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New model of magnetospheric current-voltage relationship

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FOREWORD

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AVANT-PROPOS

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VOORWOORD

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VORWORT

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Abstract

A new model based on the generalized Lorentzian velocity distribution function (VDF) is used to estimate the total current density, J_{tot}, along the magnetic field lines as a function of the field-aligned electric potential difference, V, between the ionosphere and the magnetosphere. As in the earlier kinetic models based on the Maxwellian VDF, J_{tot} is a non-linear function of V, except for values of V between 0.1 and 10 kVolts where the current-voltage relationship becomes linear. The effect of an enhanced population of suprathermal particles in the tail of the VDF is to increase the value of the applied potential for which the linear relationship is a valid approximation. The application of this current-voltage relationship to the auroral precipitation region and return current region is discussed.

Résumé

Un nouveau modèle basé sur la fonction de distribution des vitesses (FDV) Lorentzienne généralisée est utilisée pour estimer la densité de courant totale, J_{tot}, le long des lignes du champ magnétique en fonction de la différence de potentiel électrique parallèle au champ, V, existant entre l'ionosphère et la magnétosphère. Comme dans les précédents modèles basés sur la FDV Maxwellienne, J_{tot} est une fonction non-linéaire de V, excepté pour les valeurs de V comprises entre 0.1 et 10 kVolts où la relation courant-voltage devient linéaire. Une population plus importante de particules suprathermiques dans les ailes de la FDV a pour effet d'augmenter la valeur du potentiel au-delà de laquelle la relation linéaire est une bonne approximation. L'application de cette relation courant-voltage à la région des précipitations aurorales et à la région de "courant retour" est discutée.

Samenvatting

De totale stroomdichtheid J_{tot} over magnetische veldlijnen wordt bepaald met een nieuw model gebaseerd op de veralgemeende Lorentziaanse snelheidsverdelingsfunctie (SVF), in functie van het veld-gealigneerde potentiaalverschil V tussen de ionosfeer en de magnetosfeer. Zoals in oudere kinetische modellen gebaseerd op de Maxwelliaanse SVF, is J_{tot} een niet-lineaire functie van V, behalve voor waarden van V tussen 0.1 en 10 kVolt waar de stroom-voltage relatie lineair wordt. Het effect van een toegenomen populatie van suprathermische deeltjes in de staart van de SVF is een toename van de waarde van de potentiaal waarvoor de lineaire relatie een geldige benadering is. De toepassing van deze stroom-voltage relatie op de aurorale precipitatiezone en op de zone van "return current" wordt besproken.

Zusammenfassung

Aufgrund eines neuen, auf der verallgemeinerten Lorentzschen Geschwindigkeitsverteilungs-Funktion (GVF) basierenden Modells soll die gesamte Flussdichte J_{tot} entlang der magnetischen Feldlinien anhand des zwischen der Ionosphäre und der Magnetosphäre bestehenden Unterschiedes in dem zum Feld parallelen elektrischen Potential V geschätzt werden. Wie in der vorhergehenden, auf der Maxwellschen GVF basierenden Modellen ist J_{tot} eine nicht-lineare Funktion von V, mit Ausnahme der zwischen 0, 1 und 10 kVolt liegenden V-Werte, bei denen das Strom-Spannung-Verhältnis linear wird. Die Wirkund einer vermehrten Population suprathermischer Teilchen an den Rändern der GVF erhöht den Potentialwert oberhalb dessen die linearbeziehung eine gute Annäherung bildet. Die Anwendung dieses Strom-Spannung-Verhältnisses auf den Bereich der Polarniederschläge und des "Rückstromes" wird erörtert.

1 Introduction

In the auroral regions, kilovolt magnetospheric particles are injected into the Earth's ionosphere, due to the field-aligned potential difference created by electrostatic interactions between the relatively cold ionospheric plasma and the hot plasma sheet particles [*Evans*, 1974; *Chiu*, 1981]. *Knight* [1973] has been first to determine a current-voltage relationship between the field-aligned potential V and the field-aligned current J_{tot}. The expressions used by Knight are similar to those published by *Lemaire and Scherer* [1970, 1971, 1973] in their kinetic models of the polar wind and of plasma sheet particle precipitation. According to these kinetic models, cold electrons and ions evaporate out of the topside ionosphere into the collisionless ion-exosphere, while hot plasma sheet electrons and ions spiral down the magnetic field lines and precipitate into the atmosphere. The partial currents, J_i, contributed by the escaping cold ionospheric electrons and ions and by the precipitated hot electrons and protons, are functions of V. However, in Knight's model [1973], the current carried by the ions was neglected. *Lemaire and Scherer* [1974] have shown that such an omission underestimates J_{tot} and leads to erroneous results for $V < 100$ Volts.

The total field-aligned current density, $J_{\text{tot}} = \sum_i J_i$, is in general a non-linear function of V [*Knight*, 1973; *Lemaire and Scherer*, 1974 and 1983]. Nevertheless, there exists a range of potential (100 Volts $< V <$ 10 kVolts) where the current-voltage relationship is linear. The extrapolation of this linear relationship is not a valid approximation in the return current region where $J_{\text{tot}} < 0$.

The kinetic models of *Knight* [1973] and *Lemaire and Scherer* [1974] rest on the assumption that the velocity distribution functions (VDF) are truncated Maxwellians. But the VDF of space plasmas has most of the time a non-Maxwellian super-thermal tail: this VDF, $f(\vec{v})$, decreases generally as a power law of the velocity v instead of exponentially [*Bame et al.*, 1967]. A useful function to model such plasmas VDFs is the generalized Lorentzian (or Kappa) distribution [*Summers and Thorne*, 1991].

