; for Neptune we used f = 0.0259.

The planetary data for the calculations are listed in Table III, values in parenthesis where those used in Vorob'yev and Monin (1975) (T), Levine et al. (1977) (T) and Van Hemelrijck (1982a) (T and f).

As mentioned earlier, the incident solar radiation is given in Watt per square meter; insolation values expressed in calories per square centimeter per planetary day as in the papers by Vorob'yev and Monin (1975) and Levine et al. (1977) may be obtained by multiplying the unit used by a factor of about 2.065.

3. DISCUSSION OF CALCULATION

3.1. Daily solar radiation

The daily solar radiation of Uranus (and also of Neptune) is presented in the form of a contour map giving the seasonal distribution in terms of the planetocentric longitude of the Sun taken to be 0° at the northern vernal equinox. Application of expression (4) with $f = 0.033$ leads to the isopleths illustrated in Fig. 1. (The isopleths corresponding to $f = 0.022$ are only slightly different from Fig. 1).

As already stated by Levine et al. (1977), the very large obliquity of Uranus results in a position reverse of both hemispheres : the northern hemisphere lies "below" the ecliptic, the southern one "above" it.

From the Figure it can be seen that the incident solar radiation reaches its maximum at the poles around the summer solstices with values of about 2.5 W m^{-2}. The insolation at the north pole during summer solstice $(I_D)_{NP(ss)}$ is approximately equal to that of the south pole at its summer solstice $(I_D)_{SP(ss)}$ with $(I_D)_{NP(ss)} > (I_D)_{SP(ss)}$. This can easily be evaluated by computing the insolation at both poles; it follows that (Van Hemelrijck, 1982c, 1985) :

$$(I_D)_{NP(ss)}/(I_D)_{SP(ss)} = [(1 + e \sin \lambda_P)/(1 - e \sin \lambda_P)]^2 \qquad (5)$$

Hence : $(I_D)_{NP(ss)} > (I_D)_{SP(ss)}$ if $0 < \lambda_P < \pi$

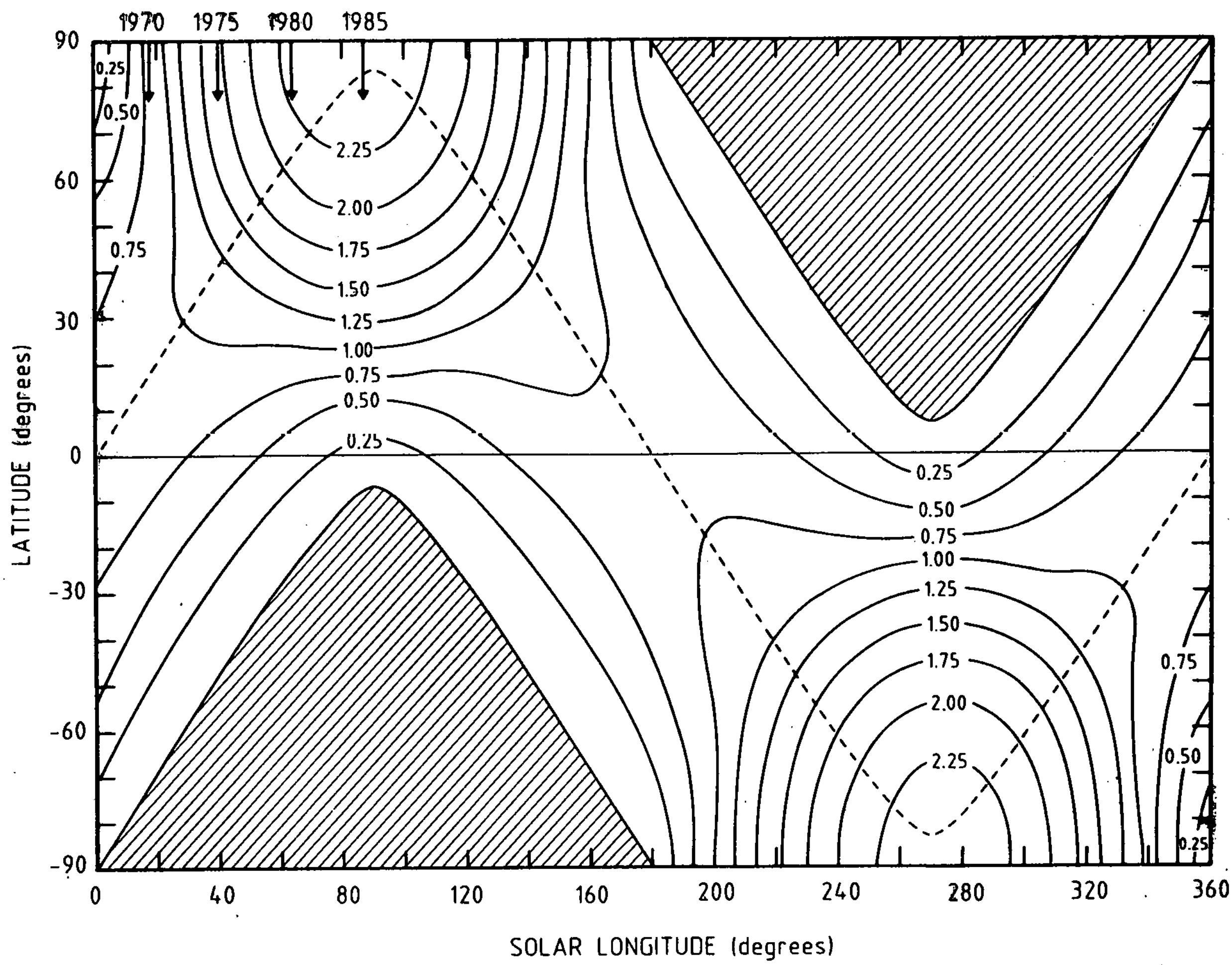

Fig. 1.- Seasonal and latitudinal variation of the daily solar radiation at the top of the atmosphere of Uranus with a period of rotation equal to 16 hr and an oblateness factor of 0.033. Solar declination is represented by the dashed line. The areas of permanent darkness are shaded. Values of the solar radiation in Watt per square meter are given on each curve.

