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3 - Avenue Circulaire
B - 1180 BRUXELLES

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# Climate predictability: a dynamical view

by

C. NICOLIS and C.L. KEPPENNE

BELGISCH INSTITUUT VOOR RUIMTE-AERONOMIE

3 - Ringlaan
B - 1180 BRUSSEL

## FOREWORD

The paper entitled : "Climate predictability : a dynamical view" is accepted for publication in Climate and Geosciences (Reidel).

## AVANT-PROPOS

L'article intitulé : "Climate predictability : a dynamical view" est accepté pour publication dans Climate and Geosciences (Reidel).

## VOORWOORD

Dit werk : "Climate predictability : a dynamical view" is voor publikatie in Climate and Geosciences (Reidel) aanvaard.

## VORWORT

Die Arbeit : "Climate predictability : a dynamical view" wurde für die Publikation im Climate and Geosciences (Reidel), akzeptiert.

# CLIMATE PREDICTABILITY : A DYNAMICAL VIEW

by

C. NICOLIS and C.L. KEPPENNE

**Abstract**

Evidence is reported that the dynamics involved in the 500 mb geopotential record is associated to deterministic chaos. The fractal dimension of the attractor and the average time beyond which predictions on individual trajectories break down are estimated. The possibility of carrying out predictions beyond this limit on the basis of a statistical approach is discussed.

**Résumé**

On montre que la dynamique associée à l'évolution temporelle de géopotentiel de 500 mb est une dynamique deterministe possédant un attracteur chaotique. La dimension fractale de celui-ci ainsi que le temps moyen de prévisibilité sont estimés. Une approche statistique est développée permettant, dans certaines conditions, d'étendre les prévisions au delà de cette limite.

**Samenvatting**

Er wordt aangetoond dat de dynamica verbonden met de tijdsevolutie van de geopotentiële hoogte van 500 mb een deterministische dynamica is met een chaotische attractor. De fractale dimensie van deze laatste en de gemiddelde voorspelbaarheidstermijn worden geschat. Een statistische benadering wordt ontwikkeld die, in bepaalde gevallen, toelaat de voorspellingen uit te breiden buiten deze grens.

**Zusammenfassung**

Es wird gezeigt dass die Dynamik verbunden mit der Zeit-evolution der 500 mb Geopotentialhöhe eine deterministische Dynamik ist mit einem chaotischen Attraktor. Die fraktale Dimension dieses letzten und der mittleren Vorhersagbarkeitstermin werden geschätzt. Einer statistische Ansatz wird entwickelt der, auf manche Fälle, zulasst die Vorhersagen zu erweitern.

## 1. INTRODUCTION

One of the basic reasons behind the well-known difficulty of long term predictions in the evolution of the atmosphere and climate, is in the fact that the principal variables undergo complex dynamics, typical features of which are aperiodicity in time and irregular patterning in space. According to the common wisdom, this is however only a temporary drawback : thanks to the availability of computational facilities, it is believed that a detailed description will eventually be achieved, leading to a theoretically unlimited predictive capability.

Our purpose in this work is to present a rather different view. First, we shall report on some data analysis suggesting that the dynamics associated with low frequency atmospheric variability corresponds to deterministic chaos. As a consequence, the underlying system will turn out to be extremely sensitive to initial conditions, in the sense that initially nearby states diverge exponentially in the course of time (Guckenheimer and Holmes, 1983). Because of the inherent uncertainty related to the finite precision of measurements, this will entail that individual trajectories cannot be predicted beyond a certain interval of time (Lorenz, 1984). We shall show that this time is an intrinsic property of the system, and give an estimate for the particular set of atmospheric data we shall analyze. Finally, we shall suggest a possibility for carrying out predictions beyond this intrisic limit, based on the idea that a description in terms of individual trajectories must be abandoned in favor of a statistical approach.

