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3 - Avenue Circulaire
B - 1180 BRUXELLES

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Fitting the AE-8 energy spectra with two maxwellian functions

by

V. Pierrard and J. Lemaire

BELGISCH INSTITUUT VOOR RUIMTE-AERONOMIE

3 - Ringlaan
B - 1180 BRUSSEL

Foreword

This work about the fitting of AE-8 spectra by two maxwellian functions was presented at the International Workshop in Dubna (Russia), in June 1993, and has been accepted for publication in *Nuclear Tracks Radiat. Meas.*

Avant-Propos

Ce travail sur l'ajustement des spectres du modèle AE-8 par la somme de deux fonctions maxwelliennes a été présenté au Workshop international de Dubna (Russie) en juin 1993 et a été accepté comme publication dans *Nuclear Tracks Radiat. Meas.*

Voorwoord

Dit werk over de fit van AE-8 spectra met twee maxwelliaanse functies wordt op de Dubna International Workshop (Russia) in Juny 1993 voorgesteld en is aanvaard voor publicatie in *Nuclear Tracks Radiat. Meas.*

Vorwort

Diese arbeit wurde in Dubna International Workshop (Russia) in Juny 1993 vorgetragen und wurde zur Veröffentlichung in *Nuclear Tracks Radiat. Meas.* angenommen.

FITTING THE AE-8 ENERGY SPECTRA WITH TWO MAXWELLIAN FUNCTIONS

Pierrard V. and Lemaire J.,
IASB,
3 av. Circulaire,
B-1180 Brussels

Résumé

Les fonctions généralement utilisées pour approcher le spectre du flux intégral $J(>E; B; L)$, au-dessus d'un seuil d'énergie E, des particules énergétiques piégées dans le champ magnétique terrestre, sont des exponentielles $\exp(-E/E_0)$ ou des lois de puissance $E^{-\gamma}$, où l'énergie constante E_0 ou l'exposant γ sont ajustés pour représenter le plus précisément possible les mesures du flux $J_i(> E_i; B; L)$. Ces fonctions ont été choisies pour leur simplicité mathématique mais ne reposent sur aucune base physique. Elles ont été utilisées depuis 1960 dans la modélisation des ceintures de radiations de Van Allen. Cependant, il existe de nombreuses fonctions mathématiques capable d'approcher les données avec une aussi bonne précision. Mais la plupart d'entre elles n'ont aucune signification physique.

Dans ce travail, les spectres d'énergie discrets $J_i(E_i; B_0; L)$ du modèle AE-8 ont été ajustés par la somme de deux fonctions maxwelliennes. La pente du flux différentiel $J_d(E)$ est alors liée à la température caractéristique de la distribution maxwellienne des vitesses et la constante de normalisation est proportionnelle à la densité de la population maxwellienne. En plus de son fondement plus physique, cette nouvelle fonction a un comportement non-singulier à la limite $E = 0$. Ce n'est pas le cas pour les lois de puissance mentionnées précédemment, qui ajustent les données sur des intervalles d'énergie limités. Les paramètres d'ajustement ont été déterminés pour des coquilles magnétiques situées entre $L = 1.2$ et $L = 10$. Leur dépendance en L est déterminée et discutée. L'ajustement analytique des spectres d'énergie est bon pour des énergies inférieures à 4 MeV, mais n'est plus satisfaisant aux énergies supérieures pour des raisons discutées plus loin.

Ce travail montre que les spectres d'énergie du modèle AE-8 peuvent être bien approchés par deux distributions maxwelliennes pour des énergies électroniques inférieures à 4 MeV, non seulement près de l'orbite géostationnaire mais aussi pour un large intervalle de valeurs de L.

Abstract

In fitting the integral flux spectrum $J(> E; B; L)$ of trapped energetic particles above the energy threshold E, it is common practice to use either an exponential function $\exp(-E/E_0)$ or a power law $E^{-\gamma}$ where the energy constant E_0 or the index γ are adjusted to obtain best fits to the measured flux values $J_i(> E_i; B; L)$. These fit functions have generally been chosen not on firm physical grounds, but because of their mathematical simplicity. They have been used since 1960 in radiation belt modelling. However, a large number of mathematical functions can be selected to fit data with an equally good accuracy. But most of them have no physical meaning or relevance.

In this work, the discrete AE-8 energy spectra $J_i(E_i; B_0; L)$ have been fitted with a sum of two maxwellian functions. The slope of the differential flux $J_d(E)$ is then related to the characteristic temperature of the maxwellian velocity distribution and the normalisation constant is proportional to the number density of the maxwellian population. Besides its greater physical relevance, this new fit function has a non-singular behaviour in the limit $E = 0$. This is not the case for the power laws mentioned above which fit the data only in limited energy ranges. The fitting parameters have been determined for drift shells parameters ranging between $L = 1.2$ and $L = 10$. Their dependence on L is determined and discussed. The analytical fit of the energy spectra is good for energies smaller than 4 MeV, but it is not satisfactory at larger energies for reasons which are discussed.

This work indicates that the energy spectra of the AE-8 model can be approximated rather well by two maxwellian distributions for electron energies smaller than 4 MeV, not only near geostationary orbit but also for a wide range of L values.