TABLE III.- Elements of the planetary orbits of Uranus and Neptune.

PARAMETERS		URANUS	NEPTUNE
Semi-major axis	$a_{\odot}$ (AU)	19.18	30.06
Eccentricity	e	0.04727	0.00859
Longitude of perihelion	λ_p(deg)	3.02	5.23
Obliquity	ϵ(deg)	97.86	29.56
Rotation period	T(Earth days)	0.66 (0.45)	0.75 (0.66)
Tropical year	T_o (Earth days)	30684.80	60190.5
Oblateness	f	0.022-0.033	0.0259
		(0.05769)	(0.02066)

For Uranus, this difference is of the order of 1%.

The equatorial summer solstice insolation, hereafter denoted as $(I_D)_{E(ss)}$ is related to the polar insolation at summer solstice $(I_D)_{P(ss)}$ by the well-known relationship :

$$(I_D)_{P(ss)}/(I_D)_{E(ss)} = \pi \tan \varepsilon \tag{6}$$

stating that the ratio of both insolations is larger than unity for $17.7° < \varepsilon < 162.3°$ and that it is exclusively dependent upon the obliquity. For Uranus, application of expression (6) yields about 22.8.

Another point of interest regards the distribution of the daily solar radiation in the equatorial region. At $\varepsilon \leq 45°$ the solar radiation as a function of latitude has two peaks : a maximum one and a minimum one (the polar night being considered as a minimum). Uranus, and also Pluto, occupies a rather exceptional position in that the spin axis lies nearly in the orbital plane. In the polar regions the day and the night are approximately half an Uranian year long (about 42 Earth years). In the equatorial region summer and winter are, roughly speaking, repeated twice a year and the two seasons are substantially more temperate than in the polar areas.

Application of equations (1) and (4) leads to the isopleths illustrated in Fig. 2 where values of constant ratio distribution are given on each curve. As already stated by Van Hemelrijck (1982) there are, in summer, two obviously distinguished regions where $I_{DO} > I_D$. The first zone coincides with the area of permanent sunlight and the isocontours parallel the lines of constant latitude. The second is limited by the seasonal march of the Sun (or the area between the equator and the subsolar point). In general, the two above mentioned parts are linked by the isocontour $I_{DO}/I_D = 1$, coinciding remarkably well with the two branches of an hyperbola symmetric with respect to the solar longitude $\lambda_\odot = 90°$. In winter, the horizon plane is always tilted away from the Sun