## 2. THE DATA SET

In previous work we have analyzed the oxygen isotope record of a deep sea core of the last million years using the above described point of view,and produced evidence of chaotic dynamics on this long time scale (Nicolis and Nicolis, 1984; 1985). Here we shall deal with variability on a shorter time scale, associated with the 500 mb geopotential record (Keppenne and Nicolis, 1988; see also Freadrich, 1986; Essex et al., 1987). The particular data set we shall consider is the daily record over a period of 24 years starting from January 1st, 1961 of the following nine western European stations : Bordeaux, De Bilt, Lisbon, London, Marseille, Paris, Reykjavik, Rome and Stockholm. Fig. 1 depicts a representative time series drawn from the data at the Marseille station.

We want to identify the salient features of the dynamical system associated with these data, independent of any modelling. To this end, we embed the evolution in phase space, the space spanned by a set of linearly independent variables taking the dominant part in the dynamics. We do not know offhand the number of these variables, but a result established in 1981 by Takens stipulates that they can be picked up from the initial time series and the hierarchy of variables obtained from it by applying successively higher shifts $\tau$, $2\tau$, ... etc.

$$
\begin{aligned}
X_1(t_i) &: X_o(t_1), \ldots\ldots\ldots, X_0(t_N) \\
X_2(t_i) &: X_o(t_1 + \tau), \ldots\ldots\ldots, X_o(t_N + \tau) \\
X_n(t_i) &: X_o(t_1 + (n-1)\tau), \ldots\ldots\ldots, X_o(t_N + (n-1)\tau)
\end{aligned}
\qquad (1)
$$

An alternative procedure is to combine Takens' method and the familiar empirical orthogonal function analysis. EOF's are just the eigenvectors of the covariance matrix i.e. the matrix of quadratic averages. They therefore describe variables that are statistically independent up to third order correlations. Using Takens' reconstruction, the covariance matrix takes the form :

$$
\Phi_{ij} = \frac{1}{N_T} \sum_{k=1}^{N_T} X_o(t_k + i\tau)\, X_o\,(t_k + j\tau) \; ; \; i,j = 0,1,\ldots,n-1 \qquad (2)
$$

where $N_T$ is the number of data points.

Let $\lambda_1 > \lambda_2 > \ldots \lambda_n$ be the eigenvalues of the covariance matrix and $c_1, \ldots\ldots, c_n$ the corresponding eigenvectors. The space spanned by these latter is referred to as singular space (Broomhead and King, 1986). Embedding our data into this space amounts therefore to switching from the state vector $\underset{\sim}{X} = (X_1, \ldots\ldots, X_n)$ to a state vector $\underset{\sim}{Y} = \underset{\sim}{X}.\underset{\approx}{c}$, where $\underset{\approx}{c}$ is an nxn matrix whose columns are the eigenvectors $c_i$.

## 3. THE WEATHER ATTRACTOR

It is now clear that in the space spanned by the time lagged variables or by the EOFs the time evolution given by our data set will

