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Comparison between methods to compensate for the secular motion of the south atlantic anomaly

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Foreword

This paper has been accepted for publication in *Nuclear Tracks Radiat. Meas.*

Voorwoord

Dit artikel is aanvaard voor publicatie in *Nuclear Tracks Radiat. Meas.*

Avant Propos

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Vorwort

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Comparison Between Methods to Compensate for the Secular Motion of the South Atlantic Anomaly

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Abstract. In order to avoid the prediction of artificially high particle fluxes, the static NASA trapped radiation models AP-8 and AE-8 should only be used with the same geomagnetic field models, for the same epochs, with which they were constructed. It is possible to correct for one aspect of the secular variation of the geomagnetic field, i.e. the secular drift of the local particle flux maximum in the South Atlantic Anomaly (SAA). The secular motion of the SAA may be compensated for by a coordinate transformation on the satellite trajectory corresponding to the inverse of the secular change of the geographic coordinates of the SAA. Three such transformations, defined by different approximations to the location of the centre of the SAA, are presented. The secular motion of the SAA is best represented by a longitudinal westward drift at a rate of 0.3°/year, in accordance with the findings of other authors. For a typical Shuttle orbit in the vicinity of the SAA, flux increases of a factor ten were found when the longitudinal correction is applied. In addition, the time of the encounter with the SAA may differ by as much as one orbital period from the time predicted without the correction.

Samenvatting. De statische NASA stralingsfluxmodellen AP-8 en AE-8 worden best alleen gebruikt met dezelfde geomagnetische veldmodellen, met dezelfde epoch, als waarmee ze opgebouwd werden, om te vermijden dat de berekende fluxen kunstmatig hoog worden. Het is mogelijk te corrigeren voor één aspect van de seculiere variatie van het geomagnetisch veld, met name de seculiere verschuiving van het locale flux maximum in de "South Atlantic Anomaly" (SAA). De seculiere beweging van de SAA kan gecompenseerd worden door een coördinatentransformatie op de satellietbaan die overeenkomt met het omgekeerde van de seculiere variatie van de geografische coördinaten van de SAA. We beschrijven drie zulke transformaties, die gedefinieerd worden door verschillende benaderingen van de positie van het midden van de SAA. De beste voorstelling van de seculiere beweging van de SAA is een longitudinale westwaardse verschuiving aan een tempo van 0.3° per jaar, wat overeenstemt met de bevindingen van andere auteurs. Voor een typische Shuttle baan in de omgeving van de SAA kunnen de stralingsfluxen met een factor 10 toenemen wanneer de longitudinale correctie wordt toegepast. Daarenboven kan het tijdstip van het doorkruisen van de SAA tot een omloopsperiode verschillen in vergelijking met het tijdstip wanneer de correctie niet wordt toegepast.

Résumé. En vue d'éviter une surévaluation du flux des particules énergétiques à partir des modèles AP-8 et AE-8 de la NASA, il a été démontré que l'on doit utiliser les mêmes modèles géomagnétiques pour le calcul des coordonnées B, L que celui qui a été utilisé pour créer les modèles de flux de particules eux-mêmes. Il est cependant possible de corriger partiellement de la variation séculaire du champ géomagnétique, et notamment de la dérive séculaire de l'anomalie magnétique de l'Atlantique sud (SAA), en effectuant une rotation du système de coordonnées géographiques de manière à compenser la dérive séculaire de la SAA vers l'ouest. Celle-ci est égale à 0.3° par année soit 6° en vingt ans. Pour une orbite typique de la navette spatiale traversant la SAA, nous montrons que les flux de particules calculés avec les anciens modèles de la NASA, peuvent varier d'un facteur 10 suivant que l'on applique ou non cette rotation de coordonnées qui dépend de la différence d'époque entre la date du vol de la navette spatiale et celle de la création des modèles AP-8 et AE-8. En plus, le temps de rencontre avec la SAA peut différer par autant qu'une période orbitale du temps prédit sans appliquer la correction longitudinale.