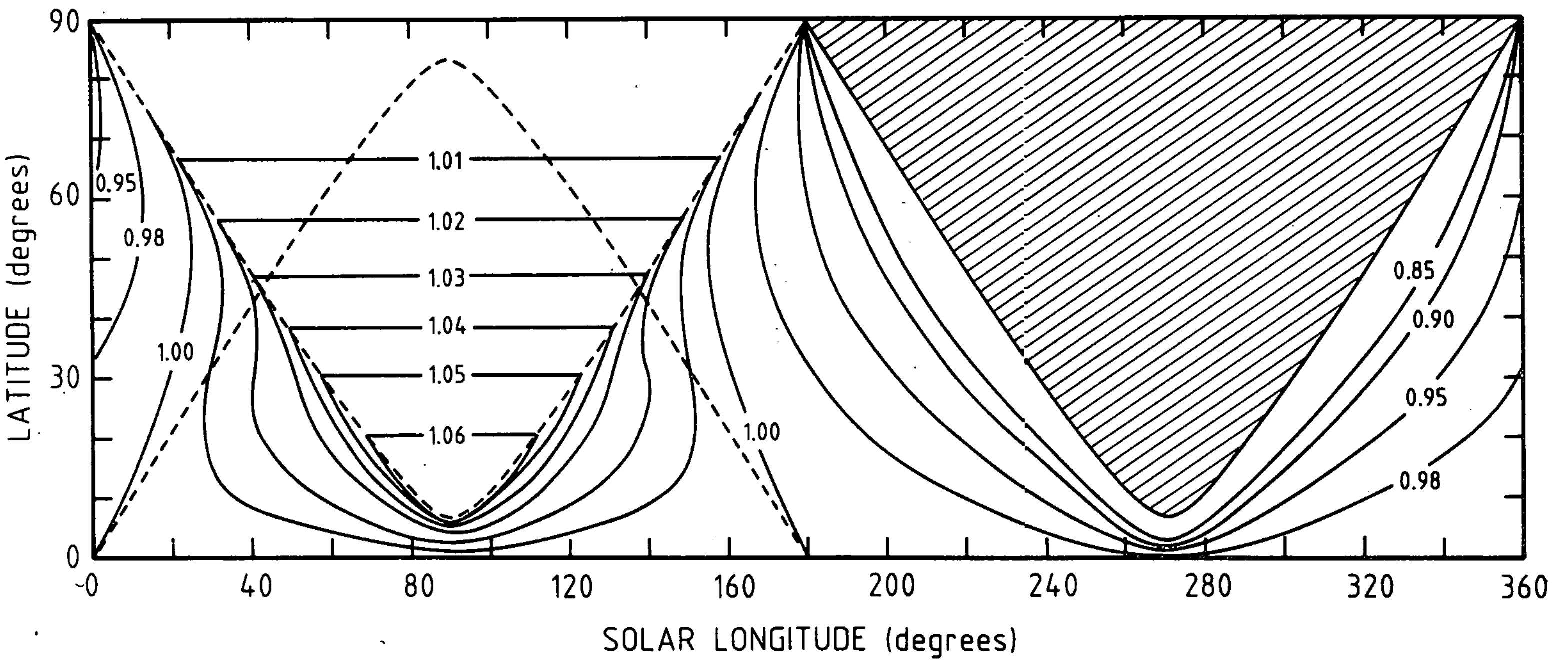

Fig. 2.- Seasonal and latitudinal variation of the ratio of the daily solar radiation with and without the oblateness effect (f = 0.033) at the top of the atmosphere of Uranus. Solar declination and the region where the Sun does not set are represented by the dashed lines. The areas of permanent darkness are shaded. Values of the ratio of both insolations are given on each curve.

(Brinkman and McGregor, 1977; Van Hemelrijck, 1982a) causing both the cosine of the zenith angle and the length of the day to decrease. Thus the insolation is reduced. This findings are clearly demonstrated in Fig. 2.

As a consequence of the very large obliquity of Uranus the solar radiation of the oblate planet is increased over practically the entire summer season. The obliquity and oblateness of Uranus cause in the vicinity of the equator a gain of insolation of approximately 7% ($f = 0.033$) and 4.5% ($f = 0.022$) decreasing systematically to about 1% at a planetocentric latitude of 70°.

In winter, as noted earlier, the insolation is decreased, the rate of change being extremely rapid near winter solstice, but less sensitive near the equinoxes. The effect of the flattening results also in a more extensive polar region; the maximum difference of the Arctic circles $I_D = 0$ and $I_{DO} = 0$ occurs at solar longitudes 225° and 315° with values of about 1.9° ($f = 0.033$) and 1.3° ($f = 0.022$). At winter solstice the differences are rather small (~ 0.5° and 0.3° respectively).

The solar radiation incident on Neptune with an oblateness factor equal to 0.0259 and a rotational period of 18 hr is given in Fig. 3. The maximum solar energy is attained at the poles near the summer solstices with a value of about 0.55 $W \cdot m^{-2}$. As for Uranus the solar energy at the north pole during summer solstice is greater than the corresponding insolation at the South pole but due to the much smaller eccentricity the difference is extremely small and reaches scarcely $3°/_{oo}$ (evidently the latest value is only valid if λ_P and e are constants as a function of time). Application of equation (6) leads to a ratio $(I_D)_{P(ss)}/(I_D)_{E(ss)}$ of 1.8. It can also be seen that there is no seasonal asymmetry in the distribution of the solar radiation.