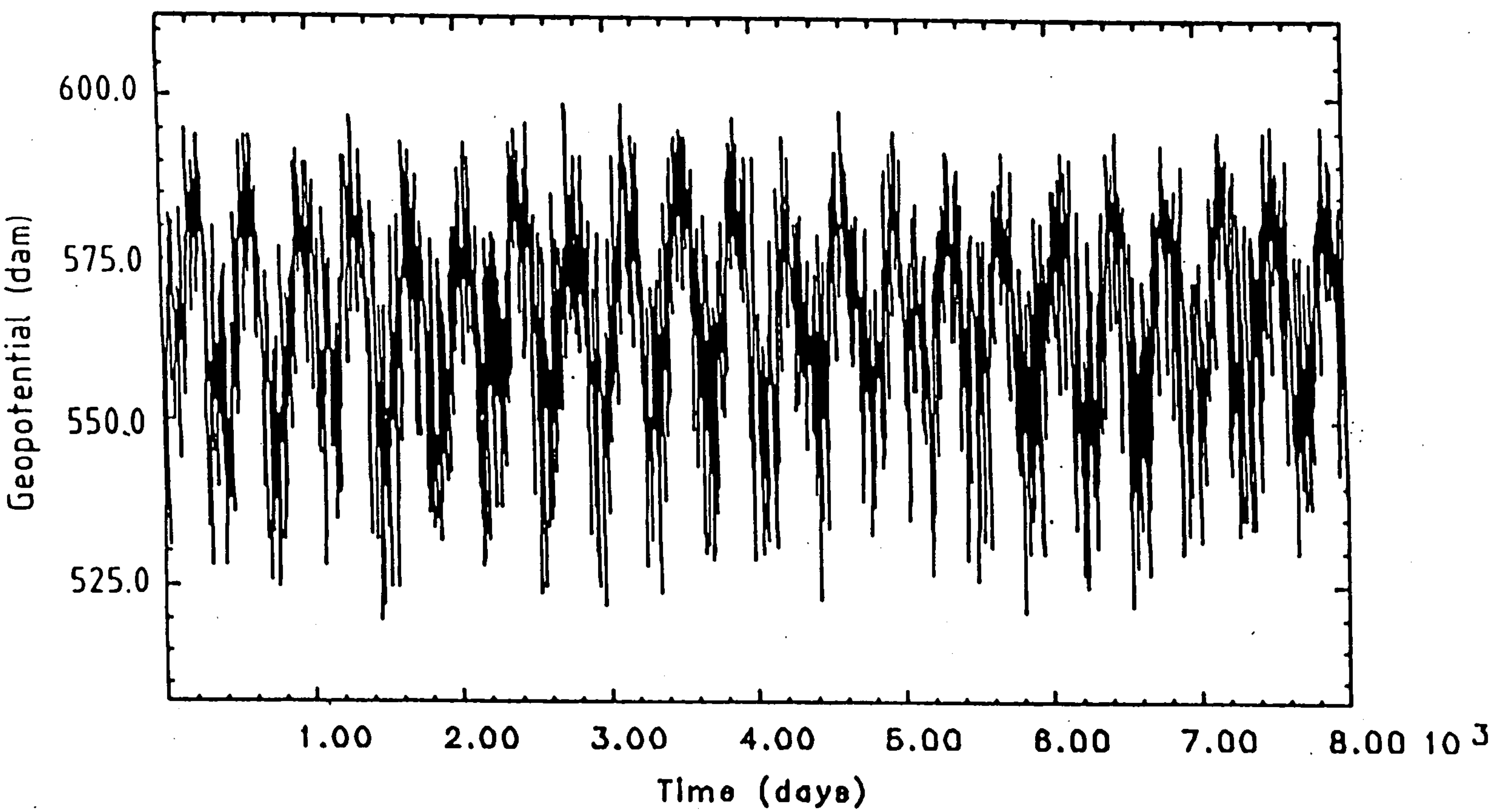



Fig. 1. Time evolution of the 500 mb geopotential height at Marseille.

lie on a curve, the phase space trajectory. Sooner or later this trajectory will converge to a certain set (or manifold), which will be referred to as the attractor (Fig. 2). An attractor describing chaotic motion with