Kappa distributions have been used to analyze spacecraft data collected in the Earth's magnetospheric plasma sheet [*Vasyliunas*, 1968; *Lui and Krimigis*, 1981; *Christon et al.*, 1988] and in the solar wind [*Scudder*, 1992 a and b]. Since many space plasmas VDFs can be better fitted by Kappa distributions than by Maxwellians or exponential functions, we developed a new

kinetic model for the current-voltage relationship based on the Kappa VDF.

We determine first the expressions of the partial current density using the Kappa VDF and compare them with those of the earlier model based on the Maxwellian VDF. We calculate the partial and total current densities in an auroral magnetic flux tube for typical temperature and number densities of the ionospheric and magnetospheric particles. We compare the results obtained for different values of the parameter κ, including values of κ deduced from observed auroral energy spectra. Finally, we discuss the current-voltage relationship in the limit of very low values of V which is relevant in the return current region.

2 Model description

The current density parallel to magnetic field lines is defined by:

$$J_{\|}(\vec{r}) = Ze \int v_{\|} f(\vec{r}, \vec{v})\, d^3\vec{v} \,, \tag{1}$$

where $\vec{r}$ and $\vec{v}$ are respectively the position and the velocity of the particle, Ze is the electric charge of the particles, $v_{\|}$ is the velocity component parallel to the magnetic field direction and $f(\vec{r}, \vec{v})$ is the velocity distribution function, normalized so that $\int f(\vec{r}, \vec{v})\, d^3\vec{v} = n(\vec{r})$ is the number density. The domain of the integration corresponds to the velocity space where $f(\vec{r}, \vec{v})$ is not equal to zero.

In the models of *Lemaire and Scherer* [1970, 1971, 1973], the VDF is assumed to be a truncated Maxwellian at the reference level r_0,

$$f(\vec{r_0}, \vec{v}) = N_0 \left(\frac{m}{2\pi k T_0}\right)^{3/2} \exp\left(-\frac{mv^2}{2kT_0}\right) \,. \tag{2}$$

For ionospheric particles, the VDF is given by (2) outside the downward loss cone and is equal to zero inside the downward loss cone (i.e. for $\pi - \theta_m < \alpha < \pi$ where α is the pitch angle of the particles). For the plasma sheet particles, the VDF is given by (2) outside the upward loss cone and is equal to zero inside the upward loss cone (i.e. for $0 < \alpha < \theta_m$). This means that we neglect the contribution of the ionospheric particles from the conjugate auroral region and consider that the precipitated plasma sheet particles with mirror points below the baropause are not reflected, but are all scattered

and lost by inelastic collisions. N_0 and T_0 are constants determined by the number density and the temperature at the altitude of the reference level r_0. k is the Boltzmann constant.

Lemaire and Scherer [1971] obtained analytical expressions of the integral (1), $J_i = J_i(V)$, for two extreme cases. When the total potential energy $U(r) = Z_i eV(r) + m_i\phi_g(r)$ (where m_i is the particle mass and ϕ_g is the gravitational potential) is positive and is a uniformly decreasing function of altitude, the current density is found to be:

$$J_i(r_0) = Z_i e N_{0i} \left(\frac{kT_{0i}}{2\pi m_i}\right)^{1/2} . \tag{3}$$

For ions and electrons whose total potential energy $U(r) = Z_i eV + m_i\phi_g$ is negative and is a uniformly increasing function of altitude, the current density is given by:

$$J_i(r_0) = Z_i e N_{0i} \left(\frac{kT_{0i}}{2\pi m_i}\right)^{1/2} \left(1 - \frac{U(r_0)}{kT_{0i}}\right) \exp\left(\frac{U(r_0)}{kT_{0i}}\right) . \tag{4}$$

In these equations, the parameters N_{0i} are determined by the actual number densities $n_i(r_0)$ at the reference level r_0. For Eq. (3) (when $U(r) > 0$), the normalization factor is given by

$$N_{0i} = 2n_i(r_0) . \tag{5}$$

For Eq. (4) (when $U(r) < 0$), it is given by

$$N_{0i} = \frac{n_i(r_0)}{\frac{1}{2}(\psi_i + \zeta_i) + (\psi_i - \zeta_i)\, K_2\left(\sqrt{\frac{-U(r_0)}{kT_{0i}}}\right)} , \tag{6}$$

where $\psi_i = 0$ and $\zeta_i = 1$ for the incoming magnetospheric particles which are confined in the downward loss cone; $\psi_i = 1$ and $\zeta_i = 0$ for the ionospheric particles which escape into the magnetotail or in the opposite hemisphere, and $K_2(x) = \frac{2}{\pi^{1/2}} \int_0^x dt \exp(-t^2)t^2 = \frac{\mathrm{erf}(x)}{2} - \pi^{-1/2} x \exp(-x^2)$.

In this model, the magnetic field at the high altitude region B_M is assumed equal to zero. In a more general magnetic model like that considered in Lemaire and Scherer [1973], the altitude of the large-scale monotonic potential distribution is limited. The magnetic field intensity is a non-zero constant B_M at this high altitude and B_I at the ionosphere's reference level.