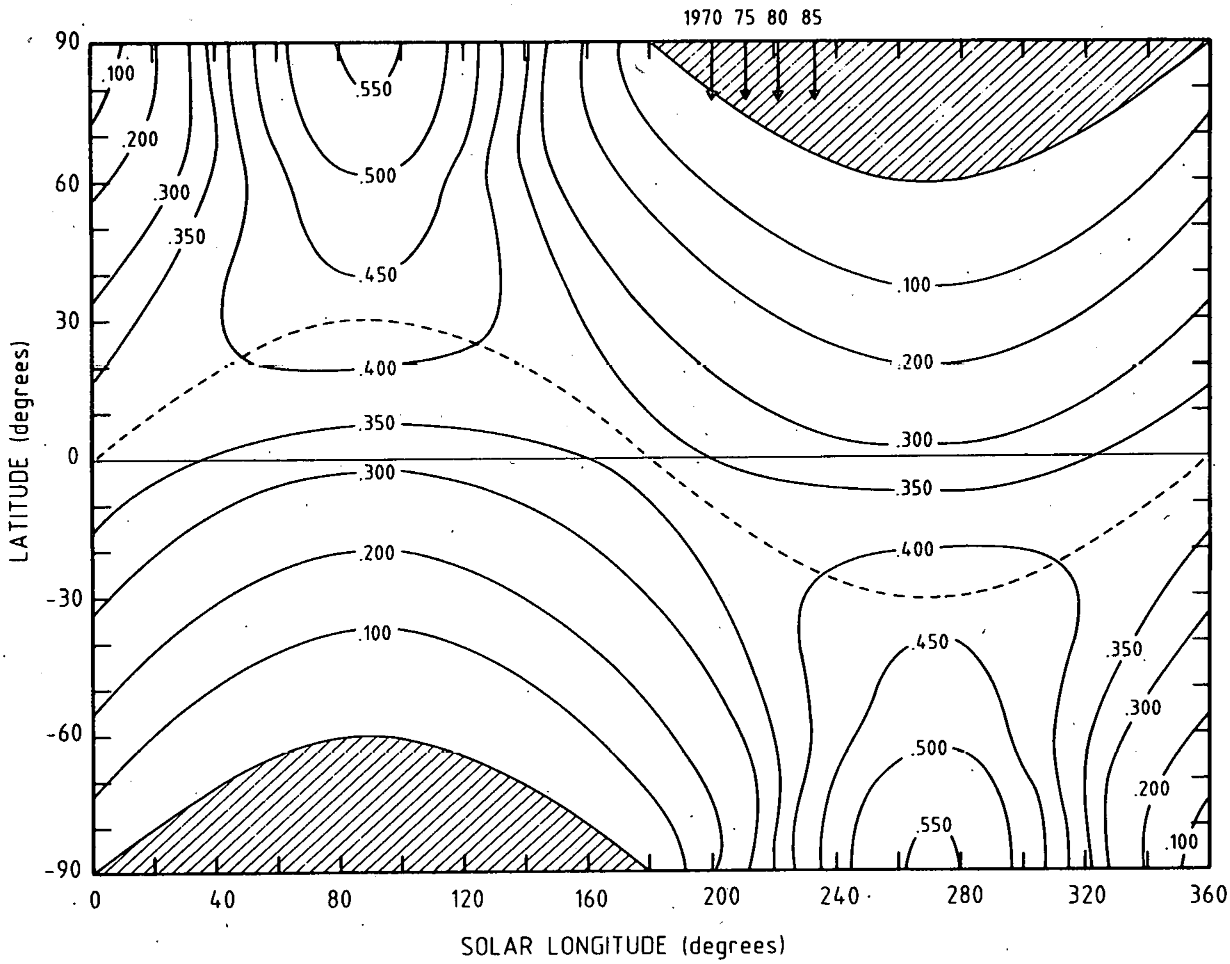

Fig. 3.- Seasonal and latitudinal variation of the daily solar radiation at the top of the atmosphere of Neptune with a period of rotation equal to 18 hr and an oblateness factor of 0.0259. See Fig. 1 for full explanation.

The obliquity of Neptune, being situated in the (0 - 45°) interval, it follows that in the equatorial region there exist only one maximum and only one minimum in the upper-boundary insolation.

The solar radiation ratio pattern (I_{DO}/I_D) of Neptune is plotted in Fig. 4. The maximum value (at a latitude of about 60° and at summer solstice) amounts to about 1.01 (1%). In winter, the solar radiation very slowly decreases with increasing latitude, this effect being ascribed to the small value of f. Another point about the curves is that the iso-contours $I_{DO} = 0$ and $I_D = 0$ practically coincide. It should also be emphasized that the results presented in Figs. 2 and 4 apply to either hemispheres.

Finally, we have indicated four epochs i.e. 1970, 1975, 1980 and 1985 in the upper part of the Figs. 1 and 3. Values of the mean anomaly (M) related to the position of the planets in January of the above mentioned years were obtained from the American Ephemeris and Nautical Almanac. The corresponding true anomaly (W) was calculated from the well-known relationship :

$$W = M + [2e - (e^3/4)] \sin M + (5/4)\, e^2 \sin 2M + (13/12)\, e^3 \sin 3M \quad (7)$$

In expression (7) we kept only terms up to the third degree in e, but this is sufficiently accurate for our computations.

According to Vorob'yev and Monin (1975) the planetocentric longitude of the perihelion may be written in terms of the argument of perihelion (ω) as :

$$\lambda_p = \omega + \Lambda \quad (8)$$

where Λ is the planetocentric longitude of the ascending node altered by 180° and may be expressed in the general form :

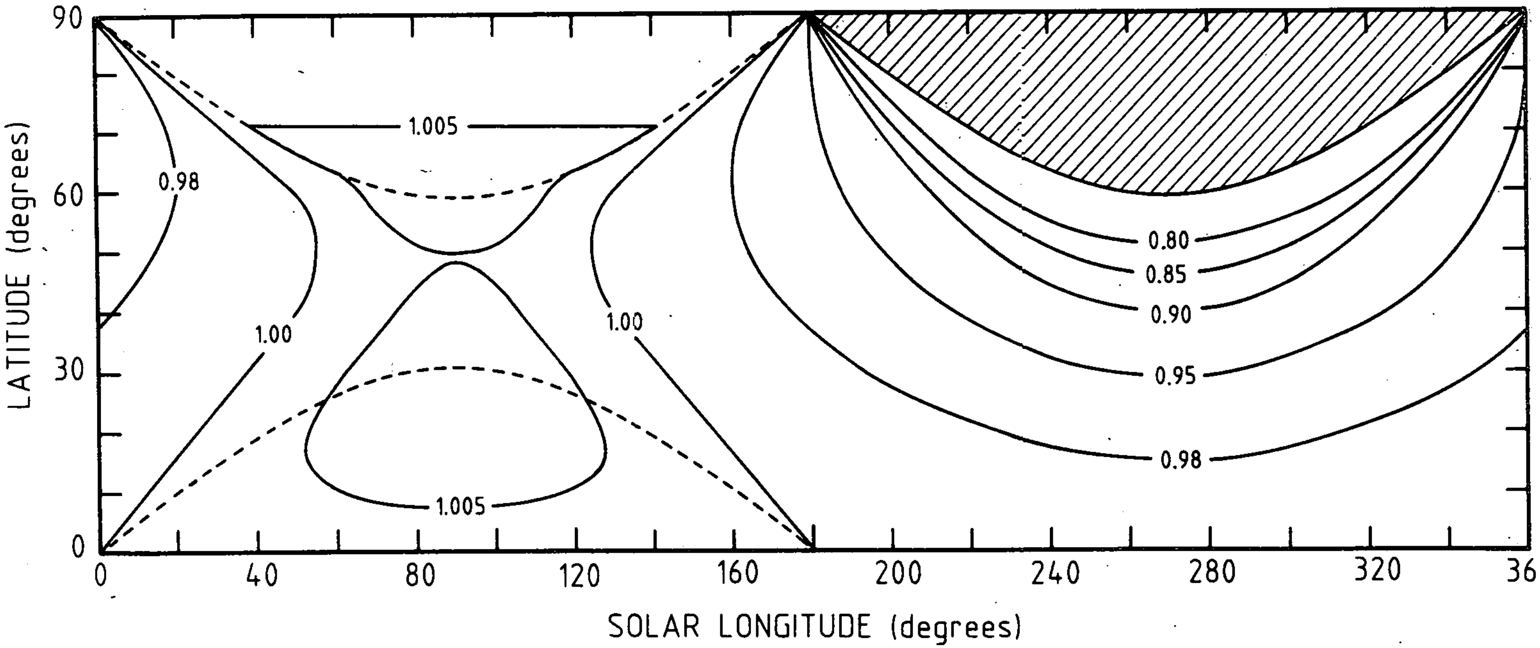

Fig. 4.- Seasonal and latitudinal variation of the ratio of the daily solar radiation with and without the obla:eness effect at the top of the atmosphere of Neptune. See Fig. 2 for full explanation.