Zusammenfassung. Um eine Überbewertung der Strömung von energetischen Teilchen aufgrund der NASA-Modelle AP-8 und AE-8 zu vermeiden, sollten erwiesenermassen für die Berechnung der Koordinaten B, L die gleichen magnetischen Modelle zum Einsatz kommen, wie bei der Schaffung der Modelle von Teilchenströmen. Es ist jedoch möglich, die Sekulärvariation des erdmagnetischen Feldes und insbesondere den Sekulärabtrieb der südatlantischen magnetischen Anomalie (SAA) zu berichtigen, indem der Sekulärabtrieb der SAA nach Westen durch eine Drehung des geographischen Koordinatensystems ausgeglichen wird. Dieser Abtrieb erreicht 0,3° pro Jahr oder 6° innerhalb von zwanzig Jahren. Für eine typische Umlaufbahn der durch die SAA fliegenden Raumfähre zeigen wir, dass die mit den alten NASA-Modellen berechneten Teilchenströme bis zu einem Zehnerfaktor schwanken können, je nachdem diese Koordinatendrehung erfolgt oder nicht und zwar in Abhängigkeit der unterschiedlichen zwischen dem Termin der Raumfahrt und dem der Schaffung der AP-8 und AE-8 Modelle vergangenen Zeitspanne. Ausserdem kann der Treffzeitpunkt mit der SAA einen Unterschied aufweisen, der, ohne Anwendung der Längskorrektur, eine gesamte vorgegebene orbitale Zeit ausmacht.

1 Introduction

The main portion of the radiation flux encountered by a satellite in a Low-Earth Orbit (LEO) is concentrated in a region called the South Atlantic Anomaly (SAA). The existence of the SAA is closely linked to the geomagnetic field distribution. Hence, its geographic location drifts over the years with the surface features of the geomagnetic field (Merrill and McElhinney 1983).

The NASA trapped radiation models AP-8 and AE-8, dating from the late 1960's and early 1970's, still are the most commonly used tools for describing the Earth's trapped radiation environment (Vette 1991). A major disadvantage of using these older models to predict present day particle populations is that the data constituting the models were ordered by means of magnetic coordinates based on the geomagnetic field model used at that time. In particular, the location of the SAA predicted by the old magnetic field models does not coincide with the current location of the SAA. Consequently, the fluxes predicted for a low-altitude spacecraft mission crossing the SAA differ from the actual fluxes in intensity and time of occurrence. These effects have been observed by recent Shuttle missions (Konradi et al. 1992).

In Sect. 2, we describe a method to compensate for the secular motion of the SAA between the epochs of the NASA models and the present. Once the positions of the centre of the SAA for both epochs are known, a suitable coordinate transformation will position the satellite trajectory correctly with respect to the location of the SAA corresponding to the epoch of the NASA model used. We present three such transformations, defined by different approximations to the location of the centre of the SAA. The three transformations are applied to a low-Earth orbit in Sect. 3, and the results are compared.

2 Determination of the location of the SAA

According to the different definitions of the SAA, one can approximate the location of the centre of the SAA in three ways:

1. as the mirror point of the geographic location of the centre of the eccentric dipole approximation to the geomagnetic field;

2. as the locus of the local minimum of the geomagnetic field at a fixed altitude in the region of the South Atlantic;

3. as the locus of the local particle flux maximum in this region.

2.1 The eccentric dipole approximation

It is customary to express the geomagnetic field as the gradient of a scalar potential V, which is usually expressed as an orthogonal expansion in spherical harmonics. In

spherical geographic coordinates r, θ, ϕ, the expansion of V is written as

$$V(r,\theta,\phi) = R_{\mathrm{E}} \sum_{n=1}^{N} \sum_{m=0}^{m=n} \left(\frac{R_{\mathrm{E}}}{r}\right)^{n+1} \left(g_n^m \cos m\phi + h_n^m \sin m\phi\right) P_n^m(\theta)\,, \tag{1}$$

where R_{E} is the mean radius of the Earth (6371.2 km), and $P_n^m(\theta)$ are the Schmidt-normalized associated Legendre functions.

The main contribution in Expansion (1) comes from the terms with $n = 1$ and can be identified with the field produced by a dipole with centre coinciding with the centre of the Earth and dipole axis inclined with respect to the rotation axis. The direction defined by the Earth-centred dipole is called the geomagnetic axis. The sum of terms with $n = 1$ in Expansion (1) reduces to one term in the spherical coordinate system with centre at the Earth's centre and polar axis coinciding with the geomagnetic axis. This new coordinate system r_1, θ_1, ϕ_1 is called the system of geomagnetic coordinates.

