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3 - Avenue Circulaire
B - 1180 BRUXELLES

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I. KH. KHABIBRAKHMANOV and F. VERHEEST

BELGISCH INSTITUUT VOOR RUIMTE-AERONOMIE
3 - Ringlaan
B - 1180 BRUSSEL

FOREWORD

This paper was made possible by the cooperation between the BISA and the Institute of Space Research (Moscow, USSR) and will be published in the *Journal of Geophysical Research (Space Physics)*

VOORWOORD

Dit artikel is mogelijk gemaakt door de samenwerking tussen het BIRA en het Instituut voor Ruimte-onderzoek (Moskou, USSR) en zal in het *Journal of Geophysical Research (Space Physics)* gepubliceerd worden

AVANT–PROPOS

Cet article a été rendu possible grâce à la coopération entre l'IASB et l'Institut de Recherches Spatiales (Moscou, URSS) et sera publié dans le *Journal of Geophysical Research (Space Physics)*

VORWORT

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I. Kh. Khabibrakhmanov* and F. Verheest**

Space Research Institute, USSR Academy of Sciences *

Profsoyuznaya 84/32, Moscow 117810, USSR

Instituut voor theoretische mechanika, Rijksuniversiteit Gent **

Krijgslaan 281, B-9000 Gent, Belgium

Abstract

A kinetic description of magnetized plasma flow with mass loading is developed and leads to a nonlinear wave equation for small scale dispersive magnetosonic plasma motions superimposed on large scale motions due to mass loading. The mass loading can drive quasistationary, finite-amplitude magnetosonic type waves in a supersonic plasma flow, even though small amplitude linear disturbances are damped, but only when the magnetosonic Mach number drops to 2 during the slowing down of the mass loaded plasma flow. Nonlinear motions with constant amplitude could be permanent features of a mass loaded plasma flow, as in the neighborhood of comets and planets with extended atmospheres but negligible magnetic fields. The relation between the wave amplitude and the local magnetosonic Mach number is computed and can easily be compared with *in situ* measurements in cometary plasma environments.

Samenvatting

Een kinetische beschrijving is ontwikkeld voor een gemagnetizeerd stromend plasma met massalading en leidt tot een niet-lineaire golfvergelijking voor kleinschalige dispersieve magnetosonische plasmabewegingen, gesuperponeerd op grootschalige bewegingen te wijten aan de massalading. Deze massalading kan quasistationaire magnetosonische golven met eindige amplitude opwekken in een supersonisch stromend plasma, zelfs al zijn lineaire golven met kleine amplitude gedempt, maar alleen als het magnetosonisch Machgetal naar 2 daalt gedurende de vertraging van het stromend plasma door de massalading. Niet-lineaire golven met konstante amplitude zouden dus blijvende kenmerken kunnen zijn van stromende plasma's met massalading, zoals in de omgeving van kometen en planeten met een uitgestrekte atmosfeer maar met verwaarloosbaar magneetveld. Het verband tussen de golfamplitude and het lokaal magnetosonisch Machgetal is berekend en kan gemakkelijk vergeleken worden met *in situ* metingen in kometaire plasma's.

Résumé

On a développé une description cinétique du flux d'un plasma magnétisé avec chargement de masse, résultant en une équation nonlinéaire pour des mouvements dispersifs et magnétosonores à petite échelle, superposés à des mouvements du plasma à grande échelle dûs au chargement de masse. Ce chargement de masse peut générer des ondes magnétosonores quasistationnaires d'amplitude finie dans un flux de plasma supersonique, bien que des perturbations de petite amplitude soient amorties, mais seulement quand le nombre de Mach magnétosonore tombe à 2 pendant la décélération du flux de plasma avec chargement de masse. Ces ondes nonlinéaires à amplitude constante pourraient être des caractéristiques permanentes d'un flux de plasma avec chargement de masse, comme au voisinage des comètes et planètes avec une atmosphère étendue mais un champ magnétique négligeable. La relation entre l'amplitude de l'onde et le nombre de Mach magnétosonore local est calculée et peut facilement être comparée aux mesures faites *in situ* dans l'environnement des plasmas cométaires.

Zusammenfassung

Eine kinetische Beschreibung eines magnetisierten Plasmaflusses mit Massenladung ist entwickelt und führt zu einer nichtlinearen Wellengleichung für dispersive, magnetosonore Plasmabewegungen auf kleinen Maßstab, superponiert auf Bewegungen mit großem Maßstab wegen der Massenladung. Diese Massenladung kann quasistationären, magnetosonoren Wellen mit konstanter Amplitude antreiben in einem Überschallfluß , eben wenn lineare, infinitesimale Störungen gedämpft werden, aber nur wenn die magnetosonore Machzahl zu 2 senkt während der Plasmafluß mit Massenladung langsamer wird. Solche nichtlineare Bewegungen mit konstanter Amplitude könnten permanente Kennzeichen des Plasmaflusses mit Massenladung darstellen, wie in der Nähe von Kometen und Planeten mit ausgedehnter Atmosphäre aber vernachlässigbarem Magnetfeld. Die Relation zwischen die Wellenamplitude und die lokale magnetosonore Machzahl ist berechnet worden und kann leicht mit *in situ* Meßwerten in kometaren Plasmen verglichen worden.

1. INTRODUCTION

There are various astrophysical situations where a magnetized plasma flows through a neutral gas. One prominent example is the interaction of the solar wind with extended cometary atmospheres, studied in detail following the recent successful cometary missions (see review by *Galeev*, 1987), or with planets having relatively extended atmospheres but small or no intrinsic magnetic fields. Among the more distant objects one could mention stellar winds, or astrophysical plasma jets from the nuclei of active galaxies, or again expanding shells of supernovae remnants passing through and interacting with the neutral component of the interstellar medium [*Petelski et al.*, 1980; *Petelski*, 1981]. Inside the Jovian magnetosphere there is yet another example of such an interaction: the Io plasma torus resulting from the ionization of material evaporating from the surface of Io [*Galeev and Khabibrakhmanov*, 1986; *Horton and Smith*, 1988].

The interaction of a plasma flow with a neutral gas gives rise to a so-called mass loading. By this one means that newly ionized particles, created by several ionization processes such as photoionization, electron impact or dissociative ionization, are incorporated into the main plasma flow. The relative motions between the original plasma and the newly ionized particles induce electric fields, which accelerate the new ions and give them drift velocities $\mathbf{v}_D = \mathbf{E} \times \mathbf{B}/B^2$. There is, of course, the feedback effect on the original plasma flow, which is decelerated so as to conserve momentum and energy in the combined system. In the case of slow electromagnetic field variations (adiabatic motions of the particles), the new plasma particles acquire a thermal velocity equal to the drift velocity, due to their motion transverse to the magnetic field.