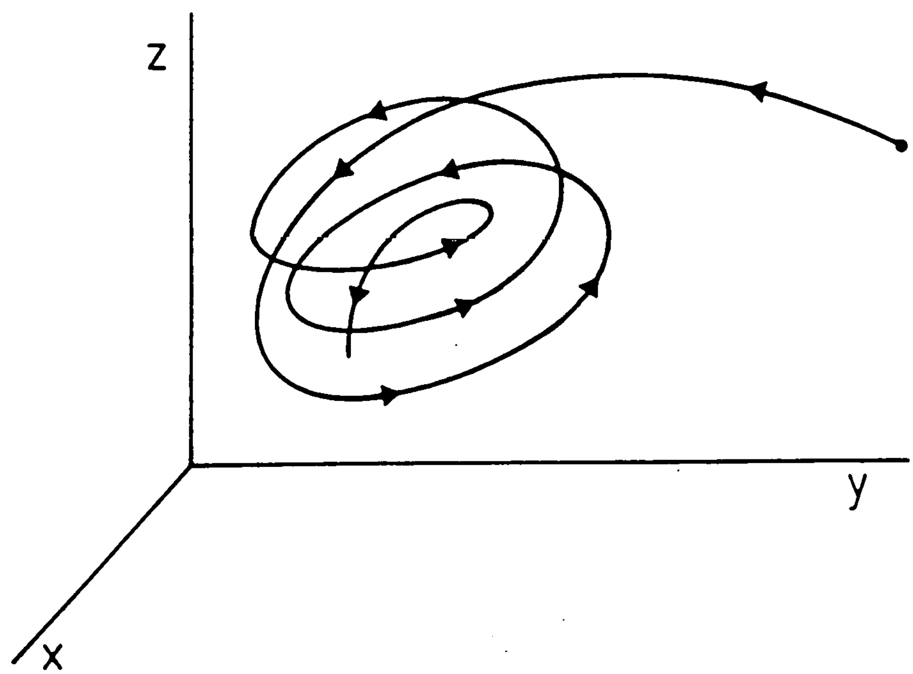



Fig. 2. Schematic representation of a phase space trajectory converging to an attractor set.

sensitivity to initial conditions is a very complex set, generally different from the familiar shapes of Euclidean geometry. A very important quantity characterizing this difference is the correlation dimension, $\nu$, generally a noninteger number for a chaotic attractor (see for instance Mayer-Kress, 1986). To determine $\nu$ one counts the number of pairs of data points which lie within a prescribed distance r (Grassberger and Procaccia, 1983). One can show that for small r, this number scales as

$$C(r) \sim r^{\nu} \quad (3)$$

Obviously for $\nu = 1, 2, \ldots.$ one obtains the familiar Euclidean manifolds (lines, surfaces, etc), but for $\nu$ non-integer and greater than 2 one deals with a fractal object.

These considerations suggest the following algorithm :

(i) Choose increasingly large values of embedding dimension n, and for each n plot the values of $\underset{\sim}{X} = (X_1, \ldots, X_n)$ for the $N_T$ data points (cf. eq. (1)).

(ii) For each n, determine the correlation dimension, $\nu$ of the above plotted data set.

(iii) Study the dependence of $\nu$ for increasing n (Fig. 3). If this dependence saturates to some value $\nu_s$ beyond a certain reasonably small $n_s$, we will conclude that our system is a deterministic dynamical system. As for $n_s$, it will represent the minimum number of variables needed to describe the dynamics.

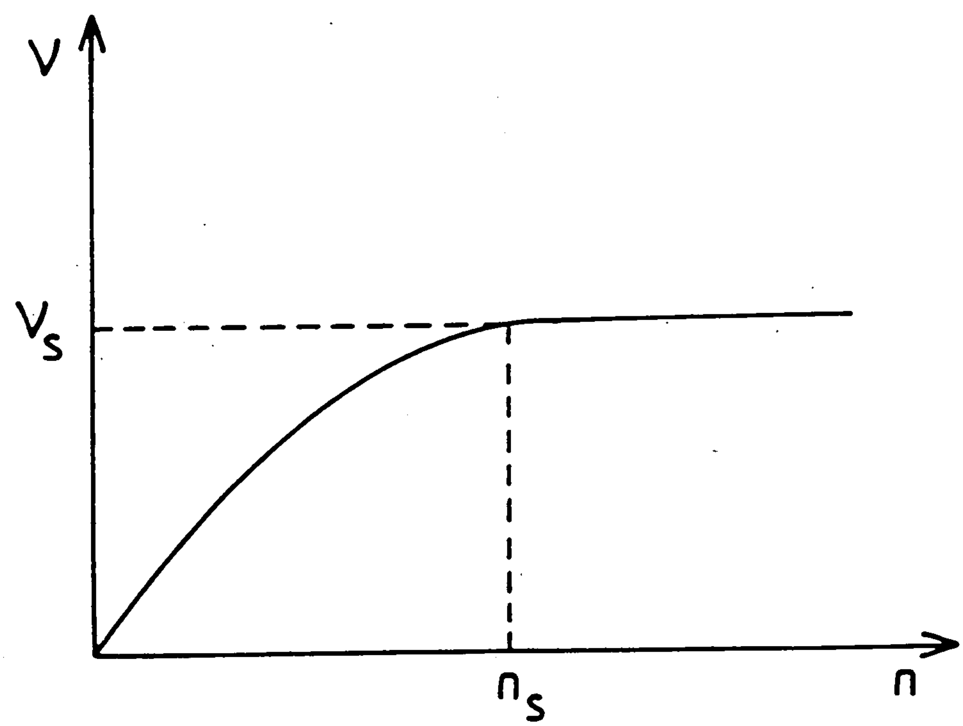



Fig. 3. Dependence of dimensionality, $\nu$ on the number of phase space variables n for a deterministic dynamical system.