The coefficients of the expansion of V in geomagnetic coordinates can be related to the coefficients g_n^m and h_n^m by transforming the spherical harmonics in Expansion (1) with Wigner's formula (Bernard et al. 1969). In geomagnetic coordinates, Expansion (1) then takes the form

$$\begin{aligned} V(r_1,\theta_1,\phi_1) &= -R_{\mathrm{E}} \left(\frac{R_{\mathrm{E}}}{r_1}\right)^2 B_0 \cos\theta_1 \\ &+ R_{\mathrm{E}} \sum_{n=2}^{N} \sum_{m=0}^{m=n} \left(\frac{R_{\mathrm{E}}}{r_1}\right)^{n+1} \left(g_n'^{\,m} \cos m\phi_1 + h_n'^{\,m} \sin m\phi_1\right) P_n^m(\theta_1)\,, \end{aligned} \tag{2}$$

where

$$B_0 = \sqrt{(g_1^0)^2 + (g_1^1)^2 + (h_1^1)^2}\,. \tag{3}$$

The geomagnetic field is often approximated by the first term in Expansion (2). This approximation is called the *centred dipole model.* Its parameters are completely specified by the coefficients g_1^0, g_1^1, and h_1^1.

Expansion (2) can be further simplified by a translation of the centre of the coordinate system over $\Delta x, \Delta y, \Delta z$ away from the centre of the Earth. In the new coordinate system r_2, θ_2, ϕ_2, Expansion (2) is reduced to

$$\begin{aligned} V(r_2,\theta_2,\phi_2) &= -R_{\mathrm{E}} \left(\frac{R_{\mathrm{E}}}{r_2}\right)^2 B_0 \cos\theta_2 \\ &+ R_{\mathrm{E}} \left(\frac{R_{\mathrm{E}}}{r_2}\right)^3 \frac{\sqrt{3}}{2} \left(g_2'^{\,2} \cos 2\phi_2 + h_2'^{\,2} \sin 2\phi_2\right) \sin^2\theta_2 \\ &+ \mathcal{O}\left(r_2^{-4}\right)\,. \end{aligned} \tag{4}$$

This expansion can be interpreted as a centred dipole in the coordinate system that has been tilted and shifted with respect to the geographic coordinate system plus perturbation terms. Note that B_0 and the coefficients $g_2'^{\,2}$ and $h_2'^{\,2}$ have not been affected by the displacement of the origin of the coordinate system. The approximation of the geomagnetic field by the first term in Expansion (4) is called the *eccentric*

Table 1. Geographic radial distance, latitude and longitude of the three approximations to the location of the centre of the SAA, obtained with the DGRF/IGRF geomagnetic field models. r Is expressed in km, λ and ϕ are given in degrees. The last line gives the coordinates for the Jensen and Cain (1962) field model.

	Eccentric dipole			Field minimum		Flux maximum	
Epoch	r_s	λ_s	ϕ_s	λ_b	ϕ_b	λ_f	ϕ_f
1945	407	−13.12	333.7	−23.7	315.8	−39.9	340.5
1950	419	−13.99	332.0	−23.5	315.8	−39.6	339.3
1955	430	−14.91	330.7	−23.5	315.5	−36.9	335.8
1960	442	−16.09	329.6	−23.7	313.3	−37.5	333.3
1965	452	−17.20	328.7	−23.8	312.3	−37.6	332.5
1970	463	−18.44	328.2	−23.8	311.3	−37.4	331.0
1975	474	−19.69	328.0	−23.7	310.2	−37.0	330.1
1980	489	−20.39	327.3	−23.8	309.4	−37.0	328.1
1985	502	−20.85	326.6	−23.8	308.6	−36.7	326.7
1990	515	−21.16	325.7	−23.8	307.9	−36.8	324.8
1995	527	−21.49	324.8	−23.8	307.1	−36.0	324.7
2000	540	−21.80	323.9	−23.8	306.3	−35.9	323.1
1960	438	−17.05	329.8	−24.3	313.6	−38.1	331.7

dipole model. The displacement vector $\Delta x, \Delta y, \Delta z$ is determined by the coefficients of degrees 1 and 2 in Expansion (1).