So the main effect of the mass loading on the plasma flow is its deceleration, although the accompanying pressure gradients can in some cases compensate the inertial forces and ensure a steady state situation. This was *e.g.* in the case of the solar wind interaction with comets originally considered by *Wallis* [1973] and *Wallis and Ong* [1975] and more recently by *Galeev, Cravens and Gombosi* [1985]. They found that smooth deceleration of the supersonic solar wind flow is possible until the mass flux of the flow reaches a critical value $(\rho u)_c = 4/3$, corresponding to a local Mach number $M = 1$. This means that a stationary picture cannot be the whole story and that a shock transition has to occur somewhere before the sonic point of the flow.

In the case of planets with intrinsic magnetic fields the interaction region between the solar wind and the magnetosphere is usually very small compared to the size of planet. This means that the braking of the solar wind flow occurs in a short distance and the characteristics of the stationary bow shock are determined almost exclusively by the properties of the solar wind far upstream. In the case of comets, however, having no magnetic field of any significance, no rigid obstacle is present to play the role of a piston generating a shock. Physically this means that the interaction region is very wide and its size is determined by the coma. Nevertheless the solar wind flow possesses another intrinsic spatial scale, the dispersive length scale, which was the only important one in the case of a magnetized planet. It is precisely the interaction of these two very different scales which determines the bow shock in the case of comets.

The ratio of the dispersive scale to the characteristic size of the coma introduces in a natural way a small parameter and one can then exploit the weak interaction between plasma motions on such vastly different scales. That is, small scale motions, on scales much shorter than the ionization scalelengths, have to influence the plasma flow. In a hot collisionless plasma the low frequency magnetosonic type wave motions exert a major influence, higher frequency disturbances being globally of much less of importance.

Galeev and Khabibrakhmanov [1989] considered the collisonless interaction of small scale dispersive motions with large scale motions induced by mass loading, in the case of solar wind interaction with cometary atmospheres. This interaction leads to finite amplitude magnetosonic modes in the region of mass loaded plasma flow with magnetosonic Mach numbers less than 2. This Mach number is defined by characteristics of the large scale motions. It is thus natural to suppose that the real cometary shock must be somewhere in this region, that is, the interaction of large and small scale motions is indeed responsible for the bowshock formation.

Numerical simulations [*Galeev, Lipatov and Sagdeev*, 1985; *Omidi and Winske*, 1986] show that the cometary bowshock Mach number is close to $1.7 \sim 2$ and is only weakly dependent on the supply rate of neutral material from the cometary nucleus. Another consequence of this interaction between motions on different scales is the damping or growth of nonlinear waves. As a result, there are quasistationary nonlinear waves, propagating with almost constant amplitudes.

In the present paper we will analyze quasistationary nonlinear motions in general, following the approach outlined by *Galeev and Khabibrakhmanov* [1989]. Both that paper and the present one run along similar lines, but here we fully take all nonlinear and mass loading effects into account, contrary to previous treatments. We expand the solution of the Vlasov equation for the plasma flow particles (Appendix C) in appropriate variables μ and ϕ, where $\mu = v_\perp^2/2B$ is the magnetic moment and ϕ the modified gyrophase angle of a particle (Appendix B). The right hand side of this Vlasov equation is determined by the ionization rate of the neutral particles (Appendix A). The expansion of the solution allows us to calculate the total current. Upon substitution of the resulting currents for each plasma species into Maxwell's equations we get a set of equations describing the motion of the magnetized plasma flow. The analytical analysis is then straightforward, as the equations are already conveniently expanded in the derivatives of the electromagnetic fields. So in Section 2 we present the kinetic description of slow motions of the mass loaded plasma flow. In Section 3 we consider the ionization scale motions of the flow and in Section 4 motions on a much shorter dispersive scale. Such a division is convenient and justified here because of the very large differences between these scales. Finally in Section 5, the interaction between motions on these two different scales determines the evolution of the quasistationary nonlinear waves.

2. KINETIC DESCRIPTION OF MAGNETIZED, MASS LOADED PLASMA FLOW

For simplicity we will consider the motion of a mass loaded plasma flow only in the plane transverse to the magnetic field direction (z–axis), magnetic field which has thus only one component. The Vlasov equation for the distribution function of the plasma particles in the variables $\mu = v_\perp^2/2B$ (the magnetic moment) and ϕ (the modified gyrophase angle, see Appendix A) is given in Appendix B. The newly ionized particles, embedded now in the main plasma flow, are considered to be initially cold, as the thermal speed of the neutral particles is very small compared to the local drift velocity of the plasma. This assumption yields a relatively simple form of the distribution function of newly ionized particles (A.1) and of the source term in the Vlasov equation for the plasma species (A.4). We consider quasistationary motions of the plasma with spatial scales much greater than the ion gyroradius r_{ci} and the electron gyroradius r_{ce}. Using this approximation we expand the solution of the Vlasov equation for the ions in the small parameter $\epsilon = O(\omega/\Omega) = O(kr_{ci})$, where Ω is the gyrofrequency of the ions and ω and k are the inverse time and spatial scales of the plasma motion, respectively. The expression for the solution of the Vlasov equation up to the third order in ϵ is given in Appendix C. With the help of this solution one calculates the total current $\mathbf{j}$:

$$\mathbf{j} = \sum_{\alpha} e_{\alpha} \int \mathbf{v} F_{\alpha}(\mathbf{r}, \mathbf{v}, t) \mathrm{d}^3\mathbf{v}, \tag{2.1}$$

where the subscript α denotes the plasma species and e_{α} is the charge of particles of species α. This expression can be used to obtain the equations of motion for the plasma flow from Maxwell's equation

$$c^2 \nabla \times \mathbf{B} = \frac{\partial \mathbf{E}}{\partial t} + \frac{1}{\varepsilon_0}\mathbf{j}. \tag{2.2}$$

The y–component of this is the equation of motion for the plasma along the x–axis. The drift velocity along the x–axis, $v_{Dx} = f = E_y/B$ is in a first approximation the bulk velocity of the plasma.