Samenvatting

Spectra van geintegreerde fluxen $J(>E, B, L)$ van gevangen energetische deeltjes met energie groter dan E worden gewoonlijk gefit met ofwel een exponentiële functie $\exp(-E/E_0)$ of een macht $E^{-\gamma}$, waarbij die waarden voor de energieconstante E_0 of the index γ gekozen worden die het best aansluiten bij de gemeten fluxen $J_i(>E_i, B, L)$. Men heeft deze fit functies ingevoerd niet op een fysische basis maar wegens hun wiskundige eenvoud. In het domein van de modellering van de stralingsgordels zijn ze reeds in gebruik sinds de jaren zestig. Het is nochtans zo dat een groot aantal wiskundige functies de gegevens met dezelfde nauwkeurigheid kunnen weergeven, maar de meeste van deze functies hebben geen fysische betekenis of relevantie.

We hebben de discrete AE-8 energie spectra $J_i(>E_i, B, L)$ gefit met een som van twee Maxwelliaanse functies. The helling van de differentiële flux is verbonden met de karakteristieke temperatuur van de Maxwelliaanse snelheidsverdeling en de normalisatieconstante is evenredig met de numerieke dichtheid van de Maxwelliaanse populatie. Naast zijn grotere fysische relevantie heeft deze nieuwe fit functie het voordeel van een niet-singulier gedrag in de limiet $E = 0$. Dit is niet het geval voor de functies van de vorm $E^{-\gamma}$ die de gegevens slechts in beperkte energie-intervallen fitten. The fit parameters werden bepaald voor "drift shell" parameters tussen $L = 1.2$ en $L = 10$. Hun afhankelijkheid van L wordt bepaald en besproken. De analytische fit van de energie spectra is goed voor energieën kleiner dan 4 MeV maar is niet bevredigend voor hogere energieën, voor redenen die later besproken zullen worden.

Onze resultaten wijzen erop dat de energie spectra van het AE-8 model vrij goed benaderd kunnen worden door de som van twee Maxwelliaanse distributies, voor electron energieën kleiner dan 4 MeV, niet alleen rond de geostationaire baan maar ook voor een groot interval in L.

Zusammenfassung

Bei der Anpassung des Spektrums des integrierten Flusses von im Strahlungsgürtel gefangenen, energetischen Teilchen über einer bestimmten Energieschwelle wird gewöhnlich entweder eine Exponentialfunktion $\exp(-E/E_0)$ oder eine Potenzfunktion $E^{-\gamma}$ verwendet. Die Energiekonstante oder der Index γ werden so angepaßt, daß die beste Übereinstimmung mit den gemessenen Werten erreicht wird. Die Anpassungsfunktionen wurden meist nicht aus physikalischen Gründen, sondern ihrer mathematischen Einfachheit wegen gewählt. Sie wurden seit dem Jahr 1960 zur Modellierung der Strahlungsgürtel benützt. Jedoch kann eine große Zahl von mathematischen Funktionen gewählt werden, um mit guter Genauigkeit an die Daten angepaßt zu werden, wobei aber viele keinen physikalischen Sinn oder Bedeutung haben.

In der vorliegenden Arbeit wurden diskrete AE8-Energiespektren mit einer Summe zweier Maxwell-Verteilungsfunktionen angepaßt. Die Steigung des differentiellen Flusses hängt mit der charakteristischen Geschwindigkeit der Maxwell'schen Geschwindigkeitsverteilung zusammen und die Normalisierungskonstante ist der Dichte der maxwell-verteilten Teilchenpopulation proportional. Neben der größeren physikalischen Relevanz besitzt diese neue Anpassungsfunktion ein nicht-singuläres Verhalten am Energielimit $E = 0$. Dies ist für die vorher beschriebenen Potenzfunktionen nicht der Fall. Sie können nur Meßwerte in einem beschränkten Parameterraum zwischen $L = 1.2$ und $L = 10$ beschreiben. Die Abhängigkeit vom Schalenparameter L wird bestimmt und diskutiert. Die analytische Anpassung der Energiespektren für Energien kleiner als 4 MeV ist zufriedenstellend, im Gegensatz zu höheren Energien.

Die vorliegende Arbeit zeigt, daß die Energiespektren des AE8-Models für Elektronenenergien kleiner als 4 MeV sehr gut mit zwei Maxwellverteilungen angenähert werden können. Dies gilt nicht nur für den geostationären Orbit, sondern auch für einen großen Bereich des Schalenparameters L.

1 Introduction

The differential or integral spectra of energetic particles fluxes ($J_d(E)$ or $J(> E)$) in the magnetosphere are often fitted by power laws $E^{-\gamma}$ or by exponential functions $\exp(-E/E_0)$. The power index γ or the characteristic energy E_0 is determined from the slope of the spectra in a limited range of energy : $E_1 < E < E_2$. But, generally, the spectrum of energetic electrons near geosynchronous orbit displays different slopes in different energy ranges. A fit with two power laws gives a significant improvement over a fit with a single power law for the whole energy range, but requires specification of continuity conditions, as well as the determination of four parameters. Furthermore, for power laws, J diverges in the limit of zero energy: $\lim_{E\to 0} J(E) = \infty$. The Kappa-like distribution (e.g. Christon *et al.*, 1988) well represents the overall shape of the observed electron distributions, but seriously underestimates the data at lowest energies. Neither the Kappa-like distribution, nor the power law distribution, are based on solid physical grounds.