The Cartesian geographic coordinates η, ζ, ξ of the eccentric dipole centre are found by transforming the vector $\Delta x, \Delta y, \Delta z$ from geomagnetic to geographic coordinates. Fraser-Smith (1987) gives expressions for η, ζ, ξ in function of the coefficients of degrees 1 and 2 in Expansion (1).

The displacement vector $\boldsymbol{S} = \eta, \zeta, \xi$ can be regarded as a first approximation to the displacement of particle drift shells. The angular components of $-\boldsymbol{S}$ then are an approximation of the angular coordinates of the centre of the SAA. We have calculated the geographic coordinates r_s,λ_s,ϕ_s ($\lambda_s = 90° - \theta_s$) of $-\boldsymbol{S}$ with the DGRF and IGRF models for epochs between 1945 and 2000, with 5 year intervals, and for the Jensen and Cain (1962) model with epoch 1960. The results are listed in Table 1 and represented graphically in Fig. 1. From Fig. 1, it appears that the secular motion of the mirror point of the eccentric dipole centre has been predominantly southward between 1960 and 1980, while from 1980 on it has been mostly westward. The westward displacement corresponds to the empirical secular motion of the SAA reported by Konradi et al. (1992) and other authors. However, as far as we know, a southward motion of the SAA has never been detected.

We now describe the procedure to correct a satellite's trajectory in order to compensate for the secular motion of the SAA. Let $\boldsymbol{S}_1 = \eta_1, \zeta_1, \xi_1$ and $\boldsymbol{S}_2 = \eta_2, \zeta_2, \xi_2$

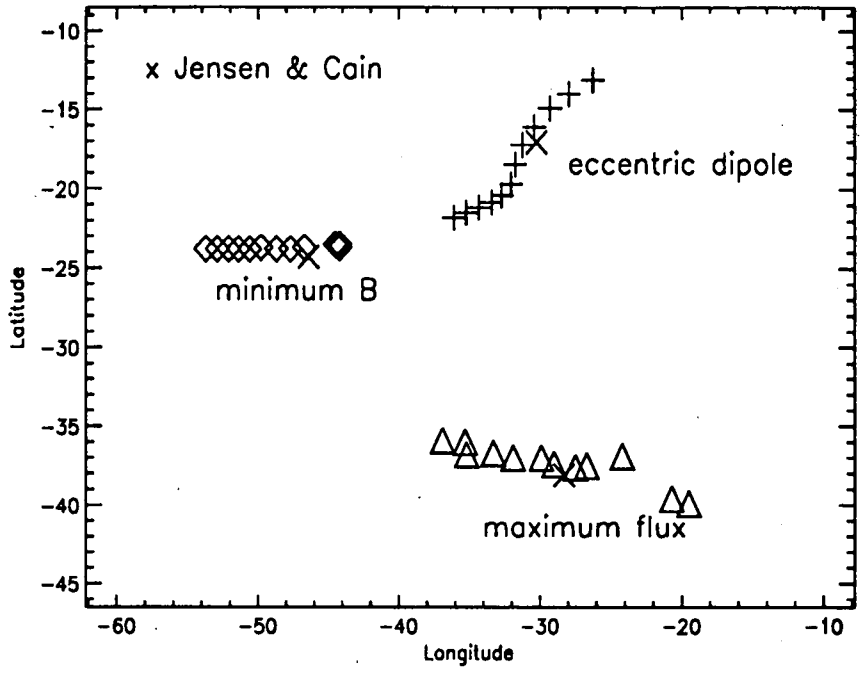

Figure 1. Geographic locations of the three approximations to the centre of the SAA discussed in the text, in function of epoch (increasing to the left).