Now we will consider only one–dimensional motions along the x–axis. This implies that the drift velocity $v_{Dy} = -g = -E_x/B$ along the y–axis is zero, although it has nonvanishing gradients. It is then easy to determine the evolution of g from the x–component of equation (2.2):

$$\begin{aligned}\frac{\partial E_x}{\partial t} + \sum \frac{eB}{\varepsilon_0 \Omega} \Bigg\{ \frac{n}{B}\widehat{T} g - \frac{1}{\Omega} \Bigg(\frac{3P}{2mB^2}\widehat{T}B_x + \frac{n}{B}\widehat{T}^2 f \\ + \frac{B}{2}\frac{\partial \widehat{T}}{\partial x}\frac{P}{mB^2} + \frac{\sqrt{2B}}{2}\widehat{T}\frac{\nu}{B}\sqrt{\frac{f^2}{2B}} \Bigg) \Bigg\} = 0.\end{aligned} \tag{2.3}$$

Here we used up to second–order terms (the orders of the terms refer to powers of $1/\Omega$), because g occurs only in the second or higher order terms in the equation for the motion along the stagnation line:

$$
\begin{aligned}
\frac{\partial E_y}{\partial t} + c^2\frac{\partial B}{\partial x} + \sum \frac{eB}{\varepsilon_0 \Omega}\Bigg\{ & \frac{n}{B}\widehat{T}f + B\frac{\partial}{\partial x}\frac{P}{mB^2} + \frac{2P}{mB^2}B_x + \frac{\nu f}{B} \\
& + \frac{B}{\Omega}\left(\frac{n}{B^2}\widehat{T}^2 g - \frac{3P}{2mB^2}g_{xx}\right) \\
& - \frac{1}{\Omega^2}\left(\frac{11P}{4mB^2}\widehat{T}^2 B_x + \frac{n}{B}\widehat{T}^3 f - \frac{3\Pi}{2m^2B^3}B_{xxx}\right) \\
& + \frac{B}{2\Omega^2}\left(\frac{\partial \widehat{T}^2}{\partial x}\frac{P}{mB^2} + B\frac{\partial^3}{\partial x^3}\frac{\Pi}{m^2B^3}\right) \\
& - \frac{\sqrt{2B}}{\Omega^2}\left(\widehat{T}^2\frac{\nu}{B}\sqrt{\frac{f^2}{2B}} - \frac{2B}{3}\frac{\partial^2}{\partial x^2}\frac{\nu}{B}\left(\frac{f^2}{2B}\right)^{3/2}\right)\Bigg\} = 0.
\end{aligned}
\tag{2.4}
$$

In equations (2.3–4) the operator $\widehat{T}$ is defined as $\widehat{T} = \partial/\partial t + f\partial/\partial x - g\partial/\partial y$. For the sake of simplicity, all variables will eventually be supposed independent of y, so that later we only consider one–dimensional motions of the plasma flow. Also, the standard designation for derivatives $d./dx \equiv ._x$ is used.

We have to add equations governing the evolution of the plasma density n,

$$n = B\int_0^\infty F_0(\mathbf{r},\mu)\mathrm{d}\mu, \tag{2.5}$$

of the ion pressure P,

$$\frac{P}{m} = B^2\int_0^\infty F_0(\mathbf{r},\mu)\mu\mathrm{d}\mu, \tag{2.6}$$

of the second moment Π (in the magnetic moment μ of the particles),

$$\frac{\Pi}{m^2} = B^3\int_0^\infty F_0(\mathbf{r},\mu)\mu^2\mathrm{d}\mu, \tag{2.7}$$

and of the magnetic field B. However, it is easier to use equations governing the changes in n/B, P/B^2 and Π/B^3, because of their invariance in the case of zero production rate ν. This is a consequence of the adiabatic conservation of the magnetic moment of the particles during slow motions:

$$\hat{T}\frac{n}{B} = \frac{\nu}{B}, \tag{2.8}$$

$$\hat{T}\frac{P}{mB^2} = \frac{\nu f^2}{2B^2}, \tag{2.9}$$

$$\hat{T}\frac{\Pi}{m^2 B^3} = \frac{\nu f^4}{4B^3}. \tag{2.10}$$

For the evolution of the magnetic field we can use Maxwell's equation $\nabla\times\mathbf{E}+\partial\mathbf{B}/\partial t = 0$, which in our notations is written as

$$\hat{T}B = (g_y - f_x)B. \tag{2.11}$$

So we have a complete set of equations (2.3–4),(2.8–11) describing the slow motions of the plasma. As can be seen from the method of expansion of the distribution function in Appendix C, slow motions here mean motions with characteristic time and space scales much greater than the ion gyroperiod and gyroradius, respectively.

3. QUASISTATIONARY PLASMA MOTIONS: LARGE SCALE MOTIONS

Let us now consider the quasistationary motions described by the set (2.3–4). As was stated in the Introduction, we take into account motions simultaneously on two very different spatial scales and it is hence advantageous to distinguish contributions on different scales from the very outset. Every variable of the system is supposed to have two parts, one purely stationary and another one with a weak time dependence. The former describes the large space scale variations of the plasma flow, whereas the latter takes dispersive effects into account. For example, the flow velocity f along the stagnation line has a stationary part $\tilde{f}(x)$ and a part $f(t,x)$ which depends only weakly on time.

Neglecting the small scale motions in the set (2.3–4),(2.8–11) yields the following magnetohydrodynamic equations:

$$
\begin{aligned}
\frac{d}{dx}\left(m\tilde{n}\tilde{f}^2 + \tilde{P} + \frac{\tilde{B}^2}{8\pi}\right) &= 0, \\
\tilde{f}\frac{d}{dx}\frac{\tilde{n}}{\tilde{B}} &= \frac{\nu}{\tilde{B}}, \\
\frac{d}{dx}\frac{\tilde{P}}{\tilde{B}^2} &= \frac{\nu m\tilde{f}}{2\tilde{B}^2}, \\
\frac{d}{dx}\frac{\tilde{\Pi}}{\tilde{B}^3} &= \frac{\nu m^2\tilde{f}^3}{4\tilde{B}^3}, \\
\frac{d\tilde{B}\tilde{f}}{dx} &= 0.
\end{aligned}
\tag{3.1}
$$

The large scale motion is precisely the one used by *Galeev, Craven and Gombosi* [1985] to describe the solar wind flow in the vicinity of comets. We can use their solutions for (3.1), in the limit of small magnetic field pressure (neglecting it in the first equation):

$$
\begin{aligned}
\tilde{P} &= mn_\infty f_\infty^2 - m\tilde{n}\tilde{f}^2, \\
\tilde{n}\tilde{f} &= n_\infty f_\infty\left(\frac{4f_\infty}{3\tilde{f}} - \frac{f_\infty{}^2}{3\tilde{f}^2}\right), \\
\frac{\tilde{n}\tilde{f}}{n_\infty f_\infty} &= 1 + \frac{\nu r}{n_\infty f_\infty} = 1 + \frac{Q_n}{4\pi V_g\tau n_\infty f_\infty}\frac{1}{r}.
\end{aligned}
\tag{3.2}
$$

Here the subscript ∞ denotes the undisturbed solar wind values. The plasma production rate ν depends on the production rate Q_n of neutral cometary gas, on a constant gas outflow velocity V_g, on a characteristic time τ for photoionization and on the radial distance r from the cometary nucleus, hence $\nu = Q_n/(4\pi V_g\tau r^2)$.

It can now be seen that the characteristic space scale for large scale plasma motions is

$$
R_L = \frac{Q_n}{4\pi V_g\tau n_\infty f_\infty}. \tag{3.3}
$$

The expressions (3.2) completely define these large scale motions, except near the point with large gradients where the local Mach number, defined through $M^2 = m\tilde{n}\tilde{f}^2/2\tilde{P} = 2(\hat{f} - 1/4)/(1 - \hat{f})$, goes to 1. This means $\hat{f} = 1/2$, with $\hat{f} = \tilde{f}/f_\infty$.