Table 1 gives the numbers obtained from five of our data sets. We see that the two methods of phase space reconstruction give comparable results for each time series as well as for computations involving the space average of all signals. All attractors analyzed have a correlation

TABLE I : Correlation dimension.

| Signal | Method of delays | Singular phase space |
|---|---|---|
| Lisbon | 7.6 | 7.4 |
| Marseille | 6.8 | 6.7 |
| Reykjavik | 7.2 | 7.4 |
| Roma | 8.3 | 8.2 |
| Stockholm | 7.8 | 8.1 |
| Space averaged | 7.7 | 8.0 |

dimension in a narrow range with a mean value of ~ 7.5 and a dispersion of 10%. Consequently it is reasonable to ascertain that the individual time series refer to a well defined dynamical system describing the short term variability of the western European climate. Since the error bar associated to these calculations is not easy to estimate despite the relative abundance of the data, let us, however, be cautious and forget about the fractal nature of these numbers until we show that our dynamical system displays the most important property of fractal attractors, namely, sensitivity to initial conditions. We carry out this analysis in the following section.

## 4. LYAPOUNOV EXPONENTS AND PREDICTABILITY

Let us formulate the problem of sensitivity to initial conditions in a quantitative manner. We imagine at time $t = 0$ a set of data included in a small n-dimensional sphere, whose center is on the attractor (see Fig. 4). The long time evolution of this sphere is subsequently monitored. We order the principal axes of this object from most rapidly to least rapidly growing and compute the mean growth rate $\sigma_i$ of the ith principal axis $p_i$ over a long period of time :

$$\sigma_i = \lim_{t\to\infty} \frac{1}{t} \int_0^t d\tau \frac{d}{d\tau} \ell n\left(\frac{p_i(\tau)}{p_i(0)}\right) = \lim_{t\to\infty} \frac{1}{t} \ell n\left(\frac{p_{i(t)}}{p_i(0)}\right) \qquad (4)$$

$p_i(0)$ being the radius of the initial sphere. The set of $\sigma_i$ is referred to as <u>Lyapounov exponents</u> of the underlying dynamical system. It can be shown that there exist as many Lyapounov exponents as phase space dimensions (Guckenheimer and Holmes, 1983). One of them is necessary equal to zero, expressing the fact that the relative distance of