In this case, the relation between the current density and the potential becomes:

$$J_i(r_0) = Z_i e N_{0i} \left(\frac{kT_{0i}}{2\pi m_i}\right)^{1/2} a^{-1} \exp\left(\frac{U(r_0)}{kT_{0i}}\right) \times \left[1 + (a-1)\exp\left(\frac{a}{1-a}\frac{U(r_0)}{kT_{0i}}\right)\right] \quad (7)$$

where $a = B_M/B_I$. The normalization factor is slightly different from Eq. (6). It is easy to show that Eq. (7) tends to Eq. (4) when $a \to 0$. For the magnetosphere-ionosphere system, the value of a turns out to be of the order of 0.02 [Lu *et al.*, 1991] or smaller (0.001 in articles of *Lemaire and Scherer* [1971, 1983]) and the Eq. (7) gives results very similar to Eq. (4) for potential lower than 10 kV. For higher potential, the current given by Eq. (7) tends to an asymptotic value depending on a (cf Figure 3 for $a = 0.02$).

We determine here the corresponding expressions in the case of a Kappa VDF defined by:

$$f_\kappa(\vec{r_0}, \vec{v}) = \frac{N_0}{2\pi(\kappa w^2)^{3/2}} A_\kappa \left(1 + \frac{v^2}{\kappa w^2}\right)^{-(\kappa+1)} \quad (8)$$

with $A_\kappa = \Gamma(\kappa+1)/[\Gamma(\kappa - 1/2)\Gamma(3/2)]$ where κ is the spectral index, w is the characteristic thermal speed of the distribution ($\overline{v} = w$), and $\Gamma(x)$ is the gamma function. At high velocities, the distribution obeys an inverse power law: $f_\kappa \sim (mv^2/2)^{-(\kappa+1)}$.

A comparison between the Maxwellian distribution and the generalized Lorentzian (Kappa) is shown on Fig. 1 for different values of the κ parameter. There are more suprathermal particles in the high-energy tail of the Kappa VDF but the difference becomes less significant as κ increases. When the spectral index $\kappa \to \infty$, the Kappa distribution tends to a Maxwellian with a temperature T_0 related to w by $kT_0 = mw^2/2$.

The current densities obtained for a Kappa distribution function (integrating Eq.(1) with the same truncated pitch angle distributions) are:

$$J_i(r_0) = \frac{1}{4} Z_i e N_{0i} \left(\frac{2kT_{0i}}{m_i}\right)^{1/2} \frac{A_\kappa \kappa^{-1/2}}{(\kappa - 1)}, \quad (9)$$

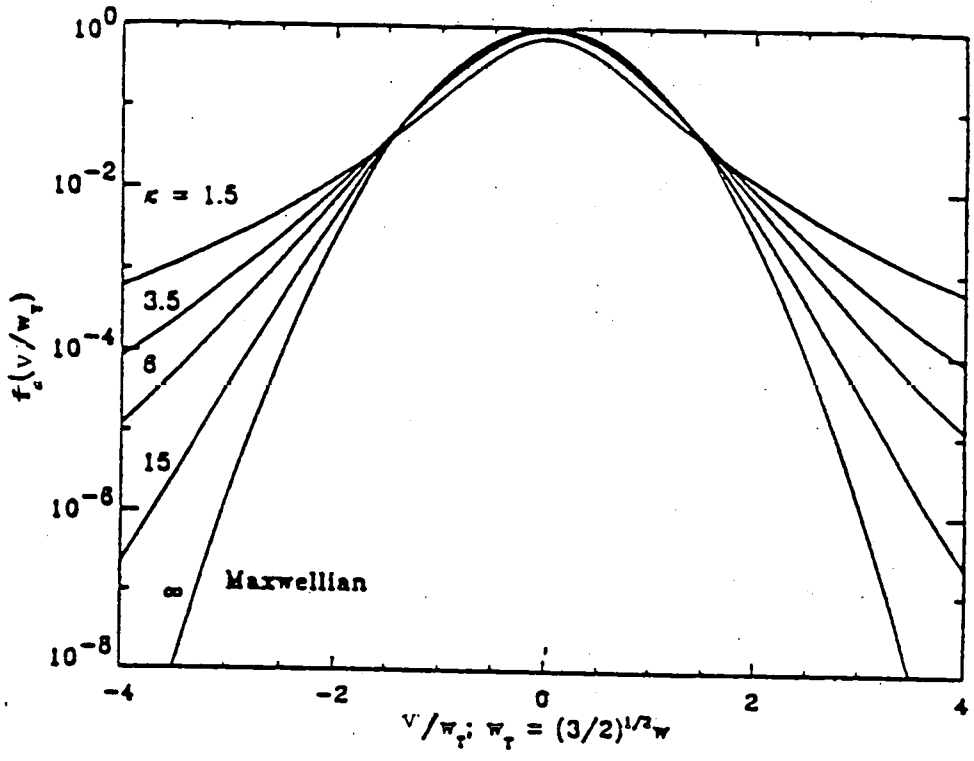

Figure 1. Comparison of generalized Lorentzian distributions for different spectral index - with the corresponding Maxwellian distribution ($\kappa = \infty$) (Scudder 1992a).

when the total potential energy $U(r) = Z_i e V(r) + m_i \phi_g(r)$ is positive and uniformly decreasing with altitude, and

$$J_i(r_0) = \frac{1}{4} Z_i e N_{0i} \left(\frac{2kT_{0i}}{m_i}\right)^{1/2} \frac{A_\kappa \kappa^{-1/2}}{(\kappa - 1)} \left(1 - \frac{U(r_0)}{kT_{0i}}\right) \left(1 - \frac{U(r_0)}{\kappa(kT_{0i})}\right)^{-\kappa}, \tag{10}$$

when $U(r)$ is negative and uniformly increasing with altitude. The current density given in Eq. (9) is independent on the parallel potential drop V while, according to Eq. (10), J_i is an increasing function of V.