4. QUASISTATIONARY PLASMA MOTIONS: DISPERSIVE SCALE MOTIONS

Now we will take into account the dispersive effects on the mass loaded solar wind flow. This is easily done by taking advantage of the fact that the ratio of the dispersive scale a_D to the plasma production scale R_L (defined in (3.3)) is so small. The spatial derivatives of the large scale motions will then be smaller than those of the small scale motions by the same ratio. The following analysis is in fact based on the expansion of the set (2.3–4),(2.8–11) in the small parameter a_D/R_L. Before doing that, however, we note that in the opposite limit ($R_L = 0$, signifying the absence of mass loading) the set (2.3–4),(2.8–11) describes nonlinear magnetosonic type motions (see *e.g. Mikhaïlovskii and Smolyakov* [1985]).

For quasistationary motions, where in the operator $\widehat{T}$ the time derivatives $\partial/\partial t$ are much smaller than the $f\partial/\partial x$ terms, one reduces the set (2.3–4),(2.8–11) to one equation for perturbations in the magnetic field. These perturbations, denoted by b and defined through $B/\tilde{B} = 1 + b(t, x - ut)$, obey a KdV–equation

$$-\frac{2}{\tilde{f} - u} b_t - \left(1 - \frac{2\tilde{P} + \tilde{B}^2/\mu_0}{\tilde{\rho}(\tilde{f} - u)^2}\right) b_x + 3bb_x + a_D^2 b_{xxx} = 0. \tag{4.1}$$

Here $\tilde{\rho} = m\tilde{n}$ is the mass density of the plasma and a_D the dispersion length for the magnetosonic wave, defined through

$$a_D^2 = \frac{\tilde{P}}{4\tilde{\rho}\Omega^2}\left\{1 + \frac{3\tilde{P}}{\tilde{\rho}(\tilde{f} - u)^2}\left(\frac{2\tilde{\Pi}\tilde{\rho}}{m\tilde{P}^2} - 3\right)\right\}. \tag{4.2}$$

We see that in the absence of mass loading ($\nu = 0$, that is, the values of n/B, P/B^2 and Π/B^3 are adiabatically conserved) the set (2.3–4),(2.8–11) describes soliton–type nonlinear motions:

$$b = \frac{b^{(0)}}{\cosh^2\left(\kappa(x - ut)\right)}, \tag{4.3}$$

where

$$
\begin{aligned}
b^{(0)} &= 1 - \frac{V^2}{(\tilde{f} - u)^2}, \\
\kappa &= \frac{1}{2a_D}\sqrt{1 - \frac{V^2}{(\tilde{f} - u)^2}}, \\
V &= \sqrt{\frac{2\tilde{P} + \tilde{B}^2/\mu_0}{\tilde{\rho}}},
\end{aligned} \tag{4.4}
$$

are the soliton amplitude, its characteristic width and the velocity of the linear magnetosonic waves, respectively.

The preceeding results are well known. They include large scale motions, describing the smooth deceleration of the solar wind flow, as well as soliton–type wave motions on the small scale (dispersive motions). It is worth noting that the description used in this paper is purely a kinetic one and thus automatically obviates problems concerning the precise calculation of the viscosity tensor in a hydrodynamic treatment (see *Kennel and Sagdeev* [1967]; *Macmahon* [1968]; *Kennel* [1968]). The kinetic description enables us to reproduce results of *Mikhailovskii and Smolyakov* [1985] derived by another approach — they used linear kinetic theory to calculate the dispersive terms and then a hydrodynamic model to describe nonlinear effects in the wave equation. Moreover, in principle only a full kinetic treatment can give an accurate computation of the effects of different warm plasma components on the nonlinear motions, in particular on the dispersive lengthscale. Although Appendix C has all the expressions needed for such calculations, it is not the aim of the present paper to pursue this aspect further.

It is also well known that if we include small dissipative terms, the soliton solution has two different kinds of evolution, depending on how the dissipation is catalogued. If the dissipative term is proportional to b ("hydrodynamic" dissipation), we have the motion of a soliton with slowly varying amplitude. In this case there is no shock. If on the other hand the dissipative term is proportional to b_{xx} ("viscosity" dissipation), then the wave equation describes a train of solitons with different amplitudes and this gives the structure of a collisionless shock [*Sagdeev* 1966]. The next section is devoted to the analysis of the wave equation, taking the effects of weak mass loading into account, that is, the above mentioned ratio of spatial scales a_D/R_L

is small. We will see that the main terms have exactly the form of "hydrodynamic" as well as of "viscosity" dissipation.

5. QUASISTATIONARY PLASMA MOTIONS: EVOLUTION OF NONLINEAR WAVES

Now we consider the slow evolution of quasistationary nonlinear waves in a hot plasma flow with mass loading, that is we are interested in the nonlinear solution of the set (2.3–4),(2.8–11) which is quasistationary in the reference frame of the neutral gas. In the absence of mass loading such a solution has the form of a soliton, as we saw in the preceeding section, with velocity u in a frame of reference moving with the plasma and with velocity $\tilde{f} - u$ relative to the cometary nucleus. If one takes weak mass loading into account we must add to (4.1) dissipative terms which can be determined from the set (2.3–4),(2.8–11). The main term, in the case where the ratio of dispersive scale to mass loading scale is small, is proportional to b and to b_{xx}. We thus find the nonlinear equation

$$
\begin{aligned}
&-\frac{2}{(\tilde{f}-u)}b_t - \left(1 - \frac{V^2}{(\tilde{f}-u)^2}\right) b_x + 3bb_x + a_D^2 b_{xxx} + \nu_v b_{xx} \\
&-\frac{\nu m \tilde{f}^2}{\rho(\tilde{f}-u)^3}\left\{\left(\left(\frac{u}{\tilde{f}}\right)^2 - 3\frac{u}{\tilde{f}} + 3\right) + \frac{\tilde{f}_x \rho}{m\nu}\left(\left(\frac{V}{\tilde{f}}\right)^2 + 2\left(1 - \frac{u}{\tilde{f}}\right)\right)\left(1 - \frac{u}{\tilde{f}}\right)\right\} b = 0,
\end{aligned}
\tag{5.1}
$$

where the viscosity dissipation term ν_v has a rather complicated dependence on the large scale motion.