$$\Lambda = f(i, \Omega, \tilde{\omega}, \varepsilon, \varepsilon_o, \alpha_o, \delta_o) \tag{9}$$

with

$$\tilde{\omega} = \Omega + \omega \tag{10}$$

where i, Ω, $\tilde{\omega}$, ε, ε_o, α_o and δ_o are respectively the inclination to the ecliptic, the mean longitude of the ascending node, the mean longitude of the perihelion, the obliquity, the angle between the Earth's spin axis an its orbit normal ($\varepsilon_o = 23.44°$) and the right ascension and declination of the north pole. i, Ω and $\tilde{\omega}$ are taken from the American Ephemeris and Nautical Almanac, whereas α_o and δ_o are the recommended values reported by Davies et al. (1983). Expressions (2), (7) and (8) allow the calculation of the planetocentric longitude of the Sun ($\lambda_\odot$) corresponding approximately to the beginning of the years 1970, 1975, 1980 and 1985.

A final remark concerns the use of a constant value for the planetocentric longitude of the perihelion (λ_P) in the calculations of the solar radiation although it is apparent from expression (8) and (9) that this parameter is a function of the osculating elements i, Ω and $\tilde{\omega}$; the eccentricity (e) also changes with time. The reason for holding constant λ_P and e at the values represented in Table III is simply due to the fact that past of 1986 the planetary elements were not avaluable to us in the form as represented in the American Ephemeris and Nautical Almanac. It follows that it was impossible to evaluate the solar radiation over a complete orbital period taking into account the exact time dependent values for λ_P and e. It has, however, to be noted that the precession of the equinoxes and the eccentricity variation cause no change in the Figs. 2, 4, 6 and 8, whereas Figs. 1, 3, 5 and 7 are only weakly dependent upon fluctuations of the above mentioned parameters over a time span of one Uranian or one Neptunian year. Moreover, the annual average solar radiation (Figs. 5 and 7) is not affected by the angle between the data of equinox and the time of perihelion passage.

3.2. MEAN DAILY SOLAR RADIATION

The mean (summer, winter and annual) daily solar radiations on Uranus are depicted in Fig. 5 for f = 0.033 and for the northern hemisphere. Due to the insufficiency of the scale adopted for the ordinate the curves corresponding to an oblateness factor equal to 0.022 coincide with those of Fig. 5. Values for the southern hemisphere are only scarcely different from the northern ones except for the mean annual daily solar radiation values which are symmetric with respect to the planet's equator. The percent differences between an oblate and a spherical planet Uranus are illustrated in Fig. 6.

From Fig. 5 it is obvious that the equatorial daily solar radiations averaged over a season or a year are approximately equal and amount to about 0.5 $W\ m^{-2}$. At the pole, the mean summer daily insolation reaches its maximum value (1.55 $W\ m^{-2}$) and the yearly averaged solar radiation is about one half of the summertime insolation. In winter, the Sun does not rise at the poles and consequently $(I_{DO})_W = 0$.

The very large obliquity of Uranus results in an increased mean summertime insolation over the entire latitudinal region (Fig. 6). The gain of insolation is of most importance between 25 and 30° with values of about 2.8% (f = 0.033) and 1.8% (f = 0.022). In winter, and at polar region latitudes, as much as 12.5% and 8.5% respectively of the mean winter daily insolation is lost through the oblateness effect. Furthermore, it is obvious that the percent differences of the mean annual daily insolations are governed by the two opposite summer and winter effects. For Uranus this results in an increase of insolation taking a maximum value approaching 1.3% and 0.9% at latitudes near 30-40°.

The mean daily solar radiations incident on Neptune are given in Fig. 7, their percent differences are plotted in Fig. 8. At the equator, the mean daily insolations are the same and amount to about 0.34 $W\ m^{-2}$. The mean summer daily insolation reaches a values of 0.36 $W\ m^{-2}$ at the