Since $\lim_{\kappa\to\infty} \left(1 + \frac{x}{\kappa}\right)^{-\kappa} = \exp(-x)$ and $\lim_{\kappa\to\infty} \kappa^{b-a} \frac{\Gamma(\kappa+a)}{\Gamma(\kappa+b)} = 1$ [*Abramowitz and Stegun*, 1968], one can verify that Eqs. (9) and (10) tend respectively to Eqs. (23) and (21) given by *Lemaire and Scherer* [1970] when $\kappa \to \infty$. The actual number density at the reference level $n_i(r_0) = \int f_\kappa(\vec{r_0}, \vec{v})\, d^3\vec{v}$ is related to N_{0i} by:

$$N_{0i} = 2n_i(r_0) \tag{11}$$

in the former case, when $U(r) > 0$, and

$$N_{0i} = \frac{n_i(r_0)}{\psi_i + \frac{(\zeta_i - \psi_i)}{2} \beta\left(\left(1 - \frac{U(r_0)}{\kappa(kT_{0i})}\right)^{-1}\right)} \tag{12}$$

in the latter case, when $U(r) < 0$. $\beta(x) = \int_0^x A_\kappa t^{\kappa-3/2}(1-t)^{1/2}\, dt$, $(0 \le x \le 1)$ is the classical incomplete Beta function given in most standard mathematical libraries.

3 Comparison of the theoretical results

Consider an auroral magnetic flux tube extending from the ionosphere up into the plasma sheet and containing both cold ionospheric plasma and hot magnetospheric electrons and protons. A quasi-stationary field-aligned electrostatic potential difference can develop between the low altitude reference level, taken as 1 000 km, where the magnetic field is B_I, and the high altitude equatorial region where the magnetic field intensity B_M is considered to be equal to zero. The temperature T_{0i} and number density $n_i(r_0)$ of the different particles species at the reference altitude 1 000 km are given in Table 1.

Table 1: Average temperature (T_{0i}) and number densities (n_i) of cold ionospheric electrons and ions and of warm magnetospheric electrons and protons at 1000 km altitude.

i	ce^-	O^+	H^+	he^-	p^+
T_{0i} (K)	4 500	1 500	4 000	10^7	5×10^7
n_i (cm^{-3})	2 200	2 000	200	0.1	0.1

Figure 2a shows the partial and total field-aligned current densities J_{he^-}, J_{p^+}, J_{ce^-}, J_{H^+} and J_{tot} as a function of the applied potential difference V for a Maxwellian VDF. The results of Fig. 2a are analogous to those of Fig. 1 in Lemaire and Scherer [1983] except that the latter give the current densities in a dayside cusp magnetic flux tubes containing magnetosheath like particles instead of in an auroral magnetic tube containing plasma sheet particles.

Figure 2b shows the same partial and total current densities as in Fig. 2a but calculated for a Kappa VDF with $\kappa = 5.5$ instead of a maxwellian VDF; the same boundary conditions were used at the exobase altitude (1 000 km) (see Table 1). From the comparison of Fig. 2a and 2b, one sees that by enhancing the suprathermal tail population of particles (i.e. by decreasing the value of κ), all partial currents densities are also enchanced.

For $V < 10$ kVolts, the hot electron current density J_{he^-}, given by Eq. (10) can be approximated by a linear function of V:

$$J_{he^-} = \frac{e^2 N_{0he^-}}{(2\pi m_e k T_{0he^-})^{1/2}} V = K^{Maxw} V . \tag{13}$$

The approximate linearity of the current-voltage characteristic curves of the current density carried by the plasma sheet electrons has already been demonstrated earlier for a Maxwellian VDF [*Lundin and Sandahl*, 1978; *Fridman and Lemaire*, 1980; *Lyons*, 1981]. K^{Maxw} is a field-aligned conductance whose value can be determined from measurements of auroral electron spectra [*Lyons et al.*, 1979; *Weimer et al.*, 1985].

We confirm here that this ohmic-like behaviour remains true when Kappa VDFs are used instead of Maxwellians. But in this case, the conductance