In the case of weak nonlinearity ($b/a_D b_{xx} \ll 1$) and weak mass loading, (5.1) can be solved by perturbation techniques (adiabatic motion of the soliton (4.3) with slowly varying amplitude). As customary for a perturbation method, the condition that the zeroth–order solution (4.3) be orthogonal to the first–order solution determines the evolution of the soliton amplitude:

$$
\begin{aligned}
\frac{2}{\tilde{f}-u}b_t^{(0)} + \frac{\nu m \tilde{f}^2}{\rho(\tilde{f}-u)^3}&\left\{\left(\left(\frac{u}{\tilde{f}}\right)^2 - 3\frac{u}{\tilde{f}} + 3\right)\right. \\
&\left. + \frac{\tilde{f}_x \rho}{m\nu}\left[\left(\frac{V}{\tilde{f}}\right)^2 + 2\left(1 - \frac{u}{\tilde{f}}\right)\right]\left(1 - \frac{u}{\tilde{f}}\right)\right\} b^{(0)} = 0.
\end{aligned}
\tag{5.2}
$$

From this equation it is evident that nonlinear waves can propagate quasistationary in a mass loaded plasma flow, provided the term between curly brackets vanishes in (5.2). This condition determines a second relation, in addition to (4.4), between the soliton amplitude and its propagation velocity. Both together give the quasistationary soliton amplitude at any point in the mass loaded plasma flow. Using the large scale motion characteristics (3.2), we can express the only parameter in the dissipative term of (5.2), which depends on these parameters, in terms of the magnetosonic Mach number of the plasma flow $M = \tilde{f}/V$:

$$\frac{\tilde{f}_x \rho}{m\nu} = \frac{3M^2}{2(1 - M^2)}. \tag{5.3}$$

One then finds that the dissipation term is proportional to

$$\frac{1}{1 - M^2}\left((1 + 2M^2)\xi^2 + \left(\frac{5}{2} - M^2\right)\xi + 1 - M^2\right), \tag{5.4}$$

where $\xi = 1 - u/\tilde{f}$. This second–order polynomial in ξ has two roots

$$\begin{aligned} \xi_1 &= -\frac{1}{2}, \\ \xi_2 &= \frac{M^2 - 1}{M^2 + 0.5}, \end{aligned} \tag{5.5}$$

that determine the propagation velocities of quasistationary localized solutions. Here we consider not only soliton–type solutions of the wave equation (5.2) but also a wider class of quasistationary solutions as a wave packet or a train of solitons, which is the limiting form of a moving collisionless shock in the case of small viscosity. All such solutions are the more stationary in a mass loaded plasma flow, the closer their propagation velocities are to the values (5.5). For *linear* magnetosonic waves ($\xi = \pm 1/M$) one can reproduce from (5.5) the result of *Galeev and Khabibrakhmanov* [1989] that magnetosonic waves propagating forward through a mass loaded plasma become unstable as the local magnetosonic Mach number decreases to $M = 2$. For backward propagating linear magnetosonic waves this value is a little less, $M = 1.74$.

CONCLUSIONS

We have shown in this paper that, although supermagnetosonic mass loaded plasma flows with $M > 2$ are stable against infinitesimal disturbances, sufficiently strong nonlinear disturbances have a definite growth rate due to mass loading. There are solutions for the wave equation of the form of $1/\cosh^2$–solitons with a threshold amplitude, which are quasistationary — their growth rates are zero — on the ionization time scale. It seems that such solitons are a permanent feature of the mass loaded plasma flow, because arbitrary disturbances of the KdV–equation split into a train of solitons, as is well known from soliton theory. According to our results, solitons with amplitudes below the threshold will be damped, whereas stronger ones will be accelerated and their amplitudes will grow until overturning. Only solitons with threshold amplitudes can be quasistationary. At the point in the plasma flow where the magnetosonic Mach number equals 2, the threshold amplitude for forward propagating waves decreases to zero. As was pointed out by *Galeev and Khabibrakhmanov* [1989], this can be indicative of the beginning of the bowshock formation process in the neighborhood of comets. Similar conclusions can be drawn for the other astrophysical occurrences of mass loaded plasma flow, of which some indicative examples were given in the Introduction. These are, however, as yet less amenable to *in situ* observational verification.

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APPENDIX A: DESCRIPTION OF NEWLY IONIZED PARTICLES

Let us suppose that the distribution function of the neutral particles in a cometocentric frame is maxwellian with a characteristic temperature $m\Delta/2$, m being the mass of the particles:

$$\Phi_0 = \frac{1}{\pi\Delta} \exp\left(-\frac{v_x^2 + v_y^2}{\Delta}\right). \tag{A.1}$$

We change variables to the magnetic moment $\mu = v_\perp^2/2B$, with $v_\perp^2 = v_x^2 + v_y^2$, and to the gyrophase $\tilde{\phi}$ of the ionized particles:

$$\begin{aligned} v_x &= \sqrt{2\mu B}\cos\tilde{\phi} + f, \\ v_y &= \sqrt{2\mu B}\sin\tilde{\phi} - g, \end{aligned} \tag{A.2}$$

where $f = E_y/B$ and $-g = -E_x/B$ are the components of the drift velocity of the plasma. One then gets the initial distribution function of the newly ionized particles in the following form:

$$\Phi_0 = \frac{1}{\pi\Delta} \exp\left(-\frac{2\mu B + g^2 + f^2}{\Delta}\right) \exp\left(-\frac{2\sqrt{2\mu B}}{\Delta}(f\cos\tilde{\phi} - g\sin\tilde{\phi})\right). \tag{A.3}$$

We now see that the description of the newly ionized particles is simpler if one replaces the gyrophase $\tilde{\phi}$ by a modified gyrophase $\phi = \tilde{\phi} + \pi + \alpha$, with $\alpha = \arctan(g/f)$ the initial gyrophase of the particle at the moment of ionization. Taking into account the following series expansion $\exp(z\cos\theta) = I_0(z) + 2\sum_{k=1}^{\infty} I_k(z)\cos k\theta$, where $I_k(z)$ are the modified Bessel functions of the first kind, and using the expansions for large arguments ($z \gg 1$), namely $I_\nu(z) = \frac{\exp z}{\sqrt{2\pi z}}\left(1 - \frac{4\nu^2 - 1}{8z} + \cdots\right)$, the distribution function for the newly ionized ions can be expressed in the small temperature limit in the form:

$$\begin{aligned} \lim_{\Delta\to 0} \Phi_0 &\to \frac{1}{\pi\sqrt{4\pi\Delta v v_0}} \exp\left(-\frac{(v - v_0)^2}{\Delta}\right)\left(1 + 2\sum_{k=1}^{\infty}\cos k\phi\right) \\ &\to \delta(v - v_0)\left(1 + 2\sum_{k=1}^{\infty}\cos k\phi\right), \end{aligned} \tag{A.4}$$

where $v = \sqrt{2\mu B}$ is the initial velocity of the particles perpendicular to the magnetic field and $v_0 = \sqrt{f^2 + g^2}$ is the local drift velocity which the newly ionized particles acquire at ionization, due to the electric field induced by the solar wind flow.