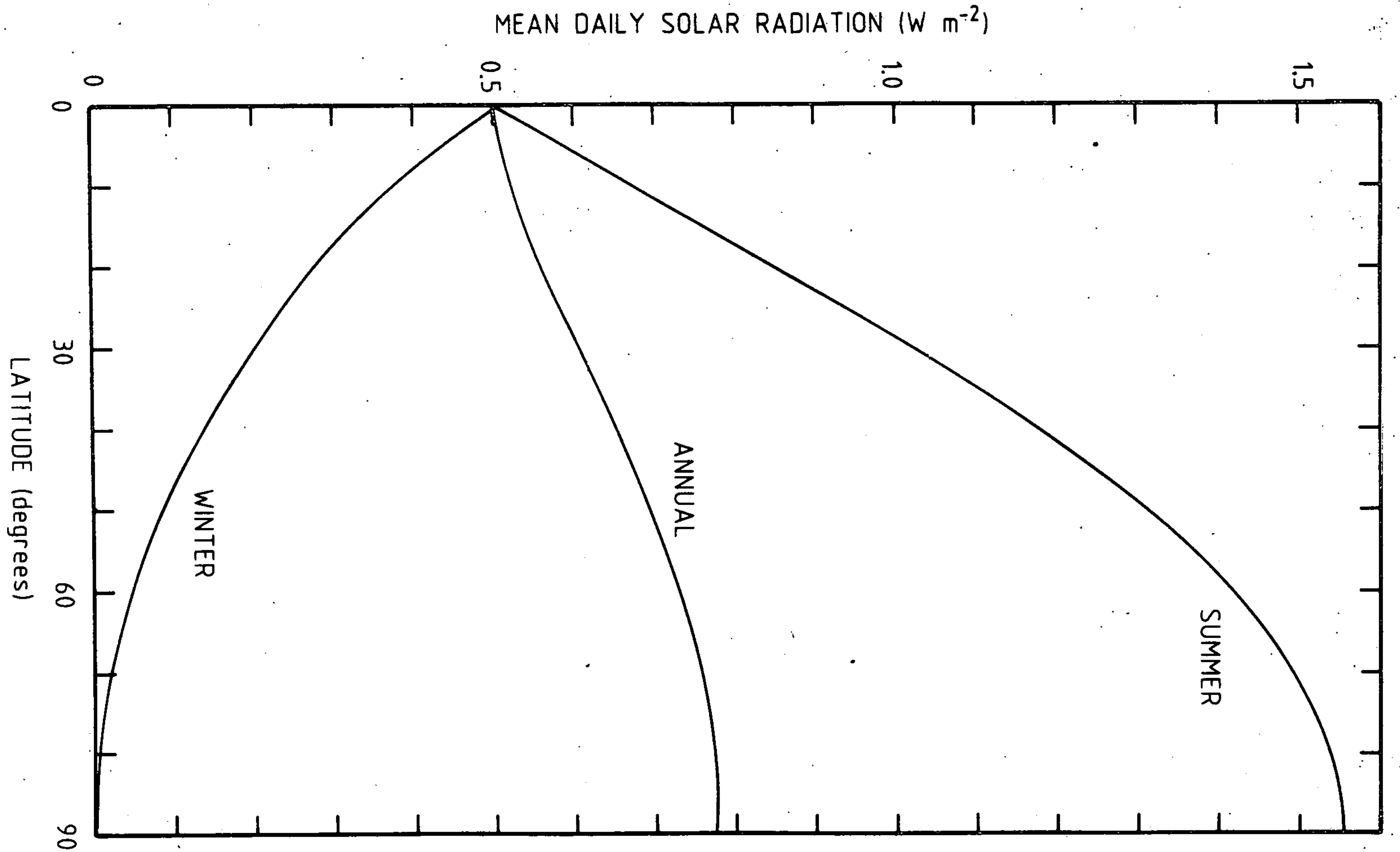

Fig. 5.- Latitudinal variation of the mean daily solar radiations at the top of the atmosphere of Uranus.

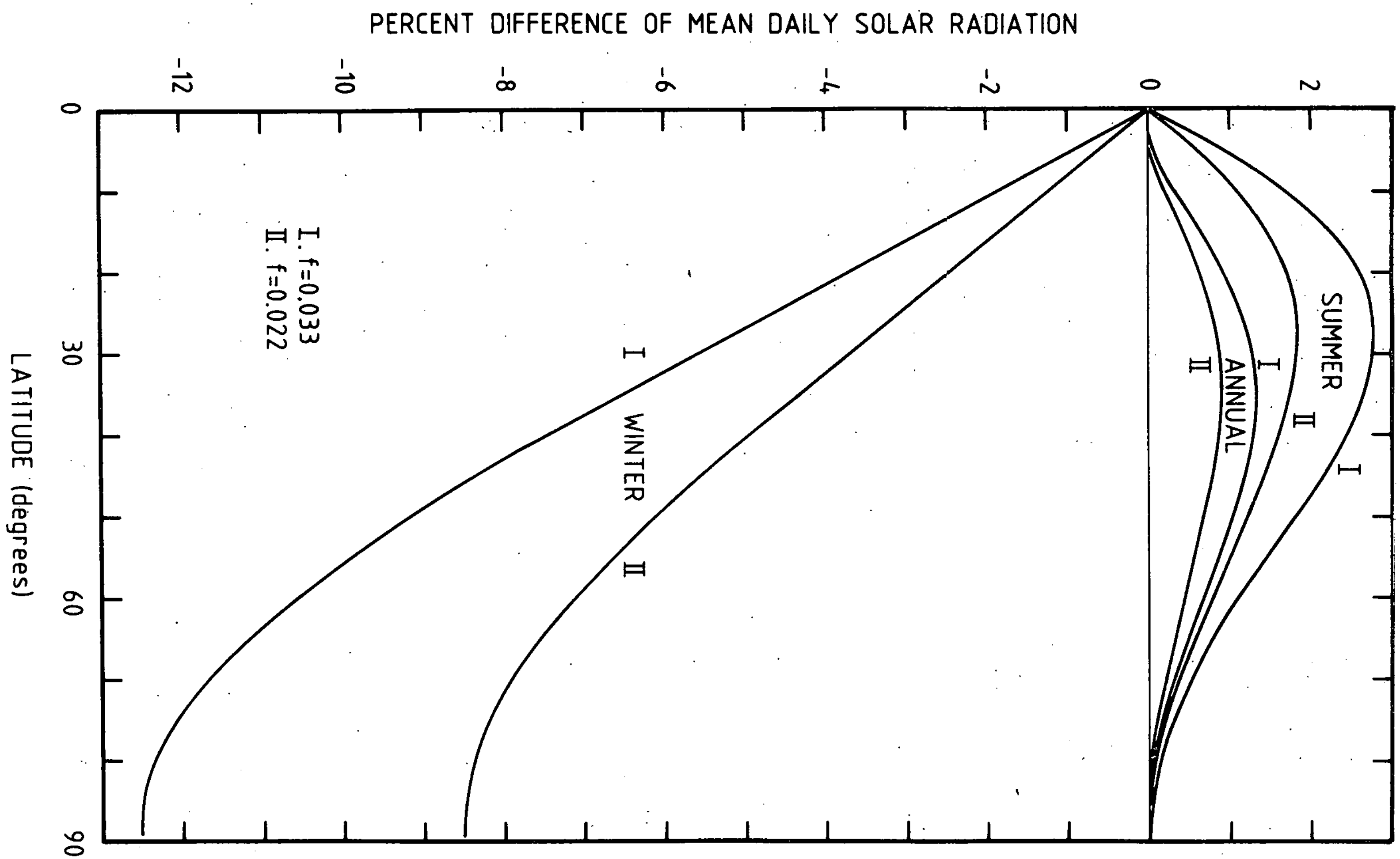

Fig. 6.- Latitudinal variation of the percent difference of the mean daily solar radiations with and without the oblateness effect on Uranus.