Parks *et al.* (1979) and McIlwain and Whipple (1986) have shown that electrons near geostationary orbit can be resolved into two or more distinct maxwellian distributions. Following the same idea, Cayton *et al.* (1989) have resolved the energy distribution functions of 30–2000 keV electrons observed simultaneously by three geostationary satellites into two distinct relativistic maxwellian components: a "hard" (300–2000 keV) electron population and a "soft" electron population (30–300 keV). Each component is fully parametrized by a density and a temperature. Note that such physical parameters (densities and temperatures) can not be deduced from the commonly used power law fits nor from the exponential fits of the particle's energy spectra. The same is true for emission lines in an optical spectrum. The Doppler temperature and abundance of an element can only be deduced if a relevant physical line profile has been used for the fit function.

In this paper, we have investigated whether the energy spectra of 0.1–4.0 MeV electrons, provided by the NASA empirical models AE-8 MIN and AE-8 MAX, can be resolved into two or more maxwellian populations. The variation of the densities and the temperatures of the different electron populations as a function of L will also be discussed.

2 The AE-8 energy spectra

The empirical AE-8 model produced by NSSDC was completed in December 1983. It provides omnidirectional fluxes in electrons/(cm^{2}sec) for a set of 18 discrete energies E ranging from 40 keV to 7.00 MeV. A comprehensive description of the AE-8 MIN and AE-8 MAX models can be found in a recent publication by Vette (1991) (see also Bilitza, 1990). AE-8 MAX and AE-8 MIN, based on satellite's data measured in the years 1960–1970, correspond respectively to solar maximum and solar minimum conditions. For $L > 5.5$, the two models are identical.

The AE-8 model is comprised in three parts. In the inner zone (i.e. for $1.2 < L < 2.4$), the model depends on the solar cycle epoch (Teague *et al.*, 1976). The AE-8 omnidirectional flux values in the outer region (i.e. for $3 < L < 11$) are independent on solar activity (Singley and Vette, 1972a and 1972b). In the transition region (i.e. for $2.4 < L < 3.0$), the AE-8 electron fluxes were interpolated between the inner and the outer regions (Vette, 1991). Figure 1 shows the integral fluxes $J(> E, L, B/B_0)$ from AE-8 MIN at $L = 2.0$ in the equatorial plane where $B/B_0 = 1$. The solid line corresponds to a best fit function which has been determined by a least-squares method[1]. The analytical equation of this best fit function is :

$$F(E) = \frac{a + c \ln E + e(\ln E)^2 + g(\ln E)^3}{1 + b \ln E + d(\ln E)^2 + f(\ln E)^3 + h(\ln E)^4} \tag{1}$$

with $a = 4.873$, $b = -0.883$, $c = -5.980$, $d = 0.473$, $e = 3.772$, $f = -0.065$, $g = -1.299$, $h = 0.022$. The quality of the fit is determined by the coefficient of determination:

$$r^2 = 1 - \frac{\sum_{i=1}^{n}(\tilde{F}_i - F_i)^2}{\sum_{i=1}^{n}(\overline{F} - F_i)^2} \tag{2}$$

where F_i are the $\log_{10} J_i$ of the AE-8 spectrum, $\tilde{F}_i$ are their estimated values, $\overline{F}$ is the mean of the F data values and n is the total number of data points. The value of r^2 is equal 0.99986, very close to unity. The standard deviation σ is less than 0.05. This indicates that $F(E)$ is a very good approximation of the discrete energy spectrum $\log_{10} J_i(> E)$. The values of the parameters a, b, c, d, e, f, g, h are different for each L-shell. If best fit functions $A(L)$, $B(L)$,

[1]The TABLECURVE^{TM} software package (Jandel Scientific) has been used here to determine among 3000 fit functions which is the best approximation to the data. This software ranks these functions according to the value of the coefficient of determination (cf. equation 2).