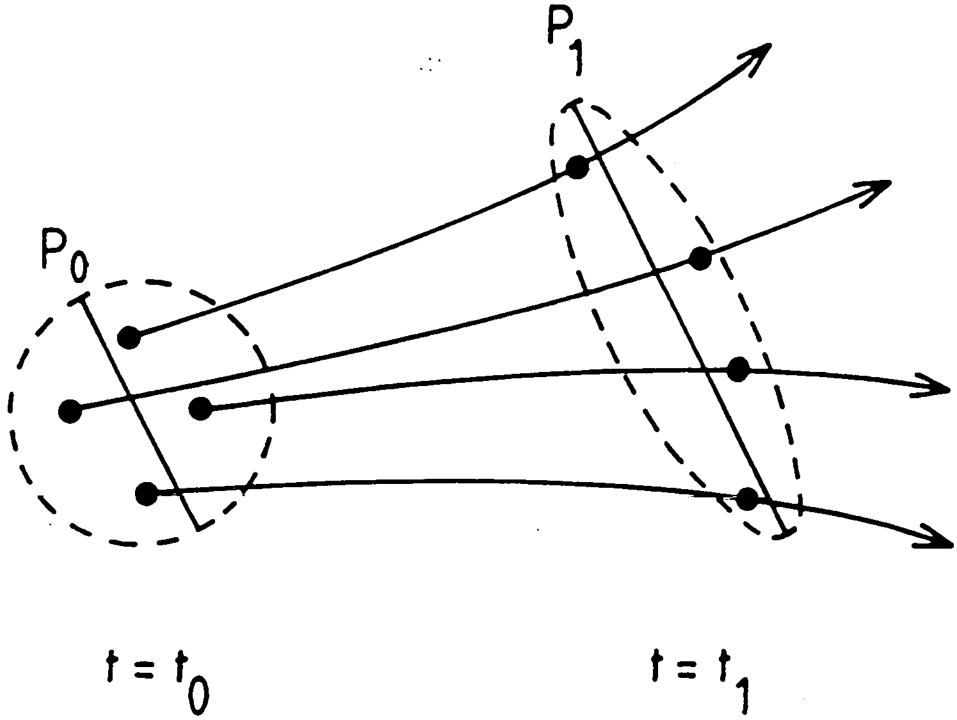



Fig. 4. Schematic representation of the exponential divergence of initially nearby states on a chaotic attractor.

initially close states on a given trajectory varies slower than exponentially. Others are negative, expressing the exponential approach of initial states to the attractor. If the dynamical system at hand is chaotic the sphere will evolve to a complex ellipsoidlike form reflecting the exponential divergence of nearby initial conditions along at least one direction on the attractor. We will conclude in this case that there is at least one positive Lyapounov exponent. Note that in a well-behaved dissipative system the sum of all exponents must be strictly negative (Guckenheimer and Holmes, 1983).

Our analysis is mainly based on the work of Eckmann et al. (1986). The algorithm developed by these authors allows the estimation of large magnitude exponents with a relatively good accuracy. Since we are interested in the predictability problem of chaotic attractors we will focus only on the largest positive $\sigma_i$'s. We found two such positive Lyapounov exponents of comparable magnitude. Therefore one deals here with a hyperchaotic attractor. The sum of the positive Lyapounov exponents gives an estimate of the Kolmogorov entropy K, whose inverse is the mean predictability time of our variable (Schuster, 1984). As it turns out

$$K^{-1} \sim 3 \text{ weeks} \tag{5}$$

meaning that this is as far as we can go to predict the 500 mb geopotential height over western Europe using the most elaborate weather prediction model.

## 5. PREDICTABILITY BEYOND THE LYAPOUNOV TIME

Obviously, there is an imperative need to extend our prediction capabilities beyond such limits. But how to reconcile this need with the intrinsic limitations imposed by chaotic dynamics ? We shall here suggest briefly a modest step toward a systematic method, based on a statistical approach.

The most familiar examples of statistical approach are linear regression or the use of Markov chain models (see for instance, Spekat et al., 1983; De Swart and Grasman, 1987; Mo and Ghil, 1987). Despite their usefulness they suffer from a fundamental flaw : the inability to select in a systematic manner those variables that are indeed likely to give rise to a well-defined stochastic dynamics, for instance of the Markov type (Feller, 1968). In contrast, in our method statistical properties emerge as the consequence of the complexity of the underlying dynamics.