denote the normalised position vectors of the eccentric dipole centre for the reference epoch T_1 and for epoch T_2. We define a new Cartesian coordinate system X, Y, Z with origin at the centre of the Earth. The Y-axis coincides with $\boldsymbol{S}_1$. The Z-axis is perpendicular to the plane defined by $\boldsymbol{S}_1$ and $\boldsymbol{S}_2$. The X-axis completes a right-handed orthogonal coordinate system. The coordinate system is then described by the unit vectors

$$\left.\begin{aligned} \boldsymbol{X} &= \boldsymbol{Y} \times \boldsymbol{Z}\,, \\ \boldsymbol{Y} &= \boldsymbol{S}_1\,, \\ \boldsymbol{Z} &= \frac{\boldsymbol{S}_2 \times \boldsymbol{S}_1}{|\sin\alpha|}\,, \end{aligned}\right\} \tag{5}$$

where α is the angle between $\boldsymbol{S}_1$ and $\boldsymbol{S}_2$. Let T represent the coordinate transformation from Cartesian geographic coordinates to the coordinate system X, Y, Z. The array elements of T are easily obtained in terms of the components of $\boldsymbol{S}_1$ and $\boldsymbol{S}_2$.

We further define the transformation R as a rotation in the X, Y plane, which contains both $\boldsymbol{S}_1$ and $\boldsymbol{S}_2$, over the angle α, so that $RT\,\boldsymbol{S}_2 = T\,\boldsymbol{S}_1$. The combined transformation $T^{-1}RT$ then lets $\boldsymbol{S}_2$ coincide with the non-transformed $\boldsymbol{S}_1$: $T^{-1}RT\,\boldsymbol{S}_2 = \eta_1, \zeta_1, \xi_1$. Therefore, we can compensate for the secular motion of the SAA by applying the transformation $T^{-1}RT$ to the position vector of a present-day satellite location in Cartesian geographic coordinates.

We investigated the feasibility of applying an additional coordinate transformation to account for the secular variation of the inclination of the dipole axis. It appears that the Z-component of the dipole axis changes very little in time. Consequently, the secular drift of the dipole axis takes place predominantly in the X, Y-plane and

it is not possible to align the position vectors of the SAA and the dipole axes at the same time.

2.2 The geomagnetic field minimum

The location of the centre of the SAA may also be approximated as the locus of the local geomagnetic field minimum in the region of the South Atlantic.

The determination of the minima of Expansion (1) is not straightforward. Therefore, we calculated B by means of Expansion (1) for a number of points in the South Atlantic at altitude 500 km. First, we defined a grid delimited in longitude by the interval $[-70°, 0°]$ and in latitude by $[-50°, -10°]$. The grid points are spaced by 0.5° in both longitude and latitude. We then defined a finer grid with a spacing of 0.1° around the point in the first grid with the smallest value of B, calculated B for the new grid points, and determined the location where B is smallest. This procedure was repeated for the field models listed in Table 1. The resulting values for the geographic latitude λ_b and longitude ϕ_b of the location of the local minimum of the geomagnetic field are given in Table 1 and represented graphically in Fig. 1.

2.3 The particle flux maximum

The third approximation to the location of the centre of the SAA is the locus of the maximum of the particle flux distribution in the region of the South Atlantic.

We calculated the integral proton flux above 50 MeV with the AP-8 MAX model for the points making up the coarser grid described in Sect. 2.2, and repeated this procedure for the finer grid around the point where the flux is highest. The values of the geographic latitude λ_f and longitude ϕ_f of the local flux maximum in the finer grid are given in Table 1 and represented in Fig. 1.

2.4 Evaluation of the three methods

From Fig. 1, it appears that the respective locations of the centre of the SAA obtained with the three methods differ substantially for a given epoch.

The location and extent of the particle SAA is dependent on the energy of the particles. The higher the energy, the closer the flux maximum lies to the point where the minimum of the geomagnetic field is situated. The locus of the flux maximum also depends on the model of the trapped radiation environment.

The mirror point of the eccentric dipole position is only a first-order approximation to the location of the SAA. According to Roederer (1972), higher-order terms in Expansion (1) influence the location and extent of the SAA.

The determination of the location of the magnetic field minimum does not depend on particle energy, nor on approximations introduced by truncating Expansion (1). Even though this point does not coincide with the locus of the maximum particle flux, even at high energies, the secular motion of the locus of the flux maximum should be linked to the secular motion of the locus of the field minimum.

From Fig. 1, it appears that the secular motion of the magnetic field minimum has no transversal component. The particle flux maximum seems to move slightly northward as well as westward. However, the loci corresponding to epochs before 1960 were derived with geomagnetic field models that were constructed without satellite measurements. Therefore, the three corresponding loci cannot be very accurately determined. If we consider only epochs later than 1960, the secular motion of the particle flux maximum is mainly longitudinal. Consequently, we assume that the secular motion of the SAA in time is westward only.