APPENDIX B: VLASOV EQUATION FOR THE DISTRIBUTION FUNCTION IN VARIABLES (μ, ϕ)

Now we consider the most simple geometry of a moving plasma. The magnetic field vector has only one component, along the z–axis, so that $\mathbf{B} = (0, 0, B)$. The electric field $\mathbf{E} = (E_x, E_y, 0)$ only has components perpendicular to $\mathbf{B}$. In this geometry the drift velocity of the plasma will have two components: $\mathbf{v}_D = (E_y/B, -E_x/B, 0) = (f, -g, 0)$.

The Vlasov equation for the distribution function $F(t, \mathbf{r}, \mathbf{v})$, describing the motion of particles in the above defined field configuration, turns out to be more tractable if one uses instead of $\mathbf{v}$ the variables μ (magnetic moment) and ϕ (modified gyrophase as defined in Appendix A) and expresses F throughout as a function $F(t, \mathbf{r}, \mu, \phi)$, so that

$$\frac{dF}{dt} = \frac{\partial F}{\partial t} + \frac{d\mathbf{r}}{dt} \cdot \frac{\partial F}{\partial \mathbf{r}} + \frac{d\mu}{dt}\frac{\partial F}{\partial \mu} + \frac{d\phi}{dt}\frac{\partial F}{\partial \phi}. \tag{B.1}$$

To calculate the derivatives $d\mu/dt$ and $d\phi/dt$ one uses the particle equations of motion

$$\begin{aligned} \frac{d^2x}{dt^2} &= \frac{eB}{m}\frac{dy}{dt} + \frac{e}{m}E_x, \\ \frac{d^2y}{dt^2} &= -\frac{eB}{m}\frac{dx}{dt} + \frac{e}{m}E_y. \end{aligned} \tag{B.2}$$

With the following rule for the change of variables:

$$\begin{aligned} \frac{dx}{dt} &= -\sqrt{2\mu B}\cos(\phi - \alpha) + f, \\ \frac{dy}{dt} &= -\sqrt{2\mu B}\sin(\phi - \alpha) - g, \end{aligned} \tag{B.3}$$

where $\alpha = \arctan(g/f)$ is the initial gyrophase of a particle (significant only for newly added particles) and using the chain rule for the derivative d/dt:

$$\begin{aligned} \frac{d}{dt} \equiv (\dot{\ }) &= \frac{\partial}{\partial t} + \dot{x}\frac{\partial}{\partial x} + \dot{y}\frac{\partial}{\partial y} \\ &= \frac{\partial}{\partial t} + f\frac{\partial}{\partial x} - g\frac{\partial}{\partial y} - \sqrt{2\mu B}\left(\cos(\phi - \alpha)\frac{\partial}{\partial x} + \sin(\phi - \alpha)\frac{\partial}{\partial y}\right), \end{aligned} \tag{B.4}$$

one obtains expressions for $\dot{\mu}$ and $\dot{\phi}$:

$$\begin{aligned}
\dot{\mu} =& \frac{\mu}{B}\left(-\widehat{T}B + B\,(g_y - f_x)\right) \\
& - \frac{\sqrt{2\mu B}}{B}\left[\cos(\phi-\alpha)\left(-\mu B_x - \widehat{T}f\right) + \sin(\phi-\alpha)\left(-\mu B_y + \widehat{T}g\right)\right] \\
& + \mu\left[\cos 2(\phi-\alpha)\,(-f_x - g_y) + \sin 2(\phi-\alpha)\,(g_x - f_y)\right],
\end{aligned} \tag{B.5}$$

$$\begin{aligned}
\dot{\phi} =& \dot{\alpha} - \Omega + \frac{1}{2}(f_y + g_x) - \frac{1}{\sqrt{2\mu B}}\left(\cos(\phi-\alpha)\widehat{T}g + \sin(\phi-\alpha)\widehat{T}f\right) \\
& + \frac{1}{2}\left[\cos 2(\phi-\alpha)\,(g_x - f_y) + \sin 2(\phi-\alpha)\,(g_y + f_x)\right].
\end{aligned} \tag{B.6}$$

Here $\Omega = eB/m$ is the gyrofrequency of a particle, and standard notations for the derivatives $d./dx \equiv ._x$ and $d./dy \equiv ._y$ are used. The operator $\widehat{T}$ is defined as $\widehat{T} \equiv \partial/\partial t + f\partial/\partial x - g\partial/\partial y$. We note that the first term in the expression for $\dot{\mu}$ vanishes due to Maxwell's equation $\nabla \times \mathbf{E} + \partial\mathbf{B}/\partial t = 0$, rewritten in our notations as

$$\widehat{T}B = B(g_y - f_x). \tag{B.7}$$

This simply means that μ is an adiabatic invariant of the motion of the particle, and this is a particular incentive to use the magnetic moment in the following expansions of the distribution function for slow motions.

The Vlasov equation for the distribution function $F(\mu,\phi)$, neglecting the motion along the magnetic field lines, looks like

$$\widehat{L}F = \mathrm{St}(F), \tag{B.8}$$

the operator $\widehat{L}$ being defined as:

$$
\begin{aligned}
\widehat{L} =& \widehat{T} - \Omega\frac{\partial}{\partial\phi} + \left(\frac{1}{2}\left(g_x + f_y\right) + \frac{f\widehat{T}g - g\widehat{T}f}{f^2+g^2}\right)\frac{\partial}{\partial\phi} \\
& - \sqrt{2\mu B}\,\cos(\phi-\alpha)\left\{\frac{\partial}{\partial x} - \frac{\mu B_x + \widehat{T}f}{B}\frac{\partial}{\partial\mu} + \left(\frac{1}{2\mu B}\widehat{T}g + \frac{fg_x - gf_x}{f^2+g^2}\right)\frac{\partial}{\partial\phi}\right\} \\
& - \sqrt{2\mu B}\,\sin(\phi-\alpha)\left\{\frac{\partial}{\partial y} - \frac{\mu B_y - \widehat{T}g}{B}\frac{\partial}{\partial\mu} + \left(\frac{1}{2\mu B}\widehat{T}f + \frac{fg_y - gf_y}{f^2+g^2}\right)\frac{\partial}{\partial\phi}\right\} \\
& + \mu\cos 2(\phi-\alpha)\left\{(-f_x - g_y)\frac{\partial}{\partial\mu} + \frac{1}{2\mu}(g_x - f_y)\frac{\partial}{\partial\phi}\right\} \\
& + \mu\sin 2(\phi-\alpha)\left\{(-f_y + g_x)\frac{\partial}{\partial\mu} + \frac{1}{2\mu}(g_y + f_x)\frac{\partial}{\partial\phi}\right\}.
\end{aligned}
\tag{B.9}
$$

APPENDIX C: EXPANSION OF DISTRIBUTION FUNCTION FOR SLOW MOTIONS

To solve equation (B.8) we suppose that terms in $\widehat{T}$ are small compared to those in Ω. The righthand side of equation (B.8), according to (A.4) and taking the Jacobian of the transformation (B.3) into account, will have the form:

$$
\mathrm{St}(F) = \frac{\nu}{B}\delta(\mu-\eta)\left(1 + 2\sum_{k=1}^{\infty}\cos k\phi\right), \tag{C.1}
$$

where ν is the production rate of the newly ionized particles and $\eta = (f^2+g^2)/2B$ is the initial value of their magnetic moment.