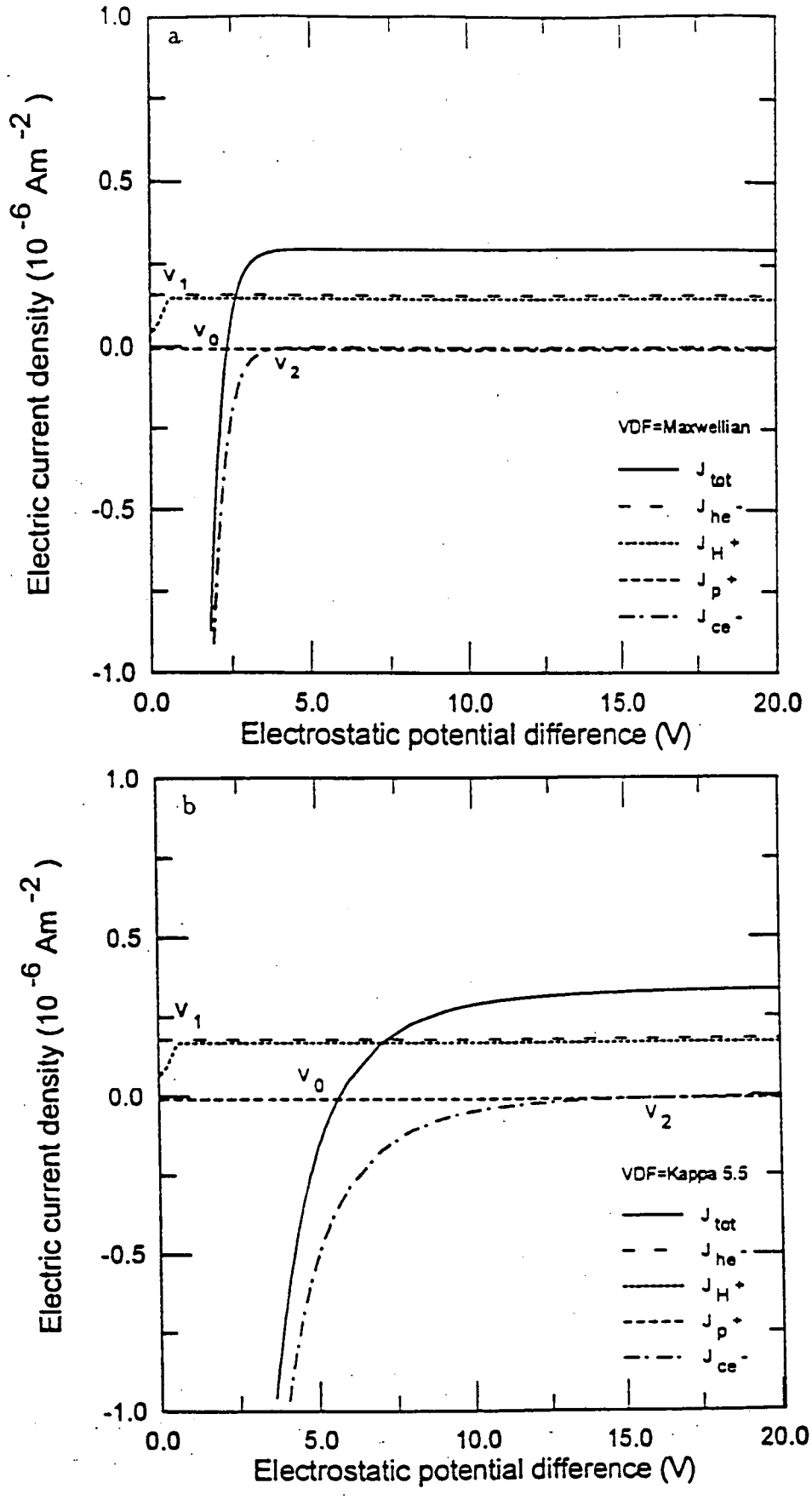

Figure 2. With a Maxwellian VDF model (a) and a Kappa VDF model with $\kappa = 5.5$ for each distribution (b), partial and total field-aligned currents in an auroral magnetic flux tube as a function of the applied field-aligned potential difference, V, between the ionosphere and the magnetosphere. Plasma densities and temperatures at 1 000 km are summarized in Table 1.

$K = J_{\mathrm{he}^-}/V$ is larger than K^{Maxw}:

$$K^{\mathrm{Kappa}} = \frac{\Gamma(\kappa+1)}{\Gamma(\kappa-1/2)\kappa^{1/2}(\kappa-1)}\, K^{\mathrm{Maxw}} \,. \tag{14}$$

It can be verified that $K^{\mathrm{Kappa}} \rightarrow K^{\mathrm{Maxw}}$ when $\kappa \rightarrow \infty$.

The flux of the plasma sheet protons, injected downward, is negligibly small in both models since their thermal velocity is much smaller than that of the electrons, unless they have a field-aligned bulk velocity. Indeed, Miura [1990] pointed out that J_{p^+} may become important when the VDF of the protons is supposed to have a cigar-like pitch-angle distribution, instead of an isotropic or pancake-like one. While most plasma sheet protons are reflected back toward the plasma sheet by the mirror force before reaching the ionosphere, a larger fraction of these protons would then have pitch angles in the loss cone and more of these protons contribute to the net downward current J_{p^+}.

The contribution of the ionospheric hydrogen current density J_{H^+} is constant (Eq. (3) except for $V < V_1 = 0.65$ Volts. V_1 is independent on κ. The importance of the ionospheric ions in the total current depends on their density $n_{\mathrm{H}^+}(r_0)$ relatively to the hot magnetospheric plasma density $n_{\mathrm{he}^-}(r_0)$ at the reference altitude.

The partial current density J_{ce^-} carried by the upward flowing cold electrons accounts for the non-linearity of J_{tot} as a function of V. There always exists a positive value V_0 of the electrostatic potential for which $J_{\mathrm{tot}} = 0$. When $V < V_0$, the downward current carried by the escaping ionospheric electrons J_{ce^-} becomes dominant. A field-aligned potential V_0 of only a few volts can inhibit these cold ionospheric electrons from escaping to infinity. Table 2 shows how V_0 increases when κ decreases.

4 Current-voltage characteristic for different values of κ

Figure 3 shows J_{tot} as a function of V for different values of κ. Logarithmic scales are used for J_{tot} and for V ranging between 1 and 10^5 Volts. One can see that the curve marked Knight, neglecting the ions contributions (J_{p^+} and J_{H^+}), underevaluates J_{tot}. When $\kappa = 100$, the current-voltage curve is almost identical to that obtained in the case of a Maxwellian VDF.