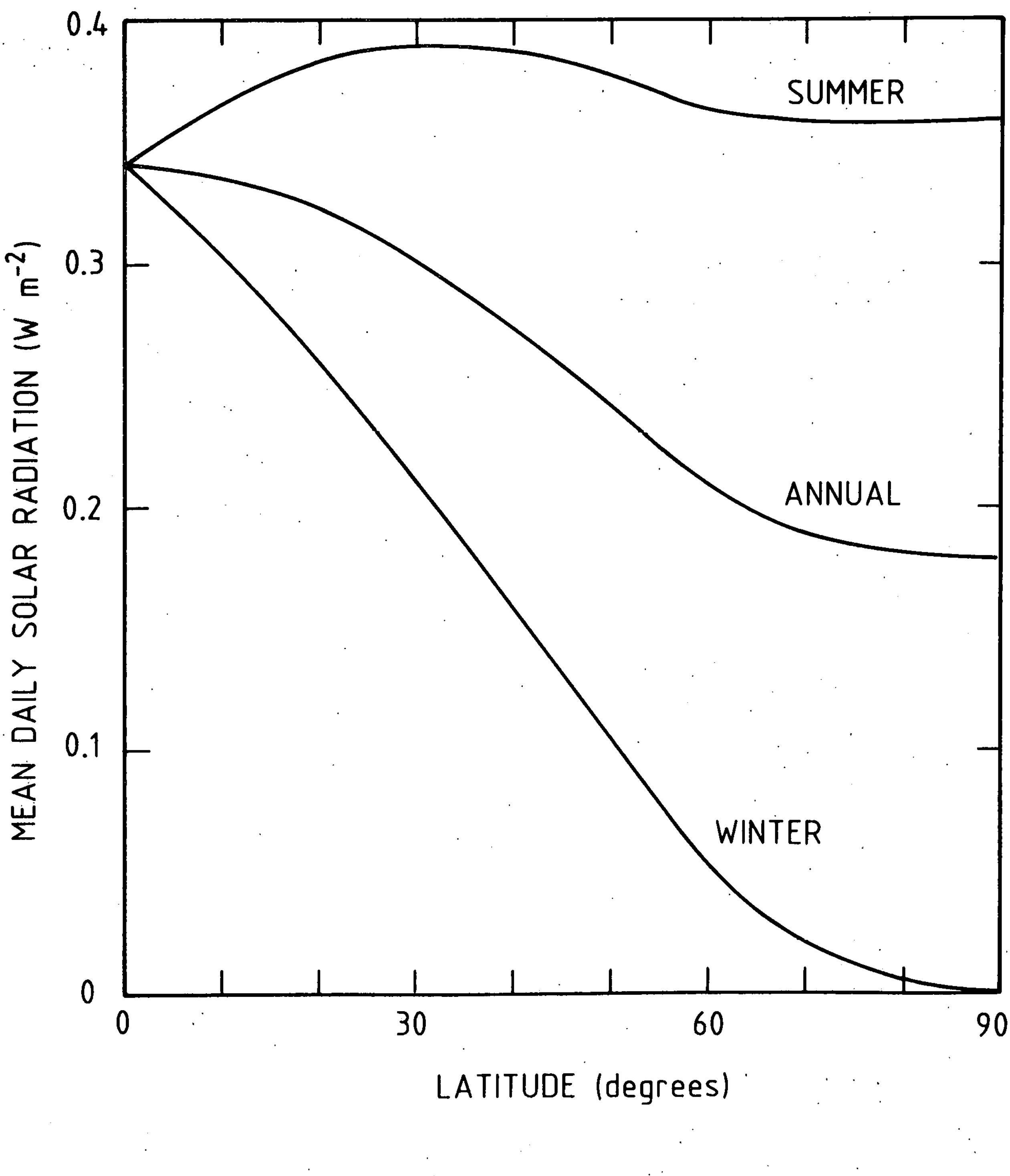

Fig. 7.- Latitudinal variation of the mean daily solar radiations at the top of the atmosphere of Neptune.