The zeroth–order approximation of equation (B.8) gives as only result that the zeroth–order distribution function F_0 is independent of ϕ. Averaging the equation for F_0 over the gyrophase ϕ according to

$$
\frac{1}{2\pi}\int_0^{2\pi} < \ldots > d\phi, \tag{C.2}
$$

we can determine the equation governing F_0:

$$
\widehat{T}F_0 = \frac{\nu}{B}\delta(\mu-\eta). \tag{C.3}
$$

For the next approximation we split the distribution function into two parts according to $F_1 = \tilde{F}_1 + F_1$, of which the first part is independent of ϕ. Averaging now the first-order equation

$$
\begin{aligned}
\widehat{T}\tilde{F}_1 - \Omega\frac{\partial F_1}{\partial\phi} &- \sqrt{2\mu B}\,\cos(\phi-\alpha)\left\{\frac{\partial F_0}{\partial x} + \frac{1}{B}\left(-\mu B_x - \widehat{T}f\right)\frac{\partial F_0}{\partial\mu}\right\} \\
&- \sqrt{2\mu B}\,\sin(\phi-\alpha)\left\{\frac{\partial F_0}{\partial y} + \frac{1}{B}\left(-\mu B_y + \widehat{T}g\right)\frac{\partial F_0}{\partial\mu}\right\} \\
&- \mu\cos 2(\phi-\alpha)(f_x + g_y)\frac{\partial F_0}{\partial\mu} + \mu\sin 2(\phi-\alpha)(-f_y + g_x)\frac{\partial F_0}{\partial\mu} \\
=&2\widehat{T}F_0\sum_{k=1}^{\infty}\cos k\phi
\end{aligned}
\tag{C.4}
$$

over ϕ, one obtains an equation for the part $\tilde{F}_1$ which is independent of ϕ:

$$
\widehat{T}\tilde{F}_1 = 0. \tag{C.5}
$$

Substituting the solution of this equation back into (C.4), we get an equation for F_1, with solution

$$
\begin{aligned}
\Omega F_1 = &- \sqrt{2\mu B}\,\sin(\phi-\alpha)\left\{\frac{\partial F_0}{\partial x} + \frac{1}{B}\left(-\mu B_x - \widehat{T}f\right)\frac{\partial F_0}{\partial\mu}\right\} \\
&+ \sqrt{2\mu B}\,\cos(\phi-\alpha)\left\{\frac{\partial F_0}{\partial y} + \frac{1}{B}\left(-\mu B_y + \widehat{T}g\right)\frac{\partial F_0}{\partial\mu}\right\} \\
&- \frac{\mu}{2}\sin 2(\phi-\alpha)(f_x + g_y)\frac{\partial F_0}{\partial\mu} \\
&- \frac{\mu}{2}\cos 2(\phi-\alpha)(-f_y + g_x)\frac{\partial F_0}{\partial\mu} - 2\widehat{T}F_0\sum_{k=1}^{\infty}\frac{\sin k\phi}{k}.
\end{aligned}
\tag{C.6}
$$

The second approximation to the distribution function can found from the equation

$$
\begin{aligned}
\Omega\frac{\partial F_2}{\partial\phi} = \widehat{T}\tilde{F}_2 + \widehat{T}F_1 &- \sqrt{2\mu B}\,\cos(\phi-\alpha)\frac{\partial(F_1 + \tilde{F}_2)}{\partial x} \\
&- \sqrt{2\mu B}\,\sin(\phi-\alpha)\frac{\partial(F_1 + \tilde{F}_2)}{\partial y}.
\end{aligned}
\tag{C.7}
$$

Again averaging this over the gyrophase, we obtain the ϕ–independent part of the second approximation to the distribution function:

$$
\Omega\tilde{F}_2 = \mu(g_x + f_y)\frac{\partial F_0}{\partial\mu} - \sqrt{2\mu B}\left(\sin\alpha\frac{\partial F_0}{\partial x} + \cos\alpha\frac{\partial F_0}{\partial y}\right). \tag{C.8}
$$

Substitution of this result back into (C.7) allows one to determine the remaining part:

$$
\begin{aligned}
\Omega^2 F_2 = \\
& \cos(\phi-\alpha)\sqrt{2\mu B}\left\{\frac{\partial \widehat{T} F_0}{\partial x} + \left(-\frac{\mu \widehat{T} B_x + \widehat{T}^2 f}{B} + \frac{\mu}{4}\left(4g_{xy} - f_{xx} + 3f_{yy}\right)\right)\frac{\partial F_0}{\partial \mu}\right\} \\
& + \sin(\phi-\alpha)\sqrt{2\mu B}\left\{\frac{\partial \widehat{T} F_0}{\partial y} + \left(-\frac{\mu \widehat{T} B_y - \widehat{T}^2 g}{B} + \frac{\mu}{4}\left(g_{yy} - 3g_{xx} - 4f_{xy}\right)\right)\frac{\partial F_0}{\partial \mu}\right\} \\
& + \frac{\mu B}{2}\cos 2(\phi-\alpha)\left\{\frac{\partial^2 F_0}{\partial y^2} - \frac{\partial^2 F_0}{\partial x^2} + \frac{1}{B}\left(\mu\left(B_{xx} - B_{yy}\right) + \frac{3}{2}\widehat{T}\left(g_y + f_x\right)\right)\frac{\partial F_0}{\partial \mu}\right\} \\
& - \frac{\mu B}{2}\sin 2(\phi-\alpha)\left\{2\frac{\partial^2 F_0}{\partial x \partial y} + \frac{1}{B}\left(-2\mu B_{xy} + \frac{3}{2}\widehat{T}\left(g_x - f_y\right)\right)\frac{\partial F_0}{\partial \mu}\right\} \\
& + \sqrt{2\mu B}\frac{\mu}{12}\frac{\partial F_0}{\partial \mu}\cos 3(\phi-\alpha)\left(f_{yy} - 2g_{xy} - f_{xx}\right) \\
& - \sqrt{2\mu B}\frac{\mu}{12}\frac{\partial F_0}{\partial \mu}\sin 3(\phi-\alpha)\left(g_{yy} + 2f_{xy} - g_{xx}\right) \\
& - 2\mu B\cos(\phi-\alpha)\left\{\cos\alpha\frac{\partial^2 F_0}{\partial y^2} + \sin\alpha\frac{\partial^2 F_0}{\partial x \partial y}\right\} \\
& + 2\mu B\sin(\phi-\alpha)\left\{\sin\alpha\frac{\partial^2 F_0}{\partial x^2} + \cos\alpha\frac{\partial^2 F_0}{\partial x \partial y}\right\} \\
& + 2\widehat{T}^2 F_0 \sum_{k=1}^{\infty}\frac{\cos k\phi}{k^2} \\
& - \sqrt{2\mu B}\left\{\sum_{k=2}^{\infty}\left(\frac{\cos((k-1)\phi+\alpha)}{k(k-1)} + \frac{\cos((k+1)\phi-\alpha)}{k(k+1)}\right) + \frac{\cos(2\phi-\alpha)}{2}\right\}\frac{\partial \widehat{T} F_0}{\partial x} \\
& + \sqrt{2\mu B}\left\{\sum_{k=2}^{\infty}\left(\frac{\sin((k-1)\phi+\alpha)}{k(k-1)} - \frac{\sin((k+1)\phi-\alpha)}{k(k+1)}\right) - \frac{\sin(2\phi-\alpha)}{2}\right\}\frac{\partial \widehat{T} F_0}{\partial y}.
\end{aligned}
\tag{C.9}
$$