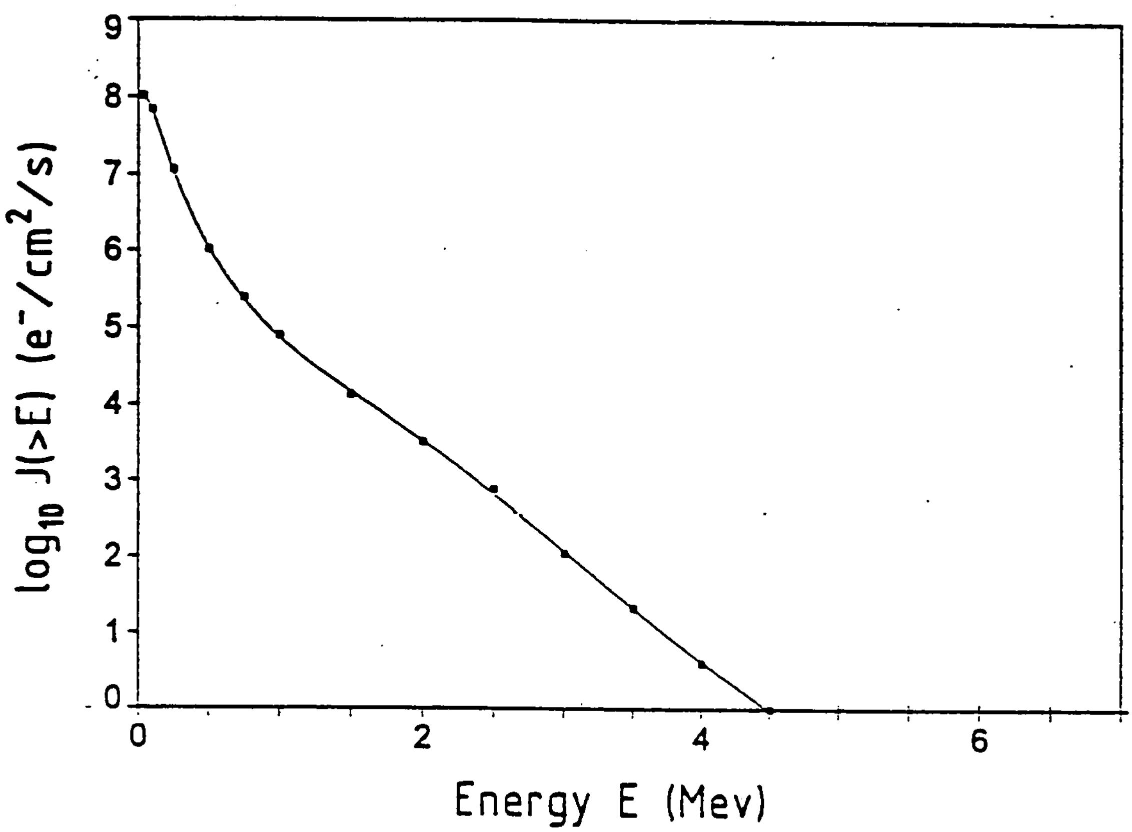


Figure 1: Integral flux $J(> E)$ of relativistic electrons trapped in the Earth's geomagnetic field in the equatorial plane ($B/B_0 = 1$) at $L = 2$. The dots are values corresponding to AE-8 MIN. The solid line is an analytical best fit by the function $F(E)$ given in the text.

$C(L)$, ... could be found for $a(L), b(L)$, ..., then a fully analytical version of AE-8 models could be obtained. But this is not the direct objective of the present study. The aim of the present paper is merely to check whether the AE-8 electrons energy distributions can be approximated by a sum of maxwellian functions for all L-shells between $L = 1.2$ and $L = 10.0$ instead of by physically meaningless fit functions.

3 The relativistic maxwellian distribution

The energy distribution function for relativistic electrons brought to thermal equilibrium by collisions and/or wave-particle interactions is described by a relativistic maxwellian distribution:

$$f^{(M)} = \frac{n(2m_0c^2)^{1/2}\exp(-E/E_0)}{(E_0)^2\alpha\exp(\alpha)K_2(\alpha)} \tag{3}$$

where n is the number density of the electron population, E_0 is the characteristic energy related to the equilibrium temperature T by $E_0 = kT$ with k the Boltzmann constant, $\alpha = m_0c^2/kT$ where $m_0c^2 = 0.511$ MeV is the electron rest energy and $K_2(\alpha)$ is the modified Bessel function of the second kind (Synge, 1957).

The energy distribution function $f(E)$ can be obtained from the measured differential flux $J_d(E)$ through the equation:

$$(2m_0)^{1/2}f(E) = \frac{2m_0J_d(E)}{p^2} \tag{4}$$

where p is the relativistic momentum:

$$p^2c^2 = E(E + 2m_0c^2) \tag{5}$$

The omnidirectional differential flux $J_d(E)$ can be deduced from the integral omnidirectional flux $J(> E)$ by numerical differentiation methods.

Figure 2 shows the logarithm of the distribution function $(2m_0)^{1/2}f(E)$ i.e. $2m_0c^2J_d(E)/[E(E + 2m_0c^2)]$ corresponding to the integral flux spectrum of Figure 1 ($L = 2.0$), obtained by this procedure, for 22 discrete energy values up to 4 MeV. The logarithm of the low energy fluxes up to about 0.7 MeV and the logarithm of the higher energy flux values (1.0 to 4.0 MeV) are well represented by two different straight lines. This fact supports the idea that the trapped electrons velocity distribution can be fitted by a sum of two maxwellian functions:

$$f(E) = f_1^{(M)}(E) + f_2^{(M)}(E) \tag{6}$$

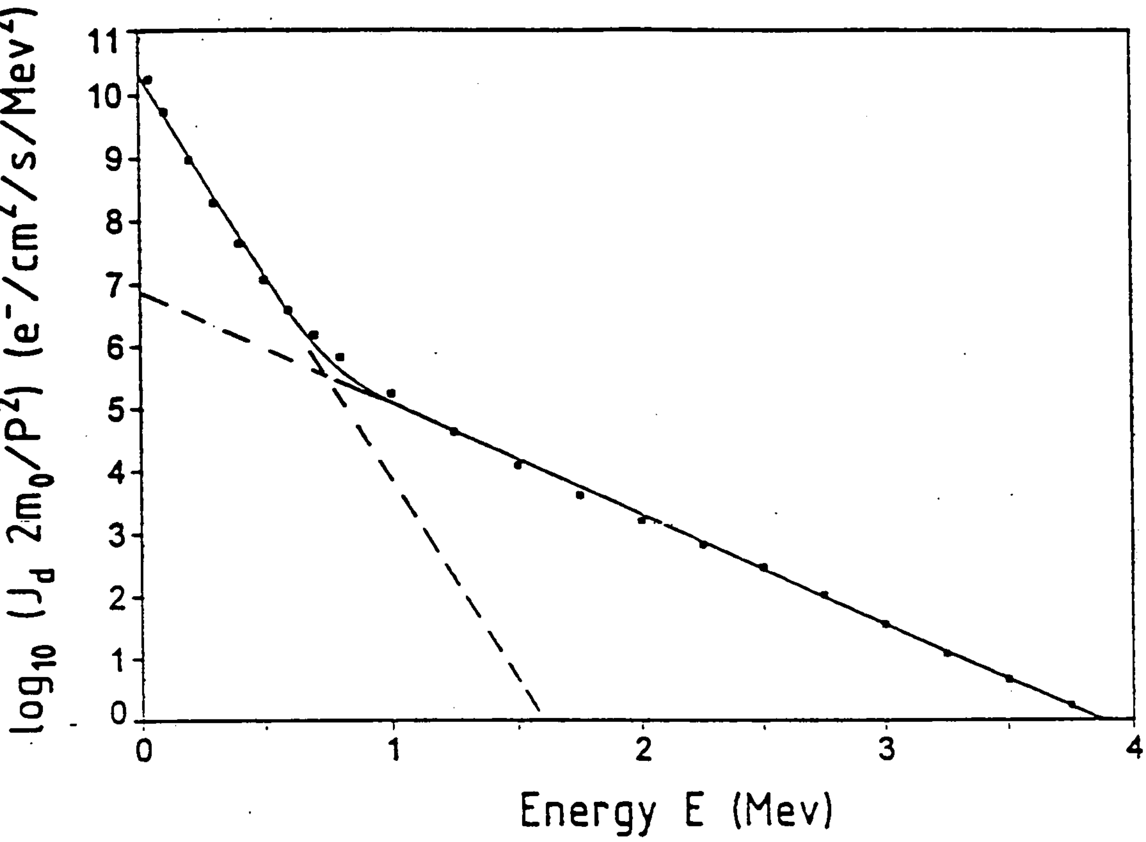


Figure 2: Electron energy distribution function deduced from the AE-8 MIN model for $L = 2.0$ and $B/B_0 = 1.0$ (dots). The solid line is the best fit approximation using a sum of two relativistic maxwellian velocity distributions. The dotted lines correspond to the asymptotes of both maxwellians taken separately.

characterized by two different temperatures T_1 and T_2 or characteristic energies $E_{0,1}$ and $E_{0,2}$ and two distinct densities n_1 and n_2. We have determined simultaneously the values of n_1, n_2, $E_{0,1}$, $E_{0,2}$ by fitting the 22 values of $\log_{10}(2m_0/p^2)J_d$ in the energy range from 40 keV to 4 MeV. Since the statistics of the flux measurements above 5–6 MeV are rather poor and the AE-8 model flux values are extrapolated, we have disregarded these values in our fitting procedure.

The values of the four fitted parameters (n_1, n_2, $E_{0,1}$, $E_{0,2}$) are given in Tables 1 and 2, for AE-8 MIN and AE-8 MAX respectively, for different L-shells. The standard error on these parameters is also given in these Tables. The quality of each fit is given by r^2 which is in all cases very close to 1. It can be seen that the standard deviation is very small. The solid line in Figure 2 corresponds to the best fit obtained with the sum of two relativistic maxwellians at $L = 2.0$. The two asymptotes in this plot correspond respectively to both maxwellians taken separetely.

The values of n_1, n_2, $E_{0,1}$ and $E_{0,2}$ at $L = 6.6$ may be compared to those obtained by Cayton *et al.* (1989) in a similar way from electron fluxes measured throughout the year 1986 aboard several geosynchronous satellites with the Los Alamos charged particle analyzer in the soft and hard energy ranges 30–300 keV and 200– 2000 keV, respectively. According to these observations, the "hard" electrons are characterized by $E_{0,2} = 200$ keV and $n_2 = 1 \times 10^{-4}\,\mathrm{cm}^{-3}$ and the lower energy "soft" electrons by $E_{0,1} = 25$ keV and $n_1 = 5 \times 10^{-3}\,\mathrm{cm}^{-3}$. It can be seen from Tables 1 and 2 that the AE-8 MIN and AE-8 MAX densities at $L = 6.6$ are comparable to those obtained in 1986 around the minimum between solar cycles 21 and 22. However, the values of the characteristic temperatures derived from AE-8 are higher by a factor 3.5 for the soft electrons and a factor 1.5 for the hard electrons than the average values obtained by Cayton *et al.* (1989). Part of the differences may be due to the extreme temporal variability of the soft component at geostationary orbit as well as to the difference between the solar activity levels in 1986 and the period when the data used in AE-8 have been collected. But the difference in temperature is also due to the fact that Cayton *et al.* (1989) fit separately over a small number of energy steps (4 channels in the range 30–140 keV) for the soft electrons and 4 energy channels in the range 400–2000 keV for the hard electrons). These least- squares fits underestimate the slope of the low energy asymptote and overestimate the slope of the high energy portion of the spectra. To illustrate this fact, we fitted two straight-lines to the AE-8 spectrum at $L = 6.6$ for the same energy ranges as Cayton (*et al.*, 1989, see their Figure 4) (40 to 300 keV for the first slope (4 points) and 1.0 to 3.0 MeV for the second one (10 points)). Following this procedure, we find $E_{0,1} = 66$ keV and $E_{0,2} = 297$ keV. On the other hand, with

Table 1: Values of the four parameters of the sum of two maxwellian distributions which best fit the AE-8 MIN energy spectra at different L-shell. The errors are given in parentheses. The coefficient of determination r^2 and the standard error on the fit are also given.