The rate of the westward drift of the magnetic field minimum and the particle flux maximum can then be estimated with a linear regression fit to the longitudes in Table 1, excluding the values for the Jensen and Cain (1962) field model and the DGRF values with epochs earlier than 1960. Epochs later than 1990 are also excluded since the associated results are based on extrapolations of the IGRF 90 model. The rate of the westward drift resulting from the regression analysis is 0.18°/year for the magnetic field minimum and 0.29°/year for the particle flux maximum, respectively. Konradi et al. (1992) have estimated the westward drift of the SAA by comparing Space Shuttle dose measurements with dose calculations based on AP-8 MAX. They report a yearly drift of 0.34°/year, which is in agreement with the value we found.

The difference between the drift rate of the magnetic field minimum and the particle flux maximum may be ascribed to the fact that the secular change in the geomagnetic field is more complex than just a westward drift. Merrill and McElhinney (1983) describe the separation of the Earth's non-dipole field into drifting and standing parts. The standing and drifting parts of the none-dipole field are of approximately the same size and intensity. The drifting field consists mainly of low harmonics ($l \leq 3$) whereas the standing field has a more complicated distribution. The drifting part of the non-dipole field moves westward at a rate slightly greater than 0.3°/year. The inclusion of the standing part of the non-dipole field in the drift rate lowers the average rate to near 0.2°/year, which corresponds to the value we found for the westward drift rate of the magnetic field minimum.

In order to correct a satellite trajectory for the westward drift of the SAA, we apply an eastward shift $\delta\phi$ to the geographic satellite longitude, so that the corrected longitude becomes $\phi + \delta\phi$. $\delta\phi$ is a function of the time t for which the satellite trajectory is calculated and is defined as

$$\delta\phi(t) = 0.3^\circ\,(t - \mathrm{EPOCH})\,. \tag{6}$$

EPOCH is the reference epoch associated with the magnetic field models used to construct AP-8 and AE-8. The epochs for the Solar minimum and maximum models are 1960 and 1970, respectively (Heynderickx et al. 1993). The coordinate transformation compensating for the westward drift has been included in our implementation of the NASA models.