From the equation for the third-order approximation to the distribution function:

$$\Omega\frac{\partial F_3}{\partial\phi} = \widehat{T}\tilde{F}_3 + \widehat{T}F_2 - \sqrt{2\mu B}\cos(\phi-\alpha)\frac{\partial(F_2+\tilde{F}_3)}{\partial x} - \sqrt{2\mu B}\sin(\phi-\alpha)\frac{\partial(F_2+\tilde{F}_3)}{\partial y}, \tag{C.10}$$

we need only the part averaged over ϕ:

$$\begin{aligned}\Omega^2\tilde{F}_3 =& \mu B\left\{\frac{\partial^2 F_0}{\partial x^2} + \frac{\partial^2 F_0}{\partial y^2} + \frac{1}{B}\left(-\frac{3\mu}{4}(B_{xx}+B_{yy}) + \frac{1}{B}\widehat{T}^2 B\right)\frac{\partial F_0}{\partial\mu}\right\}\\ &+ \sqrt{2\mu B}\left\{\frac{\partial\widehat{T}F_0}{\partial x}\cos\alpha - \frac{\partial\widehat{T}F_0}{\partial y}\sin\alpha\right\}\\ &- \frac{\mu B}{2}\left\{\left(\frac{\partial^2 F_0}{\partial x^2} - \frac{\partial^2 F_0}{\partial y^2}\right)\cos 2\alpha - 2\frac{\partial^2 F_0}{\partial x\partial y}\sin 2\alpha\right\},\end{aligned} \tag{C.11}$$

and the first harmonics in the gyrophase ϕ of the ϕ-dependent part:

$$
\begin{aligned}
\Omega^3 F_3 = \sin(\phi-\alpha)\sqrt{2\mu B}\Bigg\{ & \frac{\partial \widehat{T}^2 F_0}{\partial x} - \frac{3\mu B}{4}\left(\frac{\partial^3 F_0}{\partial x \partial y^2} + \frac{\partial^3 F_0}{\partial x^3}\right) \\
& + \cos\alpha\left(2\frac{\widehat{T}^3 F_0}{\sqrt{2\mu B}} - \frac{3\sqrt{2\mu B}}{4}\left(\frac{\partial^2 \widehat{T} F_0}{\partial x^2} + \frac{\partial^2 \widehat{T} F_0}{\partial y^2}\right)\right) \\
& + \frac{3}{2}\left(\sin 2\alpha \frac{\partial \widehat{T}^2 F_0}{\partial y} - \cos 2\alpha \frac{\partial \widehat{T}^2 F_0}{\partial x}\right) \\
& + \frac{\mu B}{2}\left(\cos 2\alpha\left(\frac{\partial^3 F_0}{\partial x^3} - \frac{\partial^3 F_0}{\partial x \partial y^2}\right) - 2\sin 2\alpha \frac{\partial^3 F_0}{\partial x^2 \partial y}\right) \\
& + \frac{1}{12}\sqrt{2\mu B}\left(\cos 3\alpha\left(\frac{\partial^2 \widehat{T} F_0}{\partial x^2} - \frac{\partial^2 \widehat{T} F_0}{\partial y^2}\right) - 2\sin 3\alpha \frac{\partial^2 \widehat{T}}{\partial x \partial y}\right) \\
& + \left(-\frac{11\mu}{8B}\widehat{T}^2 B_x - \frac{\widehat{T}^3 f}{B} + \frac{\mu^2}{2}(B_{xyy} + B_{xxx}) + \frac{3\mu}{8}\dot{T}(f_{yy} + g_{xy})\right)\frac{\partial F_0}{\partial \mu}\Bigg\} \\
- \cos(\phi-\alpha)\sqrt{2\mu B}\Bigg\{ & \frac{\partial \widehat{T}^2 F_0}{\partial y} - \frac{3\mu B}{4}\left(\frac{\partial^3 F_0}{\partial x^2 \partial y} + \frac{\partial^3 F_0}{\partial y^3}\right) \\
& - \sin\alpha\left(2\frac{\widehat{T}^3 F_0}{\sqrt{2\mu B}} - \frac{3\sqrt{2\mu B}}{4}\left(\frac{\partial^2 \widehat{T} F_0}{\partial x^2} + \frac{\partial^2 \widehat{T} F_0}{\partial y^2}\right)\right) \\
& + \frac{3}{2}\left(\cos 2\alpha \frac{\partial \widehat{T}^2 F_0}{\partial y} + \sin 2\alpha \frac{\partial \widehat{T}^2 F_0}{\partial x}\right) \\
& + \frac{\mu B}{2}\left(\cos 2\alpha\left(\frac{\partial^3 F_0}{\partial x^2 \partial y} - \frac{\partial^3 F_0}{\partial y^3}\right) - 2\sin 2\alpha \frac{\partial^3 F_0}{\partial x \partial y^2}\right) \\
& + \frac{1}{12}\sqrt{2\mu B}\left(\sin 3\alpha\left(\frac{\partial^2 \widehat{T} F_0}{\partial x^2} + \frac{\partial^2 \widehat{T} F_0}{\partial y^2}\right) - 2\cos 3\alpha \frac{\partial^2 \widehat{T}}{\partial x \partial y}\right) \\
& + \left(-\frac{11\mu}{8B}\widehat{T}^2 B_y + \frac{\widehat{T}^3 g}{B} + \frac{\mu^2}{2}(B_{xxy} + B_{yyy}) - \frac{3\mu}{8}\widehat{T}(g_{xx} + f_{xy})\right)\frac{\partial F_0}{\partial \mu}\Bigg\}.
\end{aligned}
\tag{C.12}
$$

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