L (R$_e$)	n_1 (e$^-$/cm^3)	$E_{0,1}$ (keV)	n_2 (e$^-$/cm^3)	$E_{0,2}$ (keV)	r^2	Fit st. err.
1.2	4.0×10^{-5} (4)	83 (2)	1.4×10^{-7} (2)	344 (10)	0.99845	0.09978
2	7.0×10^{-3} (9)	68 (2)	2.7×10^{-5} (4)	245 (4)	0.99882	0.10589
3	1.8×10^{-3} (4)	62 (5)	3.0×10^{-5} (6)	409 (19)	0.99145	0.19328
4	1.5×10^{-3} (2)	70 (5)	1.5×10^{-4} (2)	424 (14)	0.99454	0.13267
5	3.0×10^{-3} (3)	97 (6)	3.4×10^{-4} (5)	350 (9)	0.99710	0.10940
6.6	3.0×10^{-3} (3)	90 (5)	6.5×10^{-5} (8)	321 (10)	0.99604	0.15138
8	1.5×10^{-3} (3)	73 (4)	2.9×10^{-6} (4)	308 (13)	0.98910	0.29776
10	8.2×10^{-4} (9)	29 (3)	1.1×10^{-7} (2)	111 (14)	0.98567	0.33500

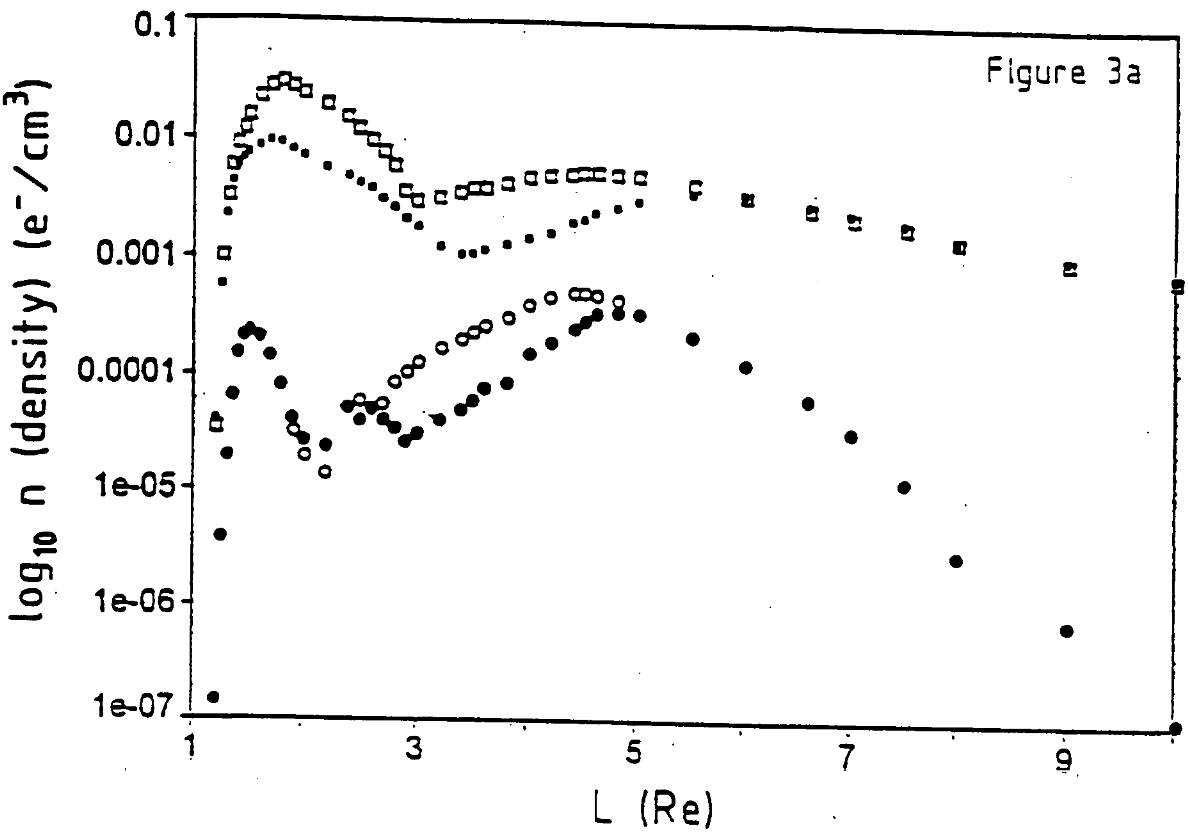
Table 2: Values of the four parameters of the sum of two maxwellian distributions which best fit the AE-8 MAX energy spectra at different L-shell. The errors are given in parentheses. The coefficient of determination r^2 and the standard error on the fit are also shown.

L (R$_e$)	n_1(e$^-$/cm^3)	$E_{0,1}$ (keV)	n_2(e$^-$/cm^3)	$E_{0,2}$ (keV)	r^2	Fit St. err.
1.2	3.8×10^{-5} (4)	87 (5)	1.2×10^{-7} (2)	355 (10)	0.99845	0.09978
2	2.6×10^{-2} (3)	78 (1)	2.0×10^{-5} (4)	259 (4)	0.99882	0.10589
3	3.2×10^{-3} (5)	68 (5)	1.3×10^{-4} (2)	398 (15)	0.99145	0.19328
4	4.9×10^{-3} (5)	76 (5)	3.9×10^{-4} (5)	393 (12)	0.99454	0.13267
5	5.2×10^{-3} (6)	87 (6)	3.7×10^{-4} (5)	345 (9)	0.99710	0.10940
6.6	3.0×10^{-3} (3)	90 (5)	6.5×10^{-5} (8)	321 (10)	0.99604	0.15138
8	1.5×10^{-3} (3)	73 (4)	2.9×10^{-6} (4)	308 (13)	0.98910	0.29776
10	8.2×10^{-4} (9)	29 (3)	1.1×10^{-7} (2)	111 (14)	0.98567	0.33500

our least-squares fitting procedure, taking into account at once all 22 energy steps between 40 keV and 4 MeV, we find $E_{0,1} = 90$ keV and $E_{0,2} = 321$ keV. This way, we are able to find a fit for the whole spectrum up to 4 MeV and not only for two limited spectral regions. However, in spite of the quality of the fit indicated by r^2 being very close to unity, it can be noted in Tables 1 and 2 that the standard error on the energies and principally on these densities are rather large. The standard deviation could be reduced with a larger number of data points.