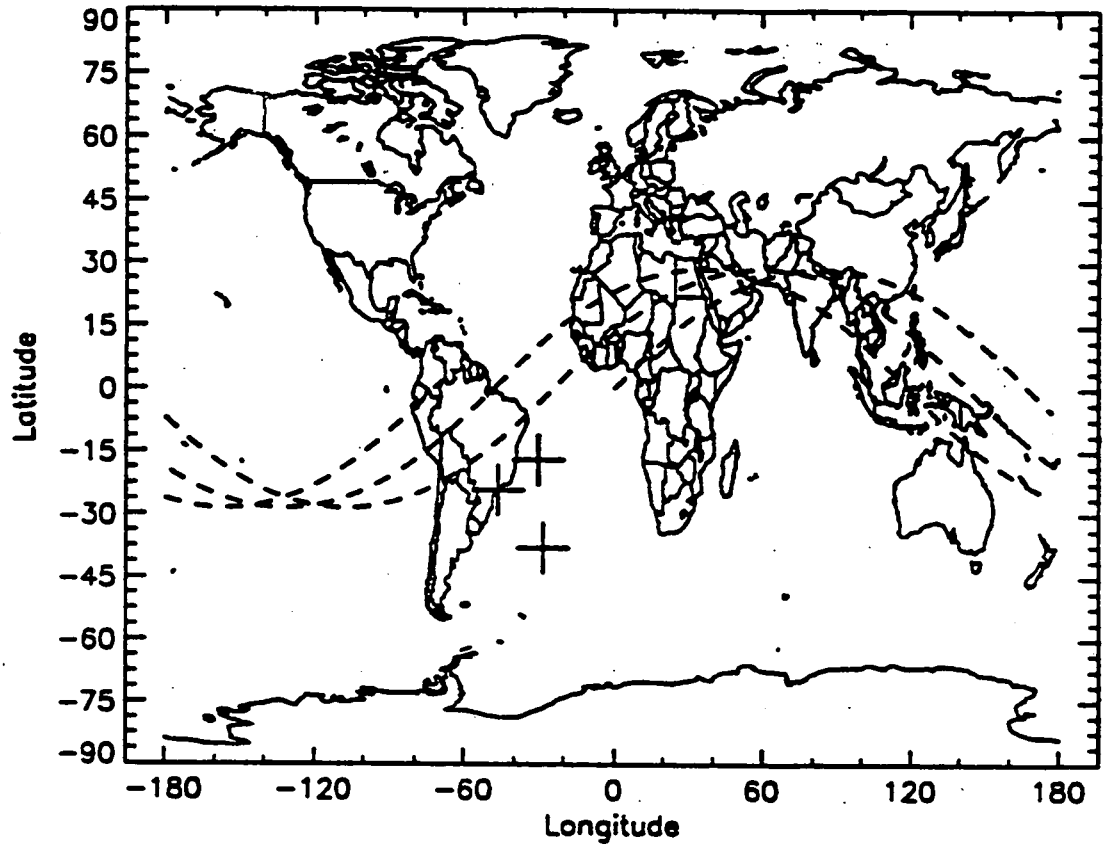

Figure 2. Representation of the LEO satellite trajectory used for the flux calculations described in the text. The loci of the three approximations to the centre of the SAA, obtained for the Jensen and Cain (1962) magnetic field model, are represented by the symbols +.

3 Application to a LEO trajectory

We illustrate the influence of the westward drift of the SAA on a typical LEO trajectory represented in Fig. 2. The orbit altitude is 350 km, the inclination is 28.5°. We considered only three orbits, in order not to average out the longitude effects.

For this trajectory, we calculated the integral proton fluxes above 5, 20, and 100 MeV with AP-8 MIN and the Jensen and Cain (1962) geomagnetic field model, applying the longitude correction in Eq. (6) for the epochs in Table 1. The resulting flux values are represented in Fig. 3. It is seen that the proton flux strongly increases with the epoch.

In addition, when the AP-8 model is used to map particle flux distributions without applying the longitude correction, the encounter with the SAA predicted for 1990 occurs about one orbit later than is actually measured. This time difference may be crucial for the planning of EVA's for vehicles like the Space Shuttle.

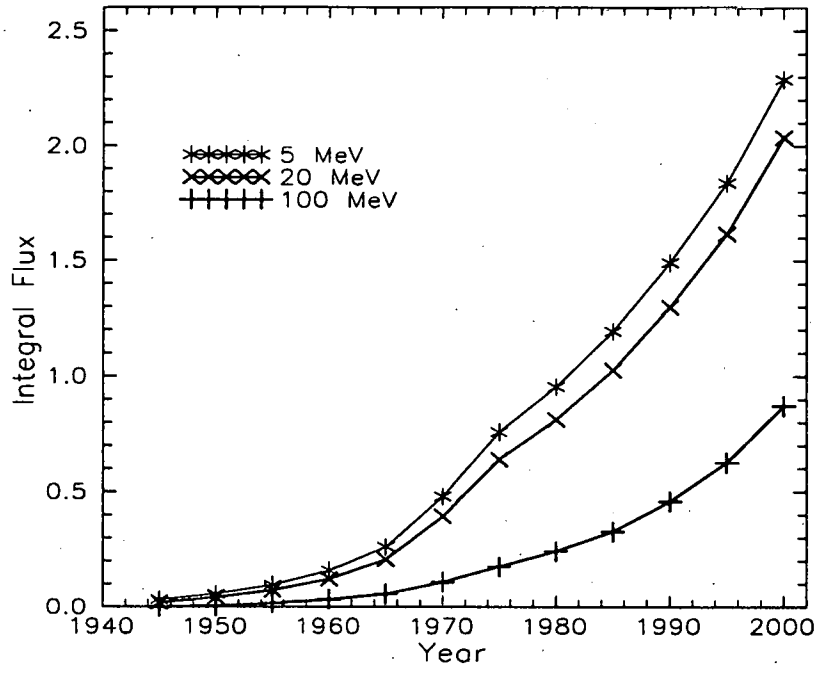

Figure 3. Integral proton fluxes above 5, 20, and 100 MeV, averaged over the trajectory represented in Fig. 2, and corrected for the westward drift of the SAA with the epochs on the abscissa.

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