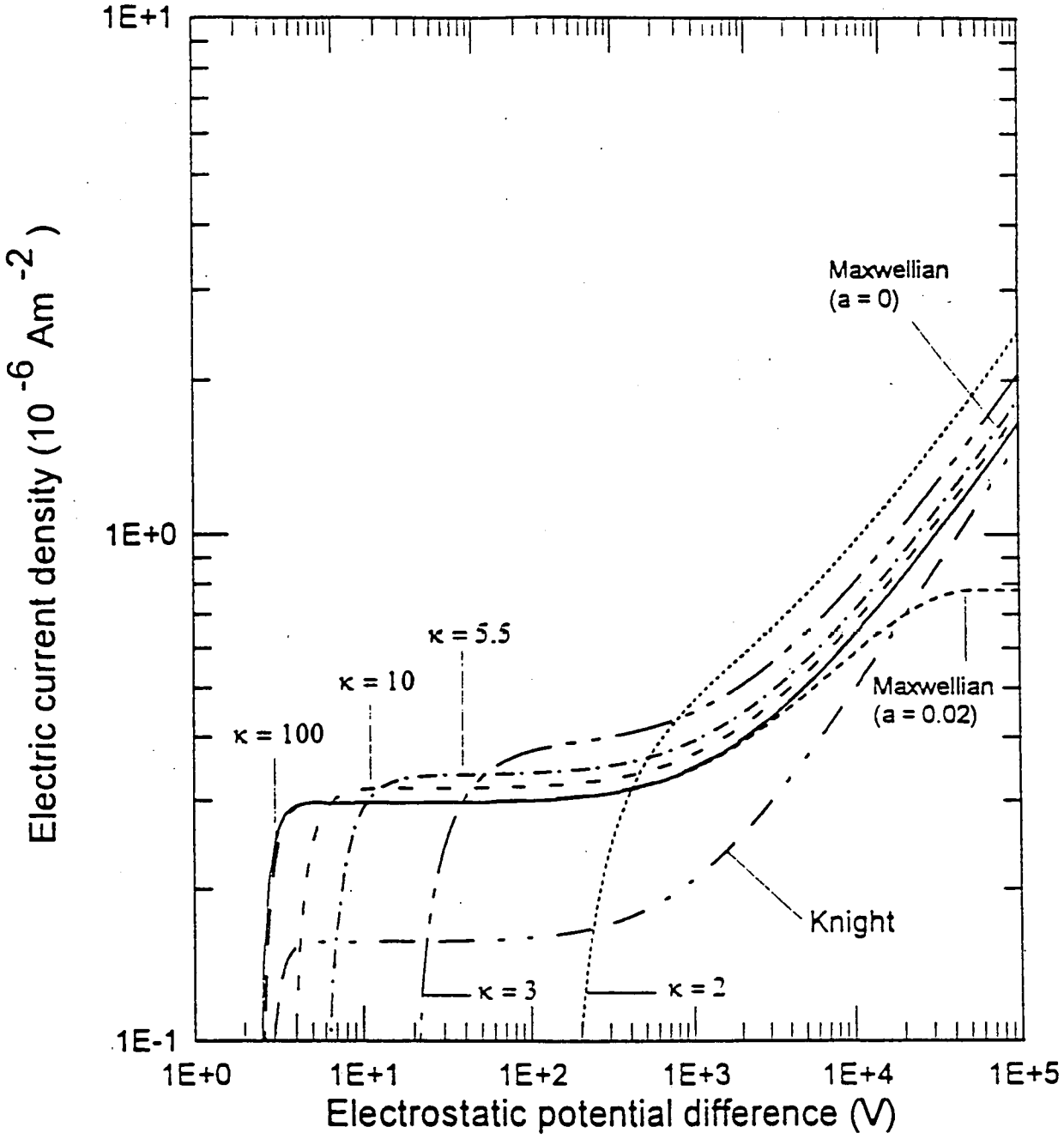

Figure 3. Influence of the κ parameter on the total current density (J_{tot}) in an auroral magnetic flux tube with characteristics summarized in Table 1. Note the non-linear trend except in a limited range of the potential difference V. Note also the similarity between the results obtained with a Maxwellian VDF (earlier models) and those obtained with a Lorentzian VDF when the parameter κ is large. Knight's curve takes into account the magnetospheric and ionospheric electrons but not the contributions of the ion currents.

Table 2: Variations of V_0, the potential for which $J_{tot} = 0$, as a function of κ.

Values of κ	V_0 (Volts)
Maxwellian ($\kappa = \infty$)	2.32
$\kappa = 100$	2.42
$\kappa = 10$	3.63
$\kappa = 5.5$	5.66
$\kappa = 3$	17.6
$\kappa = 2$	159

In the range from 0.04 to 10 kV, many authors [*Yeh and Hill*, 1981; *Burch et al.*, 1983, *Marklund and Blomberg*, 1991, *Reiff et al.*, 1988] verified that $J_{he^-} \sim J_{tot}$ is almost a linear function of V. This linearity is still applicable for Kappa functions, but the range of V for which the near linearity of the current-voltage applies is shrinking when κ decreases from $\kappa = \infty$ to $\kappa = 2$.

The linear relationship fails to be applicable in the range below 100 Volts, i.e. in the plasmasphere, plasma through, polar wind and return current regions. Indeed, below the critical value V_0, J_{tot} drops rapidly and becomes negative. For a Maxwellian ($\kappa = \infty$), the negative return current can only subsist up to $V_0 = 2.32$ Volts. But for $\kappa = 3$, it can subsist up to $V_0 = 17.6$ Volts and even up to $V_0 = 159$ Volts for $\kappa = 2$ (see Table 2). Negative values of J_{tot} are expected in the return current regions on both sides of the auroral region where the precipitated plasma sheet electrons produce a positive (upward) field-aligned current.