4 Radial distribution of the two electron populations

Figure 3 shows the equatorial densities (panel a) and characteristic energies (panel b) of the soft electrons (squares) and hard electrons (circles) for the AE-8 MIN (solid symbols) and AE-8 MAX models (open symbols), as a function of L. It can be seen that the density of the soft electrons has two peaks: the highest one $3 \times 10^{-2}\,\mathrm{cm}^{-3}$ at $L = 2$ and a more shallow one ($5 \times 10^{-3}\,\mathrm{cm}^{-3}$) between $L = 4$ and $L = 5$ for AE-8 MAX. The electron density minimum occurs near $L = 3$, sligthly above the position of the slot between the inner and outer electron zones. The position of the slot depends on the energies of the particles. The characteristic energy of the soft electron population varies between 68 keV to 92 keV in the case of AE-8 MAX. It has a minimum at $L = 3$ which coincides with the minimum of density of this population of low energy electrons. This confirms that the drift shell $L = 3$ is on the average a sink for these electrons as well as for their thermal energy. We suggest that this population of electrons is part of the Ring Current and that they are plasmaspheric electrons which have been energized in the inner magnetosphere during storm events by acceleration processes which remains to be understood and constitute one of the key unresolved issue in magnetospheric physics. The density displays a maximum around $L = 2$ which is the location of the source region of the heart of the inner electron belt. The density decrease for $L > 2$ and $L < 2$ leads to the suggestion that part of these soft electrons are diffusing inwards and part of them are diffusing outwards. A large fraction of them is scattered in the atmospheric loss cone near L =3–4 which is the location where the plasmapause is most often observed. During their outward diffusion, the mean energy of the soft electrons decreases as shown in Figure 3a. On the contrary, the electrons diffusing toward lower L-shells increase their thermal energy. The second peak in the temperature and density of the soft electron profile could be due to heating of plasmatrough electrons by wave-particle interactions or some other accel-



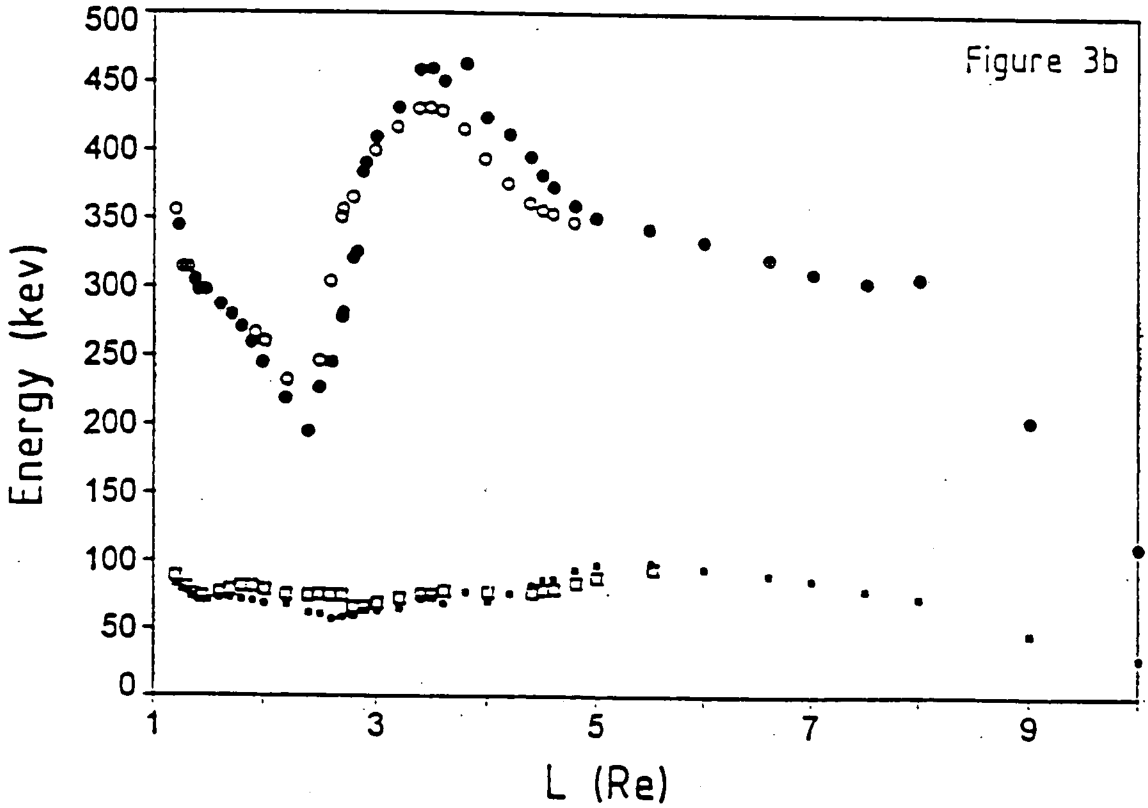


Figure 3: Radial distributions of density (a) and characteristic energy (b), respectively for soft electrons (squares) and hard electrons (circles), as a function of L. The AE-8 MIN and AE-8 MAX models are given respectively by solid and open symbols (circles or squares).