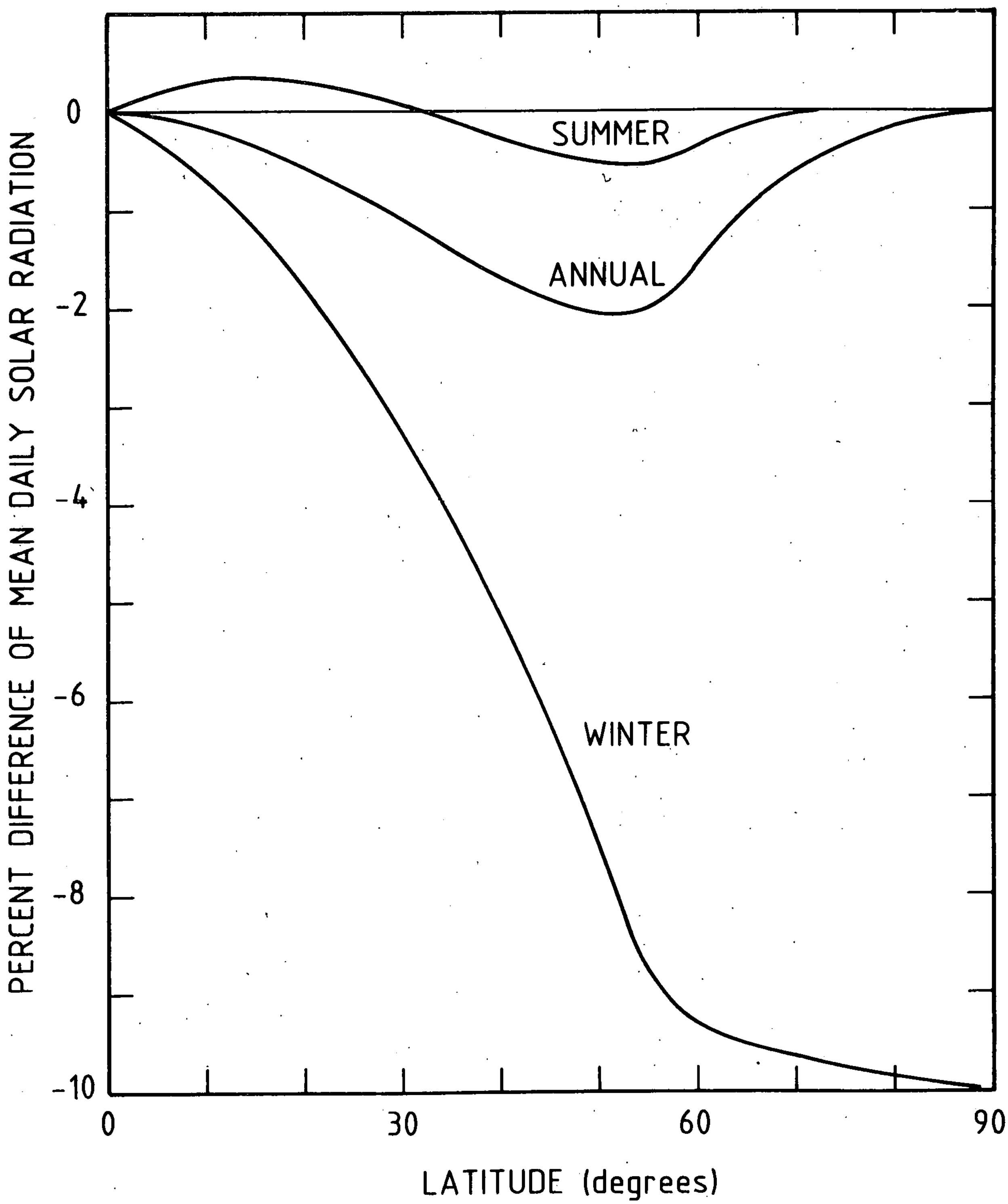

Fig. 8.- Latitudinal variation of the percent difference of the mean daily solar radiations with and without the oblateness effect on Neptune.

pole and a peak value of 0.39 W m^{-2} in the 25-40° latitudinal interval. Finally, at the poles, the daily solar radiation averaged over a Neptunian year is equal to 50% of the summertime insolation.

When comparing Fig. 5 with Fig. 7 a striking difference is noticed : the steady increase of the mean annual daily insolation of Uranus as a function of latitude - the latitudinal decrease of the corresponding insolation for Neptune. This is explained by the fact that, for the outer planets, there exists a critical obliquity ($\varepsilon \sim 54°$) (Ward, 1974; Vorob'yev and Monin, 1975; Toon et al., 1980) past which the poles receive more annual average energy than the equator (see also French et al., 1983). This situation is not only realised by Uranus but also by Pluto with a present pole position of about 118.5° (Harris and Ward, 1982; Dobrovolskis and Harris, 1983).

From Fig. 8 it is clear that the mean summer daily insolation is increased between the equator and about the subsolar point, but decreased poleward of the latter limit. The maximum values are + 0.35% and - 0.55% at about 15 and 55°. In winter, the horizon plane is always tilted away from the Sun causing the mean winter daily insolation to be reduced (maximally by 10% at polar region latitudes); the mean annual daily insolation is decreased by approximately 2% in the latitude interval 45-55°.

Finally, it should be pointed out that Figs. 6 and 8 seem to be somewhat misleading in that they might suggest that the percent difference of the mean winter daily solar radiation reaches its maximum value at the pole. This is, of course, not the case because at $\phi' = 90°$ both $(I_D)_W$ and $(I_{DO})_W$ equal zero. The curve in the 89-90° latitudinal interval has been omitted due to the fact that in this relatively small region the above mentioned curve roughly coincides with the ordinate at the right side of the two figures.

In conclusion, this paper constitutes an updating of the results obtained by Vorob'yev and Monin (1975), Levine et al. (1977) and Van Hemelrijck (1982a) for the planets Uranus and Neptune. Solar radiation values are expressed in the commonly adopted International System of Units.

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NOTE ADDED IN PROOF

Very recently, using the Voyager planetary radioastronomy and magnetometer observations at Uranus, a rotation period of 17.24 ± 0.01 h has been derived (Desch et al., 1986).

Recent determinations of the planetary oblateness were obtained from the results of the 15 June 1983 occultation by Neptune. From separate analyses of astrometric solutions for various stations, Hubbard et al. (1985) and French et al. (1985) found f = 0.022 and f = 0.0191, respectively. An intermediate value equal to 0.0208 has been deduced by Lellouch et al. (1986).

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