When κ decreases, $| J_{ce^-} |$ and $| J_{tot} |$ increase. Indeed, when κ is smaller, there are relatively more superthermal electrons in the tail of the Kappa VDF and the number of electrons which are able to escape is then larger.

5 Determination of kappa index from observations

Christon *et al.* [1988] and Williams *et al.* [1988] have fitted measured plasma sheet ion and electron differential energy spectra with Kappa functions. For

the majority of cases studied, they found that a Kappa VDF with values of κ between 4 and 7 provides a better overall description of the energy spectra than the Maxwellian VDF. The value of κ stays roughly constant, even when the temperature varies with time or with position in the plasma sheet transition regions. But the value of κ differs from one species to the other.

In our model, we assumed for simplicity that all particle populations have the same kappa index. To be able to relax this assumption, it would be necessary to have reliable observations of the energy spectra for all ions species and electron populations considered in this study. The required observations should range from well below the peak of the spectrum to far up into the tail of the VDF (cf Figure 1). This implies that detailed and high resolution energy spectra are measured simultaneously, at the same location in space and in the appropriate ranges of energies for each electron and ion populations.

From such ideal spectral measurements, one would then be able to determine by standard parameter fitting procedures the estimates of the temperature (T_{0i}), density (n_{0i}) and Kappa index (κ) for each population of particles ($\mathrm{H}^+, \mathrm{ce}^-, \mathrm{he}^-, \mathrm{p}^+ \ldots$). Usually, only the first two parameters are adjusted under the assumption that the VDF is a Maxwellian. But there is now more experimental evidence that a Maxwellian VDF is not the best zero-order approximation in most collisionless space plasmas, and that Kappa functions should be preferred in future data analysis.

An interesting alternative model would be obtained by taking Maxwellians for the ionospheric distributions of particles and Kappa for the magnetospheric distributions with different index κ determined by satellite's observations. In this case, only hot electron and proton current densities would be enhanced. However, the value of V_0 would practically not change. The large increase of the cross-over potential V_0 on Figure 3 is due to the cold electron Kappa VDF.

6 Return current intensity

Figure 4 shows the negative current densities obtained for small values of V and for different values of κ. J_{tot} is very sensitive to small changes of V. When V increases from zero to more positive values, the total potential barrier $(-m_e\phi_g + eV)$ that the ionospheric electrons must overcome in order

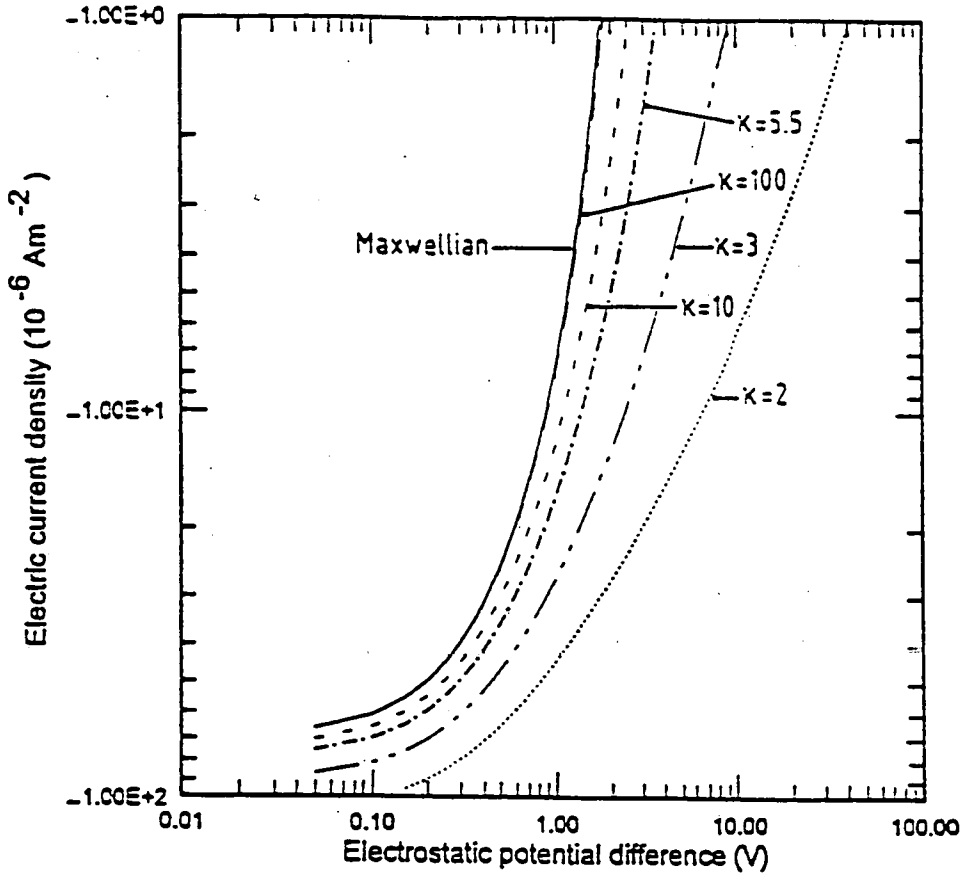

Figure 4. Negative current densities obtained for small but positive values of V and for different values of κ.

to escape and contribute to the net field-aligned current, is then increasing. In other words, the escape speed of an electron is increasing from 11 km/sec to much larger values given by $\sqrt{-2\phi_g + 2eV/m_e}$.