eration mechanism. More detailed and comprehensive observations would be needed to check these suggestions. From the observations at $L = 6.6$ by Cayton *et al.* (1989) it is clear that n_1 and $E_{0,1}$ are much more subject to fluctuations and temporal variations than the "hard" electron population (n_2, $E_{0,2}$). The recent CRRES data that we have also analyzed, lead to the same conclusion (Vampola *et al.*, 1992). The soft electron density and temperature undergo, at geostationary orbit, strong time variations during magnetospheric disturbances (i.e. periods of large K_p). The largest excursions of n_1 and $E_{0,1}$ were found to be due to large injections associated with geomagnetic storms and substorms (Baker *et al.*, 1979). These injections increase n_1 by a factor of 100 or more during a few tens of minutes to a few hours.

For the hard component, the electron density minimum occurs around $L = 2$. For the soft one, the minimum occured at $L = 3$. The dependent position of the slot on energy was already illustrated by Vette (1991, see his figure 40). The radial density distribution of the hard electron component of AE-8 MAX has a peak value of $5 \times 10^{-4}\,\mathrm{cm}^{-3}$ between $L = 4$ and $L = 5$. This may be the source region for hard electrons whose mean energy is 350 keV at this radial distance. At larger L, their temperature decreases. Conversely, for decreasing L, the energy $E_{0,2}$ increases to a maximum of 425 keV at $L = 3.5$ and then drops to 250 keV between $L = 3$ and $L = 2$. The reason for this temperature drop in the vicinity of the plasmapause is not known but must be explained in future physical models of the trapped radiation belts. Cayton *et al.* (1989) showed that the temperature of the hard electrons exhibits little variability compared with the other parameters.

5 Conclusions

The present work demonstrates that the spectra of the AE-8 model can be fitted with a sum of two relativistic maxwellian functions with very good accuracy. This fit displays two populations of electrons: a "soft" component with a mean energy $E_1 = 60\,\mathrm{keV}$ and a mean density $n_2 = 5 \times 10^{-3}\,\mathrm{cm}^{-3}$ and a "hard" component with $E_2 = 340\,\mathrm{keV}$ and $n_2 = 1 \times 10^{-4}\,\mathrm{cm}^{-3}$. The density of the soft population shows large temporal variability depending of the solar activity. Our least-squares fit takes into account the complete energy range of the spectra up to 4 MeV. The variations of densities and energies as a function of the radial distance L show clearly the difference between inner and outer zone. The analysis of these variations helps understanding the source and loss processes in the radiation belts.

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References

- Baker D.N., Stauning P., Hones E.W. Jr., Higbie P.R. and Belian R.D. (1979a) Strong electron pitch angle diffusion observed at geostationary orbit. *Geophys. Res. Letters* **6**, 205–209.
- Baker D.N., Higbie P.R., Belian R.D. and Hones E.W. Jr. (1979b) Do Jovian electrons influence the terrestrial outer radiation zone? *Geophys. Res. Letters* **6**, 531–536.
- Bilitza D. (1990) Solar terrestrial models and applications software. NSSDC / WDC-A-R&S 90-19.
- Cayton T.E., Belian R.D., Gary S.P., Fritz T.A., and Baker D.N. (1989) Energetic electron components at geosynchronous orbit. *Geophys. Res. Letters* **16**, 147–150.
- Christon S.P., Mitchell D.G., Williams D.J., Frank L.A., Huang C.Y. and Eastman T.E. (1988) Energy Spectra of Plasma Sheet Ions and Electrons From ~ 50 eV/e to ~ 1 MeV During Plasma Temperature Transitions. *J. Geophys. Res.* **93**, 2562–2572.
- McIlwain C.E. and Whipple E.C. (1986) The dynamic behavior of plasmas observed near geosynchronous orbit. *IEEE Trans. Plasma Sci.* **PS14**, 874–879.
- Parks G.K., Mauk B., Gurgiolo C. and Lin C.S. (1979) Observations of plasma injection. In: *Dynamics of the Magnetosphere* (ed. Akasofu S.-I.), pp. 371–383. D. Reidel Publishing Company, Boston.
- Paulikas G.A. and Blake J.B. (1979) Effects of the solar wind on magnetospheric dynamics: energetic electrons at the synchronous orbit. In:

Quantitative Modeling of Magnetospheric Processes, Geophys. Monograph Series **21** (ed. Olson W.P.), pp. 180–191. AGU, Washington DC.

- Singley G.W. and Vette J.I. (1972a) The AE-4 Model of the Outer Radiation Zone Electron Environment. NSSDC 72-06.
- Singley G.W. and Vette J.I. (1972b) A Model Environment for Outer Zone Electrons. NSSDC 72-13.
- Synge J.L. (1957) The relativistic gas. North-Holland Publishing Company, Amsterdam.
- Teague M.J., Chan K.W. and Vette J.I. (1976) AE-6: A Model Environment for Trapped Electrons for Solar Maximum. NSSDC / WDC-A-R&S 76-04.
- Vampola A.L., Osborn J.V. and Johnson B.M. (1992), CRRES Magnetic Electron Spectrometer AFGL-701-5A (MEA), *Journal of Spacecraft and Rockets, 29*, 4, 555–556.
- Vette J.I. (1991) The AE-8 Trapped Electron Model Environment. NSSDC / WDC-A-R&S 91-24.