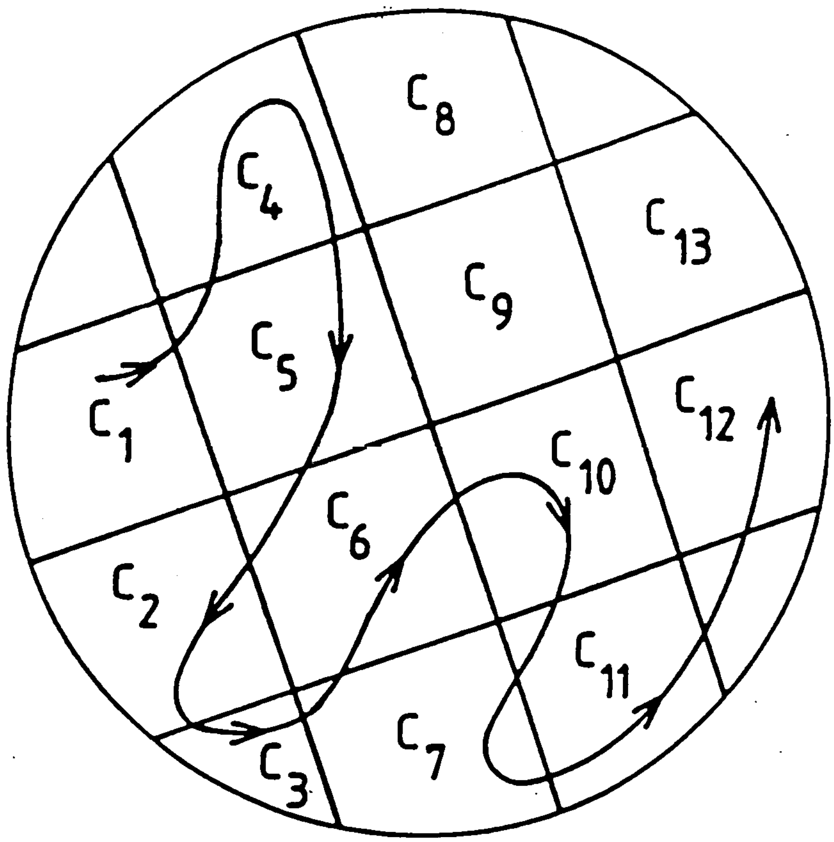
The basic idea is as follows. Operationally, because of the inherently limited precision of measurements, atmospheric or climatic states are defined in a non-local fashion (Lorenz, 1984). This "coarse-graining" or "lumping" maps the evolution based on the full set of the primitive equations into a complex sequence of transitions between cells $C_1$, $C_2$, .... in the phase space (see Fig. 5). Among the large number of ways to monitor the variables is there, then, one that leads to a clearcut stochastic process like for instance a first order Markov chain ? We have given a partial answer to this question by finding out conditions on the dynamics and on the partitioning of the state space, for which a Markov evolution is an exact image of the full dynamics described by the primitive equations (Nicolis and Nicolis, 1988).

Our method is based on the concept of Markov partitions ($C_i$) for which the boundaries between cells are preserved by the dynamics. We have identified some classes of dynamical systems, which include low order atmospheric circulation models, for which the probabilities $\{P_n(j)\}$, for being in cell $C_j$ of such a partition at time n evolve according to the Markovian master equation (Nicolis, 1988)

$$P_n(j) = \sum_i W_{ij} P_{n-1}(i) \tag{6}$$

The conditional probability matrix W is a positive matrix, whose row sums are equal to unity. Its structure can be explicitly determined from the dynamical system, once a Markov partition has been determined.

Eq. (6) allows us to estimate the statistical state of the system at time n on the sole basis of the time series (assumed stationary) and of the initial data. Now, eq. (6) is a linear equation possessing a unique




$$C_1 \rightarrow C_5 \rightarrow C_4 \rightarrow C_5$$
$$\rightarrow C_6 \rightarrow C_2 \rightarrow C_3 \rightarrow C_7 \rightarrow \cdots$$

Fig. 5. Time evolution of a phase space trajectory viewed as a sequence of transitions between the cells of a "coarse-graining" partition.

stationary solution. It can be shown that starting from any initial condition, the system converges to this unique stationary solution as $n \rightarrow \infty$. Put differently, small changes in the (probabilistic) initial conditions will give rise to only slight differences in the time evolution of the variable of interest. This fact, to be contrasted with the sensitivity to initial conditions of the underlying dynamical system, guarantees the fiability of the statistical forecasting. Notice that sensitivity to initial conditions has been instrumental in allowing us to cast the dynamics in a Markovian form.

It should be realized that the increased fiability of the statistical description is obtained at the cost of a partial loss of information, arising from the coarseness of the cells $C_i$. In this view an "optimal" forecasting would therefore amount to reconciling, in a judicious manner, these opposing trends.

## 6. CONCLUSIONS

Let us summarize our main results.

(i) The dynamics involved in the 500 mb geopotential record stems from a deterministic dynamical system of few degrees of freedom.

(ii) This system shares two essential features of deterministic chaos : fractal attractor dimension and at least two positive Lyapounov exponents.

(iii) Despite the limited number of variables involved the system is intrinsically unpredictable beyond a time of the order of a few weeks.

(iv) It is possible to set up a systematic statistical description, which is an exact image of the dynamics. This description allows one to extend our predictive capabilities beyond the limits imposed by chaotic dynamics.

We believe that these approaches are likely to shed some new light into the basic problem of prediction in meteorology and climatology.

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