The effect of a higher magnetospheric plasma density is to increase the hot electron current density. But the field-aligned current density remains negative without a reversal sign of the field-aligned potential difference. To compensate for the cold electron downward current density (which is around $-100\,\mu\mathrm{Am}^{-2}$ for $V = 0$), it would be necessary to have very large density of magnetospheric particles, lacking in consistency with the auroral particle data.

It should be emphasized that according the different kinetic models, negative currents are obtained without reversing the sign of V. In some applications and models of ionosphere-magnetosphere coupling [*Blomberg and Marklund*, 1991], the linear relationship (Eq. (13)) is extrapolated outside its range of validity. For instance, the same value of the conductance K is used in the return current region where $J_{\mathrm{tot}} < 0$ as in the auroral precipitation region where $J_{\mathrm{tot}} > 0$. This abusive extrapolation of the relationship $J_{\mathrm{tot}} = KV$ would imply that $V < 0$ in the return current region. But, large-scale reversed field-aligned potential difference would lead to unreasonably large escape fluxes of the cold ionospheric electrons (cf Figure 4). When $V = 0$, almost all cold electrons would blow out of the topside ionosphere because their thermal speed (=3 500 km/sec for $T_0 = 3\,000$ K) is much larger than 11 km/s (i.e. the gravitational escape velocity for neutral particles). This would produce a downward field-aligned current density of more than $100\,\mu\mathrm{A/m}^2$ and unreasonably large transverse magnetic field perturbations.

A large-scale reversed potential difference would also lead to downward parallel electric field $E_{||}$ which would be opposite to the ambipolar electric field required to maintain the quasi-neutrality of the ionospheric and magnetospheric plasma. A downward directed parallel electric field, $E_{||} = -[dp_e/dr]/en_e$, would necessarily imply an electron pressure p_e increasing with altitude. Since this is physically unrealistic, it can be concluded that negative field-aligned potential difference between the ionosphere and magnetosphere must be excluded in the return current region as well as elsewhere. In other words, to drive a negative current, it suffices to reduce the positive value of the field-aligned potential difference below the value of V_0 given in Table 2.

Note that we only consider here a large-scale potential distribution between the ionosphere and the magnetosphere, though localized structures with small-scale size may well be imbedded in the larger-scale regions. Small-scale downward electric fields has been observed at high altitudes [Burch *et al.*, 1983]. Since they are confined well above the ionosphere, they don't lead to large runaway outflows of ionospheric electrons.

7 Conclusions

Field-aligned currents (FAC) can be driven upward or downward magnetospheric field lines by a quasi-stationary (DC) potential created by the temperature difference between the cold ionospheric and the warmer magnetospheric plasma. In earlier kinetic models [*Lemaire and Scherer*, 1970, 1971 and 1973; *Knight*, 1973], truncated Maxwellian velocity distributions (VDF) have been assumed to determine the total FAC in an auroral magnetic flux tube. But there is more and more evidence that the VDF of collisionless space plasma is closer to a Kappa (or Lorentzian) function than to a Maxwellian [*Christon et al.*, 1988]. The spectra of the super thermal particles decrease as power law of the energy E with a slope which can be related to the value of the kappa index.

Since the particles with largest velocity contribute most to the escape or precipitated flux, we determined the FAC for a Kappa VDF. The comparison between the results obtained with Maxwellian and Kappa VDFs shows that

1) In both cases, one obtains non-linear current-voltage characteristic curves.

2) When the index κ increases, the total FAC intensities decrease (increase) at large (small) values of V.

3) In the limit $\kappa \rightarrow \infty$, the current-voltage for Maxwellian VDF is recovered.

4) For values of V between 0.1 and 10 kV, corresponding to field-aligned potentials observed in auroral arcs or strong plasma sheet precipitation, the total current is positive (upward) and can be approximated by a linear current-voltage relationship.

5) The slope of the current-voltage linear relationship corresponds to a conductance K, whose value depends on the index κ of the Kappa VDF. When κ increases indefinitely, the value of K^{Kappa} decreases toward the

Maxwellian values K^{Maxw}.

6) The current carried upward by the ionospheric hydrogen ions J_{H^+} is constant but not always negligible, although it is ignored in some studies of ionosphere-magnetosphere coupling [*Knight*, 1973 and *Fridman and Lemaire*, 1980].

7) For $V > 10\,\mathrm{kVolts}$, the current density mainly transported by the plasma sheet electrons increases exponentially with V for the Maxwellian VDF as well as for the Kappa VDF, when the magnetic field intensity in the magnetosphere B_{M} is assumed to be equal to zero. In a more general magnetic model where B_{M} is a non-zero constant ($a \neq 0$), the current is limited.

8) For $V < 100\,\mathrm{Volts}$, the current-voltage curves cease to be well approximated by a linear relationship like Eq. (13). The total current drops rapidly due to the negative current transported downwards by the escaping cold electrons of the ionosphere.

9) The value of the thermoelectric charge separation potential, V_0, for which $J_{\mathrm{tot}} = 0$, increases when κ decreases, in models where the kappa index is identical for each particle distribution.

10) To obtain a negative total FAC density in a return current region, it is not necessary to reverse the sign of V. Negative (downward) current can be arbitrarily large when $V < V_0$ (see Fig. 1). For $V = 0$, all the ionospheric electrons would be blown out the gravitational potential due to their large thermal velocity.

11) To determine appropriate values of κ for each particle population contributing to the total FAC, high resolution energy spectra must be measured over an energy range from below the peak flux to far up into the tail of the VDF. This should serve as a general recommendation for designing particle spectrometers to be flown in future missions in Earth's magnetosphere.

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